Topical Revision Paper 10

Term 3 Unit 10: Points, Lines and Shapes

Question 1:

The equation of a circle C_1 is $x^2 + y^2 + 20x - 4y + 100 = 0$. Find

(i) the radius and the coordinates of the centre of the circle, [2]

(ii) the value(s) of k if the circle touches the line $y = k$. [2]

(iii) the equation of the another circle, C_2 that has the same centre as C_1, and passes through the point $(-11, 2)$. [2]

Solution:

(i)

$x^2 + y^2 + 20x - 4y + 100 = 0$

$(x+10)^2 - 100 + (y-2)^2 - 4 + 100 = 0$

$(x+10)^2 + (y-2)^2 = 4$

Centre : $(-10, 2)$ and Radius = 2 units

(ii) Since the centre of the circle C_1 is $(-10, 2)$ and the radius is 2 units, k = 0 or 4

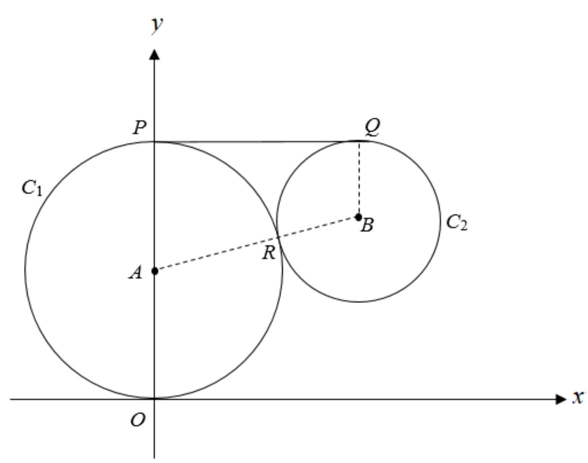
(iii) Radius of circle C_2 = 1 unit

Equation of circle C_2: $(x+10)^2 + (y-2)^2 = 1$ [Standard Form]

Or

Equation of circle C_2: $x^2 + y^2 + 20x - 4y + 103 = 0$ [General Form]

Question 2:



The diagram shows two circles C_1 and C_2 centred at A and B respectively. C_1 passes through O and P and touches C_2 at R. Q is on C_2 such that QB is parallel to the y-axis. The length of PQ is 12 units and PQ is a tangent to both circles.

Given that the equation of C_1 is $x^2 + y^2 - 18y = 0$, find

(i) the centre and radius of C_1, [3]

(ii) the equation of C_2, [4]

(iii) the equation of the perpendicular bisector of AB. [4]

Answer:

Solution:

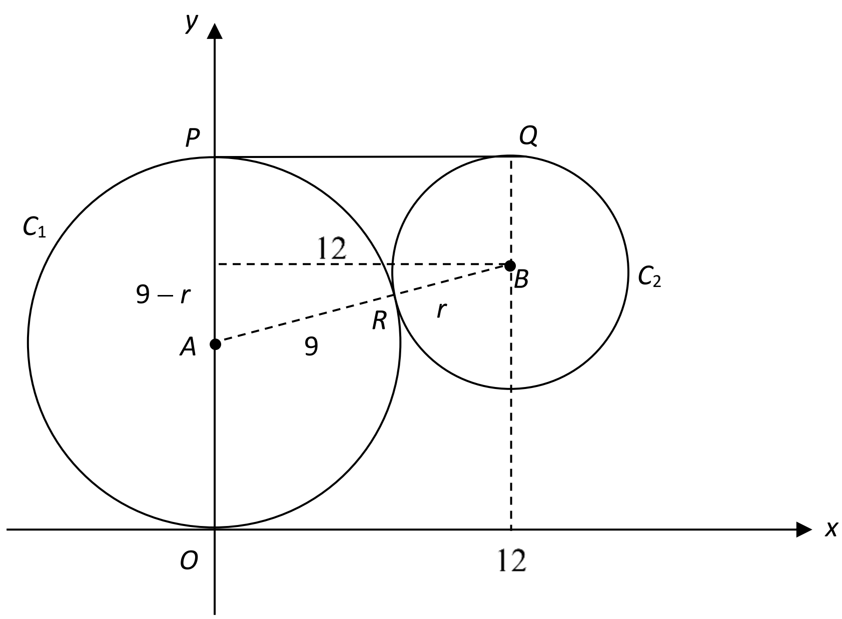
(i)

$$x^2 + y^2 - 18y = 0$$
$$x^2 + y^2 - 18y + (-9)^2 = 0 + (-9)^2$$
$$(x - 0)^2 + (y - 9)^2 = 9^2$$

The centre of C_1 : $(0,\ 9)$.

The radius of $C_1 =$ 9 units.

(ii)



Let the radius of C_2 be r units.

By Pythagoras' Theorem,

$$(9 - r)^2 + 12^2 = (9 + r)^2$$
$$81 - 18r + r^2 + 144 = 81 + 18r + r^2$$
$$36r = 144$$
$$r = 4$$

The radius of $C_2 = 4$ units.
The centre of C_2 : $(12, 14)$.

$(x - 12)^2 + (y - 14)^2 = 4^2$

The equation of C_2 is $(x - 12)^2 + (y - 14)^2 = 4^2$.

(iii)

Midpoint of AB $= \left(\frac{0+12}{2}, \frac{9+14}{2}\right)$

$= \left(6, 11\frac{1}{2}\right)$

Gradient of AB $= \frac{14-9}{12-0}$

$= \frac{5}{12}$

Gradient of the perpendicular bisector of AB

$= -\frac{12}{5}$

$= -2\frac{2}{5}$

$$y - 11\frac{1}{2} = -2\frac{2}{5}(x-6)$$

$$y - 11\frac{1}{2} = -\frac{12}{5}x + 14\frac{2}{5}$$

$$y = -\frac{12}{5}x + 25\frac{9}{10}$$

The equation of the perpendicular bisector of AB is

$y = -\frac{12}{5}x + 25\frac{9}{10}$.

Question 3:

The equation of a circle, C, is given by $x^2 + y^2 - 4x - 10y + m = 0$ where m is a constant.

A line $y = 2x - 4$ is a tangent to the circle.

(i) Write down the coordinates of the centre, O, of the circle. [1]

(ii) Find the radius of the circle and the value of m. [5]

(iii) The circle, C, is reflected in the line $y = 2x - 4$. Find the equation of the reflected circle. [3]

Solution:

(i)

(2,5)

(ii)

Since $y=2x-4$ is a tangent to circle, $x^2+y^2-4x-10y+m=0$

$x^2+(2x-4)^2-4x-10(2x-4)+m=0$

$5x^2-40x+56+m=0$

Discriminant = 0

$40^2-(4)(5)(56+m)=0$

$480m-20m=0$

$m=24$

Radius = $\sqrt{2^2+5^2-24}$

$=\sqrt{5}$ units

(iii)

Since (4, 4) is the midpoint between two centres of the circles:

$\frac{x_1+x_2}{2}$ = 5 and $\frac{y_1+y_2}{2}$ = 4

$\frac{2+x_2}{2}$ = 5 and $\frac{5+y_2}{2}$ = 4

(6, 3) is the centre of reflected circle

Hence equation:

$(x-6)^2+(y-3)^2$ = 10

or

$x^2+y^2-16x-6y$ + 63 = 10

Question 4:

(i) The equation of a circle is given by $x^2+y^2-10x+2y+1=0$.
Find the centre and the radius of the circle. [2]

(ii) The tangent to the circle at a point A is parallel to the straight line $x-y+2=0$.
Find the equation of the diameter of the circle through A. [2]

(iii) The circle is then reflected about the y-axis.
State the equation of the new circle formed. [1]

Solution:

(i) Centre is (5, -1) and the radius is $r = \sqrt{5^2 + (-1)^2 - 1} = 5$ units

(ii) The gradient of the straight line $x - y + 2 = 0$ is m = 1 so the gradient of the diameter through A is -1.

The equation of the diameter passes through A is

$$y - 5 = -1(x + 1)$$

$$x + y = 4$$

$$y = -x + 4$$

(iii)

The coordinates of the centre of the circle after the reflection about y-axis : (-5, -1)

Hence the equation of the new circle formed is $(x+5)^2 + (y+1)^2 = 5^2$