Chapter 7: Gravitational Field Solutions

1 **A**

$$g = \frac{GM}{r^2}$$
$$= \frac{G\left(\frac{4}{3}\pi r^3 \rho\right)}{r^2}$$
$$= \frac{4\pi G\rho}{3} r$$

Hence, when density is constant, g is proportional to r (or d).

2 **D**

$$\phi = -\frac{GM}{r}$$
$$\frac{\phi_Q}{\phi_P} = \frac{r_P}{r_Q} = \frac{X}{2X}$$
$$\phi_Q = -800 \times \frac{1}{2} = -400 \text{ kJ kg}^{-1}$$

Work done on 1-kg mass by gravitational force from P to Q
= initial PE – final PE
= –800 – (–400)
= –400 kJ

Remarks:
1. The negative answer makes sense because the gravitational force on the mass acts towards the planet, which is in the opposite direction to the displacement from P to Q.
2. It is useful to remember that work done by gravitational force = loss in GPE and loss in GPE = initial PE – final PE.

3 **B**

From $F = \frac{GMm}{r^2}$: Gravitational force decreases with a larger radius.

From $U = -\frac{GMm}{r}$: GPE becomes less negative and hence increases with a larger radius.

From $\frac{mv^2}{r} = \frac{GMm}{r^2}$, we have $v = \sqrt{\frac{GM}{r}}$: Linear speed decreases with a larger radius.

From $\omega = \sqrt{\frac{GM}{r^3}}$: Angular velocity decreases with a larger radius.

4 **D** Since gravitational force on the satellite provides the centripetal force, $a_c = g$

$$g = r\left(\frac{2\pi}{T}\right)^2$$
$$T^2 = 4\pi^2 \frac{r}{g}$$
$$\left(\frac{T_M}{T}\right)^2 = \frac{\frac{R/4}{g/6}}{R/g} = \frac{3}{2}$$
$$T_M = \sqrt{\frac{3}{2}}T$$

5 **(a)**

$$\omega = \frac{2\pi}{T}$$

$$= \frac{2\pi}{24 \times 3600}$$ M1

$$= 7.27 \times 10^{-5} \text{ rad s}^{-1}$$ A1

(b)

$$v = r\omega$$

$$= 4.23 \times 10^{7} \times 7.27 \times 10^{-5}$$ M1

$$= 3.08 \times 10^{3} \text{ m s}^{-1}$$ A1

(c)

$$a = r\omega^2$$

$$= 4.23 \times 10^{7} \times 7.27 \times 10^{-5}$$ M1

$$= 0.226 \text{ m s}^{-2}$$ A1

(d) Gravitational attraction provides the centripetal force:

$$F_g = ma$$ M1

$$= 2400 \times 0.226$$

$$= 537 \text{ N}$$ A1

(e)

$$F_g = \frac{GMm}{r^2}$$

$$M = \frac{F_g \times r^2}{Gm}$$

$$= \frac{537 \times \left(4.23 \times 10^{7}\right)^2}{6.67 \times 10^{-11} \times 2400}$$ M1

$$= 6.0 \times 10^{24} \text{ kg}$$ A1

6 **(a)** **(i)** Gravitational field strength is numerically equal to the potential gradient at the point. B1
The direction of the gravitational field strength is towards lower potential. B1

(ii) The potential at infinity is defined to be zero. B1
Since gravitational force is attractive in nature, to bring a mass from infinity to a point in the gravitational field, the direction of the external force is opposite to the direction of displacement of the mass. This results in negative work done by the external force. B1

(b) **(i)** $E_p = \phi_p \times m = \left(-20 \times 10^{6}\right) \times 2.0 = -4.0 \times 10^{7} \text{ J}$ A1

(ii)

$$\text{Gravitational field stength} = \frac{\left[-10 - \left(-34\right)\right] \times 10^{6}}{\left(30 - 5\right) \times 10^{6}}$$ M1

$$= 0.96 \text{ N kg}^{-1}$$ A1

(iii) Difference in potential $= -40 \text{ MJ kg}^{-1}$ A1

(iv) To escape, the total energy $E_T \geq 0$

$$E_P + E_K \geq 0$$

$$-\frac{GMm}{R_{\text{Earth}}} + \frac{1}{2}mv^2 \geq 0$$ M1

$$v \geq \sqrt{\frac{2GM}{R_{\text{Earth}}}} \Rightarrow \text{escape velocity} = \sqrt{\frac{2GM}{R_{\text{Earth}}}}$$ A1

7 **(a)** **(i)**

$$\text{angular velocity} = \frac{2\pi}{24 \times 3600}$$

$$= 7.272 \times 10^{-5} \text{ rad s}^{-1}$$ B1

$$= 7.27 \times 10^{-5} \text{ rad s}^{-1}$$

(ii)

$$\text{centripetal acceleration} = R\omega^2$$

$$= 6.38 \times 10^6 \times \left(7.27 \times 10^{-5}\right)^2$$ M1

$$= 0.0337 \text{ m s}^{-2}$$ A1

(b) **(i)** An object at the equator is undergoing circular motion with radius 6.38×10^6 m. The effective acceleration of free fall is thus given by the difference between the gravitational field strength and the centripetal acceleration: B1

$$g_{eff} = \frac{GM}{R^2} - a_c$$

At the poles, the radius of the circular motion is zero. Hence a_c = 0 and B1

$$g_{eff} = \frac{GM}{R^2}$$

(ii) The difference is a_c which has a value of 0.0337 m s^{-2} which is small compared to 9.81 m s^{-2}. A1