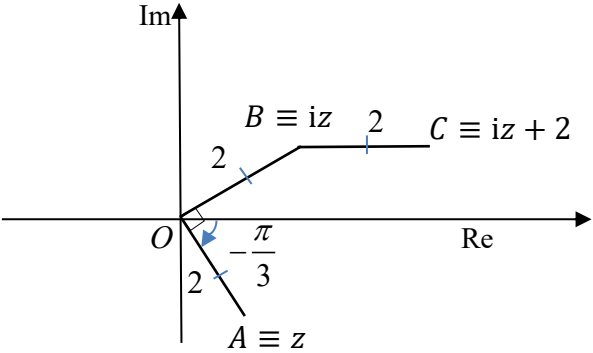
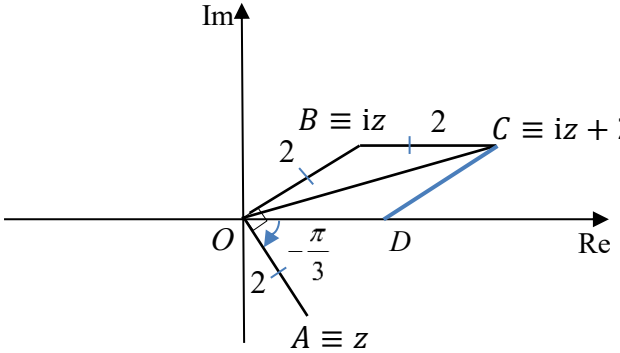


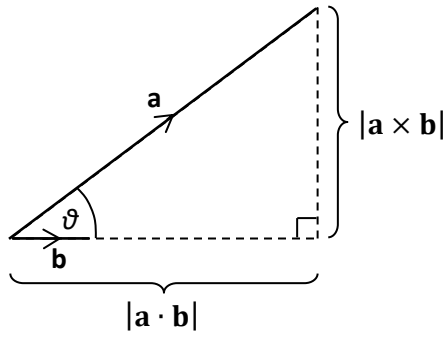
Solutions for 2026 JC2 H2 MA Preliminary Examination Paper 2

1 (a)	$ z = 1 - \sqrt{3}i $ $= \sqrt{1^2 + (\sqrt{3})^2}$ $= 2$ Basic angle, $\alpha = \tan^{-1} \left \frac{-\sqrt{3}}{1} \right = \frac{\pi}{3}$ Since z is in 4th quadrant, $\arg(z) = -\alpha$ $= -\frac{\pi}{3} \text{ or } -60^\circ$	Read the question, calculators are not allowed and hence clear workings need to be shown. Common mistake for z: $\sqrt{1^2 + (\sqrt{3}i)^2}$ *Know the formula for z. While correct, it is a long-winded way to do the following: $ z = \sqrt{(1 + \sqrt{3}i)(1 - \sqrt{3}i)}$ Common mistakes for $\arg z$: 1. $\arg(z) = -\tan^{-1} \frac{1}{\sqrt{3}}$ 2. Taking basic angle as a negative angle $(\alpha = \tan^{-1} \left(-\frac{\sqrt{3}}{1} \right))$ *Basic angle is the acute angle $\alpha = \tan^{-1} \left \frac{y}{x} \right$. Then based on the quadrant, we can then find $\arg(z)$ While still getting the correct answer, workings such as $\arg(z) = \tan^{-1} \left(-\frac{\sqrt{3}}{1} \right)$ may not have clear understanding of finding argument.
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(b)		The vertical axis should be the Im axis and the horizontal axis should be the Re axis. To show the geometrical relationship: 1. $\angle AOB$ should be labelled with 90° 2. OA and OB should be labelled as equal length 3. B and C should be horizontally aligned. (BC parallel to Real axis and C must be on the right of B) 4. BC should be indicated as length 2 There is no need to find the cartesian form of each of the complex numbers. You can utilise the z and $\arg z$ together with the geometrical relationships to label the points. To label A, ensure that the modulus 2 and the $\arg z$ of $-\frac{\pi}{3}$ are labelled. $\arg z$ should be labelled with a directed arrow. Common mistake: - A and B are drawn as reflections of each other in the Real axis. This is for conjugates which is not the case for A and B. - B and C are not horizontally aligned - $\angle AOB$ does not look like a right angle.
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(c)	 $\arg(iz) = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ Since $OBCD$ is a rhombus, $\arg(iz + 2) = \frac{1}{2} \arg(iz)$ $= \frac{1}{2} \left(\frac{\pi}{6} \right)$ $= \frac{\pi}{12}$ or 15°	Majority did not use the diagram from (b) to find $\arg(iz + 2)$ which is not answering question requirement of “Hence”. Workings must be shown to indicate understanding of the use of diagram to find the respective angles. Knowledge of property of Rhombus where the diagonal bisects the angle. ($OBCD$ is a Rhombus because $OB = BC$ and we can then proceed to indicate a point D which will form the rhombus) At no point in the working should there be the use of $\arg _ = \tan^{-1}(_)$.
2(a)	$\mathbf{a} \times \mathbf{c} = n(\mathbf{a} \times \mathbf{b})$ for some (non-zero) constant n $\mathbf{a} \times \mathbf{c} = \mathbf{a} \times n\mathbf{b}$ $\mathbf{a} \times \mathbf{c} - \mathbf{a} \times n\mathbf{b} = \mathbf{0}$ $\mathbf{a} \times (\mathbf{c} - n\mathbf{b}) = \mathbf{0}$ Given non-zero $\mathbf{a}$, $\mathbf{c} - n\mathbf{b} = \mathbf{0}$ or $\mathbf{c} - n\mathbf{b} // \mathbf{a}$ $\mathbf{c} - n\mathbf{b} = \mathbf{0}$ or $\mathbf{c} - n\mathbf{b} = m\mathbf{a}$ for some constant $m \neq 0$ Hence $\mathbf{c} = m\mathbf{a} + n\mathbf{b}$, m and n are constants.	It is important to add in the scalar multiple n since $\mathbf{a} \times \mathbf{c}$ and $\mathbf{a} \times \mathbf{b}$ are parallel. Many students did the following $(\mathbf{a} \times \mathbf{c}) \times (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$ And then <u>wrongly</u> manipulated in the following manner : $\mathbf{a} \times (\mathbf{c} + \mathbf{b}) = \mathbf{0}$. where did this wrongly come from?

2(b)



Let θ be the acute angle between vectors $\mathbf{a}$ and $\mathbf{b}$

$$\tan \theta = \frac{2}{2.5}$$

$$\theta = \tan^{-1}\left(\frac{4}{5}\right) \text{ (shown)}$$

$$|\mathbf{a}| = \sqrt{2^2 + 2.5^2}$$

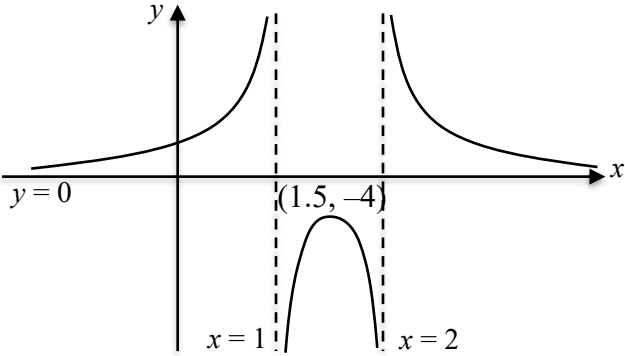
$$= \sqrt{\frac{41}{4}}$$

$$= \frac{\sqrt{41}}{2}$$

Some students missed out on showing $\theta = \tan^{-1}\left(\frac{4}{5}\right)$.

Note that the question asks for the 'exact' value of $|\mathbf{a}|$.

<u>Alternative</u> Let θ be the acute angle between vectors $\mathbf{a}$ and $\mathbf{b}$ $ \mathbf{a} \times \mathbf{b} = \mathbf{a} \mathbf{b} \sin \theta \text{ or } \mathbf{a} \sin \theta,$ $ \mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos \theta \text{ or } \mathbf{a} \cos \theta$ $\frac{ \mathbf{a} \times \mathbf{b} }{ \mathbf{a} \cdot \mathbf{b} } = \frac{2}{2.5}$ $\frac{ \mathbf{a} \sin \theta}{ \mathbf{a} \cos \theta} = \frac{4}{5}$ $\tan \theta = \frac{4}{5}$ $\theta = \tan^{-1} \left(\frac{4}{5} \right) \text{ (shown)}$ $ \mathbf{a} \times \mathbf{b} ^2 + \mathbf{a} \cdot \mathbf{b} ^2 = (\mathbf{a} \sin \theta)^2 + (\mathbf{a} \cos \theta)^2$ $2^2 + 2.5^2 = \mathbf{a} ^2 (\sin^2 \theta + \cos^2 \theta)$ $ \mathbf{a} ^2 = 10.25$ $ \mathbf{a} = \frac{\sqrt{41}}{2}$ Alternatively, $\tan \theta = \frac{4}{5} \Rightarrow \cos \theta = \frac{5}{\sqrt{41}}$ $ \mathbf{a} \cos \theta = 2.5 \Rightarrow \mathbf{a} = \frac{2.5}{5/\sqrt{41}} = \frac{\sqrt{41}}{2}$ Alternatively, $\tan \theta = \frac{4}{5} \Rightarrow \sin \theta = \frac{4}{\sqrt{41}}$ $ \mathbf{a} \sin \theta = 2 \Rightarrow \mathbf{a} = \frac{2}{4/\sqrt{41}} = \frac{\sqrt{41}}{2}$	Some students missed out on showing $\theta = \tan^{-1} \left(\frac{4}{5} \right)$. Note that the question asks for the ‘exact’ value of $\mathbf{a}$.
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3(a)		Students are advised to label the asymptote $y=0$ to leave no doubt that the x-axis is one of the asymptotes. They should also realise that the graph is symmetrical, so the turning point is exactly at $x=1.5$.
(b)	$R_f = (-\infty, -4] \cup (0, \infty)$	There were several errors in writing the interval notation. If students are not confident, they can use alternative forms e.g. $\{y \in \mathbb{R} : y \leq -4 \text{ or } y > 0\}$.
(c)	$R_f = (-\infty, -4] \cup (0, \infty) \quad D_f = \mathbb{R} \setminus \{1, 2\}$ Since $R_f \not\subseteq D_f$, f^2 does not exist.	Many students thought the explanation has to do with the horizontal line test. If they knew that f^2 is the composite function $f \circ f$, then they should use the usual ‘range of first function must be a subset of the domain of the second function’ idea.
(d)	Largest value of $a = \frac{3}{2}$	This is the largest a that still leaves a one-one function. However, many students were ‘overthinking’ and chose the value 1 due to the vertical asymptote there. They forgot that the original domain already <i>omitted</i> $x=1$.

(e)	Let $y = f(x)$ $y = \frac{1}{x^2 - 3x + 2}$ $x^2 - 3x + 2 = \frac{1}{y}$ $\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 2 = \frac{1}{y}$ $\left(x - \frac{3}{2}\right)^2 = \frac{1}{y} + \frac{1}{4}$ $x - \frac{3}{2} = \pm \sqrt{\frac{1}{y} + \frac{1}{4}}$ $x = \frac{3}{2} \pm \sqrt{\frac{1}{y} + \frac{1}{4}}$ Reject $x = \frac{3}{2} + \sqrt{\frac{1}{y} + \frac{1}{4}}$ as $D_f = \left(-\infty, \frac{3}{2}\right]$ $\therefore f^{-1}(x) = \frac{3}{2} - \sqrt{\frac{1}{x} + \frac{1}{4}}$ $D_{f^{-1}} = (-\infty, -4] \cup (0, \infty)$ Alternatively Let $y = f(x)$ $y = \frac{1}{x^2 - 3x + 2}$ $x^2 - 3x + 2 = \frac{1}{y}$ $x^2 - 3x + 2 - \frac{1}{y} = 0$ $x = \frac{-(-3) \pm \sqrt{9 - 4\left(2 - \frac{1}{y}\right)}}{2}$ $= \frac{3}{2} \pm \frac{1}{2} \sqrt{1 + \frac{4}{y}}$ Reject $x = \frac{3}{2} + \frac{1}{2} \sqrt{1 + \frac{4}{y}}$ as $D_f = \left(-\infty, \frac{3}{2}\right]$	The majority chose to complete the square while the rest used the quadratic formula. There were a handful of students who committed algebraic errors such as $(x-1)(x-2) = \frac{1}{y} \Rightarrow x-1 = \frac{1}{y}$ or $x-2 = \frac{1}{y}$; these students need to learn $ab = c \Rightarrow a = c$ or $b = c$ only when $c = 0$. Students must also learn that $a^2 = c \Rightarrow a = \pm\sqrt{c}$; a significant number of them omitted the $\pm$ sign. The rejection of the '+' case must then be accompanied by a reason, especially since this is a 'hence' question. Finally, $D_{f^{-1}} = R_f$, which is generally recognised by the students.
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	$\therefore f^{-1}(x) = \frac{3}{2} - \frac{1}{2} \sqrt{1 + \frac{4}{x}}$ $D_{f^{-1}} = (-\infty, -4] \cup (0, \infty)$	
4 (a)	$\int x e^{2x^2} dx = \frac{1}{4} \int 4x e^{2x^2} dx$ $= \frac{1}{4} e^{2x^2} + C$	Many students did not recognise the standard form $\int f'(x) e^{f(x)} dx$ and wrongly used “by parts” instead.
(b)	$\frac{dw}{dx} = 2x$ [Do not write but this is useful for the substitution: $dw = 2x dx$] $\int x^3 \cos(x^2) dx = \frac{1}{2} \int x^2 \cos(x^2) (2x) dx$ $= \frac{1}{2} \int w \cos w dw$ $= \frac{1}{2} (w \sin w) - \frac{1}{2} \int (1) \sin w dw$ $= \frac{1}{2} (w \sin w) - \frac{1}{2} (-\cos w) + C$ $= \frac{1}{2} x^2 \sin(x^2) + \frac{1}{2} \cos(x^2) + C$ $\int x^3 \cos^2\left(\frac{1}{2}x^2\right) dx = \int x^3 \frac{\cos(x^2) + 1}{2} dx$ $= \frac{1}{2} \int x^3 \cos(x^2) + x^3 dx$ $= \frac{1}{2} \left(\frac{1}{2} x^2 \sin(x^2) + \frac{1}{2} \cos(x^2) + \frac{1}{4} x^4 \right)$ $= \frac{1}{8} (2x^2 \sin(x^2) + 2 \cos(x^2) + x^4) + C$ (shown)	<u>Common errors/misconcepts:</u> <u>Substitution:</u> 1) did not change dx or change wrongly like $dx \rightarrow 2x dw$ 2) did not change back to x <u>By parts:</u> 3) chose the wrong “u” function 4) integrated $\cos w$ wrongly like $\frac{dv}{dw} = \cos w \rightarrow v = -\sin w$ <u>last part</u> 5) Some did not use the “hence...” method, but used substitution $w = x^2$ first.

(c)	Volume $= \pi \int_0^{\sqrt{\pi}} \left(x^{\frac{1}{2}} e^{x^2} \right)^2 dx - \pi \int_0^{\sqrt{\pi}} \left[x^{\frac{3}{2}} \cos \left(\frac{1}{2} x^2 \right) \right]^2 dx$ $= \pi \int_0^{\sqrt{\pi}} x e^{2x^2} dx - \pi \int_0^{\sqrt{\pi}} x^3 \cos^2 \left(\frac{1}{2} x^2 \right) dx$ $= \pi \left[\frac{1}{4} e^{2x^2} \right]_0^{\sqrt{\pi}} - \pi \left[\frac{1}{8} \left(2x^2 \sin(x^2) + 2 \cos(x^2) + x^4 \right) \right]_0^{\sqrt{\pi}}$ $= \frac{\pi}{4} \left[e^{2\pi} - 1 \right] - \frac{\pi}{8} (0 - 2 + \pi^2 - 0 - 2 - 0)$ $= \frac{\pi}{4} \left[e^{2\pi} - 1 \right] - \frac{\pi}{8} (-4 + \pi^2)$ $= \frac{\pi}{4} e^{2\pi} + \frac{\pi}{4} - \frac{\pi^3}{8}$	<u>Common errors:</u> 1) did not use π 2) used 2π instead of π 3) did not put in the limits or wrong limits 4) used $\pi \int_0^{\sqrt{\pi}} (y_1 - y_2) dx$ or $\pi \int_0^{\sqrt{\pi}} (y_1 - y_2)^2 dx$ 5) $\pi \int_0^{\sqrt{\pi}} \left(x^{\frac{1}{2}} e^{x^2} \right)^2 dx = \pi \int_0^{\sqrt{\pi}} x e^{x^4} dx$														
5(a)	<table><tr><td>Base area</td><td>Cost of base</td></tr><tr><td>$= \frac{1}{2} x^2 \sin 60^\circ$ or</td><td>$= 2 \times \frac{\sqrt{3}}{4} x^2$</td></tr><tr><td>$\frac{1}{2} x \sqrt{x^2 - \left(\frac{1}{2} x \right)^2}$</td><td>$= \frac{\sqrt{3}}{2} x^2$</td></tr><tr><td>$= \frac{\sqrt{3}}{4} x^2$</td><td></td></tr><tr><td>Each side area</td><td>Cost of 3 sides</td></tr><tr><td>$= xh$</td><td>$= 5 \times 3xh$</td></tr><tr><td></td><td>$= 15xh$</td></tr></table> $\frac{\sqrt{3}}{2} x^2 + 15hx = C$ $h = \frac{1}{15x} \left(C - \frac{\sqrt{3}}{2} x^2 \right)$ $V = \text{Base area} \times \text{Height}$ $= \frac{\sqrt{3}}{4} x^2 \times \frac{1}{15x} \left(C - \frac{\sqrt{3}}{2} x^2 \right)$ $= \frac{\sqrt{3}}{60} x \left(C - \frac{\sqrt{3}}{2} x^2 \right) \text{ (shown)}$	Base area	Cost of base	$= \frac{1}{2} x^2 \sin 60^\circ$ or	$= 2 \times \frac{\sqrt{3}}{4} x^2$	$\frac{1}{2} x \sqrt{x^2 - \left(\frac{1}{2} x \right)^2}$	$= \frac{\sqrt{3}}{2} x^2$	$= \frac{\sqrt{3}}{4} x^2$		Each side area	Cost of 3 sides	$= xh$	$= 5 \times 3xh$		$= 15xh$	For a ‘Show that’ question, all steps must be shown, and the given expression cannot be obtained from wrong working in any step. The more efficient and safer way to find the base area is to use the area of a triangle ABC as $\frac{1}{2} ab \sin C$ rather than using Pythagoras’ theorem to find the height and hence the area of the triangle. The use of Pythagoras’ theorem was a source of errors. Note that the question specified the container is ‘an open container’ i.e. it has only one base. It is hence a mistake to multiply the base area by 2 twice. Otherwise, it is standard procedure to use the restriction to form an equation relating x and h, make h the subject, and then carry out a substitution to obtain the given expression for V.
Base area	Cost of base															
$= \frac{1}{2} x^2 \sin 60^\circ$ or	$= 2 \times \frac{\sqrt{3}}{4} x^2$															
$\frac{1}{2} x \sqrt{x^2 - \left(\frac{1}{2} x \right)^2}$	$= \frac{\sqrt{3}}{2} x^2$															
$= \frac{\sqrt{3}}{4} x^2$																
Each side area	Cost of 3 sides															
$= xh$	$= 5 \times 3xh$															
	$= 15xh$															

(b)	$V = \frac{\sqrt{3}}{60} \left(Cx - \frac{\sqrt{3}}{2} x^3 \right) \text{ or } \frac{\sqrt{3}}{60} Cx - \frac{1}{40} x^3$ $\frac{dV}{dx} = \frac{\sqrt{3}}{60} \left(C - \frac{3\sqrt{3}}{2} x^2 \right) \text{ or } \frac{\sqrt{3}}{60} C - \frac{3}{40} x^2$ $\frac{dV}{dx} = 0 \qquad \frac{dV}{dx} = 0$ $\Rightarrow C - \frac{3\sqrt{3}}{2} x^2 = 0 \text{ or } \Rightarrow \frac{3}{40} x^2 = \frac{\sqrt{3}}{60} C$ $\Rightarrow x^2 = \frac{2}{3\sqrt{3}} C \qquad \Rightarrow x^2 = \frac{2\sqrt{3}}{9} C$ $\frac{d^2V}{dx^2} = \frac{\sqrt{3}}{60} (-3\sqrt{3}x) < 0 \text{ or } \frac{d^2V}{dx^2} = -\frac{3}{20} x < 0 \text{ for } x > 0$ $\text{Cost of base} = \frac{\sqrt{3}}{2} x^2 = \frac{\sqrt{3}}{2} \left(\frac{2}{3\sqrt{3}} C \right) \text{ or } \frac{\sqrt{3}}{2} \left(\frac{2\sqrt{3}}{9} C \right) = \frac{C}{3}$ Hence V is maximum when cost of base is $\frac{C}{3}$.	Observe that for part (a), the given expression is to be obtained at the end of your working. Similarly for part (b), which is another ‘Show that’ question, the given expression, namely $\frac{C}{3}$, must be obtained at or near the end of your working. Your working must arrive at $\frac{C}{3}$ rather than use it at or near the start of your working. Recall the standard procedure for solving maxima and minima problems. Differentiate the expression for V to be maximised and set the derivative to 0 to obtain the value of x or x^2. To justify that V is maximised, carry out the second derivative test, which is more efficient and safer than the first derivative test in this case, since C is an unknown constant. In particular, it must be stated that $x > 0$ for the second derivative test to be valid. Finally, substitute the value of x obtained, which is in terms of C, to find that the cost of the base is $\frac{C}{3}$ cents. Note how this $\frac{C}{3}$ is arrived at in the end.
6(a)	No. of ways = ${}^{11}P_8$ (or ${}^{11}C_8 \times 8!$ or $\frac{11!}{3!}$) = 6 652 800	This rather basic question was not well answered. The wrong answers given include ${}^{11}C_8$ (ignoring the fact that the people need to be <i>arranged</i>) and $8!$ (ignoring the fact that there are 11 seats).
(b)	No. of ways = ${}^5P_5 \times {}^6P_3 + {}^5P_3 \times {}^6P_5$ or $5! \times {}^6C_3 \times 3! + {}^5C_3 \times 3! \times {}^6C_5 \times 5!$ = 57 600	Most students knew that there were two possible cases. This time round, the common errors were writing ${}^5C_5 \times {}^6C_3 + {}^5C_3 \times {}^6C_5$ (mixing up permutations and combinations) or ${}^5P_5 + {}^6P_3 + {}^5P_3 + {}^6P_5$ (mixing up the addition and multiplication principles).
(c)	Method 1 No. of ways = $5! \times {}^6P_3$ (or $5! \times {}^6C_3 \times 3!$) = 14 400 Method 2 (weaker) No. of ways without restriction = $8! = 40\,320$	This part is equivalent to arranging the 8 people with no two boys adjacent – the use of the ‘slotting’ method should then be evident. The minority who attempted Method 2 (count

	No. of ways with 3 boys answering consecutive questions = 6! × 3! = 4320 No. of ways with exactly 2 boys answering consecutive questions = ${}^3C_2 \times 2! \times 6! \times {}^5C_1 = 21\,600$ No. of ways = 40 320 – 4320 – 21 600 = 14 400	complementary cases) were unsuccessful in counting the ‘exactly 2 boys answering consecutive questions’ case.								
(d)	Method 1 No. of ways = ${}^3C_1 \times {}^5C_3$ (or ${}^3C_2 \times {}^5C_2$) = 30 Method 2 No. of ways without restriction = $\frac{{}^8C_4 \times {}^4C_4}{2!} = 35$ No. of ways with all boys in a group = ${}^3C_3 \times {}^5C_1 = 5$ Required no. of ways = 35 – 5 = 30	Students generally failed to realise that once one group of 4 is formed, the other group is automatically formed (since it consists of all the remaining people). As such, they might do something like ${}^3C_1 \times {}^5C_3 + {}^3C_2 \times {}^5C_2$. Those who used the ‘complement’ method must be careful to divide by 2! As the two groups are of the same size.								
7(a)	F is the event that the cable is faulty N is the event that the cable is not faulty $P(X = 3) = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = \frac{1}{10}$ (shown) OR $\frac{1}{{}^5C_3} = \frac{1}{10}$ $P(X = 4) = P(\text{NFFF, FNFF, FFNF})$ $= \frac{3!}{2!} \times \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{3}{10}$ OR $\frac{{}^3C_2}{{}^5C_3} = \frac{3}{10}$ $P(X = 5)$ = P(NNFFF, NFNFF, NFFNF, FNNFF, FNFNF, FFNNF) $= \frac{4!}{2!} \times \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3} \times \frac{2}{2} \times \frac{1}{1} = \frac{6}{10}$ OR $\frac{{}^4C_2}{{}^5C_3} = \frac{6}{10}$ OR $1 - \frac{1}{10} - \frac{3}{10} = \frac{6}{10}$ <table border="1"><tr><td>x</td><td>3</td><td>4</td><td>5</td></tr><tr><td>P(X = x)</td><td>$\frac{1}{10}$</td><td>$\frac{3}{10}$</td><td>$\frac{6}{10} = \frac{3}{5}$</td></tr></table>	x	3	4	5	P(X = x)	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{6}{10} = \frac{3}{5}$	Some students think it is binomial and use np and npq. But the probabilities change as you choose the next cable so it is NOT binomial. (probability is not the same for every event/choice) For P(X=4), some students didn’t fix the last cable as F, and used $\frac{4!}{3!} \times \dots$ instead of $\frac{3!}{2!} \times \dots$ to arrange FFFN in a straight line. Actually when we fix the last position as F, we only permute the first 3 FFN These students continued on to make another mistake for P(X=5), writing $\times 5 \times \dots$ just because their P(X=4) had a $\times 4$ which is a mistake. Should check answers by ensuring that the sum of all probabilities = 1. Some students didn’t draw table. Make it a good habit to
x	3	4	5							
P(X = x)	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{6}{10} = \frac{3}{5}$							

		draw the probability distribution table.												
(b)	P(exactly 4 tests required first cable tested faulty) $= \frac{P(\text{FNFF, FFNF})}{\frac{3}{5}}$ $= \frac{2 \times \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2}}{\frac{3}{5}}$ $= \frac{2}{\frac{10}{3}} = \frac{1}{3}$	Surprisingly many wrote $P(X=4)$ which does not answer the question. The correct way would be to start off by writing $P(X=4 \mid \text{first cable faulty})$ $= \frac{P(X=4 \cap \text{First Cable Faulty})}{P(\text{First Cable Faulty})}$ There are only 2 cases that fulfil the numerator of both events happening at the same time (which many students can find). This led to illogical presentation like $P(X=4) = \frac{2}{10}$ which made no sense and was inconsistent with the $P(X=4)$ value in (a) The denominator: there is a short cut: just consider the first cable is faulty only (3 out of 5 cables).												
(c)	By GC, $\text{Var}(X) = 0.45$ $\text{Var}(C) = 10^2 \text{Var}(X)$ $= 45$ Alternative method (longer, not recommended) <table><tr><td>x</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$P(X=x)$</td><td>$\frac{1}{10}$</td><td>$\frac{3}{10}$</td><td>$\frac{6}{10} = \frac{3}{5}$</td></tr><tr><td>$c$</td><td>35</td><td>45</td><td>55</td></tr></table> Using GC, $\text{Var}(C) = 45$	x	3	4	5	$P(X=x)$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{6}{10} = \frac{3}{5}$	c	35	45	55	Mistake A: Not binomial, $np(1-p)$ is wrong Mistake B: $\text{Var}(C) = 5 + 10X$ 1. $\text{Var}(C)$ is variance. X is a random variable. They cannot equate together 2. Constants like 5 have a variance of 0 3. 10 should be 10^2 4. X should be $\text{Var}(X)$ Mistake C: Students who write $E(5 + 10X)$ and $\text{Var}(5 + 10X)$ add to their confusion (less likely to get it correct). Those who write $E(C)$ & $\text{Var}(C)$ and draw a table of values had a better chance of getting it right (See method 2). Some students had a negative variance and didn't realise it is
x	3	4	5											
$P(X=x)$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{6}{10} = \frac{3}{5}$											
c	35	45	55											

		wrong. Variance is always positive.
(d)	By GC, $E(X) = 4.5$ $E(C) = 5 + 10(4.5) = 50$ Since $n = 80$ is sufficiently large, by Central Limit Theorem, $\bar{C} \sim N\left(50, \frac{45}{80}\right)$ approximately. Therefore, $P(\bar{C} > 51.5) = 0.0228$ (3 sf).	Poor r.v. definitions. Need to read question thoroughly. X and C were previously defined in the question and should NOT be redefined to something else. Mean of C is understood as $\bar{C}$. No need to redefine another letter Steps: 1. Calculate required vars like μ 2. Justify the use of N distribution by saying $n = 80$ is ... and by central limit theorem... 3. Declare the N distribution using the correct representation symbol. Not C, not X, but $\bar{C}$ 4. Add the word "approximately" 5. Now you are ready to use the N distribution
8(a)		
(b)	Context: A linear model would mean that the number of active users will continue to decrease linearly, resulting in the number of users to eventually be negative, which is impossible.	
(c)	When a is positive, then as x increases, $\frac{a}{x}$ decreases. From scatter diagram, <u>since when x increases, y decreases, thus a must be positive.</u>	

	<u>Alternative</u> : From the scatter diagram, we note that as x increases, y decreases $\Rightarrow \frac{dy}{dx} < 0$ $y = \frac{a}{x} + b \Rightarrow \frac{dy}{dx} = -\frac{a}{x^2}$ $\therefore$ Since $x^2 > 0$, a has to be positive, so that $\frac{dy}{dx}$ can be negative.	
(d)	$r = 0.998$ (3 s.f.) Equation of regression line: $y = \frac{66.305}{x} + 19.833$ $y = \frac{66.3}{x} + 19.8 \text{ (3 s.f.)}$	
(e)	When $x = 9$, $y = \frac{66.305}{9} + 19.833$ $= 27.2002$ $= 27.2$ number of active users = 27 200 (3 s.f.) or 27.2 thousands. Since $x = 9$ is within the data range of x and $r \approx 0.998$ is close to 1, the estimate is reliable.	

9 (a)	1. A plastic bottle being non-recyclable is independent of any other plastic bottle being non-recyclable. 2. The probability of a plastic bottle being non-recyclable is constant.	This is a standard question on the assumptions for a binomial model to be appropriate, as such the answer is standard and standardised, with this template to be completed in the context of the question: 1. A (success) is independent of any other (success). 2. The probability of a (success) is constant. ‘Fixed number of trials’ and ‘two mutually exclusive outcomes’ are not assumptions for a binomial model to be appropriate. There is difference between ‘finding a non-recyclable bottle’ and ‘finding a bottle to be non-recyclable’. Specifically, the assumptions pertain not to the act or action of finding / selecting / collecting / checking a non-recyclable bottle, but to a property of each bottle itself i.e. being non-recyclable. Independence is not about probability. Specifically, it is wrong to state ‘The probability of a bottle being non-recyclable is independent of any other bottle being non-recyclable’. Use the appropriate terminology respectively. 1. You should not be attempting to ‘define’ independence and/or omit this keyword (concept) when you state this first assumption. In addition, your statement must specify ‘independent of (what)’. 2. ‘Fixed’, ‘chance’, ‘likely’ are not appropriate terminology.
(b)	$X \sim B(40, 0.06)$ $P(X \geq 4) = 1 - P(X \leq 3)$ $= 0.21726$ $= 0.217 \text{ (3 sf)}$	Generally answered well.
(c)	Let Y be the number of days, out of 5, with at least 4 non-recyclable bottles. $Y \sim B(5, 0.21726)$ $P(Y \leq 2) = 0.928 \text{ (3 sf)}$	Again, generally answered well. Remember to use another letter to represent and define the random variable since it is different from X as provided by the question. You should use a greater accuracy value of the probability from part (b) since the distribution of Y is intermediate working. Premature approximation may result in loss of accuracy and hence loss of the final mark.

(d)	P(at most 1 non-recyclable item) $= P(0 \text{ non-r bottle and } 1 \text{ non-r box})$ $+ P(1 \text{ non-r bottle and } 0 \text{ non-r box})$ $+ P(0 \text{ non-r bottle and } 0 \text{ non-r box})$ $= 0.94^3 \left[{}^2C_1 p(1-p) \right] + \left[{}^3C_1 (0.06)(0.94)^2 \right] (1-p)^2 + 0.94^3 (1-p)^2$ <u>Another presentation</u> Let U be the number of non-recyclable bottles found out of 3 bottles and V the number of non-recyclable boxes out of 2 boxes $U \sim N(3, 0.06)$ and $V \sim B(2, p)$ P(at most 1 non-recyclable item) $= P(U=0) \times P(V=1) + P(U=1) \times P(V=0) + P(U=0) \times P(V=0)$ $= 0.830584 \left[{}^2C_1 p(1-p) \right] + 0.159048(1-p)^2 + 0.830584(1-p)^2$ $1.661168p(1-p) + 0.989632(1-p)^2 = 0.9$ Using GC, $p = 0.199$ (3 sf)	Understand the structure of the situation. Specifically, for the sample to contain at most 1 non-recyclable item, there are 3 distinct cases as shown in the solution. A common error was to omit the case of 0 non-recyclable bottle and 0 non-recyclable box. Note that each case must consider the numbers of both bottle and box e.g. the case of 1 non-recyclable bottle must have 0 non-recyclable box. The probability of each case would hence involve a product. By first principles or using $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$ as provided in MF27, express the probability of each case in terms of p, and hence form the equation. Finally, use your GC to solve this equation. You should not be trying to form a combined distribution, whether binomial or normal.
10 (a)	Let A g be the mass of a randomly chosen Type A component and B g the mass of a randomly chosen Type B component $A \sim N(250, 3^2)$ and $B \sim N(240, 4^2)$ $E(A+B) = 250 + 240$ $= 490$ $\text{Var}(A+B) = 3^2 + 4^2$ $= 25$ $A+B \sim N(490, 25)$ $P(A+B > 480) = 0.977$ (to 3 s.f.)	The variables A and B must be defined clearly as “the mass of ...” since they have not been defined in the question. Working for finding mean and variance should be clearly shown to get method marks. This new distribution $A+B \sim N(490, 25)$ must be indicated clearly.
(b)	Let $S = A_1 + A_2 - 3B$ $E(S) = 2(250) - 3(240)$ $= -220$ $\text{Var}(S) = 2(3^2) + (3^2 \times 4^2)$ $= 162$ $S \sim N(-220, 162)$ $P(S < 0) = 1.00$ (to 3 s.f.)	Working for finding mean and variance should be clearly shown to get method marks. The new distribution $S \sim N(-220, 162)$ must be indicated clearly. If $S = 3B - (A_1 + A_2)$, this must be followed through with $P(S > 0)$.

(c)	$P(250 < A < 255) = 0.45221$ (to 5 s.f.) Let X be the number of components, out of 10, having mass between 250 g and 255 g $X \sim B(10, 0.45221)$ At least 25% of 10 components $\Rightarrow X \geq 2.5$ or $X \geq 3$ $P(X \geq 3) = 1 - P(X \leq 2)$ $= 0.903$	A new binomial variable must be defined clearly. The probability used should be at least 5sf. It is advisable to convert to integer values before using binomcdf.
(d)	Let C g be the mass of a randomly chosen Type C component $C \sim N(\mu, \sigma^2)$ $P(C - \mu < 7) = 0.8$ $P(-7 < C - \mu < 7) = 0.8$ $P\left(-\frac{7}{\sigma} < Z < \frac{7}{\sigma}\right) = 0.8$ $P\left(Z > \frac{7}{\sigma}\right) = 0.1$ $\frac{7}{\sigma} = 1.28155$ $\sigma = 5.4621\dots$ $P(C < 312) = 0.35$ $P\left(Z < \frac{312 - \mu}{5.4621}\right) = 0.35$ $\frac{312 - \mu}{5.4621} = -0.38532$ $\mu = 314.1\dots$ $\mu = 314$ grams, $\sigma = 5.46$ grams (to 3 s.f.)	Students should sketch out a simple normal distribution curve to visualise the required probability. A common mistake is $P\left(-\frac{7 - \mu}{\sigma} < Z < \frac{7 - \mu}{\sigma}\right) = 0.8$ if students have not understood the question fully. Invnorm does not work when there are 2 probability values on one side of the equation e.g. $P\left(Z < \frac{7}{\sigma}\right) - P\left(Z < -\frac{7}{\sigma}\right) = 0.8$. Invnorm values should be at least 5sf as they are part of intermediate working. Final answers for μ and σ should be in 3sf.
11 (a)	unbiased estimate of population mean: $\bar{x} = \frac{\sum(x - 50)}{40} + 50 = \frac{80.5}{40} + 50 = 52.0125$ unbiased estimate of population variance: $s^2 = \frac{1}{39} \left[\sum(x - 50)^2 - \frac{(\sum(x - 50))^2}{40} \right]$ $= \frac{1}{39} \left(2415 - \frac{80.5^2}{40} \right)$ $= 57.7691$ ≈ 57.8 (3 s.f.)	Students are reminded that they must answer the question by stating “Unbiased estimate of population mean” and not simply write $\bar{x}$. DO NOT round off exact values to 3 s.f. To calculate s^2, simply make reference to MF27 formula $\frac{1}{n-1} \left[\sum x^2 - \frac{(\sum x)^2}{n} \right]$ and replace x by $(x - 50)$

(b)	Let μ be the population mean mass of chilli sauce in each sachet. $H_0 : \mu = 50$ $H_1 : \mu > 50$ Test at 5% level of significance. Under H_0, since the sample size $n = 40$ is large, by Central Limit Theorem $\bar{X} \sim N\left(50, \frac{57.7691}{40}\right)$ approximately. Using GC, p-value = 0.0470 Since p-value = 0.0470 < 0.05, reject H_0 and conclude that there is sufficient evidence at 5% significance level that the population mean mass of chilli sauce in each sachet is more than 50 grams.	Define μ in the context of the question Use the standard steps for such hypo testing questions
(c)	As the <u>sample size</u> $n=40$ is <u>large</u>, by the <u>Central Limit Theorem</u>, the <u>sample mean mass of chilli sauce in each sachet</u> ($\bar{X}$) is <u>approximately normal</u>.	Many students mentioned that by Central Limit Theorem, it will be approximately normal without stating what “it” refers to. Need to be specific in the answer and elaborate on what CLT does. Please note that CLT is used to approximate the distribution of sample mean or the sum to a normal distribution when n is large. CLT will not be able to approximate the distribution of X to a normal distribution.

(d)	Let Y be the mass of chilli sauce in each sachet after the machine is adjusted. $Y \sim N(50, 6^2)$ $H_0 : \mu = 50$ $H_1 : \mu \neq 50$ Test at 10% level of significance. Under H_0, $\bar{Y} \sim N\left(50, \frac{6^2}{n}\right)$. Critical region : $z \leq -1.64485$ or $z \geq 1.64485$ Since H_0 is rejected, $\frac{51.7 - 50}{\frac{6}{\sqrt{n}}} \leq -1.64485 \quad \text{or} \quad \frac{51.7 - 50}{\frac{6}{\sqrt{n}}} \geq 1.64485$ $1.7\sqrt{n} \leq -9.8691 \quad (\text{rejected as } \sqrt{n} \geq 0) \quad \text{or} \quad 1.7\sqrt{n} \geq 9.8691$ $\sqrt{n} \geq 5.8054$ $n \geq 33.7$ $\therefore n \geq 34$ $\therefore \{n \in \mathbb{Z} : n \geq 33.7\}$ or $\{n \in \mathbb{Z} : n \geq 34\}$	Should define a new r.v. since the distribution of the mass has been changed due to the machine adjustment. Write the standard steps Note that n is an integer
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