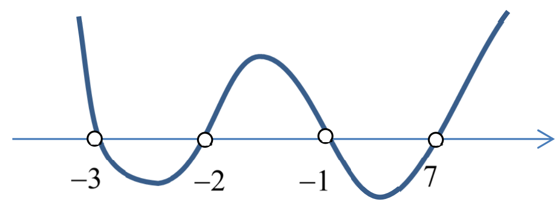
RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Additional Practice Questions for Chapter 3: Equations and Inequalities

1 **9740/2007/01/Q1**

Show that $\dfrac{2x^2-x-19}{x^2+3x+2}-1=\dfrac{x^2-4x-21}{x^2+3x+2}$. [1]

Hence, without using a calculator, solve the inequality $\dfrac{2x^2-x-19}{x^2+3x+2}>1$. [4]

Solution:
$\text{LHS}=\dfrac{2x^2-x-19}{x^2+3x+2}-1$ $=\dfrac{2x^2-x-19-x^2-3x-2}{x^2+3x+2}$ $=\dfrac{x^2-4x-21}{x^2+3x+2}=\text{RHS (Shown)}$  $\dfrac{2x^2-x-19}{x^2+3x+2}>1 \Leftrightarrow \dfrac{2x^2-x-19}{x^2+3x+2}-1>0$ $\Leftrightarrow \dfrac{x^2-4x-21}{x^2+3x+2}>0$ $\Leftrightarrow (x+3)(x-7)(x+2)(x+1)>0, \quad x\neq -2,-1.$ $\Leftrightarrow x<-3 \quad \text{or} \quad -2<x<-1 \quad \text{or} \quad x>7$

2 Sketch the graphs of $y=\lvert 4-2x\rvert$ and $y=x(4-x)$ on a single diagram. [3]

Hence solve $\lvert 4-2x\rvert<x(4-x)$, giving your answer in exact form. [4]

Solution:
From the sketch $y=\lvert 4-2x\rvert=\begin{cases}2x-4, & x\geq 2\\ 4-2x, & x<2\end{cases}$ At point A, $4-2x=x(4-x)$ $x^2-6x+4=0$ $x=\dfrac{6\pm\sqrt{6^2-4(4)}}{2}=3\pm\sqrt{5}$ Since $x<2$, $x=3-\sqrt{5}$.

At point B,

$$2x-4=x(4-x)$$
$$x^2-2x-4=0$$
$$x=\frac{2\pm\sqrt{2^2-4(-4)}}{2}=1\pm\sqrt{5}$$

Since $x>2$, $x=1+\sqrt{5}$.

Hence the solution for the inequality $|4-2x|<x(4-x)$ is $3-\sqrt{5}<x<1+\sqrt{5}$.

OR :
After finding the x co-ordinate at point A:
We note the symmetry in the line $x=2$, so the graphs intersect at $x=2+\left[2-\left(3-\sqrt{5}\right)\right]=1+\sqrt{5}$
Hence the solution for the inequality $|4-2x|<x(4-x)$ is $3-\sqrt{5}<x<1+\sqrt{5}$.

3 Without using a calculator, solve the inequality $\dfrac{x^2-2x-5}{x(1-x)}\le\dfrac{1}{x}$. [4]

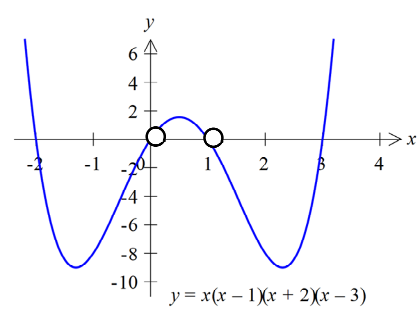
Hence find the exact solution for each of the following:

(i) $\dfrac{e^{2x}-2e^x-5}{e^x\left(1-e^x\right)}\le e^{-x}$

(ii) $\dfrac{x^2-2|x|-5}{|x|-x^2}\le\dfrac{1}{|x|}$ [6]

Solution:

$$\frac{x^2-2x-5}{x(1-x)}\le\frac{1}{x}$$
$$\frac{x^2-2x-5-(1-x)}{x(1-x)}\le 0$$
$$x(x-1)(x+2)(x-3)\ge 0,\quad x\ne 0 \text{ and } x\ne 1$$
$$x\le -2 \text{ or } 0<x<1 \text{ or } x\ge 3$$



(i)

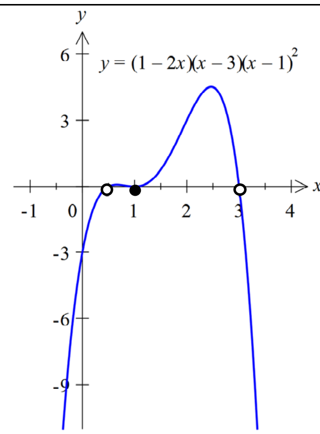
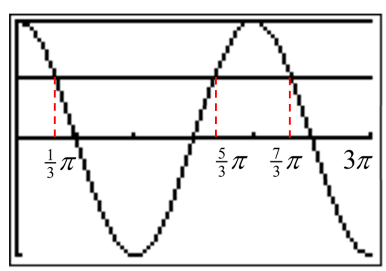
$$\frac{e^{2x}-2e^x-5}{e^x\left(1-e^x\right)}\le e^{-x}$$
$$\frac{\left(e^x\right)^2-2\left(e^x\right)-5}{e^x\left(1-e^x\right)}\le\frac{1}{e^x}$$

Replace x with e^x,

	$e^x \le -2$ (no solution since $e^x > 0\ \forall x \in \mathbb{R}$) or $0 < e^x < 1$ or $e^x \ge 3$ $x < 0$ or $x \ge \ln 3$
(ii)	$\dfrac{x^2 - 2\lvert x\rvert - 5}{\lvert x\rvert - x^2} \le \dfrac{1}{\lvert x\rvert}$ $\dfrac{\lvert x\rvert^2 - 2\lvert x\rvert - 5}{\lvert x\rvert(1-\lvert x\rvert)} \le \dfrac{1}{\lvert x\rvert}$ Replace x with $\lvert x\rvert$, $\lvert x\rvert \le -2$ (no solution since $\lvert x\rvert \ge 0\ \forall x \in \mathbb{R}$) or $0 < \lvert x\rvert < 1$ or $\lvert x\rvert \ge 3$ $x \le -3$ or $-1 < x < 0$ or $0 < x < 1$ or $x \ge 3$

4 **(i)** Solve $\dfrac{(x-1)^2}{(1-2x)(x-3)} \le 0$. [4]

(ii) Hence find the exact solution of $\dfrac{(1-\cos\theta)^2}{(1-2\cos\theta)(\cos\theta-3)} \le 0$, for $0 \le \theta \le 3\pi$. [4]

Solution:	
(i)	$\dfrac{(x-1)^2}{(1-2x)(x-3)} \le 0$ $(1-2x)(x-3)(x-1)^2 \le 0,\quad x \ne \frac{1}{2},\ x \ne 3$ $x < \frac{1}{2}$ or $x = 1$ or $x > 3$ 
(ii)	$\dfrac{(1-\cos\theta)^2}{(1-2\cos\theta)(\cos\theta-3)} \le 0$ $\dfrac{(\cos\theta-1)^2}{(1-2\cos\theta)(\cos\theta-3)} \le 0$ Replace x with $\cos\theta$ $\cos\theta < \frac{1}{2}$ or $\cos\theta = 1$ or $\cos\theta > 3$ $\frac{1}{3}\pi < \theta < \frac{5}{3}\pi$ $\quad\theta = 0, 2\pi\quad$ (n.a. $\because -1 \le \cos\theta \le 1$ or $\frac{7}{3}\pi < \theta \le 3\pi$ $\therefore \theta = 0$ or $\frac{1}{3}\pi < \theta < \frac{5}{3}\pi$ or $\theta = 2\pi$ or $\frac{7}{3}\pi < \theta \le 3$ 

5 ACJC Promo 9740/2012/Q5

Without the use of a graphing calculator, solve the inequality $\dfrac{2x^2-7x+6}{x^2-x-2}\geq 1$. [3]

Deduce the range of values of x such that

(a) $\dfrac{2(\ln x)^2-7(\ln x)+6}{(\ln x)^2-(\ln x)-2}>1$, [2]

(b) $\dfrac{2-7x+6x^2}{1-x-2x^2}\geq 1$, [2]

Solution:

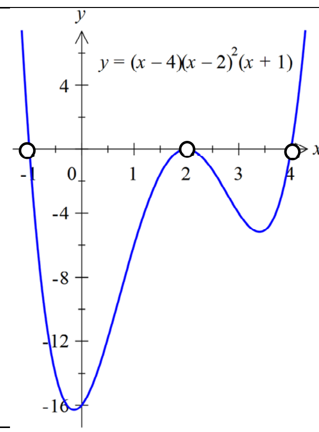
$$\frac{2x^2-7x+6-(x^2-x-2)}{x^2-x-2}\geq 0$$

$$\frac{x^2-6x+8}{x^2-x-2}\geq 0$$

$$\frac{(x-4)(x-2)}{(x-2)(x+1)}\geq 0$$

$$(x-4)(x-2)^2(x+1)\geq 0,\ \ x\neq 2,\ -1$$

$$x<-1 \quad \text{or} \quad x\geq 4$$



(a)

$$\frac{2(\ln x)^2-7(\ln x)+6}{(\ln x)^2-(\ln x)-2}>1$$

Replace x with $\ln x$,

$$\ln x<-1 \quad \text{or} \quad \ln x>4$$

$$0<x<\mathrm{e}^{-1} \quad \text{or} \quad x>\mathrm{e}^{4}$$

Note: there is no equal sign in the inequality in part (a)

(b)

$$\frac{2-7x+6x^2}{1-x-2x^2}\geq 1$$

$$\frac{2\left(\frac{1}{x}\right)^2-7\left(\frac{1}{x}\right)+6}{\left(\frac{1}{x}\right)^2-\left(\frac{1}{x}\right)-2}\geq 1 \quad \text{where } x\neq 0$$

Replace x with $\frac{1}{x}$, $\quad \frac{1}{x}<-1$ or $\frac{1}{x}\geq 4$

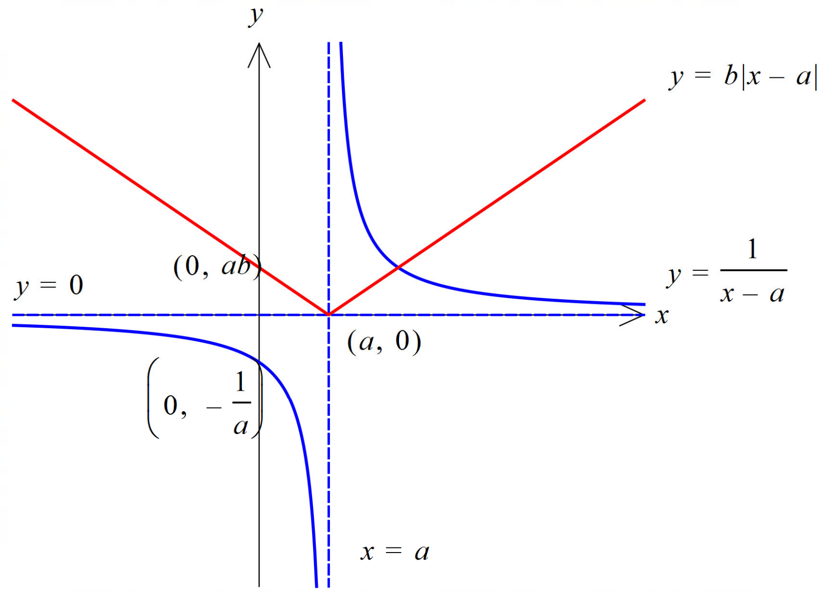
$$-1<x<0 \quad \text{or} \quad 0<x\leq \frac{1}{4}$$

Check $x=0$: $\dfrac{2-7x+6x^2}{1-x-2x^2}=2\geq 1 \Rightarrow x=0$ satisfy the inequality. $\quad \therefore -1<x\leq \dfrac{1}{4}$

6 9758/2017/01/Q2

(i) On the same axes, sketch the graphs of $y=\dfrac{1}{x-a}$ and $y=b|x-a|$, where a and b are positive constants. [2]

(ii) Hence, or otherwise, solve the inequality $\dfrac{1}{x-a}<b|x-a|$. [4]

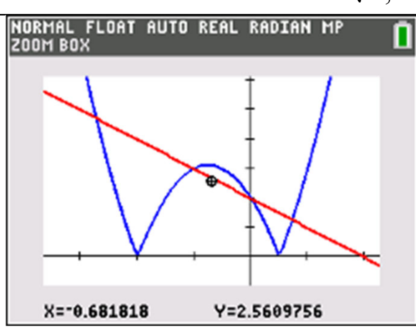
Solution:	
(i)	
(ii)	Solve for point of intersection between the two curves. $b(x-a)=\dfrac{1}{x-a}\Rightarrow(x-a)^2=\dfrac{1}{b}$ $\Rightarrow x=a\pm\dfrac{1}{\sqrt{b}}$ $\Rightarrow x=a+\dfrac{1}{\sqrt{b}}\quad(\because x>a)$ Hence, the solution for the inequality is $x<a$ or $x>a+\dfrac{1}{\sqrt{b}}$.

7 9758/2018/01/Q4

(i) Find the exact roots of the equation $\left|2x^2+3x-2\right|=2-x$ [4]

(ii) On the same axes, sketch the curves with equations $y=\left|2x^2+3x-2\right|$ and $y=2-x$.

Hence solve exactly the inequality $\left|2x^2+3x-2\right|<2-x$. [4]

Solution:	
(i)	$\left|2x^2+3x-2\right|=\begin{cases}(2x-1)(x+2), \text{ for } x\le -2 \text{ } or \text{ } x\ge \frac{1}{2} \\ -(2x-1)(x+2), \text{ for } -2\le x\le \frac{1}{2}\end{cases}$ Consider $2-x=2x^2+3x-2$, for $x\le -2$ *or* $x\ge \frac{1}{2}$ $2-x=2x^2+3x-2$ $x^2+2x-2=0$ $x=\frac{-2\pm\sqrt{2^2-4(-2)}}{2}$ $=\frac{-2\pm 2\sqrt{3}}{2}$ $=-1\pm\sqrt{3}$ [valid solutions] Consider $2-x=-2x^2-3x+2$, for $x-2\le x\le \frac{1}{2}$ $2-x=-2x^2-3x+2$ $2x^2+2x=0$ $x=0$ or -1 [valid solutions] Therefore the roots are $-1-\sqrt{3}$, -1, 0 and $-1+\sqrt{3}$.
(ii)	 For $\left|2x^2+3x-2\right|<2-x$, $-1-\sqrt{3}<x<-1$ or $0<x<-1+\sqrt{3}$.

8 ACJCCommonTest9758/2021/2

Given that $3x^3-7x^2+8x-2=(Ax+B)(x^2-2x+2)$, where A and B are constants to be determined, solve algebraically the inequality $3x^2-10x+18\geq\dfrac{20}{x+1}$. [4]

Hence solve, exactly, the inequality $3(x+1)^2-10|x+1|+18\geq\dfrac{20}{|x+1|+1}$. [2]

Solution:

$3x^3-7x^2+8x-2=(Ax+B)(x^2-2x+2)$

Comparing coefficients of x^3, $A=3$.
Comparing constant term, $B=-1$.

$$3x^2-10x+18\geq\frac{20}{x+1}\quad\Rightarrow\quad 3x^2-10x+18-\frac{20}{x+1}\geq 0$$

$$\Rightarrow\quad\frac{(3x^2-10x+18)(x+1)-20}{x+1}\geq 0$$

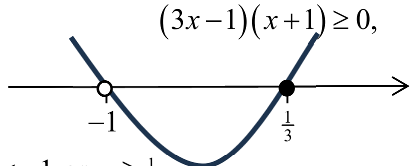
$$\Rightarrow\quad\frac{3x^3-7x^2+8x-2}{x+1}\geq 0$$

$$\Rightarrow\quad\frac{(3x-1)(x^2-2x+2)}{x+1}\geq 0$$

$$\Rightarrow\quad(3x-1)(x+1)(x^2-2x+2)\geq 0,\quad x\neq -1$$

Since $x^2-2x+2=(x-1)^2+1\geq 1>0$ for all $x\in\mathbb{R}$, hence

$$(3x-1)(x+1)\geq 0,\qquad x\neq -1.$$



Therefore $x<-1$ or $x\geq\frac{1}{3}$.

$$3(x+1)^2-10|x+1|+18\geq\frac{20}{|x+1|+1}$$

$$3|x+1|^2-10|x+1|+18\geq\frac{20}{|x+1|+1}$$

Replace x by $|x+1|$ in the previous part,

$|x+1|<-1$ or $|x+1|\geq\dfrac{1}{3}\ \Rightarrow\ x\leq-\dfrac{4}{3}$ or $x\geq-\dfrac{2}{3}$

(No solution)

Hence, $x\leq-\dfrac{4}{3}$ or $x\geq-\dfrac{2}{3}$.

9 JPJC Promo 9758/2021/Q2(i)

A curve C has equation $y=\dfrac{a}{x+3}+bx+c$, where a, b and c are constants. It is given that C passes through the point with coordinates (1, –3) and has a stationary point (–5, –21). Find the values of a, b and c. [4]

Solution:
Subst (1, –3) into C, $\dfrac{a}{4}+b+c=-3----(1)$ Subst (–5,–21) into C, $\dfrac{a}{-2}-5b+c=-21----(2)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{a}{(x+3)^2}+b$ At $x=-5$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0 \quad \Rightarrow \quad -\dfrac{a}{4}+b=0---(3)$ Using GC, $a=8$, $b=2$, $c=-7$

10 9740/2007/02/Q1

Four friends buy three different kinds of fruit in the market. When they get home they cannot remember the individual prices per kilogram, but three of them can remember the total amount that they each paid. The weights of the fruits and the total amounts paid are shown in the following table.

	Suresh	Fandi	Cindy	Lee Lian
Pineapples (kg)	1.15	1.20	2.15	1.30
Mangoes (kg)	0.60	0.45	0.90	0.25
Lychees (kg)	0.55	0.30	0.65	0.50
Total amount paid in $	8.28	6.84	13.05	

Assuming that, for each variety of fruit, the price per kilogram paid by each of the friends is the same, calculate the total amount that Lee Lian paid. [6]

Solution:
Let x, y and z be the price of pineapples per kilogram, mangoes per kilogram and lychees per kilogram. For Suresh: $1.15x+0.6y+0.55z=8.28$ For Fandi: $1.2x+0.45y+0.3z=6.84$ For Cindy: $2.15x+0.9y+0.65z=13.05$ Using GC, $x=3.5, y=2.6, z=4.9$ Total price paid by Lee Lian = \$($3.5\times1.3+2.6\times0.25+4.9\times0.5$) = \$ 7.65

11 A housewife bought at least one box of laundry detergent powder, one tube of toothpaste and one packet of sweets at a supermarket. Given that a box of laundry detergent powder costs \$10, a tube of toothpaste costs \$3 and a packet of sweets costs \$0.50 and the housewife bought a total of 100 such items with \$100 at the supermarket, how many of each item did she buy? [6]

Solution:

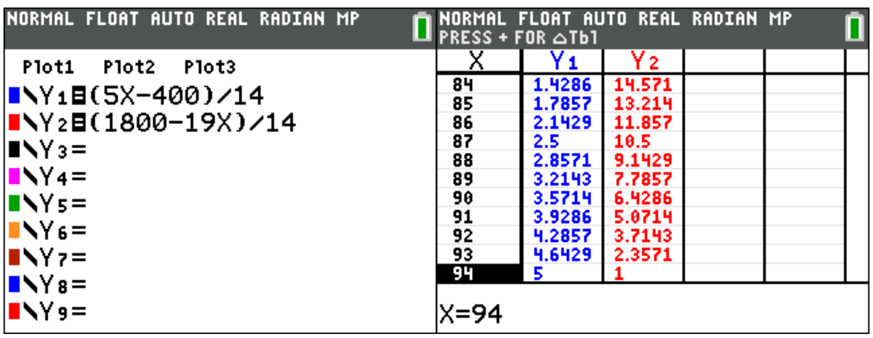
Let D, T and S denote the number of box(es) of laundry detergent powder, number of tube(s) of toothpaste and number of packet(s) of sweets bought by the housewife.
Then

$$10D+3T+(0.5)S=100$$
$$D+T+S=100$$

Method 1

From GC, using Plysmlt2, $D=\dfrac{5S-400}{14}$ and $T=\dfrac{1800-19S}{14}$.

Since D and T are positive integers, $D=\dfrac{5S-400}{14}\geq 1$ and $T=\dfrac{1800-19S}{14}\geq 1$ gives $83\leq S\leq 94$.



X	Y1	Y2
84	1.4286	14.571
85	1.7857	13.214
86	2.1429	11.857
87	2.5	10.5
88	2.8571	9.1429
89	3.2143	7.7857
90	3.5714	6.4286
91	3.9286	5.0714
92	4.2857	3.7143
93	4.6429	2.3571
94	5	1

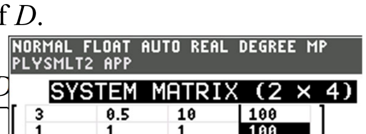
Using the gc, from table of values of $D=\dfrac{5S-400}{14}$ and $T=\dfrac{1800-19S}{14}$, it can be observed by scrolling down the values in the range $83\leq S\leq 94$ that $D=5$, $T=1$ and $S=94$ is a unique answer.

Thus, the housewife bought 94 packets of sweets, 5 boxes of laundry detergent powder and 1 tube of toothpaste at the supermarket.

Method 2

Detergent is the most expensive, so we should solve in terms of D.

From GC.



$$T=20-\frac{19D}{5}$$
$$S=80+\frac{14}{5}D$$
$$\because T\geq 1\Rightarrow D\leq\frac{95}{19}=5$$

$\dfrac{19D}{5}$ must be an integer, so $D=5$

$\Rightarrow T=1,\ S=94.$

Method 3

A non-GC method.

$10D+3T+(0.5)S=100$ multiply by 2

$20D+6T+S=200 \qquad (1)$

$D+T+S=100 \qquad (2)$

$(1)-(2)\Rightarrow 19D+5T=100$

$19D=5(20-T)$

$20-T$ must be a multiple of 19, so $T=1$.

$D=5,\ T=1\Rightarrow S=94$

12 A corporation wants to lease a fleet of 12 airplanes with a combined carrying capacity of 220 passengers. The three available types of planes carry 10, 15 and 20 passengers. The monthly cost of leasing each of these types of planes is \$8000, \$14000 and \$16000 respectively. What are the possible combinations of the different types of planes that should be leased and which of these combinations would minimise the monthly leasing cost? [6]

Solution:

Let x, y and z represent the number of planes with 10, 15 and 20 passenger seats respectively, then using the information in the question we have

$$x+y+z=12$$

$$10x+15y+20z=220$$

On solving we obtain $x=z-8, y=20-2z$.

Since x and y are both non-negative integers, we have $8 \le z \le 10$.

The possible combinations for (x, y, z) are $(0,4,8),(1,2,9)$ and $(2,0,10)$.

Possible combination (x, y, z)	Total rental charge in thousands: $8x+14y+16z$
$(0,4,8)$	184
$(1,2,9)$	180
$(2,0,10)$	176

We conclude that leasing two 10-passenger planes and ten 20-passenger planes is the least at \$176 000.

13 JPJC Prelim 9758/2020/01/Q4

The usual selling price of a bottle of Vitamin C, Grape Seed Extract and Super Antioxidant is \$365 in total. During the year-end sale, two companies, Union and Vatson, offered the following discounts to their customers:

Company	Discounts given for each item			Total price after the discount
	Vitamin C	Grape Seed Extract	Super Antioxidant	
Union	10%	15%	15%	\$314.75
Vatson	8%	25%	15%	\$301.55

(i) Find the usual selling price of each bottle of Vitamin C, Grape Seed Extract and Super Antioxidant. [3]

(ii) The employees from Union are offered a further 5% discount on the usual selling price of each bottle of Vitamin C and Grape Seed Extract. Determine if this additional discount will make it more attractive for the employees to purchase all the 3 items from their own company rather than from Vatson. [2]

Solution:	
(i)	Let x represent selling price of a bottle of Vitamin C y represent selling price of a bottle of Grape Seed Extract z represent selling price of a bottle of Super Antioxidant $x+y+z=365$ $0.9x+0.85y+0.85z=314.75$ $0.92x+0.75y+0.85z=301.55$ Hence x = \$90, y =\$150, z = \$125
(ii)	$0.85x + 0.8y + 0.85z$ = \$302.75> \$301.55 Not attractive.

14 RI Prelim 9758/2020/01/Q2

Mr. Li invested a total of \$30000 and divided this sum into three accounts, which paid 2%, 3% and 5% annual interest respectively.
At the end of the first year, Mr. Li withdrew all the money out from the 2% and 5% accounts and gave the interest earned to his son. The amount in the 3% account, including interest, was re-invested in the same account for another year.
At the end of the second year, Mr. Li withdrew all the money out of the 3% account and gave the interest earned to his son.
The total interest received by the son from the three accounts was \$1423.50.
Given that the amount invested in the 2% account was \$1000 more than the amount invested in the 5% account, find the amounts invested in each of the three accounts. [4]

Solution:

Let x, y and z be the amounts he invested into the 2%, 3% and 5% accounts respectively. $x+y+z=30000$ ------- (1) $0.02x+(1.03^2-1)y+0.05z=1423.50$ ------- (2) $x-z=1000$ ------- (3) From GC, solving the 3 equations, $x=8000$, $y=15000$, $z=7000$

15 MI Promo 9758/2018/Q2

A Water Show sells tickets at three different prices, depending on the age of the customers. The age categories are under 12 years, between 12 and 62 years, and over 62 years. The cost of the tickets for each group are given in the following table.

Age Group	Cost per ticket
Under 12 years	\$6
Between 12 and 62 years	\$9
Over 62 years	\$5

MI Residence Committee spends \$800 to sponsor a group of 100 people from these three age groups to attend the event. By writing down a system of equations, find the range of the possible number of people over the age of 62 sponsored. [4]

Solution:
Let x, y and z be the number of people in the group under 12 years, between 12 and 62 years and over 62 years respectively. $6x+9y+5z=800$ $x+y+z=100$ Using GC, $x=-\frac{4}{3}z+\frac{100}{3}$ $y=\frac{1}{3}z+\frac{200}{3}$ Since x, y and z are non-negative integers, $-\frac{4}{3}z+\frac{100}{3}\geq 0 \Rightarrow z\leq 25$ $\frac{1}{3}z+\frac{200}{3}\geq 0 \Rightarrow z\geq -200$ $\therefore -200\leq z\leq 25$ $\because z\geq 0,\ \therefore 0\leq z\leq 25$ Checking the feasibility of the solution using GC table of values (refer to screenshots), possible values of z are 1, 4, 7, 10, 13, 16, 19, 22 and 25.