NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME	

CT CLASS	**2**	**4**			Centre Number/ Index Number					/				

MATHEMATICS 9758/02

Paper 2

17 September 2025

3 hours

Additional Materials: Printed Answer Booklet

List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

Section A: Pure Mathematics [40 marks]

1 **(a)** Find the set of values of x for which $3|x-3| \le |1-2x|$. [2]

(b) Without using a calculator, solve $\dfrac{x+18}{x^2+5x-14} \ge -1$. [4]

2 **(a)** It is given that the roots to the equation $-\mathrm{i}x^3 + 5\mathrm{i}x^2 + ax + b = 0$, where a and b are purely imaginary, are $2+\mathrm{i}$, $2-\mathrm{i}$ and 1. Explain why the complex roots occur in conjugate pairs. [2]

(b) By using **(a)** and an appropriate substitution, find the roots of the equation $\mathrm{i}x^3 + 5\mathrm{i}x^2 + a^*x + b = 0$, where the complex conjugate of a is denoted by a^*. [4]

3 **(a)** Given that f is a continuous function, explain, with the aid of a sketch, why the value of

$$\lim_{n\to\infty} \frac{2}{n}\left\{ \mathrm{f}\left(1+\frac{2}{n}\right) + \mathrm{f}\left(1+\frac{4}{n}\right) + \ldots + \mathrm{f}\left(1+\frac{2n}{n}\right) \right\}$$

is $\displaystyle\int_1^3 \mathrm{f}(x)\,\mathrm{d}x$. [3]

(b) Hence evaluate $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \ln\left(1+\frac{2k}{n}\right)\frac{2}{n}$ exactly. [4]

4 The equation of a curve C is $x^3 + xy + 2y^3 = k$, where k is a constant.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [2]

It is given that C has a tangent which is parallel to the y-axis.

(b) Show that the y-coordinate of the points of contact of the tangent with C must satisfy $216y^6 + 4y^3 + k = 0$. Hence show that $k \le \dfrac{1}{54}$. [6]

(c) Find the possible values of k in the case where the line $x = -6$ is a tangent to C. [2]

5 Two planes p_1 and p_2 have respective cartesian equations given by

$$4x+y+z=8 \text{ and } 4x+3y-z=0.$$

(a) Find the sine of the acute angle between p_1 and p_2 in the form $\frac{m}{\sqrt{n}}$ where m and n are positive integers to be determined. [3]

(b) Verify that the point A with coordinates (1,0,4) lies on p_1 and p_2. Hence, without the use of a calculator, find the vector equation of the line l formed by the intersection of p_1 and p_2. [3]

(c) It is given that B is a point on p_1 with coordinates (1,3,1). Show that AB is perpendicular to l and hence use **(a)** to deduce exactly the shortest distance from B to p_2. [3]

(d) Find the cartesian equation of the plane p_3 which contains B and is perpendicular to both p_1 and p_2. [2]

Section B: Probability and Statistics [60 marks]

6 Two fair six-sided dice are thrown. Events A, B and C are defined as follows.

A: sum of the two scores is odd

B: at least one of the two scores is greater than 4

C: the two scores are equal

(a) Find, giving your reasons clearly in each case, which pair of the events are

(i) mutually exclusive, [1]

(ii) independent. [3]

(b) Find $\mathrm{P}(C \mid B)$. [2]

7 The random variable S is the number of successes in 5 independent trials of an experiment in which the probability of success in any trial is $\frac{1}{3}$. The random variable D is the difference between the number of successes and the number of failures in 5 such trials.

(a) State the values that D can take. [1]

(b) Show that $\mathrm{P}(D=1)=\frac{40}{81}$. [1]

(c) By finding the probability distribution of D, find the exact value of $\mathrm{E}(D^2)$. [3]

(d) Find the exact values of $\mathrm{E}(S)$ and $\mathrm{E}(S^2)$. [2]

(e) Hence, by showing that $D^2=4S^2-20S+25$, verify the correctness of the value of $\mathrm{E}(D^2)$ found in part **(c)**. [2]

8 Helen is a conservationist who monitors the health of fish in a river. She is investigating whether a new water-treatment plant has affected the mass of adult trout in the river. Previously, the mean mass of adult trout in the river has been w kg. Helen carries out a test, at the 10% level of significance to find out whether there has been a change in the mean mass of adult trout in the river.

Helen catches 45 adult trout and measures the mass, X, kilograms, of each fish. Her summarised data are as follows.

$$n = 45 \qquad \sum x = 80.1 \qquad \sum (x - \bar{x})^2 = 24.29$$

(a) Calculate unbiased estimates of the population mean and variance of the mass of adult trout. [2]

(b) Use an algebraic method to calculate the set of values of w for which there is insufficient evidence to conclude that there has been a change in the mean mass of adult trout in the river. You should state your hypotheses and define any symbols you use. [6]

(c) Explain why there is no need for Helen to know anything about the population distribution of the mass of adult trout. [2]

9 A small ceramic workshop produces 100 plates each working day. Some of the plates turn out to be faulty.

(a) State, in the context of the question, two assumptions needed for the number of faulty plates made in a day to be well modelled by a binomial distribution. [2]

Assume now that the number of faulty plates produced each working day has the distribution B(100, p).

(b) Show that the probability that exactly 3 faulty plates are produced on a randomly chosen working day is $161700p^3(1-p)^{97}$. [1]

(c) Given that the most likely number of faulty plates produced on a working day is 3, find the possible range of values of p, leaving your answer in exact form. [4]

The workshop also produces bowls on each working day. The number of faulty bowls also follows a binomial distribution. The probability that a bowl is faulty is q. Faults on plates are independent of faults on bowls. The plates and bowls are sold in sets of 2 randomly chosen bowls and 2 randomly chosen plates. In the case where $p = 0.01$, the probability that a set contains at most 1 faulty item is 0.88.

(d) Write down an equation satisfied by q. Hence find the value of q. [4]

10 The director of an art gallery wishes to find out the relationship between the time spent, t minutes, in the art gallery and the age, a (in years), of the visitors. Ten visitors were randomly selected and the data are summarised as follows:

t (minutes)	20	70	56	75	k	42	50	52	60	60
a (years)	21	56	42	77	22	34	40	41	48	47

(a) A linear model is proposed for the relationship between t and a. Given that the least squares regression line of t on a is

$$t = 0.8957914a + 14.16013,$$

show that $k = 40$. [2]

(b) Sketch a scatter diagram of t against a for the data given in the table and draw the line given in **(a)** on the same diagram. [2]

(c) Use the least squares regression line of t on a in **(a)** to estimate the time spent in the art gallery by a 60-year old visitor. [1]

(d) A regression line $t = ma + c$ where m and c are constants is used to model the above set of data. The residual for a data point (a_i, t_i) where $i = 1, 2, \ldots, 10$ which is plotted on a scatter diagram is defined to be the number

$$t_i - (ma_i + c).$$

The sum of the squares of these residuals, denoted by S, is given by

$$S = \sum_{i=1}^{10} \left[t_i - (ma_i + c) \right]^2 .$$

Find an inequality that is satisfied by S. [2]

(e) The director wishes to explore the following two models which represent the relationship between t and a, where p and q are constants.

(A) $t = p + \dfrac{q}{a}$, (B) $t = pe^{qa}$.

For each model, find the product moment correlation coefficient and hence explain with a reason which model is more appropriate. [3]

11 In this question you should state the parameters of any normal distributions you use.

A machine grades apples according to their mass. Apples with mass exceeding 120g are labelled as “large” and apples with mass less than 80g are labelled as “small”. A large batch of apples is graded and it is found that 15% are “large” and 10% are “small”. It is known that the masses of the apples are normally distributed.

(a) Find the mean mass of a randomly chosen apple from the batch and show that the standard deviation is 17.3 g, correct to 3 significance figures. [3]

(b) Two apples are chosen in random. Find the probability that the difference in their masses is at least 20g. [3]

The apples are labelled and packed into boxes. Each box contains 20 apples.

(c) Apples with mass between 110g and 140g are labelled as “premium”. Find the expected number of “premium” apples in a randomly chosen box. [2]

(d) Find the probability that the mean mass of apples in a randomly chosen box is at most 105g. [3]

(e) The boxes of apples are sold by weight at a price of $2.90 per kilogram. Find the probability that the total price of 3 randomly chosen boxes of apples exceeds $18. [3]

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