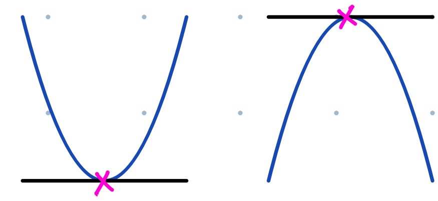
Chapter 1 : Quadratic Functions.

Quadratic Functions.



$a>0$ (min. point) $a<0$ (max. point).

→ General form : $y = ax^2 + bx + c$

→ Factorised form: $y = a(x-p)(x-q)$

→ Completed square form: $y = a(x-h)^2 + k$

Completing the square

$[y = a(x-h)^2 + k] \Rightarrow$ Turning point. (h, k)

↳ To make $x^2 + bx$ into a perfect square, add $\left(\frac{b}{2}\right)^2$ into the expression.

↳ e.g. Express each of the following expressions in the form $a(x-h)^2+k$.

(a) $x^2 + 12x$ (b) $2x^2 - 6x + 5$

(a) $x^2 + 12x = x^2 + 12x + \left(\frac{12}{2}\right)^2 - \left(\frac{12}{2}\right)^2$

$= x^2 + 12x + 6^2 - 6^2$

$= (x+6)^2 - 36$

(b) $2x^2 - 6x + 5 = 2\left(x^2 - 3x + \frac{5}{2}\right)$

$= 2\left[x^2 - 3x + \left(-\frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right)^2 + \frac{5}{2}\right]$

$= 2\left[\left(x - \frac{3}{2}\right)^2 + \frac{1}{4}\right]$

$= 2\left(x - \frac{3}{2}\right)^2 + \frac{1}{2}$ #

Chapter 2 : Equations & Inequalities

Methods for solving quadratic equations.

① Factorisation

② Completing the square

③ Quadratic formula. ↗ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Determining nature of roots of quad. eq$^{\underline{ns}}$.

$b^2 - 4ac > 0$	2 real & distinct roots	Cut at 2 points	Line cuts circle
$b^2 - 4ac = 0$	2 real & equal roots	Touches 1 point	(90°) Line tangent to circle
$b^2 - 4ac < 0$	No real roots	Does not touch	Line misses circle.

Chapter 3: Surds

Surd.

↳ It is an irrational number comprising the square root / cube root.

Conjugate Surds.

$$\frac{1}{p+q\sqrt{a}} \begin{matrix}\times (p-q\sqrt{a}) \\ \times (p-q\sqrt{a})\end{matrix}$$ } Rationalising of surds.

$$= \frac{p-q\sqrt{a}}{(p+q\sqrt{a})(p-q\sqrt{a})} = \frac{p-q\sqrt{a}}{p^2-q^2(a)}$$

Law of Surds.

Law 1 $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

Law 2 $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Law 3 $\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$

or $(\sqrt{a})^2 = \sqrt{a^2} = a$

Equality of surds.

↳ $a+b\sqrt{n} = p+q\sqrt{n}$

$\therefore a=p,\ b=q.$

Chapter 4: Polynomials, Cubic Equations & Partial Fractions.

Dividing polynomials — Long Division.

E.g. (i) Divide x^3+2x-7 by $x-2$ and state the remainder.

(ii) Hence, express x^3+2x-7 in terms of $x-2$ using the Division Algorithm.

(i)

$$\begin{array}{r|l} & x^2+2x+6 \leftarrow \text{quotient} \\ \text{divisor} \rightarrow x-2 & x^3+0x^2+2x-7 \leftarrow \text{dividend} \\ & -(x^3-2x^2) \downarrow \\ \hline & 2x^2+2x \\ & -(2x^2-4x) \downarrow \\ \hline & 6x-7 \\ & -(6x-12) \\ \hline & 5 \leftarrow \text{remainder} \end{array}$$

$\therefore$ Remainder $= 5$ #

(ii) $x^3+2x-7 = (x-2)(x^2+2x+6)+5$ #

dividend = divisor × quotient + remainder

Remainder Theorem.

E.g. Find the value of k if $4x^7+5x^3-2kx^2+7k-4$ leaves a remainder of 12 when divided by $x+1$.

Let $f(x) = 4x^7+5x^3-2kx^2+7k-4$

sub $f(-1)=12$: $4(-1)^7+5(-1)^3-2k(-1)^2+7k-4 = 12$

$5k-13 = 12$

$5k = 25$

$k = 5$ #

Factor Theorem

E.g. Find the value of k for which $x-2$ is a factor of $f(x)=3x^3-2x^2+5x+k$.

$f(x)=3x^3-2x^2+5x+k$

By Factor Theorem, since $x-2$ is a factor of $f(x)$, then $f(2)=0$.

$$3(2)^3-2(2)^2+5(2)+k=0$$

$$k=-26 \#$$

Cubic Identities

Sum of cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

Difference of cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$

used to factorise!

Partial Fractions

case 1: $\dfrac{px+q}{(Ax+B)(Cx+D)}=\dfrac{a}{Ax+B}+\dfrac{b}{Cx+D}$

case 2: $\dfrac{px+q}{(Ax+B)^2}=\dfrac{a}{Ax+B}+\dfrac{b}{(Cx+D)}$

case 3: $\dfrac{px^2+qx+r}{(Ax+B)(x^2+C^2)}=\dfrac{a}{Ax+B}+\dfrac{bx+c}{x^2+C^2}$

Proper fractions only!
(if it is improper fraction, do long division first!)

Chapter 5: Binomial Theorem.

General Term

$T_{r+1}=\binom{n}{r}a^{n-r}b^r$

Factorial

$0!=1$

$n!=n\times(n-1)\times(n-2)\times\ldots\times3\times2\times1$

Binomial Coefficients

$\binom{n}{0}=1$

$\binom{n}{r}=\dfrac{n!}{r!(n-r)!}$

The term independent of x refers to the constant term.

General Rule

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\binom{n}{r}a^{n-r}b^r+\ldots+\binom{n}{n-r}ab^{n-1}+b^n$$