Name: _____ ( ) Class: _____ Date: _____

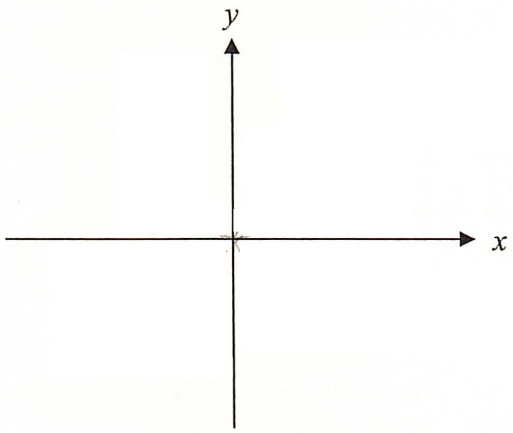
MA302.4.X2 – Power Functions with Rational Indices

The graph of $y = ax^{\frac{1}{n}}$, where n is an integer

Earlier you have sketched the graphs of linear, quadratic and cubic as well as reciprocal functions. Such graphs are part of the family of curves of $y = ax^k$, where k is an integer.

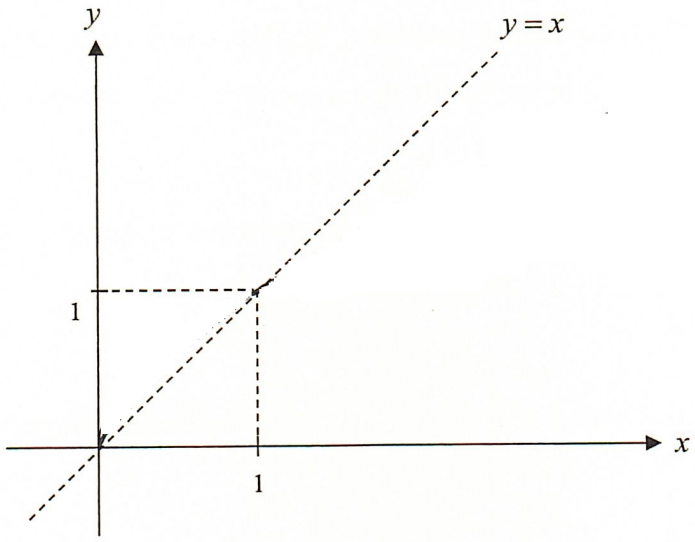
In this section, you will explore the shape of the graphs when k takes the fraction $\frac{1}{n}$, where n is a positive integer.

X1. How are the graphs of $y = x^2$ and $x = y^2$ related? Try sketching them on the **same axes**.



X2. Make x the subject in the equation $y = x^{\frac{1}{2}}$. What do you notice for the graph of $y = x^{\frac{1}{2}}$?

X3. Sketch the graphs of $y = x^{\frac{1}{6}}$, $y = x^{\frac{1}{4}}$ and $y = x^{\frac{1}{2}}$ on the same axes below.

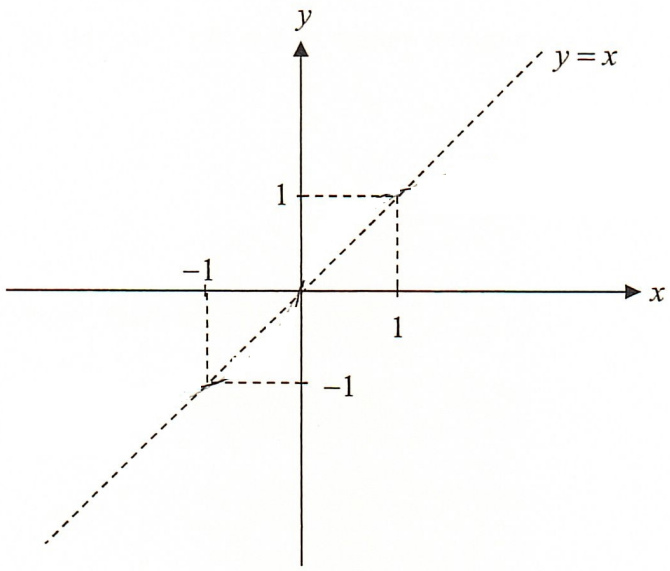


(a) What similarities do you observe from the above about the three graphs? How do the three graphs differ?

(b) Why cannot x be negative?

(c) What can you infer about the graphs of $y = x^{\frac{1}{n}}$ where n is even?

S1. Sketch the graphs of $y = x^{\frac{1}{5}}$ and $y = x^{\frac{1}{3}}$ on the same axes below.



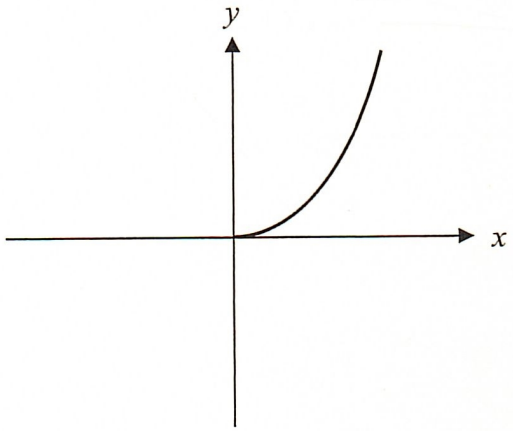
(a) What similarities do you observe from the above about the graphs? How do the two graphs differ?

(b) What can you infer about the graph of $y = x^{\frac{1}{n}}$ where n is odd?

S2. Sketch the graph of $y = -2x^{\frac{1}{5}}$.

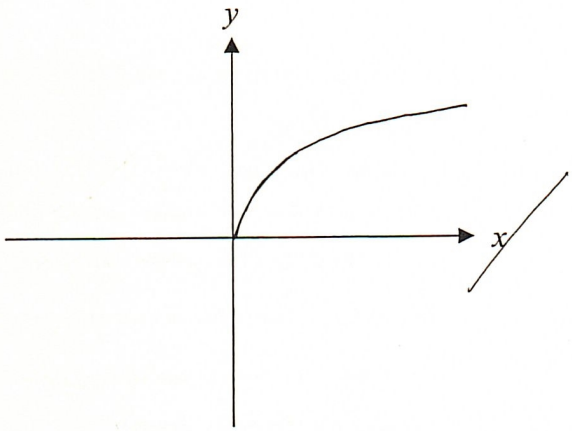
The graph of $y = ax^n$

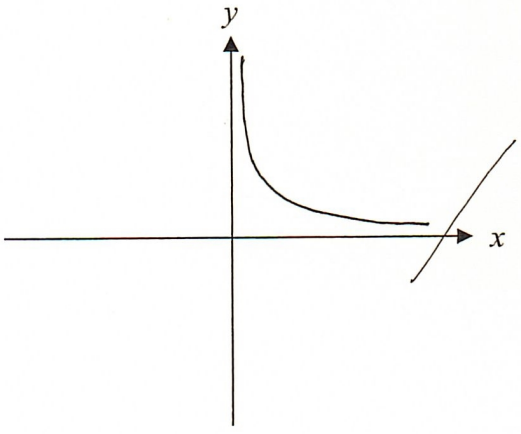
In Standard Graphs, the graphs of $y = x^n$ have been discussed when n takes integer values. What about values of n which are rational numbers? Broadly, they fall into three types.



When $n > 1$, the graph increases at an increasing rate. For example, $y = x^{\frac{4}{3}}$.

When $0 < n < 1$, the graph increases at a decreasing rate.





When $n < 0$, the graph follows that of a reciprocal shape, decreasing at a decreasing rate towards the x-axis.

In all these graphs, negative values of x are not considered. Do you know why?

S3. Sketch the following graphs on the same pair of axes, for $x > 0$, showing clearly the points of intersection.

(a) $y = 3x^{\frac{3}{2}}$ and $y = 3x^{\frac{4}{3}}$

(b) $y = -2x^{\frac{2}{3}}$ and $y = -2x^{\frac{3}{4}}$

(c) $y = x^{-2.5}$ and $y = x^{-0.4}$

The graph of $y^2 = kx$

X4. What is the difference between $y = \sqrt{x}$ and $y^2 = x$, if the latter is just the square of the former?

X5. Sketch the graphs of $y^2 = 4x + 1$.

S4. Sketch the graph of $y^2 + 9x = 0$.