

1 Evaluate, without the use of calculators, showing your working clearly.

(a) $81^{\frac{3}{4}}$

(b) $64^{\frac{-2}{3}}$

(c) $5^{\frac{3}{4}} \div 5^{\frac{1}{4}} \times 5^0$

(d) $\left(\frac{1}{8}\right)^{-\frac{2}{3}}$

(e) $\left(\frac{4^2}{2^3}\right)^{-2}$

(f) $\left(\frac{1}{8}\right)^{\frac{4}{3}} \times (-2)^{-2}$

(g) $32^{-1.2} \times \left(\frac{1}{16}\right)^{-\frac{5}{4}}$

(h) $\left(\frac{2}{5}\right)^{-1} \times \left(\frac{\sqrt[5]{31}}{4}\right)^0 \div \left(\frac{5}{3}\right)^2$

(i) $32^{-0.6} \times \left(\frac{3}{2}\right)^{-3}$

(j) $(-5)^0 \div \left(\frac{8}{27}\right)^{\frac{1}{3}} \times \sqrt{2\frac{1}{4}}$

2 Evaluate the following without the use of calculators.

(a) $4^3 + 4^3 \times 4^{-2}$

(b) $\left(\frac{4}{9}\right)^0 + \left(\frac{3}{5}\right)^{-2}$

(c) $\frac{2^3 \times 25}{5^{-3} \times 2 \times 10^2}$

(d) $\frac{27 \times 625}{5^{-2} \times 15^{-1} \times 5^6}$

(e) $3^{\frac{1}{3}} \times 15^{\frac{2}{3}} \times \sqrt[3]{5}$

(f) $\frac{4^2 \times 2^{-3}}{3^{-3} \times 6^2}$

(g) $\left(\frac{9}{25}\right)^{-\frac{1}{2}} + \left(\left[\frac{8}{125}\right]^{\frac{1}{3}}\right)^0$

(h) $\sqrt{8} \times \sqrt[3]{4} \times \sqrt[6]{2} \div \sqrt[3]{2}$

(i) $\frac{3^4 \times 49}{7^{-1} \times 9}$

(j) $\left(\frac{4}{3}\right)^{-2} - \left(\frac{1}{8} \div \frac{2}{3}\right)$

(k) $16^{\frac{3}{4}} + \left(\frac{1}{27}\right)^{\frac{2}{3}} \times \left(\frac{2}{3} - \frac{3}{4}\right)^0$

(l) $32^{\frac{2}{5}} - 27^{\frac{2}{3}} \times 16^{-0.75}$

(m) $\left(\frac{4}{5} \times 125^{-5} \div 25\right)^0 - \left(\frac{2}{3}\right)^{-2}$

(n) $5^{-2} + (64)^{\frac{2}{3}} + \left(\frac{3}{4}\right)^0$

(o) $\left(\frac{8}{27}\right)^{-\frac{1}{3}} \times \left(\frac{9}{4}\right)^{\frac{1}{2}} - (-2)^{-3}$

(p) $10^3 + 10^2 + 10^0 + 10^{-2}$

3 Solve the following equations:

(a) $2^d = 1$

(b) $10^c = 0.001$

(c) $81^b = 3$

(d) $2^a \div 2^3 = 16$

(e) $(243)^x = \frac{1}{81}$

(f) $7(7^x) = 1$

(g) $3^{2x} \times 27^{x-2} = 81$

(h) $2x^3 = 250$

(i) $102^{2x+3} = 1$

(j) $x^{-\frac{1}{4}} = 4$

(k) $5^{1-x} = \frac{1}{125^x}$

(l) $9^{x+1} \times 27^{x-1} = 3^4$

(m) $9^{2x-3} = 27^{2x-3} \div 3^2$

(n) $4\left(\frac{1}{8}\right)^{1-3x} = 32^{x+5}$

(o) $9^{2x-1} = 27^{\frac{x}{3}}$

(p) $81^{2x-1} = \left(\frac{1}{3}\right)^{-3}$

(q) $\sqrt{a^5} \sqrt{a^3} = a^x$

(r) $16^x = 128$

(s) $2^x \times 4^{x+3} = 8^{2x-1}$ (t) $4^x \times (4^3)^{-2} = 2^{x+2}$ (u) $\sqrt{4^x} = 2 \div \sqrt{8^x}$
 (v) $16x^{\frac{1}{3}} + 4^3 = 2^7$ (w) $2^x + 2^{x+1} = 12$

*4 Given that $(x^2 - 4x - 4)^{x^2+x-6} = 1$, find all the possible values of x .

5 Simplify the following, giving your answers in positive indices only:

(a) $5m^7 \times 2m^{-4} \times (4m)^{-1}$

(b) $ab^{-1} \times (2ab^3)^2$

(c) $3a^2 \times 4a^{-5} \div (2a)^3$

(d) $(2a^2)^3 \div (ab^{-1})^2$

(e) $(-pq^2)^{-3} \times (-12p^2)$

(f) $[x^{\frac{1}{3}}y^{-\frac{3}{4}}] \div [x^{-\frac{1}{3}}y^{\frac{1}{4}}]$

(g) $(a^2b^3)^2 \div (ab^{-2})^{-2}$

(h) $3a^2 \times 7a^{-3} \div (2a^2)^2$

(i) $\frac{(a^{-4}b^3)^3}{a^5b}$

(j) $\frac{6(cd^3)^3}{(2c^{-2}d^8)^2}$

(k) $\frac{(5a^2b^{-4})^2}{30a^{-3}b^2}$

(l) $\frac{8b^3}{7a^5} \div \frac{4b^2}{35a^3}$

(m) $\frac{a^3b^{-4}c^0}{a^{-2}b} \div \frac{(a^2b^{-1})^3}{ac^{-3}}$

(n) $\frac{2xy^2}{(-3x^3yz)^2}$

(o) $\frac{(-3xy^2)^2}{2x^3y^5} \div \frac{9y^3}{7x^2}$

(p) $\frac{(-2xy^2)^2}{3x^3} \div \frac{8y^3}{7x^2}$

(q) $(p^4q^3)^3 \div \frac{(pq^{-1})^2}{(p^2q^3)^{-2}}$

(r) $9b^2 \div 3ab \times \frac{b}{a^2}$

(s) $\frac{\sqrt{25m^6n^2}}{8} \times \left(\frac{n^2}{m}\right)^2$

(t) $\frac{3ab}{4} \div 12b^2 \times \sqrt{25a^2}$

(u) $\frac{4b}{3a} \times \frac{7a^3}{15c} \div \frac{3b^2}{10c^4}$

(v) $\left(\frac{a}{b}\right)^2 \times \left(\frac{c}{b}\right)^{-2} \div \left(\frac{a}{c}\right)^{-3}$

(w) $\sqrt{\frac{9a^2}{4b^6}} \div 3ab \times \frac{b^2}{2a}$

(x) $\sqrt{\frac{64a^4}{49b^8}} \times \frac{3b}{ac} \div \frac{3b^3}{5c^2}$

(y) $\frac{(ab)^{-3}}{(a^2b)^2(ab^3c)^4} \div \frac{(a^3b^2)^{-2}}{(a^2b)}$

(z) $\frac{4a^2b^3}{(-2ab^2)^3} \times \frac{6a^3b}{25ab} \div \frac{3b}{(-5a^2b)^2}$

6 Simplify the following:

(a) $\frac{(-ab^2)^3}{d^2} \times \frac{d^{16}}{b^5} \div (a^2 - a)$

(b) $\frac{9^{x+1} + 6(3^{2x})}{3^{1-x}27^{x+1}}$

(c) $(ab^2)^3 \times \frac{c^6d^{-7}}{(-ab)^2}$

(d) $\frac{(ab^2)^3}{a^2} \times a^0b^2$

(e) $(a^3)^{\frac{2}{3}} \times \left(\frac{2}{5}\right)^{-2} \times \frac{2}{5}$

- 7 Given that a and b are positive whole numbers, simplify the following, giving your answers in terms of a and/or b where necessary:
(a) $1^a + 1^b$ (b) $(-b)^{-1}$ (c) $(-5a)^3$
- 8 If $a^{2x} = 4$ and $b^y = 3$, evaluate
(a) $a^{4x} + b^{3y}$ (b) a^{3x}
- 9 If $3^x = 2$ and $3^y = 5$, find the value of 3^{y-2x} .
- 10 If $3^x = 5$ and $3^y = 8$, find the value of 3^{3x-2y} .
- *11 If $3^a = 5^b = 15^x$, express x in terms of a and b .