RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

2024 H2 Math Year 5 Timed Practice (Modified) Solutions with Comments

1 A smoothie shop sells three different types of smoothie blends, Berrylicious, Tropicality and Pumptein. The smoothie blends are dispensed from machines and each serving is prepared by blending protein powder, mixed fruits and milk.

Relative to a serving of Berrylicious, a serving of Tropicality uses 50% the amount of protein powder, 30% more the amount of mixed fruits and 75% the amount of milk. Relative to a serving of Berrylicious, a serving of Pumptein uses 50% more the amount of protein powder, 60% the amount of mixed fruits and 25% more the amount of milk.

A serving of Berrylicious, Tropicality and Pumptein weighs 360 grams, 350 grams and 355 grams respectively.

Find the amount of protein powder, mixed fruits and milk required to make a serving of Berrylicious, in grams. [4]

Solutions	**Comments**
Let the amount of protein powder, mixed fruits and milk required to make a serving of Berrylicious be *x*, *y* and *z* respectively. Total mass of a serving of Tropicality: $0.5x+1.3y+0.75z=350$ -- (1) Total mass of a serving of Pumptein: $1.5x+0.6y+1.25z=355$ -- (2) $x+y+z=360$ -- (3) Solving equations (1), (2) and (3) with the GC, we have $x=10, y=150$ and $z=200$	Very often the last line in the SLE question gives a clue to the unknowns to define.

2 A graphic calculator is **not** to be used in answering this question.

Given that $3-4\mathrm{i}$ is a root of the equation

$$5z^3+az^2+bz+25=0,$$

find the values of the real numbers *a* and *b* and the remaining roots of the equation. [4]

<table>
<tr><th>Solutions</th><th>Comments</th></tr>
<tr><td>
Since the coefficients are real, $z=3+4\mathrm{i}$ is another root of the equation.
<u>Method 1</u>
$(z-3+4\mathrm{i})(z-3-4\mathrm{i})=(z-3)^2-(4\mathrm{i})^2$
$=z^2-6z+9+16$
$=z^2-6z+25$
So, $5z^3+az^2+bz+25=(z^2-6z+25)(5z+1)$ (By inspection)
Comparing coefficients of z^2, $a=1-30=-29$
Comparing coefficients of z, $b=125-6=119$
The other roots are $z=3+4\mathrm{i}$ and $z=-\frac{1}{5}$.
</td><td>
Most students use Method 1, which is the recommended method, to do this question.
</td></tr>
<tr><td>
<u>Method 2 (Not recommended)</u>
Substitute $z=3-4\mathrm{i}$ (or $z=3+4\mathrm{i}$) into the given eqn,
$5(3-4\mathrm{i})^3+a(3-4\mathrm{i})^2+b(3-4\mathrm{i})+25=0$
$5(27-108\mathrm{i}-144+64\mathrm{i})+a(9-24\mathrm{i}-16)+b(3-4\mathrm{i})+25=0$
$5(-117-44\mathrm{i})+a(-7-24\mathrm{i})+b(3-4\mathrm{i})+25=0$
$(-560-7a+3b)-4(55+6a+b)\mathrm{i}=0$
Comparing the real parts, $7a-3b=-560$ --- (1)
Comparing the imaginary parts, $6a+b=-55$ ---- (2)
(1) + (2) × 3: $7a+18a=-560-55\times3$
$25a=-725$
$a=-29$
From (2): $b=-55+29\times6=119$
$\therefore\ a=-29$, $b=119$
$(z-3+4\mathrm{i})(z-3-4\mathrm{i})=(z-3)^2-(4\mathrm{i})^2=z^2-6z+9+16=z^2-6z+25$
$25z^3+119z^2-29z+5=0$
$(z^2-6z+25)(5z+1)=0$
The other roots are $z=3+4\mathrm{i}$ and $z=-\frac{1}{5}$.
</td><td>
As graphic calculator is not allowed in answering this question, students are to expand out $(3-4\mathrm{i})^3$ and $(3-4\mathrm{i})^2$
</td></tr>
</table>

3 With reference to the origin O, the points A and C have position vectors $\mathbf{a}$ and $\mathbf{c}$ respectively, where $\mathbf{a}$ and $\mathbf{c}$ are non-zero and non-parallel vectors. The point E lies on AC such that $AE : AC = \lambda - 2 : \lambda$, where $\lambda > 2$.

(a) Find the area of triangle OAE in terms of λ, $\mathbf{a}$ and $\mathbf{c}$. [3]

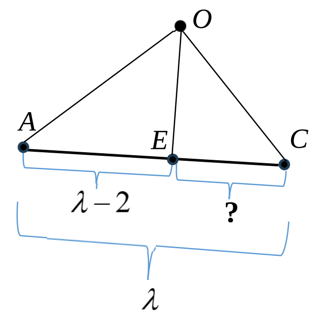
(b) Given that the area of triangle OAE is $\frac{3}{10}|\mathbf{a}\times\mathbf{c}|$, solve for the value of λ. [1]

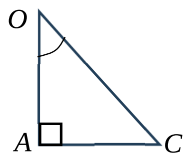
(c) Given that $\mathbf{a}$ is a unit vector, give the geometrical interpretation of

(i) $|\mathbf{a}.\mathbf{c}|$, [1]

(ii) $|\mathbf{a}\times\mathbf{c}|$. [1]

(d) If $\angle OAC = 90^\circ$, explain why $\mathbf{a}.\mathbf{c}$ is positive. [1]

Solutions		**Comments**
(a) **[3]**	Using ratio theorem, $\overrightarrow{OE} = \dfrac{2\mathbf{a}+(\lambda-2)\mathbf{c}}{\lambda}$ Area of $\Delta OAE = \frac{1}{2}\left|\overrightarrow{OA}\times\overrightarrow{OE}\right|$ $= \frac{1}{2}\left|\mathbf{a}\times\dfrac{2\mathbf{a}+(\lambda-2)\mathbf{c}}{\lambda}\right|$ $= \frac{1}{2}\left|\mathbf{a}\times\dfrac{(\lambda-2)\mathbf{c}}{\lambda}\right|$ since $\mathbf{a}\times\mathbf{a} = \mathbf{0}$ $= \frac{1}{2}\left|\dfrac{\lambda-2}{\lambda}\right||\mathbf{a}\times\mathbf{c}|$ $= \dfrac{\lambda-2}{2\lambda}|\mathbf{a}\times\mathbf{c}|$ since $\lambda > 2$	Use a diagram to interpret the ratio $AE : AC = \lambda - 2 : \lambda$  The vector product gives a vector. This is the zero vector.
(b) **[1]**	Area of $\Delta OAE = \dfrac{\lambda-2}{2\lambda}|\mathbf{a}\times\mathbf{c}| = \dfrac{3}{10}|\mathbf{a}\times\mathbf{c}|$ By comparison, $\dfrac{\lambda-2}{2\lambda} = \dfrac{3}{10}$ $10\lambda - 20 = 6\lambda$ $4\lambda = 20$ $\lambda = 5$	
(c)(i) **[1]**	$|\mathbf{a}.\mathbf{c}|$ gives the length of projection of $\mathbf{c}$ onto $\mathbf{a}$.	Note that $\mathbf{a}$ is a unit vector, so $|\mathbf{a}.\mathbf{c}| = \left|OC\cos A\hat{O}C\right|$.
(c)(ii) **[1]**	$|\mathbf{a}\times\mathbf{c}|$ gives the perpendicular distance from point C to OA.	Note that $\mathbf{a}$ is a unit vector, so $|\mathbf{a}\times\mathbf{c}| = OC\sin A\hat{O}C$.

	Alternatively, $\lvert\mathbf{a}\times\mathbf{c}\rvert$ gives area of parallelogram with *OA* and *OC* being the adjacent sides. Or, $\lvert\mathbf{a}\times\mathbf{c}\rvert$ is equivalent to twice the area of triangle *OAC*.	The description must accurately convey that it is the distance from a *point* to a *line*.
(d) **[1]**	Since $\angle OAC = 90^\circ$, $\overrightarrow{OA}.\overrightarrow{AC} = 0$ $\mathbf{a}.(\mathbf{c}-\mathbf{a}) = 0$ $\mathbf{a}.\mathbf{c} = \lvert\mathbf{a}\rvert^2$ which is positive since **a** is a nonzero vector  <u>Alternatively,</u> since $\angle OAC = 90^\circ$, the angle between **a** and **c**, $\angle AOC$ is less than 90°, i.e., it is acute. Hence $\mathbf{a}.\mathbf{c}$ is positive.	You can draw a diagram to explain the relationship between the angles.

4 The line l_1 contains the points A and B with coordinates $(-1, 3, 7)$ and $(a, 0, -2)$ respectively, where a is a constant. The line l_2 has equation $-\dfrac{x+7}{2} = \dfrac{y-10}{3}$, $z = 10$. It is given that l_1 and l_2 cross at the point C.

(a) Find the value of a and the coordinates of C. [4]

(b) The point P has coordinates $(-2, 9, 8)$. Find the point on l_2 which is closest to P and hence find the exact perpendicular distance between P and l_2. [4]

Solutions

(a) [4]

$$l_1 : \mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} a+1 \\ -3 \\ -9 \end{pmatrix}, \quad \lambda \in \mathbb{R} \text{ and}$$

$$l_2 : -\frac{x+7}{2} = \frac{y-10}{3}, z = 10 \text{ i.e., } l_2 : \mathbf{r} = \begin{pmatrix} -7 \\ 10 \\ 10 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}, \quad \mu \in \mathbb{R}$$

When l_1 and l_2 intersect/cross,

$$\begin{pmatrix} -1 \\ 3 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} a+1 \\ -3 \\ -9 \end{pmatrix} = \begin{pmatrix} -7 \\ 10 \\ 10 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} \lambda(a+1) + 2\mu = -6 \text{ ----(1)} \\ -3\lambda - 3\mu = 7 \text{ ----(2)} \\ -9\lambda = 3 \text{ ----(3)} \end{cases}$$

Solving (3), $\lambda = -\frac{1}{3}$. Then solving (2), $1 - 3\mu = 7 \Rightarrow \mu = -2$

Solving (1), $-\frac{1}{3}(a+1) - 4 = -6 \Rightarrow a = 5$

Hence C has coordinates $(-3, 4, 10)$.

<u>Alternative method:</u>

$$\begin{pmatrix} -1 \\ 3 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} a+1 \\ -3 \\ -9 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 10 \end{pmatrix}$$

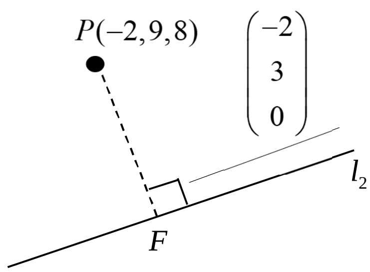
$$\Rightarrow \begin{cases} -1 + \lambda(a+1) = x \text{ ----(1)} \\ 3 - 3\lambda = y \text{ ----(2)} \\ 7 - 9\lambda = 10 \text{ ----(3)} \end{cases}$$

Comments

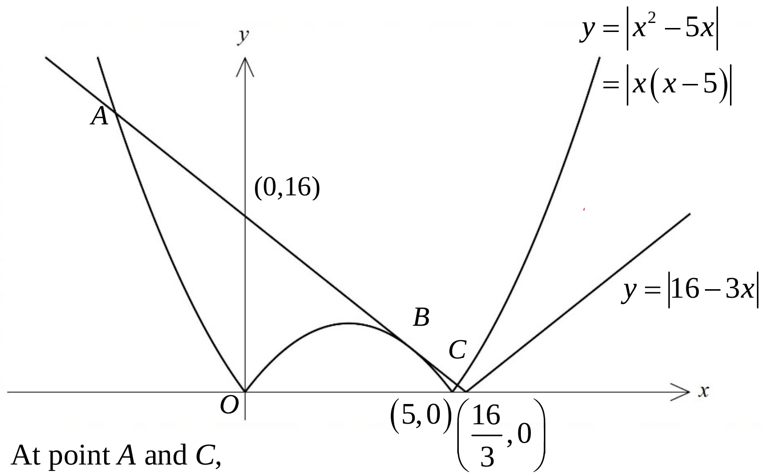
Revise on your skill in converting from cartesian form to vector form. Careless mistakes can be costly as wrong equations will impact on working for part (b).

Vector equation of a line is incomplete without "**r =**". Vectors should also be properly presented with the right notations such as "~" like $\underset{\sim}{a}$.

Question asked to give your answer in coordinates, not as a position vector.

	Solving (3), $\lambda = -\frac{1}{3}$. Then solving (2), $y = 4$ Sub into $-\frac{x+7}{2} = \frac{y-10}{3}$, we get $x = -3$ Solving (1), $-1-\frac{1}{3}(a+1) = -3 \Rightarrow a = 5$ Hence C has coordinates $(-3, 4, 10)$.	
(b) **[4]**	Let the point which is closest to P be F. $\overrightarrow{OF} = \begin{pmatrix}-7\\10\\10\end{pmatrix} + \mu\begin{pmatrix}-2\\3\\0\end{pmatrix}$, for some $\mu \in \mathbb{R}$ $\overrightarrow{PF} = \overrightarrow{OF} - \overrightarrow{OP} = \begin{pmatrix}-5-2\mu\\1+3\mu\\2\end{pmatrix}$  Since $\overrightarrow{PF} \perp l_2$, $\begin{pmatrix}-5-2\mu\\1+3\mu\\2\end{pmatrix} \cdot \begin{pmatrix}-2\\3\\0\end{pmatrix} = 0$ $10+3+(4+9)\mu = 0$ $\mu = -1$ Hence $\overrightarrow{OF} = \begin{pmatrix}-5\\7\\10\end{pmatrix}$ or coordinates of F are $(-5, 7, 10)$. Required distance $PF = \left\|\begin{pmatrix}-5-2(-1)\\1+3(-1)\\2\end{pmatrix}\right\| = \left\|\begin{pmatrix}-3\\-2\\2\end{pmatrix}\right\| = \sqrt{17}$	Calculation of PF needs to involve $\overrightarrow{PF}$ since question states "hence".

5 On the same axes, sketch the curves with equations $y=\left|x^2-5x\right|$ and $y=\left|16-3x\right|$, stating the exact coordinates of any points of intersection with the axes. Hence solve exactly the inequality $\left|x^2-5x\right| \geq \left|16-3x\right|$. [8]

Solutions		**Comments**
[8]	 At point *A* and *C*, $x^2-5x=16-3x$ $x^2-2x-16=0$ $x=\frac{2\pm\sqrt{(-2)^2-4(-16)}}{2}$ $=\frac{2\pm\sqrt{68}}{2}$ $=1\pm\sqrt{17}$ Hence $x=1-\sqrt{17}$ at *A* and $x=1+\sqrt{17}$ at *C*. At point *B*, $-\left(x^2-5x\right)=16-3x$ $x^2-8x+16=0$ $(x-4)^2=0$ $x=4$ $\therefore$ Solution to $\left\|x^2-5x\right\| \geq \left\|-3x+16\right\|$ is $x \leq 1-\sqrt{17}$ or $x \geq 1+\sqrt{17}$ or $x=4$.	Many students missed out on the *y*-intercept (0,16) Many students missed out on $x=4$ being a solution. Note $y=\left\|16-3x\right\|$ is tangential to $y=\left\|x^2-5x\right\|$ at *x*=4. When reading the graphs on your graphing calculator, do learn to zoom in or zoom out so that you can see features of the graph. The question asked for "exact coordinates", so you are expected to show the algebraic working to arrive at the exact answers. Note the use of the conjunction 'or'. It is not acceptable to use comma or 'and'.

6 The curve C has equation

$$x^2 - ae^x \tan^{-1} y = 0, \qquad \text{for } x > 0,$$

where a is a positive constant.

(a) Show that $ae^x \dfrac{dy}{dx} = \left(2x - x^2\right)\left(1 + y^2\right)$. [2]

It is given that C has one stationary point.

(b) Find the x-coordinate of this stationary point and determine its nature, showing your working clearly. [3]

Solutions		**Comments**
(a) **[2]**	$x^2 - ae^x \tan^{-1} y = 0$ Differentiating wrt x, $2x - \left(ae^x \tan^{-1} y + ae^x \cdot \dfrac{1}{1+y^2}\dfrac{dy}{dx}\right) = 0$ $2x - x^2 - ae^x \cdot \dfrac{1}{1+y^2}\dfrac{dy}{dx} = 0$, since $ae^x \tan^{-1} y = x^2$ $ae^x \dfrac{dy}{dx} = \left(2x - x^2\right)\left(1+y^2\right)$ (shown) **Alternative** Rewrite $x^2 - ae^x \tan^{-1} y = 0$ as $y = \tan\left(\dfrac{x^2 e^{-x}}{a}\right)$. Differentiating wrt x, $\dfrac{dy}{dx} = \sec^2\left(\dfrac{x^2e^{-x}}{a}\right) \cdot \dfrac{1}{a}\left(-x^2e^{-x} + 2xe^{-x}\right)$ Since $\sec^2\left(\dfrac{x^2e^{-x}}{a}\right) = 1 + \tan^2\left(\dfrac{x^2e^{-x}}{a}\right) = 1 + y^2$, $\dfrac{dy}{dx} = \left(1+y^2\right) \cdot \dfrac{1}{ae^x}\left(-x^2 + 2x\right)$ $ae^x \dfrac{dy}{dx} = \left(2x - x^2\right)\left(1+y^2\right)$ (shown)	
(b) **[3]**	For stationary points, let $\dfrac{dy}{dx} = 0$ Since $\left(1+y^2\right) \neq 0$, $\quad 2x - x^2 = 0$ Thus $x = 2$ since $x > 0$. To determine the nature of the stationary point at $x = 2$,	

Method 1 : Second Derivative Test

$$a\mathrm{e}^{x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+a\mathrm{e}^{x}\frac{\mathrm{d}y}{\mathrm{d}x}=\left(2x-x^2\right)\cdot 2y\frac{\mathrm{d}y}{\mathrm{d}x}+\left(2-2x\right)\left(1+y^2\right)$$

At $x=2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$.

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{-2\left(1+y^2\right)}{a\mathrm{e}^2}<0 \text{ since } a>0 \text{ and } 1+y^2>0$$

Maximum point at $x=2$.

Method 2 : First Derivative Test

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x\left(2-x\right)\left(1+y^2\right)}{a\mathrm{e}^{x}}$$

For x in the neighbourhood of 2, $x>0$.

Now $a>0$, $\mathrm{e}^{x}>0$ and $1+y^2>0$, so $2-x$ will determine the sign of $\frac{\mathrm{d}y}{\mathrm{d}x}$ around $x=2$.

x	2^-	2	2^+
$\left(2-x\right)$	$+$ve	0	$-$ve
$\frac{\mathrm{d}y}{\mathrm{d}x}$	$+$ve	0	$-$ve

Maximum point at $x=2$.

You should factorise the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and clearly justify how each factor is positive or negative.

The row with $\left(2-x\right)$ is necessary to show the sign changes.

7 A curve C has parametric equations

$$x=(1+t)^2, \quad y=(1-t)^2.$$

The point A is on C such that the tangent to C at A has gradient 2.

(a) Find the equation of this tangent and hence show algebraically that this tangent never meets C again. [5]

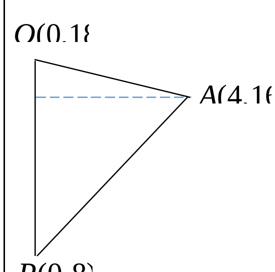
(b) The tangent and the normal to C at A meet the y-axis at points P and Q respectively. Find the area of the triangle APQ. [3]

<table>
<tr><th colspan="2">Solutions</th><th>Comments</th></tr>
<tr><td>(a)
[5]</td><td>$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2(1-t)}{2(1+t)}=\frac{t-1}{t+1}$

Solving $\frac{t-1}{t+1}=2$, we get $t=-3$.
Coordinates of A = $(4,16)$

Equation of tangent at A: $y-16=2(x-4)$,
ie $y=2x+8$
$C: x=(1+t)^2, y=(1-t)^2$

Solving simultaneously,
$(1-t)^2-16=2\left[(1+t)^2-4\right]$
$t^2+6t+9=0$
$(t+3)^2=0$
$t=-3$ is the only solution, meaning that the tangent meets C only at point A. Thus the tangent never meets C again.</td><td></td></tr>
<tr><td>(b)
[3]</td><td>Equation of normal at A: $y-16=-\frac{1}{2}(x-4)$
Sub $x=0$, $P\equiv(0,8)$ and $Q\equiv(0,18)$

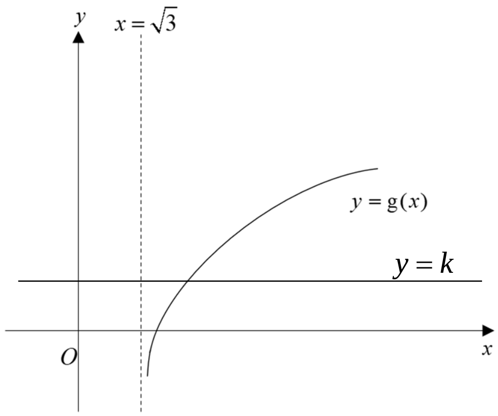
Area of triangle APQ = $\frac{1}{2}\times10\times4=20$ unit^2</td><td>
</td></tr>
</table>

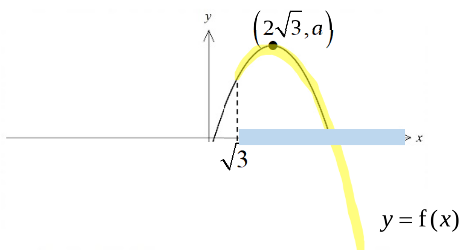
8 The function g is defined by $\mathrm{g}: x \mapsto \ln(x^2-3)+1$, for $x \in \mathbb{R}$, $x > \sqrt{3}$.

(a) Explain why g^{-1} exists. [2]

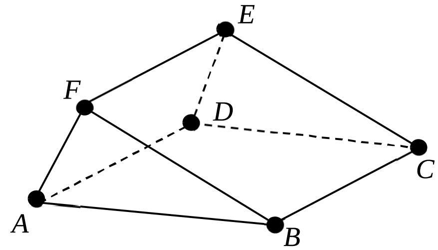
(b) Find $\mathrm{g}^{-1}(x)$ and write down the domain and range of g^{-1}. [4]

(c) The function f is defined by $\mathrm{f}: x \mapsto a-\left(x-2\sqrt{3}\right)^2$, for $x \in \mathbb{R}$, $x > 0$ for some real constant a. Show that the composite function fg^{-1} exists. Find $\mathrm{fg}^{-1}(x)$ and its range in terms of a. [4]

Solutions		**Comments**
(a) **[2]**	 Since every horizontal line $y=k$, $k \in \mathbb{R}$ cuts the graph of g at one and only one point, g is a 1-1 function. Hence, g^{-1} exists.	You should draw a graph of the function and draw the horizontal line to support your explanation. You need to be familiar with this explanation for inverse of g to exist.
(b) **[4]**	$y=\ln(x^2-3)+1,\ x>\sqrt{3}$ $\mathrm{e}^{y-1}=x^2-3$ $x^2=\mathrm{e}^{y-1}+3$ $x=\sqrt{\mathrm{e}^{y-1}+3}$ since $x>\sqrt{3}$ $\mathrm{g}^{-1}(x)=\sqrt{\mathrm{e}^{x-1}+3}$ $\mathrm{D}_{\mathrm{g}^{-1}}=\mathrm{R}_\mathrm{g}=\mathbb{R}$ $\mathrm{R}_{\mathrm{g}^{-1}}=\mathrm{D}_\mathrm{g}=\left(\sqrt{3},\infty\right)$	Do explain the choice of $x=\sqrt{\mathrm{e}^{y-1}+3}$ instead of $x=-\sqrt{\mathrm{e}^{y-1}+3}$ by considering domain of g.

(c) **[4]**	$\mathrm{R}_{\mathrm{g}^{-1}} = \mathrm{D}_{\mathrm{g}} = \left(\sqrt{3}, \infty\right)$ $\mathrm{D}_{\mathrm{f}} = \left(0, \infty\right)$ Since $\mathrm{R}_{\mathrm{g}^{-1}} \subseteq \mathrm{D}_{\mathrm{f}}$, hence fg^{-1} exists. $\mathrm{fg}^{-1} : x \mapsto a - \left(\sqrt{\mathrm{e}^{x-1}+3} - 2\sqrt{3}\right)^2, \quad x \in \mathbb{R}$ Using the graphs of g and f,  $\mathrm{D}_{\mathrm{g}^{-1}} = \mathbb{R} \xrightarrow{\mathrm{g}^{-1}} = \mathrm{R}_{\mathrm{g}^{-1}} \left(\sqrt{3}, \infty\right) \xrightarrow{\mathrm{f}} \left(-\infty, a\right]$. So $\mathrm{R}_{\mathrm{fg}^{-1}} = \left(-\infty, a\right]$	It is important to state the range of g^{-1} and domain of f explicitly.

9 An architect is designing the structure of a slide for an indoor children's playground. As shown in the diagram below, part of the structure has a rectangular base *ABCD* and a rectangular slide surface *BCEF* and two sides *ABF* and *DCE* which are perpendicular to the base *ABCD*.



It is given that the base *ABCD* is part of the plane with equation $-2x-6y+7z=7,$ and the line *BF* has equation $\mathbf{r}=\begin{pmatrix}-1\\18\\5\end{pmatrix}+\lambda\begin{pmatrix}-1\\8\\1\end{pmatrix},\ \lambda\in\mathbb{R}.$

(a) Find the position vector of *B*. [3]

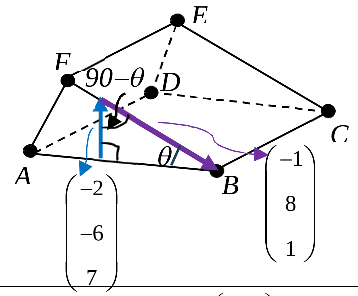
(b) Find the angle between the slide surface *BCEF* and its base *ABCD*. [3]

(c) Show that the cartesian equation of the plane containing side *ABF* is $62x+5y+22z=138$. [3]

It is now given that the other side *DCE* is part of the plane with equation $62x+5y+22z=200$.

(d) The playground's theme requires the slide surface width to be between 80 cm and 100 cm. Taking each unit in the architect's design to be a metre, determine if the slide meets the requirement. [3]

Solutions		Comments
(a) **[3]**	Point *B* is the point of intersection between $\pi_{ABCD}:\mathbf{r}\cdot\begin{pmatrix}-2\\-6\\7\end{pmatrix}=7$ and $l_{BF}:\mathbf{r}=\begin{pmatrix}-1\\18\\5\end{pmatrix}+\lambda\begin{pmatrix}-1\\8\\1\end{pmatrix},\ \lambda\in\mathbb{R}.$ $$\left[\begin{pmatrix}-1\\18\\5\end{pmatrix}+\lambda\begin{pmatrix}-1\\8\\1\end{pmatrix}\right]\cdot\begin{pmatrix}-2\\-6\\7\end{pmatrix}=7$$ $$2-108+35+(2-48+7)\lambda=7$$ $$\lambda=-2$$ $\overrightarrow{OB}=\begin{pmatrix}-1\\18\\5\end{pmatrix}-2\begin{pmatrix}-1\\8\\1\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$	To solve for a point, you need to ask yourself how the point is defined. Here it is defined by the intersection of a line and a plane.

(b) **[3]**	The angle between the surface *BCEF* and the base of the slide is given by the acute angle between the plane and line *BF* since the sides are perpendicular to the base. Let θ be the required angle. $\sin\theta = \dfrac{\left\lvert \begin{pmatrix}-2\\-6\\7\end{pmatrix}\bullet\begin{pmatrix}-1\\8\\1\end{pmatrix} \right\rvert}{\sqrt{4+36+49}\sqrt{1+64+1}} = \dfrac{39}{\sqrt{89}\sqrt{66}}$  Therefore $\theta = 30.6^\circ$ (to 1d.p.)	Alternatively, since the angle between direction vector of line *BF* and normal of plane *ABCD* is $90-\theta$, $\cos(90-\theta) = \dfrac{\left\lvert \begin{pmatrix}-2\\-6\\7\end{pmatrix}\bullet\begin{pmatrix}-1\\8\\1\end{pmatrix} \right\rvert}{\sqrt{4+36+49}\sqrt{1+64+1}}$
(c) **[3]**	Since π_{ABF} is perpendicular to π_{ABCD}, π_{ABF} is // to $\begin{pmatrix}-2\\-6\\7\end{pmatrix}$ and it contains the line *BF*. Hence, a vector equation of π_{ABF} is $\mathbf{r} = \begin{pmatrix}-1\\18\\5\end{pmatrix} + \alpha\begin{pmatrix}-1\\8\\1\end{pmatrix} + \beta\begin{pmatrix}-2\\-6\\7\end{pmatrix}, \alpha,\beta\in\mathbb{R}$ A vector perpendicular to π_{ABF} is $\begin{pmatrix}-1\\8\\1\end{pmatrix}\times\begin{pmatrix}-2\\-6\\7\end{pmatrix} = \begin{pmatrix}62\\5\\22\end{pmatrix}$ Equation of π_{ABF} is $\mathbf{r}\cdot\begin{pmatrix}62\\5\\22\end{pmatrix} = \begin{pmatrix}-1\\18\\5\end{pmatrix}\cdot\begin{pmatrix}62\\5\\22\end{pmatrix} = -62+90+110 = 138$ Hence cartesian equation of π_{ABF} is $62x+5y+22z=138$	You should show your steps clearly since this is a "show" question. The working for the dot product must be shown.
(d) **[3]**	Width of the slide surface is given by distance between the two sides *ABF* and *DCE* $\pi_{ABF}: 62x+5y+22z=138$, $\pi_{DCE}: 62x+5y+22z=200$ Distance between the two parallel planes is $\dfrac{\lvert 138-200\rvert}{\left\lVert\begin{pmatrix}62\\5\\22\end{pmatrix}\right\rVert} = 0.93972$ Since width is $0.940\text{m} = 94.0\text{cm}$, the slide meets requirement.	Refer to Chapter 4C lecture notes Example 5 for illustration of distance between two planes.