RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Tutorial C6A: Differentiation Techniques

Section A (Basic Questions)

1 Differentiate each of the following with respect to x:

(a) $x(x^3+1)^{\frac{1}{3}}$

(b) $\dfrac{3x}{(x+3)(x+1)}$

(c) $\sec^3(2x-3)$

(d) $\sin^{-1}\left(\dfrac{x}{2}\right)$

(e) $\cos^{-1}\sqrt{x^3}$ giving your answer in non-trigonometric form, **CJC Promo 9758/2021/Q3(a)**

(f) $\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$

(g) $\left(e^{2x}-e^{-2x}\right)^2$

(h) $\ln\left((x^3+2)^3(x^2+2)^2\right)$

(i) $\log_2(3x^4-e^x)$

(j) 7^{x^2}

Solution:	
(a)	$\dfrac{d}{dx}\left(x(x^3+1)^{\frac{1}{3}}\right)=(x^3+1)^{\frac{1}{3}}+x\left[\dfrac{1}{3}\left(x^3+1\right)^{-\frac{2}{3}}(3x^2)\right]$ $=\dfrac{1}{3}\left(x^3+1\right)^{-\frac{2}{3}}\left[3\left(x^3+1\right)+x(3x^2)\right]$ $=\dfrac{1}{3}\left(x^3+1\right)^{-\frac{2}{3}}\left(6x^3+3\right)$ $=\dfrac{2x^3+1}{\left(x^3+1\right)^{\frac{2}{3}}}$

(b)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{3x}{(x+3)(x+1)}\right)=\dfrac{3(x+3)(x+1)-(2x+4)3x}{(x+3)^2(x+1)^2}$ $=\dfrac{3(x^2+4x+3)-6x^2-12x}{(x+3)^2(x+1)^2}$ $=\dfrac{9-3x^2}{(x+3)^2(x+1)^2}$ $=\dfrac{3(3-x^2)}{(x+3)^2(x+1)^2}$
(c)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sec^3(2x-3)\right)=3\sec^2(2x-3)\cdot\sec(2x-3)\tan(2x-3)\cdot 2$ $=6\sec^3(2x-3)\tan(2x-3)$
(d)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1}\left(\dfrac{x}{2}\right)\right)=\dfrac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\cdot\dfrac{1}{2}$ $=\dfrac{1}{2}\dfrac{2}{\sqrt{4-x^2}}$ $=\dfrac{1}{\sqrt{4-x^2}}$
(e)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\cos^{-1}\sqrt{x^3}\right)=\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\cos^{-1}\left(x^{\frac{3}{2}}\right)\right)$ $=-\dfrac{1}{\sqrt{1-\left(x^{\frac{3}{2}}\right)^2}}\left(\dfrac{3}{2}x^{\frac{1}{2}}\right)$ $=-\dfrac{3\sqrt{x}}{2\sqrt{1-x^3}}$
(f)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\tan^{-1}\left(\dfrac{1+x}{1-x}\right)\right)=\dfrac{1}{1+\left(\dfrac{1+x}{1-x}\right)^2}\cdot\dfrac{1-x-(1+x)(-1)}{(1-x)^2}$ $=\dfrac{(1-x)^2}{(1-x)^2+(1+x)^2}\cdot\dfrac{2}{(1-x)^2}$ $=\dfrac{1}{1+x^2}$
(g)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left((\mathrm{e}^{2x}-\mathrm{e}^{-2x})^2\right)=2(\mathrm{e}^{2x}-\mathrm{e}^{-2x})(2\mathrm{e}^{2x}+2\mathrm{e}^{-2x})$ $=4(\mathrm{e}^{4x}-\mathrm{e}^{-4x})$

<table>
<tr><td>(h)</td><td colspan="3">$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln((x^3+2)^3(x^2+2)^2)\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(3\ln(x^3+2)+2\ln(x^2+2)\right)\\ &= \frac{3(3x^2)}{x^3+2}+\frac{2(2x)}{x^2+2}\\ &= \frac{9x^2}{x^3+2}+\frac{4x}{x^2+2}\\ &= \frac{13x^4+18x^2+8x}{(x^3+2)(x^2+2)}\end{aligned}$$</td></tr>
<tr><td>(i)</td><td colspan="2">Let $y=\log_2(3x^4-\mathrm{e}^x)$
$=\dfrac{\ln(3x^4-\mathrm{e}^x)}{\ln 2}$
$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12x^3-\mathrm{e}^x}{(\ln 2)(3x^4-\mathrm{e}^x)}$</td><td>Recall: $\log_a b=\dfrac{\ln b}{\ln a}$ for $a,b>0$</td></tr>
<tr><td>(j)</td><td>Let $y=7^{x^2}$
$=\mathrm{e}^{\ln 7^{x^2}}$
$=\mathrm{e}^{x^2\ln 7}$
$\dfrac{\mathrm{d}y}{\mathrm{d}x}=2x(\ln 7)\mathrm{e}^{x^2\ln 7}$
$=2x(\ln 7)7^{x^2}$</td><td>Use implicit differentiation
$y=7^{x^2}$
$\ln y=x^2\ln 7$
Differentiate w.r.t x,
$\dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x}=2x\ln 7$
$\dfrac{\mathrm{d}y}{\mathrm{d}x}=(2x\ln 7)7^{x^2}$</td><td>Recall formula:
$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(a^{\mathrm{f}(x)}\right)=\mathrm{f}'(x)\ln a\left(a^{\mathrm{f}(x)}\right)$
$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(7^{x^2}\right)=2x(\ln 7)7^{x^2}$</td></tr>
</table>

2 Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y for each of the following.

(a) $x+y+\sin y=2$

(b) $y=\sin\left((x+y)^2\right)$

(c) $\sin x\cos y=\dfrac{1}{2}$

<table>
<tr><td colspan="2">Solution:</td></tr>
<tr><td>(a)</td><td>$x+y+\sin y=2$
Differentiate w.r.t x,
$1+\dfrac{\mathrm{d}y}{\mathrm{d}x}+\cos y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$
$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{1}{1+\cos y}$</td></tr>
</table>

(b)	$y=\sin\left((x+y)^2\right)$ Differentiate w.r.t x, $\frac{\mathrm{d}y}{\mathrm{d}x}=\cos\left[(x+y)^2\right]2(x+y)\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x}-\cos\left[(x+y)^2\right]2(x+y)\frac{\mathrm{d}y}{\mathrm{d}x}=\cos\left[(x+y)^2\right]2(x+y)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2(x+y)\cos\left[(x+y)^2\right]}{1-2(x+y)\cos\left[(x+y)^2\right]}$
(c)	$\sin x\cos y=\frac{1}{2}$ Differentiate w.r.t x, $\cos y\cos x+\sin x\left(-\sin y\frac{\mathrm{d}y}{\mathrm{d}x}\right)=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\cos x\cos y}{\sin x\sin y}=\cot x\cot y$

3 Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t for each of the following.

(a) $x=\frac{2-3t}{1+t}$, $y=\frac{3+2t}{1+t}$

(b) $x=\frac{1}{2}(\mathrm{e}^t-\mathrm{e}^{-t}), y=\frac{1}{2}(\mathrm{e}^t+\mathrm{e}^{-t})$

Solution:			
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}(\frac{2-3t}{1+t})$ $=\frac{(1+t)(-3)-(2-3t)}{(1+t)^2}$ $=-\frac{5}{(1+t)^2}$	$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}(\frac{3+2t}{1+t})$ $=\frac{(1+t)(2)-(3+2t)}{(1+t)^2}$ $=-\frac{1}{(1+t)^2}$	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}$ $=[-\frac{1}{(1+t)^2}]\div[-\frac{5}{(1+t)^2}]$ $=\frac{1}{5}$
(b)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\mathrm{e}^t-\mathrm{e}^{-t})$ $=\frac{1}{2}(\mathrm{e}^t+\mathrm{e}^{-t})$	$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\mathrm{e}^t+\mathrm{e}^{-t})$ $=\frac{1}{2}(\mathrm{e}^t-\mathrm{e}^{-t})$	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}$ $=\frac{1}{2}(\mathrm{e}^t-\mathrm{e}^{-t})\div\frac{1}{2}(\mathrm{e}^t+\mathrm{e}^{-t})$ $=\frac{\mathrm{e}^t-\mathrm{e}^{-t}}{\mathrm{e}^t+\mathrm{e}^{-t}}$