Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2016
Secondary Three

INTEGRATED MATHEMATICS 2

2 hours

Tuesday **11 Oct 2016** **0845-1045**

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Write your name, register number and class in the spaces at the top of this page.
2. Answer questions 1 - 11 before attempting question 12 (Bonus Question).
3. Write your answers and working on the separate writing paper provided.
4. Omission of essential working will result in loss of marks.
5. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 80.
3. You are reminded of the need for clear presentation in your answers.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

3. MENSURATION

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2} r^2 \theta$, where θ is in radians

1 A cup of hot water is left on a table to cool. After t minutes, the temperature of the water, T °C is given by $T = 27 + 30e^{-0.02t}$.

(i) State the initial temperature of the cup of water. [1]

(ii) Find the time it takes for the cup of water to cool to 41°C. [2]

(iii) The temperature of the cup of water will not go below x °C. Write down the largest value of x given that x is an integer. [1]

2 The graph of $y = f(x)$ cuts the coordinate axes at the points (4, 0) and (0, –6). Find the coordinates of the points where the following graphs cut the coordinate axes:

(a) $y = f^{-1}(x)$, [2]

(b) $y = f(-2x)$, [2]

(c) $y = \left|\frac{3}{2}f(x)\right|$. [2]

3 **(a)** It is given that $y = 3x^2 - 4x + 5$.

(i) Express y in the form $a(x-h)^2 + k$. [2]

(ii) Hence,

(a) write down the equation of the line of symmetry of $y = 3x^2 - 4x + 5$, [1]

(b) state the value of c, given that the line $y = c$ is tangent to the curve $y = 3x^2 - 4x + 5$. [1]

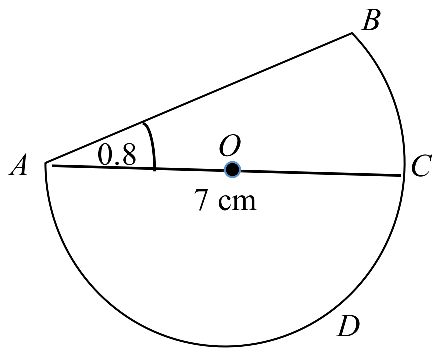
(b) Find the range of values of b for which $6x^2 - 7bx + 2b^2 + 6 \geq 0$ for all real values of x. [3]

4 The roots of the quadratic equation $3x^2 + 6x + 5 = 0$ are α and β. Find a quadratic equation whose roots are $\frac{\alpha^2}{2\beta}$ and $\frac{\beta^2}{2\alpha}$. [5]

5 Solve the equation $3 + 8\cot^2 x = 18\operatorname{cosec} x$, for $0° \leq x \leq 360°$. [5]

[Turn over

6 In the diagram, A, B, C and D are points on a circle with centre O and diameter AC. It is given that $AC = 7$ cm and $\angle CAB = 0.8$ radians.



(i) Explain briefly why $\angle BOC = 1.6$ radians. [1]

(ii) Find the perimeter of the region bounded by the arc BC and the lines AB and AC. [3]

(iii) Find the area of the major segment $ABCD$. [3]

7 It is given that $\cos A = -\frac{1}{\sqrt{3}}$, where $180° \le A \le 360°$. Find, without the use of calculators, the value of

(i) $\tan A$, [2]

(ii) $\text{cosec } A$, [1]

(iii) $\sin(A + 60°)$, giving your answer in the form $a + b\sqrt{6}$. [3]

8 **(a)** Find the remainder when $4x^3 + 3x^2 - 3x + 14$ is divided by $2x - 1$. [2]

(b) **(i)** Given that $4x^3 + 3x^2 - 3x + 14 = (x + 2)(4x^2 + Bx + C)$ find the value of B and of C.has only one real root. Hence, show that $4x^3 + 3x^2 - 3x + 14$ has only one real root. [3]

(ii) Express $\dfrac{x^2 - 5x + 19}{4x^3 + 3x^2 - 3x + 14}$ as partial fractions. [5]

[Turn over

9 **(a)** Solve the following equations.

(i) $4^{5-3x} = 6^{2x}$, [3]

(ii) $3\ln y + \lg 2y = 4$. [3]

(b) Solve the equation $\sqrt{2z} + \sqrt{z} = 3$, giving your answer in the form $a + b\sqrt{2}$ where a and b are integers. [4]

10 It is given that $\mathrm{f}(x) = 6\cos^2 x - 2\sin^2 x$ for $0 \le x \le \frac{3}{2}\pi$.

Show that f(x) can be written as $\mathrm{f}(x) = 2 + 4\cos 2x$. [2]

Hence,

(i) write down the amplitude and period of f(x). [2]

(ii) solve the equation f(x) = 0 for $0 \le x \le \frac{3}{2}\pi$. [3]

(iii) sketch the curve y = f(x) for $0 \le x \le \frac{3}{2}\pi$, showing clearly all the intercepts. [2]

(iv) given that $\mathrm{g}(x) = \frac{1}{2}\mathrm{f}(x - \pi)$, state the range of g($x$) for the domain $0 \le x \le \frac{3}{2}\pi$. [1]

11 The function f is given by f: $x \mapsto |x-2| + 3$, $x \in \mathbb{R}$.

(i) Find $\mathrm{f}^2(-4)$. [2]

(ii) Sketch the graph of y = f(x), showing clearly the coordinates of the highest/lowest point and the intercept(s). [2]

(iii) State the range of f. [1]

(iv) Write down the range of values of c for which the equation $|x-2| + 3 = x + c$ has exactly one solution. [1]

The function g is given by g: $x \mapsto |x-2| + 3$, $x \le k$.

(v) Write down the largest value of k for which the function g^{-1} exists. [1]

(vi) Find the function g^{-1} in a similar form and state its domain. [3]

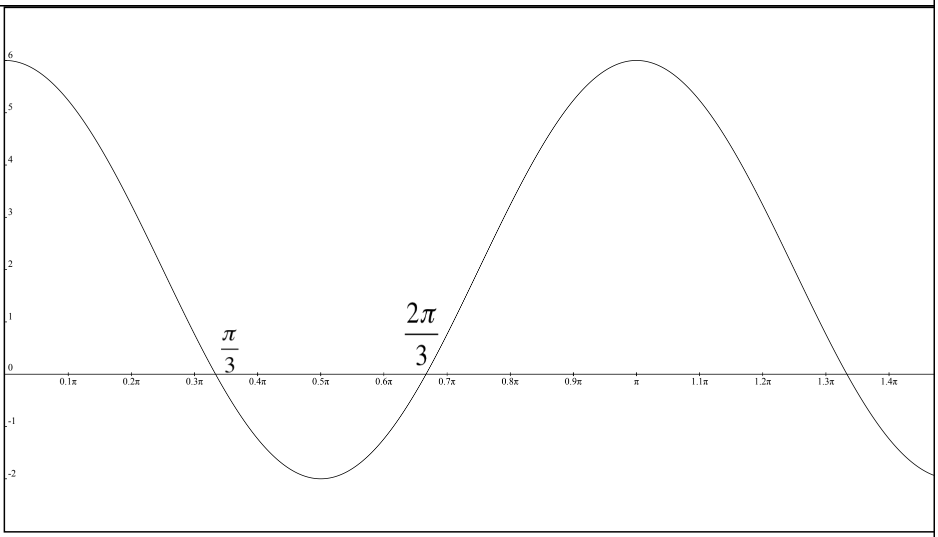
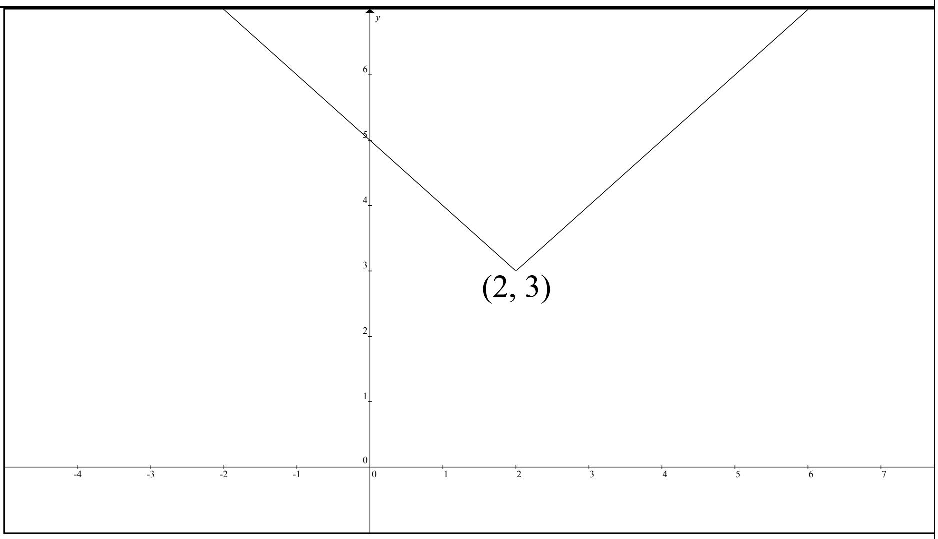
Bonus Question

12 Find an expression for $\mathrm{f}(x)$ for which $4\mathrm{f}(2x) - 5\mathrm{f}\left(\frac{2}{x}\right) = 3x$. [3]

End of Paper

2016 Sec 3 IM2 EOY Answers

1(i)	57°C
1(ii)	38.1 minutes
1(iii)	27°C
2(i)	$(-6,0)$ and (0, 4)
2(ii)	$(-2,0)$ and (0, –6)
2(iii)	(4, 0) and (0, 9)
3(a)(i)	$y=3\left(x-\frac{2}{3}\right)^2+3\frac{2}{3}$
3(a)(ii)(a)	$x=\frac{2}{3}$
3(a)(ii)(b)	$c=3\frac{2}{3}$
3(b)	$-12\le b\le 12$
4	$60x^2-36x+25=0$
5	x = 23.6° or 156.4°
6(i)	$\angle BOC=2(0.8)$ = 1.6 rad (angle at centre is twice angle at circumference)
6(ii)	17.5 cm
6(iii)	35.2 cm^2
7(i)	$\sqrt{2}$
7(ii)	$-\frac{\sqrt{6}}{2}$
7(iii)	$-\frac{1}{2}-\frac{1}{6}\sqrt{6}$
8(a)(i)	$13\frac{3}{4}$
8(b)(i)	$B=-5$ $C=7$ Since the discriminant is -87 which, $4x^2-5x+7=0$ has no real roots. Thus f(x) = 0 has one real root.
8(b)(ii)	$\frac{x^2-5x+19}{4x^3+3x^2-3x+14}=\frac{1}{x+2}+\frac{6-3x}{4x^2-5x+7}$
9(a)(i)	0.895
9(a)(ii)	2.94
9(b)	$z=27-18\sqrt{2}$

10	$f(x) = (6\cos^2 x - 2\sin^2 x)$ $= 6\left(\frac{1+\cos 2x}{2}\right) - 2\left(\frac{1-\cos 2x}{2}\right)$ $= 3 + 3\cos 2x - 1 + \cos 2x$ $= 2 + 4\cos 2x$
10(i)	Amplitude = 4 Period = π
10(ii)	$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$
10(iii)	
10(iv)	$-1 \le g(x) \le 3$
11(i)	10
11(ii)	
11(iii)	$f(x) \ge 3$
11(iv)	$c > 1$
11(v)	The largest value of k is 2 .
11(vi)	$g^{-1}: x \mapsto 5 - x, x \ge 3$.

Bonus 12	$f(x) = -\frac{2}{3}x - \frac{10}{3x}$

[Turn over