| Class | Register Number | Name |
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南洋女子中學校

Nanyang Girls' High School

**End-of-Year Examination 2024**
**Secondary 3**

# INTEGRATED MATHEMATICS 2

**2 hours**

**Monday 7 October 2024** **1145 – 1345**

**READ THESE INSTRUCTIONS FIRST**

1. Write your name, register number and class on all the work you hand in.
2. Write in dark blue or black ink.
3. You may use an HB pencil for any diagrams or graphs.
4. Do not use staples, paper clips, glue or correction tape/ fluid.
5. Write your answers and working on the separate writing paper provided, unless otherwise stated.
6. Answer **all** questions.
7. Omission of essential working will result in loss of marks.
8. The use of an approved scientific calculator is expected, where appropriate.
9. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degree to one decimal place. For $\pi$, use either your calculator value or 3.142, unless the question requires the answer in terms of $\pi$.
10. At the end of the examination, fasten all your work securely together.
11. The number of marks is given in brackets [ ] at the end of each question or part question.
12. The total of the marks for this paper is 80.

This document consists of **5** printed pages and **1** blank page.

# Mathematical Formulae

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \, .$$

**1** Express $\dfrac{5x^2+x-13}{(x-3)^2(2x+1)}$ in partial fractions. [5]

**2** By rationalising the denominator, express $\dfrac{\sqrt{2}+4\sqrt{5}}{2+\sqrt{10}}$ in the form of $a\sqrt{2}+b\sqrt{5}$,

where $a$ and $b$ are integers. [4]

**3** Evaluate $\dfrac{3^{2x-1}\times 81^{x+1}}{8\left(3^{6x}\right)-3^{6x}}$. [4]

**4** Given that $\sin\theta = \dfrac{7}{25}$ and $\tan\theta < 0$, find, without the use of a calculator, the exact value of the following.

**(a)** $\tan\theta$ [2]

**(b)** $\cos\left(-\theta\right)$ [2]

**(c)** $\sec\left(90°-\theta\right)$ [3]

**5** **(a)** The line $y = kx+1$ is a tangent to the curve $y = 2x^2+9$. Find the possible values of $k$. [3]

**(b)** Find the range of values of $k$ for which the graph of $y=(3k-5)x^2+2(kx-1)+k$ lies entirely above the line $y=-4$. [6]

**6** It is given that $\mathrm{f}(x)=4x^3-8x^2+kx+18$.

**(a)** Given that the remainder when f($x$) is divided by (2$x$ – 1) is 12, show that $k$ = –9. [2]

**(b)** Solve $\mathrm{f}(x)=0$. [5]

**(c)** Using your results from **part (b)**, solve $-4y^3-8y^2+9y+18=0$. [2]

**7** Solve the following equations, leaving your answers in three significant figures, where necessary.

**(a)** $5^{x+1} - 6\left(5^{-x}\right) = 7$ [5]

**(b)** Solve the equation $\log_3 \sqrt{2-x} = 1 - \log_9 (x+8)$ [5]

**8** A piece of meat was placed in a freezer. After $t$ minutes in the freezer, the temperature, $\theta°C$, of the meat is given by $\theta = 50e^{-kt} - 18$, where $k$ is a constant. When the meat was taken out for cooking after 20 minutes in the freezer, the chef observed that the temperature was at $-8°C$.

**(a)** Find the initial temperature of the meat the moment it was placed in the freezer. [2]

**(b)** Find the temperature of the meat if it had stayed in the freezer continuously for one hour. [4]

**(c)** If the meat stays in the freezer for a long time, the temperature of the meat will be equal to the temperature of the freezer. Deduce the temperature of the freezer. Explain your answer clearly. [2]

**9** The function f is defined by $\mathrm{f}(x) = 2\sin\frac{x}{2}$ for $0 \le x \le 4\pi$.

**(a)** State the amplitude and period of f. [2]

**(b)** Sketch the graph of $y = \mathrm{f}(x)$ for $0 \le x \le 4\pi$. Indicate clearly the intercepts and the turning points. [4]

**(c)** A line $y = k$, where $k > 0$, is drawn on the same axes as the curve $y = \mathrm{f}(x)$ for $0 \le x \le 4\pi$. Given that graphs meet at $x = p$ and $x = q$, where $p < q$, express $q$ in terms of $p$. [1]

**10** **(a)** Sketch the graph of $y = 4^x - 1$, showing clearly, if any, the asymptote(s) and the intercept(s) with the axes. [3]

**(b)** Karen claims that the equation $\log_4(4 - 2x) = x$ has two solutions.

By adding a suitable straight line on the same axes as the graph of $y = 4^x - 1$, determine if Karen is correct. [4]

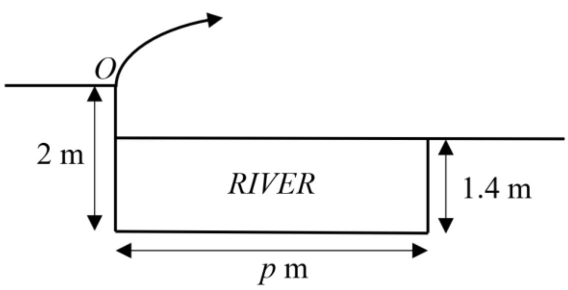
**(c)** The graph of $y = 4^x - 1$ undergoes two successive transformations.

**I** A reflection in the $x$–axis.

**II** A translation parallel to the $x$–axis by 2 units.

Write down the equation of the resulting graph. [2]

**11** In the diagram below, a dog leaps off the edge of a river, at point $O$, to the other side. The two sides of the river are 2 metres and 1.4 metres above the riverbed.




The height, $y$ metres, of the dog's jump can be modelled by the equation

$y = -\frac{2}{9}x^2 + x$, where $x$ is the horizontal distance, in metres, from the point $O$.

**(a)** Express $y = -\frac{2}{9}x^2 + x$ in the form $y = a\left(x + b\right)^2 + c$. [3]

**(b)** Hence, find the maximum height the dog jumps above the surface of the river, and the corresponding horizontal distance from $O$. [2]

**(c)** Assume the dog was able to reach the other side. Find the maximum integer value of $p$, where $p$ m is the width of the river. [3]

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**2024 IM2 EOY Answer Key**

1 $\dfrac{3}{x-3}+\dfrac{5}{(x-3)^2}-\dfrac{1}{2x+1}$

2 $3\sqrt{2}-\sqrt{5}$

3 $\dfrac{27}{7}$

4 (a) $-\dfrac{7}{24}$ (b) $-\dfrac{24}{25}$ (c) $\dfrac{25}{7}$

5 (a) $k=\pm 8$ (b) $k>2$

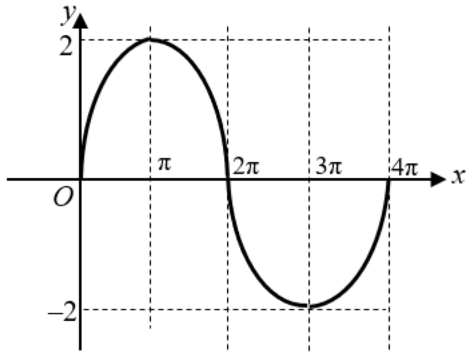
6 (b) $x=-\dfrac{3}{2},\ \dfrac{3}{2}$ or $2$ (c) $y=\dfrac{3}{2},-\dfrac{3}{2}$ or $-2$

7 (a) 0.431 (b) $x=-7$ or $1$

8 (a) 32°C (b) −17.6°C (c) −18°C

9 (a) Amplitude = 2 units, period = $\dfrac{2\pi}{\left(\frac{1}{2}\right)}=4\pi$

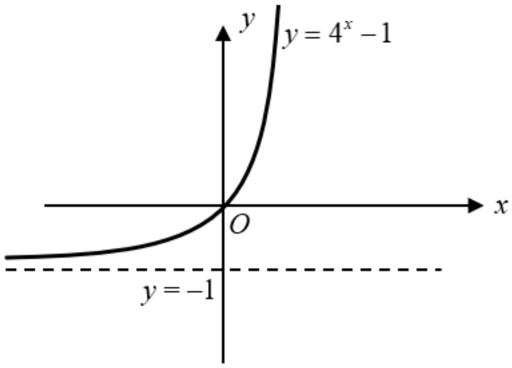
(b)




(c) $q=2\pi-p$

[Turn over

10 (a)




(b) Line to add: $y = 3 - 2x$

One intersection point $\Rightarrow$ one solution only.

We say Karen is wrong.

(c) $y = 1 - 4^{x-2}$

11 (a) $y = -\frac{2}{9}\left(x - \frac{9}{4}\right)^2 + \frac{9}{8}$

(b) Max height = 1.725 m; horizontal distance 2.25 m from O

(c) Maximum integer $p$ = 5