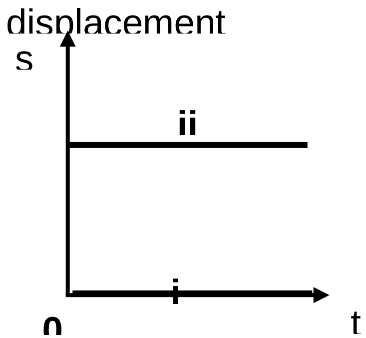
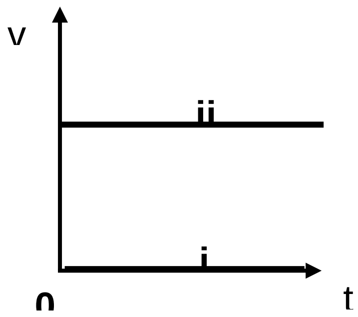
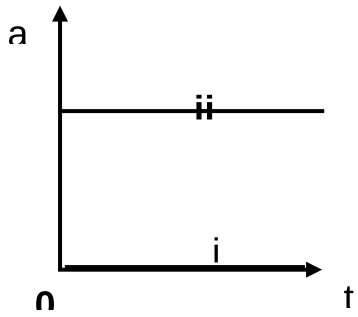
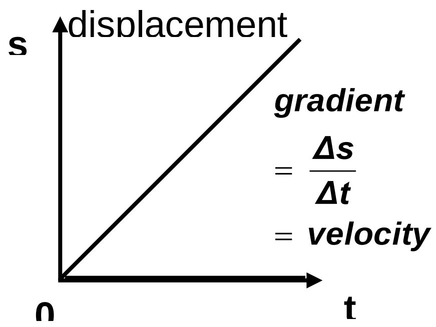
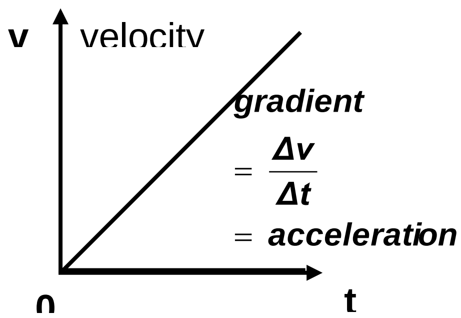
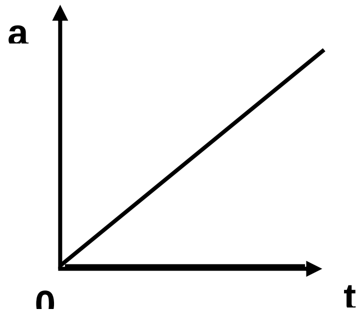
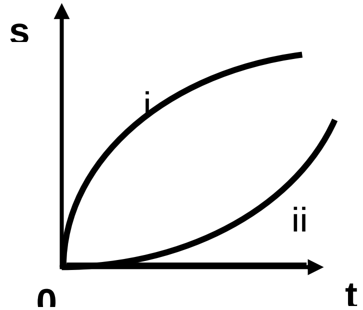
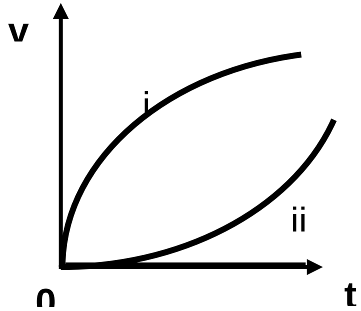
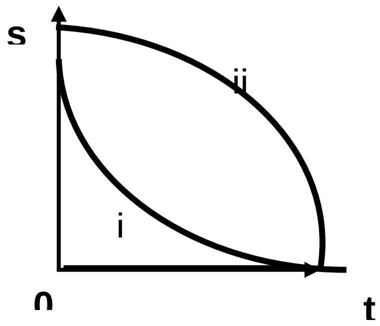
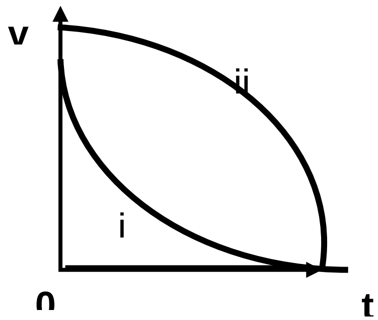
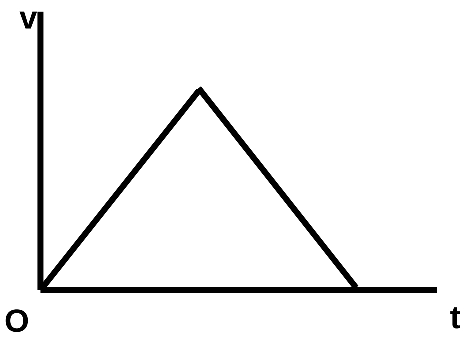
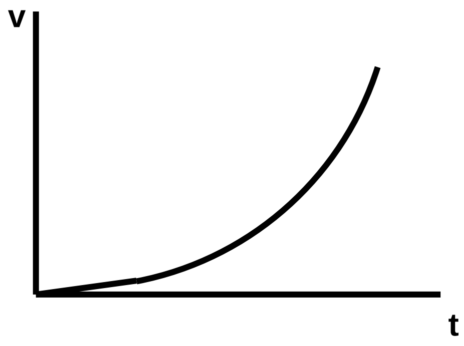
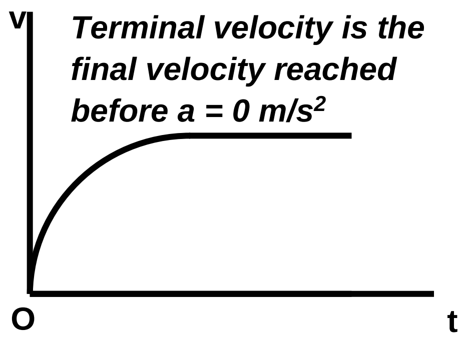
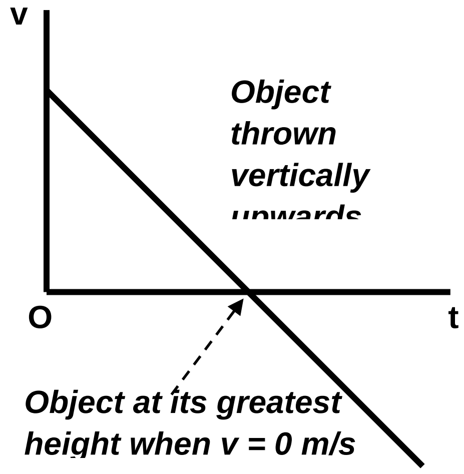
2	Kinematics				
	Term	**Definition**	**SI unit & Symbol**	**Formula**	**Remarks/Examples**
2.1	Speed, v	Is the rate of change of distance	m/s	*Average speed* $= \dfrac{distance}{time}$	*A car moving round a circular track takes 240 s to do a lap of 8 km. What is the speed of the car in* *(a) km/h* *(b) m/s* *(a) Average speed* $= \dfrac{distance}{time} = \dfrac{8\ km}{\frac{240}{3600}\ h} = 120\ km/h$ *(b) Average speed* $= \dfrac{distance}{time} = \dfrac{8000\ m}{240\ s} = 33.3 m/s$
	Velocity, v	Is the rate of change of displacement *Displacement is distance travelled in a specified direction.*	m/s	*Average velocity* $= \dfrac{displacement}{time}$	*A car has a velocity of 72 km/h. How far does it travel in half a minute ?* $velocity = \dfrac{displacement}{time}$ $Displacement = velocity\ x\ time$ $= 72\ km/h\ \ x\ \dfrac{1}{2} minute$ $= \dfrac{72\,000\ m}{3600\ s} \times \dfrac{1}{2} \times 60 = 600\ m$

	Quantities	Definition	SI unit & symbol	Formulae	Worked Examples/Remarks
	Acceleration, a	**Is the rate of change of velocity**	**m/s^2**	$Accleration = \dfrac{velocity\ change}{time\ taken\ to\ make\ the\ change}$ $\boxed{a = \dfrac{v-u}{t}}$ ***where a is the acceleration in m/s^2*** ***v is the final velocity in m/s*** ***u is the initial velocity in m/s*** ***t the time in seconds*** ***Rearranging the above equation,*** $\boxed{v = u + at}$	***Worked example 1*** ***A car traveling at 10 m/s accelerates uniformly at 2 m/s^2. Find its velocity in 5 s time.*** ***Solution*** ***Initial velocity, u = 10 m/s*** ***acceleration, a = 2 m/s^2*** ***time, t = 5 s*** ***v = u + at*** ***v = 10 + 2 x 5 = 20 m/s*** ***Worked example 2*** ***A train slows from 20 m/s with a uniform deceleration of 2m/s^2. How long will it take to reach 5 m/s ?*** ***Solution*** ***Initial velocity, u = 20 m/s*** ***Final velocity, v = 5 m/s*** ***Deceleration = 2 m/s^2*** ***Meaning acceleration = $-$2m/s^2*** $a = \dfrac{v-u}{t}$ ***Rearranging the equation,*** $t = \dfrac{v-u}{a}$ $= \dfrac{5-20}{-2} = \dfrac{-15}{-2} = 7.5\ s$

2.2	Graphical Analysis of motion			
	Displacement-time graph / distance-time graph	**Velocity-time graph Speed-time graph**	**Acceleration-time graph**	**Remarks**
				***1. The <u>gradient of displacement-time</u> graph <u>tells us</u> about the <u>velocity</u>.*** ***2. The <u>gradient of the velocity-time</u> graph <u>tells us</u> the <u>acceleration</u>.*** 3. ***The <u>area under the velocity-time graph</u> <u>tells us</u> the <u>distance</u> travelled.***
	Both (i) and (ii) show that the <u>object is at rest, not moving</u> <u>Gradient</u> is <u>zero</u>, <u>distance not changing</u> with time.	***(i) shows object at rest, <u>Speed</u> , v = 0.*** ***(ii)shows object moving at <u>constant</u> or <u>uniform speed</u>. <u>gradient</u>=<u>acceleration</u> = <u>zero</u>.***	***(i) Particle not accelerating. <u>Acceleration</u> is <u>zero</u>.*** ***(ii) Particle moves with <u>constant acceleration</u>***	
				***A very important concept here is the <u>gradient</u> of a graph.*** ***It tells us how the <u>vertical quantity changes with the horizontal quantity</u>.***
	<u>Constant gradient</u> shows <u>constant speed</u>.	***<u>Constant gradient (or slope)</u> shows <u>constant acceleration</u>.***	***The <u>acceleration is increasing at a constant rate</u>.***	

	displacement 	***velocity*** 		
	(i) ***decreasing gradient shows decreasing speed*** ***(ii)*** ***increasing gradient shows increasing speed***	***(i)*** ***decreasing gradient shows decreasing acceleration*** ***(ii)*** ***increasing gradient shows increasing acceleration***		
	 ***Object returns to starting point*** ***(i)*** ***at decreasing speed*** ***(ii)*** ***at increasing speed***	 ***The speed is decreasing*** ***(i)*** ***at decreasing rate*** ***(ii)*** ***at increasing rate***		

	 *Object <u>accelerates uniformly from rest</u> and <u>decelerates uniformly to rest</u> travelling between 2 floors or 2 train stations*		 *Object falling from a great height through air or through a viscous liquid such as glycerine*	
	This is <u>the typical velocity-time graph showing</u> *(i)* *a train traveling between 2 stations* *(ii)* **A lift moving between two floors.** *As it leaves one floor for another<u>, it accelerates until it reaches midway</u> between the 2 floors <u>whereby it starts to decelerate</u> until it reaches the other floor.*	*(i)* *This velocity-time graph shows a puck <u>accelerating at an increasing rate</u>.*	*This is <u>the typical velocity-time graph showing</u>* *(i) an object falling from a great height such as from an aeroplane.* *(ii) It <u>accelerates at a decreasing rate</u> because of <u>air resistance which increases as speed increases</u>, <u>causing</u> the <u>net force</u> and <u>hence</u> the <u>acceleration to decrease to zero</u>.* *(iii)It could also represent an object falling through a viscous liquid.*	*(i)* *This is the <u>typical velocity-time graph</u> showing <u>an object thrown vertically upwards</u> with a certain speed.* *(ii)* *It travels with <u>uniform deceleration due to gravity</u>.* *(iii)* *<u>velocity = 0</u> when the <u>object reaches its greatest height</u>.*

2.3	Free-fall	The 4 equations of linear motion (motion in a straight line)
	Experiment shows that a body falling freely under gravity, accelerates uniformly at 10 m/s^2 , This means that its velocity increases by 10 m/s every second of its fall. An object thrown vertically upwards decelerates at 10 m/s^2 . This means its velocity decreases by 10 m/s every second as it rises until it reaches its maximum height. Thereafter it will accelerate uniformly as it falls freely under gravity.	1) v = u + at 2) s = ½ (u + v) t 3) s = ut + ½ at^2 4) v^2 = u^2 +2as where v is the final velocity u is the initial velocity s is the distance travelled a is the acceleration The 4 equations above are the basic relations for motion under uniform acceleration.

To show the first equation of motion

By definition of acceleration,

$$a = \frac{v - u}{t}$$

$$at = v - u$$

$$v = u + at$$

To show the second equation of motion

Distance travelled $=$ average speed x time

$$s = \left(\frac{v + u}{2}\right)t$$

$$= \left(\frac{u + at + u}{2}\right)t$$

$$s = \left(\frac{2u + at}{2}\right)t$$

$$s = ut + \frac{1}{2}at^2$$

Distance travelled is given by area under the velocity-time graph

To show the second equation of motion

Distance travelled $=$ average speed x time

$$s = \left(\frac{v + u}{2}\right)\left(\frac{v - u}{a}\right)$$

$$= \frac{v^2 - u^2}{2a}$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as$$

	The two worked examples on the right show how the equations of motion can be used to solve kinematics problems.	***<u>Worked example 1 on linear motion</u>*** ***A ball is thrown vertically upwards from the ground with a velocity of 20 m/s. Calculate*** ***(a) the maximum height reached*** ***(b) the time to reach the maximum height*** ***(c) the time to reach the ground again after the ball is thrown up*** ***(d) the velocity reached half-way to the maximum height. (Assume acceleration due to gravity, g = 10 m/s^2.)*** ***<u>Solution</u>*** ***(a)*** $u = 20$ m/s ; $v = 0$ m/s ; $a = 10$ m/s^2 $v^2 = u^2 + 2as$ $20^2 = 0^2 + 2 \times 10 \times s$ $400 = 20\,s$ $s = 400/20 = \underline{20\text{ m}}$ ***The maximum height reached is <u>20 m</u>*** ***(b) By definition of acceleration,*** $a = \dfrac{v-u}{t}$ $t = \dfrac{v-u}{a}$ $= \dfrac{0-20}{-10} = 2$ s ***The time to reach the maximum height is 2 seconds.*** ***(c) The time to reach the ground again after the ball is thrown up is also 2 seconds.*** ***(d)*** $v^2 = u^2 + 2as = 20^2 - 2x10x10 = 400 - 200$ $v = \sqrt{200} = 14.1$m/s	***<u>Worked example 2 on linear motion</u>*** ***A car travels with a velocity of 18 km/h. It then accelerates uniformly and travels a distance of 50 m. If the velocity reached is 54 km/h, find the acceleration and the time to travel this distance.*** ***<u>Solution</u>*** ***Initial velocity,*** $u = 18$ km/h $= 18\,000$m/3600 s $= 5$ m/s ***Distance travelled = 50 m*** ***Final velocity ,*** $v = 54$ km/h $= 54\,000$m/3600s $= 15$ m/s $v^2 = u^2 + 2as$ $15^2 = 5^2 + 2 \times a \times 50$ $200 = 100a$ $a = 200/100 = \underline{2\text{ m/s}^2}$ ***The acceleration is 2 m/s^2*** $a = \dfrac{v-u}{t}$ $t = \dfrac{v-u}{a}$ $= \dfrac{15-5}{2} = 5$ s

1 Fig. 1.1 shows a gymnast on a trampoline. He reaches a height of 3.2 m above the trampoline.

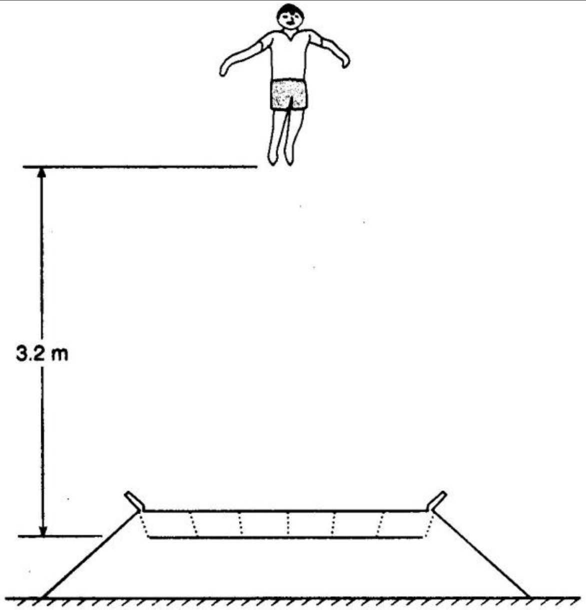
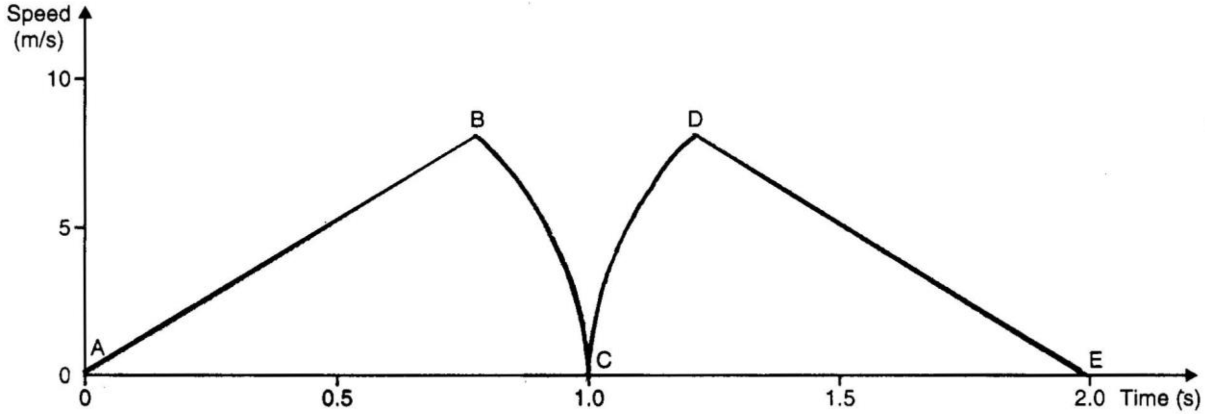


Fig. 1.2 shows how the speed of the gymnast changes as he exercises on the trampoline.



(a)Describe in words what is <u>happening to the gymnast</u> between points

(i) C and D

Between C and D**, the **<u>gymnast is being pushed up by the trampoline</u>**; and he **<u>accelerates at</u> a <u>decreasing rate</u>**. **[1],[1]

(ii) D and E

Between D and E**, the **<u>gymnast is rising</u>**, but **the deceleration due to gravity causes him to <u>decelerate at a constant rate</u>.[1],[1]

(b) At which point of the graph is the gymnast at his lowest position ?

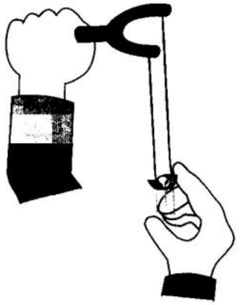
<u>At C (where his kinetic energy which he gained as he fell, is converted to elastic potential energy in the trampoline</u>

(c) Calculate the acceleration of the gymnast when he is falling.

His <u>acceleration</u> when he is falling is (5 - 0)/ (0.5 - 0) m/s^2 = <u>10 m/s^2</u> [2]

2 A stone is <u>catapulted vertically upwards from the ground with a speed of 20 m/s</u> (see the Figure below).

(a) Neglecting air resistance and taking acceleration due to gravity to be 10 m/s^2, <u>calculate the time taken</u> by the stone <u>to rise to its maximum height</u>.

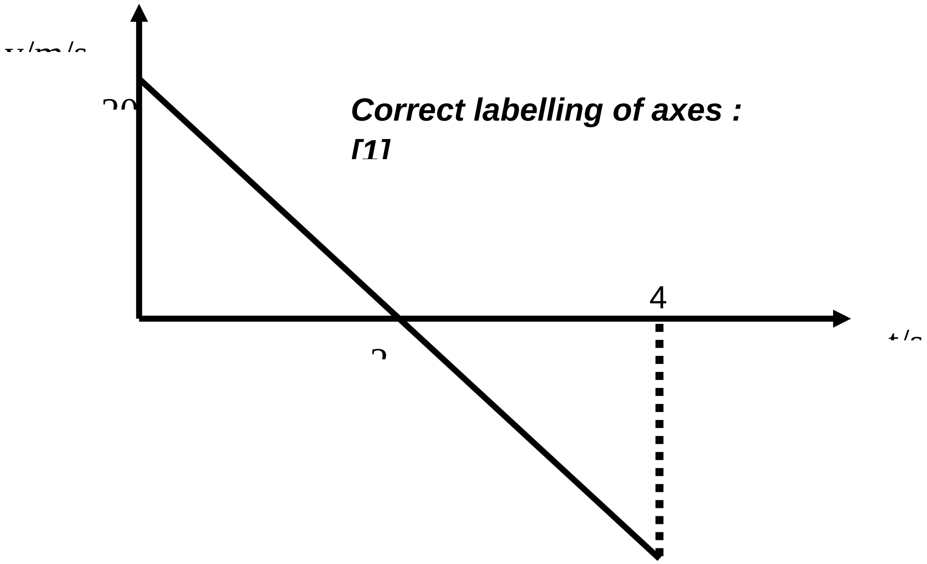


$$a = \frac{v - u}{t}$$

$$-10 = \frac{0 - 20}{t} \qquad [1]$$

$$t = \frac{-20}{-10} = 2\ s \qquad [1]$$

(b) Sketch the velocity-time graph to show the motion of the stone from its starting point till it returns to the ground.



Correct labelling of axes : [1]

(c) ***Find the maximum height reached by the stone.***

Maximum height reached by the stone

= *Distance travelled under v - t graph between t = 0 and t = 2 seconds*

$= \frac{1}{2} \times 2 \times 20 = 20\ m$ [2]

(d) ***With what speed does the stone hit the ground ?***

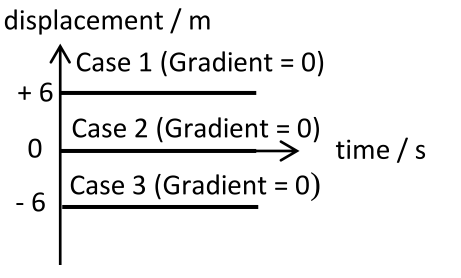
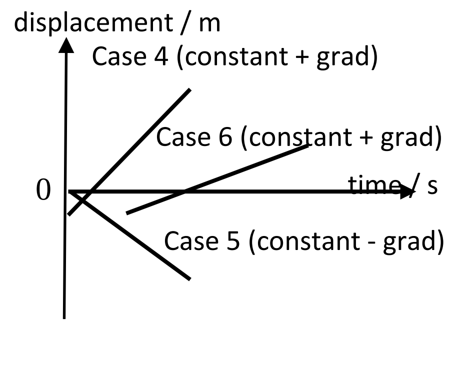
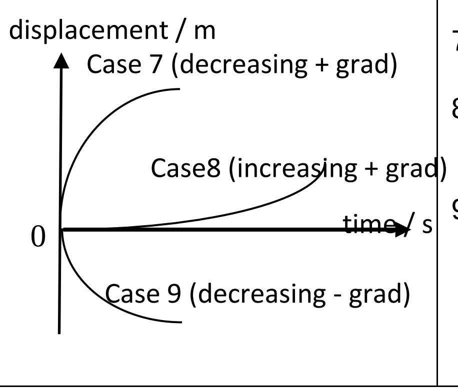
$v = u + at$

$= 0 + 10 \times 2$

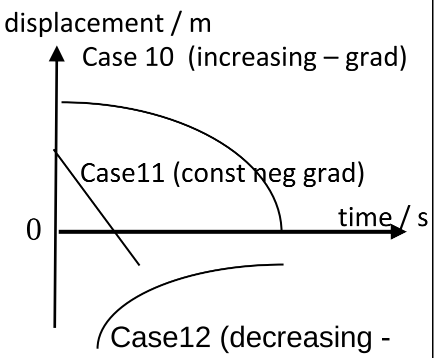
$= 20\ m / s$ [2]

Graphical Analysis of s-t & v-t graphs

DISPLACEMENT –TIME GRAPH

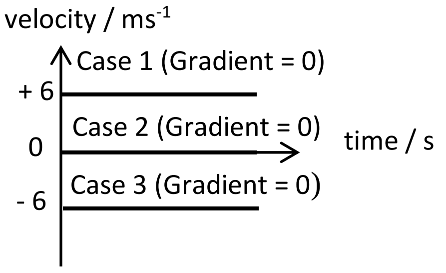
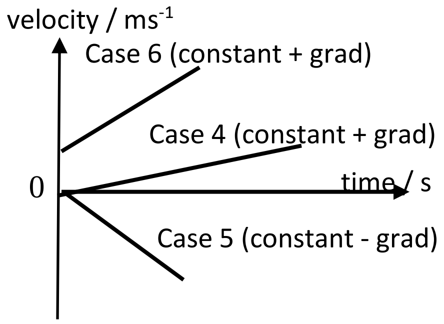
GRAPH	Case	Displacement	Velocity	Acceleration	Remarks
	1 2 3	●Δs = 0 *s* is **positive 6 m**________ ●Δs = 0 *s* is _zero________ ●Δs = 0 *s* is negative 6 m_________	0 _0___ _0___	0 __0____ ___0___	●Object is stationary__ and _0 distance_ from origin. ●Object is stationary and +6 m from origin. ●Object is stationary_ and -6m from origin.
	4 5 6	●*s* increases at constant rate in the positive direction. Δs is positive. ●*s* increases at constant rate in the negative__ direction. Δs is negative_____. ●*s* increases at constant _ rate in the positive direction. Δs is positive.	●constant (positive____) ●constant (negative______) ●constant (positive______)	0 0____ _0____	●Object moves away___ from the origin in the positive direction. ●Object moves away______ from the origin in the negative________ direction ●Object moves pass origin towards the positive________ direction
	7 8 9	*s* increases at decreasing____ rate. Δs is positive. *s* increases at increasing___ rate. Δs is positive. *s* increases at decreasing__ rate in the negative____ direction. Δs is negative________.	decreasing in the + direction. increasing in the + direction. decreasing in the - direction.	declerating__ accelerating_ _____ decelerating	Object slows down as it moves away from the origin in the positive direction. Object starts from rest and accelerates away from the origin. Object slows______ down as it moves __away______ from the origin in the negative direction.

DISPLACEMENT –TIME GRAPH

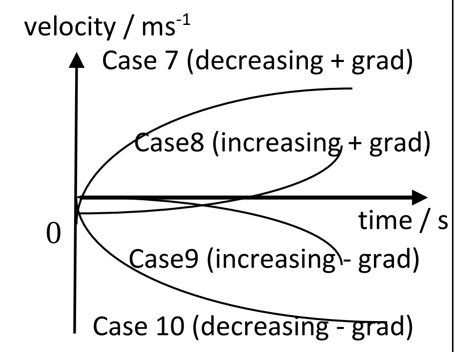
GRAPH	Case	Displacement	Velocity	Acceleration	Remarks
	10	*s* decreasing_ at increasing rate. Δ*s* is negative.	increasing in the negative direction.	accelerating_ in the negative direction	Object moves back__ towards the origin.
	11	*s* changes at constant rate. Δ*s* is positive.	constant in the negative direction.	0	Object moves past origin towards the negative direction.
	12	*s* changes at a decreasing rate in the positive direction. Δ*s* is positive.	decreasing in the positive direction.	decelerating_ in the positive direction	Object moves back towards the origin.

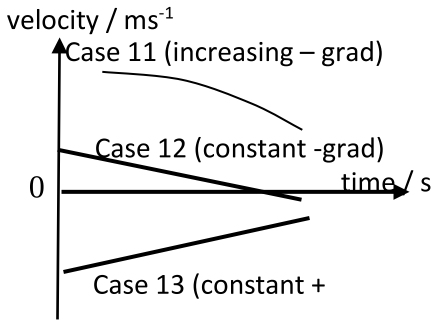
1.2 Kinematics Summary

VELOCITY – TIME GRAPH

GRAPH	Case	Displacement	Velocity	Acceleration	Remarks
	1 2 3	*s* increases at constant rate in the positive direction. s = 0 m. At rest. *s* increases at constant rate in the negative direction.	Constant positive 6 ms^{-1} Zero Constant negative 6 ms^{-1}	0 0 0	Object moves in the positive direction. Object is stationary and at origin. Object moves in the negative direction.
	4 5 6	Positive negative Positive (greater increase compared to case 4 due to steeper gradient)	increases in the positive direction at a constant rate increases in the negative direction at a constant rate. increases in the positive direction at a greater constant rate compared to case 4.	Constant acceleration (positive) Constant negative acceleration Constant acceleration (positive)	Object starts from rest and has constant acceleration in the positive direction Object starts from rest and has constant acceleration in the negative direction. Object has velocity at t = 0 s.

1.2 Kinematics Summary

GRAPH	Case	Displacement	Velocity	Acceleration	Remarks
	7	Positive	increases non constantly in the positive direction.	+ acceleration decreases to zero (decreasing acceleration)	Object moves in the + direction and reaches constant speed.
	8	Positive	increases non constantly in the positive direction.	Increasing acceleration	Object moves in the + direction with increasing speed.
	9	Negative	increases non constantly in the negative direction.	Increasing negative acceleration/ Increasing deceleration	Object moves in the negative direction with increasing speed.
	10	Negative	increases non constantly in the negative direction.	Decreasing - acceleration / retardation decreases to zero	Object moves in the negative direction and reaches constant speed.

GRAPH	Case	Displacement	Velocity	Acceleration	Remarks
	11	Positive	Decreases in the positive direction at non constant rate	Deceleration increasingly in the positive direction.	Object moving in the positive direction and slowing down.
	12	Positive	decreases to zero.	Constant negative acceleration to 0 m/s	Object moving in the positive direction at constant deceleration.
	13	Negative	Decreases constantly in the negative direction.	Constant positive acceleration	Object moving in the negative direction at constant positive acceleration.