Topical Revision Paper 5

Term 2 Unit 5: Exponents & Logarithms

Question 1:

(i) Show that $\log_{\sqrt{3}}(x-1)+\log_{\sqrt{3}}\sqrt{3}=\log_3 3(x-1)^2$. [4]

(ii) Hence, solve the equation $\log_3(x+1)-\log_{\sqrt{3}}(x-1)=\log_{\sqrt{3}}\sqrt{3}$.

[4]

Solution:

(i)

$$\log_{\sqrt{3}}(x-1)+\log_{\sqrt{3}}\sqrt{3}$$

$$=\log_{\sqrt{3}}\sqrt{3}(x+1)$$

$$=\frac{\log_3\sqrt{3}(x+1)}{\log_3\sqrt{3}}$$

$$=2\log_3\sqrt{3}(x+1)$$

$$=\log_3 3(x+1)^2 \text{ (shown)}$$

(ii)

$$\log_3(x+1)-\log_{\sqrt{3}}(x-1)=\log_{\sqrt{3}}\sqrt{3}$$

$$\log_3(x+1)=\log_{\sqrt{3}}(x-1)+\log_{\sqrt{3}}\sqrt{3}$$

$$\log_3(x+1)=\log_3 3(x-1)^2$$

$$x+1=3(x-1)^2$$

$$3(x-1)^2-x-1=0$$

$$3x^2-7x+2=0$$

$$(3x-1)(x-2)=0$$

$$x=\frac{1}{3} \text{ (n.a.)} \quad \text{or} \quad x=2$$

Question 2:

Solve the simultaneous equations

$$\log_2 xy=7$$
$$\log_4 x=2\log_8 y$$

[4]

Solution:

$\frac{\log_2 x}{\log_2 4} = 2\frac{\log_2 y}{\log_2 8}$ $\log_2 x = \frac{4}{3}\log_2 y \quad \cdots(1)$ $\log_2 x + \log_2 y = 7 \quad \cdots(2)$ Sub (1) into (2), $\frac{7}{3}\log_2 y = 7$ $\log_2 y = 3$ $y = 8$ $\log_2 x = 4$ $x = 16$
Question 3: Solve for x, given that $3^{2x-1} + 3^{x-2} = 3^x - \frac{4}{9}$. [4]
Solution: $3^{2x} \cdot 3^{-1} + 3^x \cdot 3^{-2} = 3^x - \frac{4}{9}$ Let $u = 3^x$ $\frac{1}{3}u^2 + \frac{1}{9}u = u - \frac{4}{9}$ $3u^2 + u = 9u - 4$ $3u^2 - 8u + 4 = 0$ $(3u-2)(u-2) = 0$ $u = \frac{2}{3}$ or 2 $3^x = 2$ $3^x = \frac{2}{3}$ $x = \frac{\lg 2}{\lg 3}$ or 0.631 $x = \frac{\lg \frac{2}{3}}{\lg 3}$ or -0.369
Question 4: Solve the following equations a) $\log_8[2 + \log_2(x-4)] = \log_{125} 5$ (b) $e^{2\ln y} = \lg 1.6$

Solution:

(i)

$$\log_8[2+\log_2(x-4)] = \log_{125} 5 \Rightarrow \log_8[2+\log_2(x-4)] = \frac{1}{3}$$

$$\sqrt[3]{8} = 2+\log_2(x-4)$$

$$\log_2(x-4) = 0$$

$$x-4 = 1$$

$$x = 5$$

(ii)

$$e^{2\ln y} = \lg 1.6 \Rightarrow 2\ln y = \ln(\lg 1.6)$$

$$\ln y = \frac{\ln(\lg 1.6)}{2}$$

$$y = e^{\frac{\ln(\lg 1.6)}{2}}$$

$$y = 0.452 \text{ (3 s.f.)}$$

Question 5:

The mass, m grams, of a radioactive substance after t years is given by the formula $m = ae^{-kt}$, where a and k are constants. At the beginning, the mass of radioactive substance is 83 g. After 35 years, there was only 26 g of the radioactive substance left.

(i) Find the value of a and of k. [4]

(ii) Sketch the graph of m against t. [2]

Solution:

(i)

$$m = ae^{-kt}$$

$$83 = ae^{0}$$

$$a = 83$$

$26 = 83e^{-35k}$

$\frac{26}{83} = e^{-35k}$

$-35k = \ln\frac{26}{83}$

$k = \frac{\ln\frac{26}{83}}{-35} = 0.0332$

(ii)