Class	Index Number	Name

ANG MO KIO SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY FOUR EXPRESS/5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS **4049/02**

Paper 2

Setter: Mrs D Sharav

Wednesday **14 September 2022** **2 hour 15 minutes**

Answer on the question paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **90**.

For Examiner's Use
90

This document consists of **20** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Prove that $\sec\theta - \dfrac{\cos\theta}{1+\sin\theta} = \tan\theta$. [4]

(b) Hence solve the equation $\sec\theta - \dfrac{\cos\theta}{1+\sin\theta} = 2\sin\theta$ for $0 < \theta < 2\pi$. [4]

2 A circle with centre C at $(5, 7)$ touches the y-axis at point P.

(i) State the radius of the circle and find its equation. [2]

(ii) The line $4x + 3y = 16$ is a tangent to the circle at the point Q.
Find the equation of the line CQ. [3]

(iii) Hence find the coordinates of the point Q. [2]

(iv) Given that line $4x+3y=k$ is another tangent to the circle, find the value of k. [3]

3 **(i)** Show that $2x - 3$ is a factor of $2x^3 - 3x^2 + 2x - 3$. [1]

(ii) Hence factorise $2x^3 - 3x^2 + 2x - 3$. [2]

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(iii) Using results from **(ii)**, express $\dfrac{8+7x-x^2}{2x^3-3x^2+2x-3}$ in partial fractions. [5]

4 **(a)** Solve the equation $\log_2(3x+2)-\log_{\frac{1}{2}}(x-1)=1$. [5]

(b) It is given that $a^2+b^2=7ab$. Show that $2\lg\frac{1}{3}(a+b)=\lg a+\lg b$. [4]

5 **(i)** Find all the angles between $0°$ and $360°$ which satisfy the equation
$9\tan^2 x = 24\sec x - 25$. [4]

(ii) Solve the equation $3\operatorname{cosec}^2\left(2x+\frac{\pi}{4}\right)=4$ for $0 \le x \le 2$. Leave your answers in terms of π. [4]

6 The trees in a certain forest are dying because of an unknown virus.

The number of trees, N, surviving t years after the onset of the virus is shown in the table below.

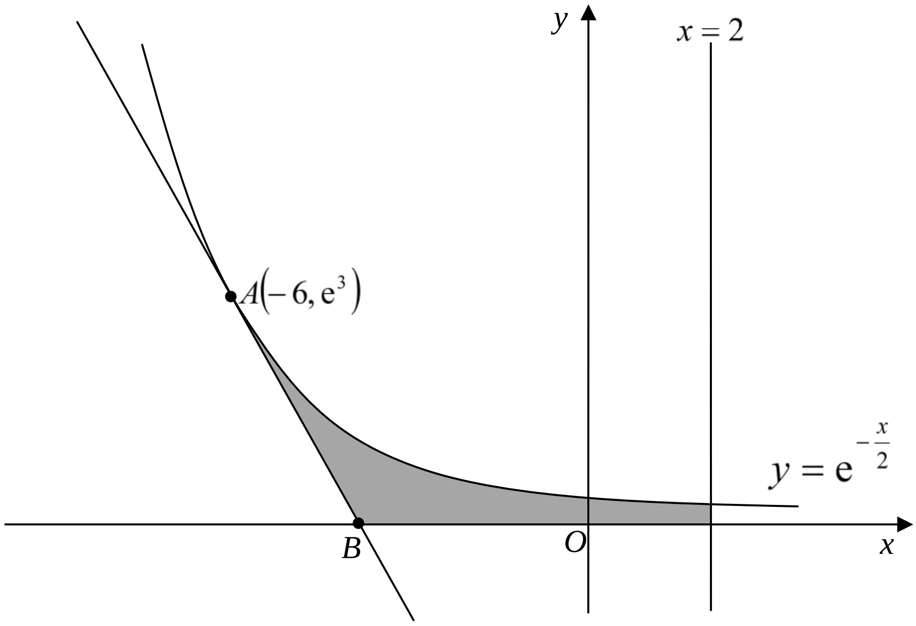
t	1	2	3	4	5	6
N	2000	1300	890	590	395	260

The relationship between N and t is thought to be of the form $N = Ab^{-t}$ where A and b are constants.

(i) Draw the graph of ln N against t and hence estimate the value of each of the constants A and b. [6]

(ii) Using the values of A and b from **(i)**, estimate the number of trees surviving after 15 years. [2]

7



The diagram shows part of the curve $y = e^{-\frac{x}{2}}$. The tangent to the curve at $A(-6, e^3)$ meets the x-axis at B.

(i) Find the coordinates of B. [4]

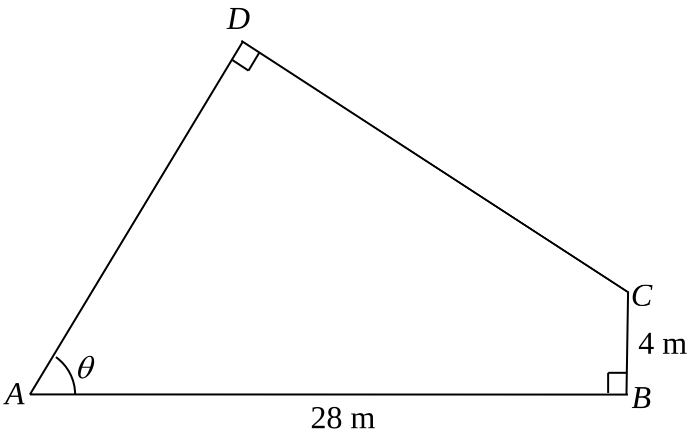
(ii) Find the area of the shaded region bounded by the curve, the tangent to the curve at A, the line $x = 2$ and the x-axis. [6]

8 **(i)** Express $\frac{10x}{3-5x}$ in the form $a+\frac{b}{3-5x}$, where a and b are integers. [2]

(ii) Differentiate $x\ln(3-5x)^2$ with respect to x. [3]

(iii) Using the results of parts **(i)** and **(ii),** determine $\int \ln(3-5x)\ \mathrm{d}x$. [5]

9



In the figure, *ABCD* is part of a piece of land, where $AB = 28\,\text{m}$, $BC = 4\,\text{m}$, angle $ABC =$ angle $ADC = 90°$ and angle $BAD = \theta$, where θ is measured in degrees. A farmer wants to put fences around the perimeter of the quadrilateral *ABCD*.

(i) Show that, L m, the length of the fencing needed, is given by
$L = 32 + 32\sin\theta + 24\cos\theta.$ [3]

(ii) Express L in the form $32 + R\sin(\theta + \alpha)$, where R is positive and α is an acute angle. [3]

(iii) Given that the farmer uses exactly 70 m of fencing, find the size of θ. [2]

(iv) Explain why the total length of fencing needed will not exceed a certain value and state this value. [2]

10 $\mathrm{f}(x)$ is such that $\mathrm{f}''(x) = 3\cos 3x - 4\sin 2x$. Given that $\mathrm{f}(0) = 0$ and $\mathrm{f}\left(\frac{\pi}{2}\right) = \frac{5}{6}$,

show that $\mathrm{f}\left(\frac{\pi}{3}\right) = 1 + \frac{\sqrt{3}}{2}$. [9]

END OF PAPER