Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2018
Secondary Three

INTEGRATED MATHEMATICS 2

2 hours

Tuesday **2 October 2018** **1130 – 1330**

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Answer questions 1 – 11 before attempting the Bonus Question.
2. Write your answers and working on the separate writing paper provided.
3. Write your name, register number and class on each separate sheet of paper that you use and fasten the separate sheets together with the string provided. Do not staple your answer sheets together.
4. Omission of essential steps will result in loss of marks.
5. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 80.
3. The use of an electronic calculator is expected, where appropriate.
4. You are reminded of the need for clear presentation in your answers.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The spread of a disease in a small town is modelled by the equation
$N = \dfrac{A}{1+34e^{-0.2t}}$ where A is a constant. In the equation, N is the number of people infected by the end of Day t. It is known that 25 people were infected by the end of Day 0 before the disease started spreading.

(i) Find the value of A. [2]
(ii) Hence, determine if the disease is expected to have infected more than 95 people by the end of Day 7. [2]

2 It is given that $\operatorname{cosec}\theta = k$, where $0 < \theta < \pi$. Express the following in terms of k,

(i) $\sin\theta$, [1]

(ii) $\sec^2\theta$, [2]

(iii) $\tan\theta$, given that $\cos\theta < 0$. [2]

3 **(a)** Prove the identity $\dfrac{\cos x + \cot x}{\cos^2 x} \equiv \dfrac{\cot x}{1-\sin x}$. [3]

(b) Solve the equation $3\sin(2\theta - 40°) + 1 = 0$ for $0° < \theta < 360°$. [4]

4 In triangle ABC, $AB = \left(1+\sqrt{3}\right)$ cm, $\angle ACB = 120°$ and $\angle CAB = 45°$.

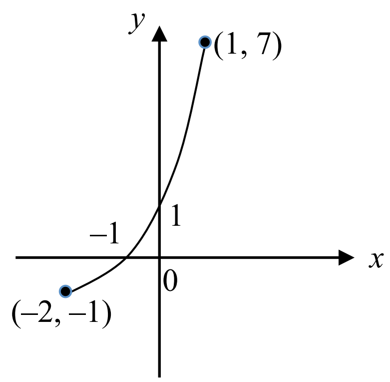
Given that $BC = \left(p\sqrt{2} + q\sqrt{6}\right)$ cm, find the value of p and of q where p and q are rational numbers. [4]

5 The function $f : x \mapsto 1 + \sqrt{2}\cos x$ is defined for $0 \le x \le 3\pi$.

(i) State the amplitude and period of f. [2]
(ii) Sketch the graph of $y = f(x)$ clearly indicating the exact values of all the intercepts and the coordinates of the turning points. [5]

[Turn over

6 The diagram shows the graph of $y = \mathrm{g}(x)$ for $-2 \leq x \leq 1$.



The curve cuts the coordinate axes at the points (–1, 0) and (0, 1). State

(i) the maximum value of $|\mathrm{g}(x)|$, [1]

(ii) the coordinates of the point where the graph of $y = \mathrm{g}^{-1}(x)$ cuts the x-axis, [1]

(iii) the value of x for which $\mathrm{g}(-x) = 0$, [1]

(iv) the coordinates of the maximum point of $y = \frac{3}{2}\mathrm{g}(x + 6)$. [2]

7 Solve the following equations

(a) $\left|3x^2 - 4\right| + x = 0$, [4]

(b) $10^{5y} = 20^{3y+4}$, [3]

(c) $\log_4(5 - 12w) - 2\log_4 w = \log_2 3$. [6]

8 It is given that $y = 2x^2 + bx + c$.

(a) In the case where $b = -6$ and $c = -3$, express y in the form $a(x - h)^2 + k$ where a, h and k are constants. State the minimum value of y. [3]

(b) In the case where $c = 2$, find the range of values of b for which the equation $y = 0$ has real and distinct roots. [3]

(c) In the case where $b = -4$ and $c = -7$, find the quadratic equation whose roots are α^3 and β^3 given that the roots of the equation $y = 0$ are α and β. [4]

9 **(a)** The function g is defined as g : $x \mapsto \log_x 5$. Write down the largest possible domain for the function g(x). [1]

(b) **(i)** Sketch the graph of $y = 1 + 3e^{-2x}$ labelling the asymptote and intercept(s) clearly. [2]

(ii) Find the equation of the straight line to be added onto the same axes so as to solve the equation $2x + \ln\frac{4-4x}{3} = 0$. Add this line to your sketch in **(i)** to determine the number of solutions to the equation $2x + \ln\frac{4-4x}{3} = 0$ [4]

10 The term containing the highest power of x in the polynomial f(x) is $8x^3$. It is known that f(x) leaves a remainder of –63 when divided by $x + 2$. It is also given that $(2x + 1)^2$ is a factor of f(x).

(i) Find the other factor of f(x). [3]

(ii) Express $\frac{f(x)+8}{f(x)}$ as partial fractions. [6]

11 The function h is defined by

$$h : x \mapsto \begin{cases} 2x+4, & x < -2, \\ (x+2)^2, & x \geq -2. \end{cases}$$

(i) Find $h^2\left(-2\frac{1}{2}\right)$. [2]

(ii) Sketch the graph of $y = h(x)$, [3]

(iii) Find in similar form the function h^{-1} and state its domains. [4]

12 **Bonus**

The roots of the quadratic equation $\left(4\sqrt{3}+7\right)x^2 + \left(\sqrt{3}+2\right)x = 2$ are α and β, where $\alpha < \beta$. Find the value of $\alpha - \beta$ giving your answer in the form $a\sqrt{3}+b$ where a and b are integers. [4]

End of Paper