JURONG PIONEER JUNIOR COLLEGE
9478 H2 PHYSICS/8867 H1 PHYSICS

QUANTITIES AND MEASUREMENT TUTORIAL SOLUTIONS

Self-Check Questions

S1 *Not in syllabus

base quantity	SI base unit
mass	kg
length	m
time	s
electric current	A
thermodynamic temperature	K
amount of substance	mol
Luminous intensity*	candela

S2 The SI base units are the standard units of measurement defined by the International System of Units for the seven base quantities of what is now known as the International System of Quantities. They are a basic set from which all other SI units can be derived.

Derived units are defined in terms of the base units, which are as stated in S1.

S3 Incorrect constants/signs, e.g. $s = ut - 3at^2$

Extra/missing terms, e.g. $v = u + at + \frac{s}{t}$

S4 Systematic errors result in readings taken being either always consistently larger or smaller than its true or actual value by fixed amount.

Random errors result in a scatter of readings about a mean value, such that the readings have an equal chance of being larger or smaller than its true or actual value.

S5

systematic errors	how to eliminate
forget to tare or "zero" an electronic balance gives mass measurements that always consistently larger or small than the true value by fixed amount	tare or "zero" the electronic balance
measuring radioactivity in the presence of background radiation	before measuring the radioactivity of a specific sample, assess and record the background radiation level at the measurement location, and subtract the background radiation reading from the measurement to obtain the true radioactivity of the sample
clock running consistently fast or slow	regularly synchronise or calibrate the clock with timing instrument with minimal time drift

random errors	how to reduce
non-uniform variations in the diameter of wire	take multiple measurements of the diameter along the length of the wire, and calculate the average value
random parallax error when reading the volume of water in a measuring cylinder	always position eye level at the bottom of the meniscus of the water when taking measurement on the cylinder
radio frequency interference affecting electronic measurement instruments	shield instruments from external interference, use proper grounding, and relocate instruments away from potential sources of interference

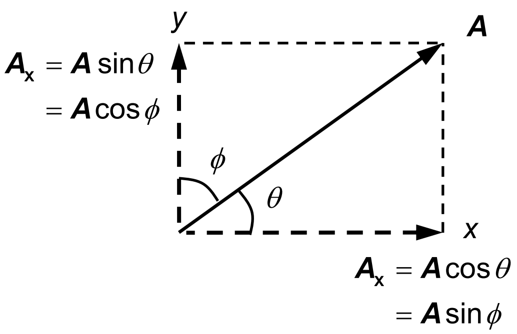
S6 Accuracy is the degree of closeness of the average value of the measurements to the true value. It is affected by systematic error (i.e. correct calibration of instruments).

Precision is the degree of agreement of repeated measurements of the same quantity. It is affected by random error.

S7 A scalar quantity has a magnitude only.

A vector quantity has both a magnitude and a direction.

S8 Orientate the perpendicular x-y axis suitably and place the origin at the start of the 2-dimensional vector. Draw a line from the tip of the vector to each of the axis, such that a right-angle is formed on each of the axis. Apply trigonometric functions (sine or cosine) to determine the respective components.



Discussion Questions

1 $\sigma = \dfrac{ak^4}{h^3c^2}$

$h = \left(\dfrac{ak^4}{\sigma c^2}\right)^{\frac{1}{3}}$

Units of h = Units of $\left(\dfrac{ak^4}{\sigma c^2}\right)^{\frac{1}{3}}$

$$= \left(\frac{\text{J}^4\ \text{K}^{-4}}{\text{W m}^{-2}\ \text{K}^{-4}\ \text{m}^2\ \text{s}^{-2}}\right)^{\frac{1}{3}}$$

$$= \left(\frac{\text{kg}^4\ \text{m}^8\ \text{s}^{-8}}{\text{kg m}^2\ \text{s}^{-3}\ \text{s}^{-2}}\right)^{\frac{1}{3}}$$

$$= \text{kg m}^2\ \text{s}^{-1}$$

Therefore, SI base units of h is $\text{kg m}^2\ \text{s}^{-1}$.

2 Units of Q $= \dfrac{\text{m}^3\ (\text{kg m s}^{-2}\text{m}^{-2})}{\text{m}}\left(\dfrac{\text{kg mol}^{-1}}{\text{J K}^{-1}\ \text{mol}^{-1}\ \text{K}}\right)^{\frac{1}{2}}$

$$= \frac{\text{m}^3\ (\text{kg m}^{-1}\ \text{s}^{-2})}{\text{m}}\left(\frac{\text{kg}}{(\text{kg m s}^{-2}\ \text{m})}\right)^{\frac{1}{2}}$$

SI base units of $\text{Q} = \text{kg s}^{-1}$

3 SI base units of F = kg m s^{-2}
SI base units of $kA\rho v^x$ = $(\text{m}^2)(\text{kg m}^{-3})(\text{m s}^{-1})^x$
= $\text{kg m}^{-1+x}\ \text{s}^{-x}$

Comparing the power for s,
$x = 2$

4 **(a)** $756 + 37.2 + 0.83 + 2.5$
$= 796.53$
$= 797$ (follow smallest d.p.)

(b) $1.6523 - 0.015$
$= 1.6373$
$= 1.637$ (follow smallest d.p.)

(c) 3.2×3.563
$= 11.4016$
$= 11$ (follow smallest s.f.)

(d) $\dfrac{5.6}{\pi}$

$= 1.7825...$

$= 1.8$ (follow smallest s.f.)

5 $\Delta v = \dfrac{3}{100} \times 326.15 = 9.7845$

$\Delta v = 10 \text{ m s}^{-1}$ (1 s.f.)

Therefore, $v = (330 \pm 10) \text{ m s}^{-1}$

6 $\rho = \dfrac{M}{V}$

$$\rho = \frac{M}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + 3\frac{\Delta d}{d}$$

percentage error in ρ, $\dfrac{\Delta\rho}{\rho} \times 100\%$ = 1% + 3(3%)

= 10%

7 $\rho = \dfrac{\pi r^2 R}{\ell} = \dfrac{\pi\left(3.15 \times 10^{-5}\right)^2 \times 939}{2.71} = 1.0801 \times 10^{-6} \ \Omega \text{ m}$

$$\frac{\Delta\rho}{\rho} = 2\frac{\Delta r}{r} + \frac{\Delta R}{R} + \frac{\Delta\ell}{\ell} = 2\left(\frac{0.05}{3.15}\right) + \frac{3}{939} + \frac{0.01}{2.71} = 0.0386$$

$\Delta\rho = (0.0386)\rho = 4 \times 10^{-8} = 0.04 \times 10^{-6} \ \Omega \text{ m}$

$\therefore \ \rho = (1.08 \pm 0.04) \times 10^{-6} \ \Omega \text{ m}$

8 $r = \dfrac{1}{8}(Y - X)$

$= \dfrac{1}{8}(5.0 - 1.0)$

$= 0.50 \text{ cm}$

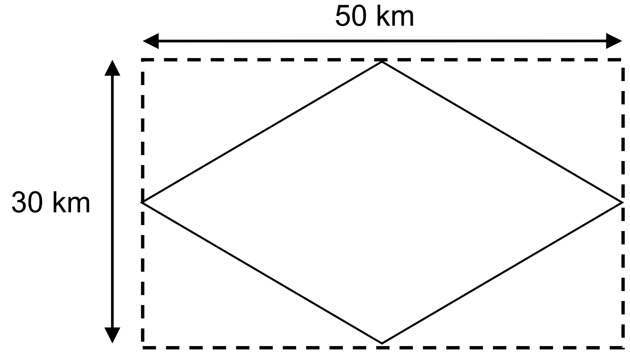
$\Delta r = \dfrac{1}{8}(\Delta Y + \Delta X)$

$= \dfrac{1}{8}(0.1 + 0.1)$

$= 0.025 = 0.03 \text{ cm}$

Therefore, $r = (0.50 \pm 0.03)$ cm.

9 **(a)** Singapore has the shape of a rhombus, approximately 50 km east to west and 30 km north to south, as shown in diagram.



Estimate area

$= \frac{1}{2} \times 50 \times 30$

$= 750 \text{ km}^2$

(b) Consider Downtown line in Singapore MRT in which the train consists of 3 cabins, each of which has an approximate length of 20 m.

Starting from rest at the station,
$u = 0$

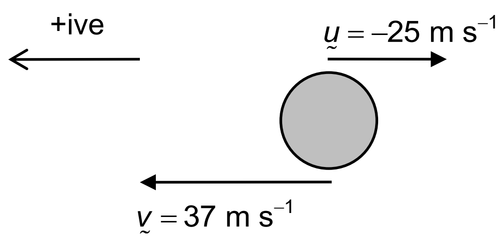
It takes about $t = 10$ s for the whole train to leave the station platform,
$s = 20 \times 3 = 60$ m

Using $s = ut + \frac{1}{2}at^2$,

$60 = \frac{1}{2}a(10)^2$

$\therefore \; a = 1.2 \text{ m s}^{-2}$

10 Taking leftwards as positive,



Change in velocity of tennis ball

$\Delta\underset{\sim}{v} = \underset{\sim}{v} - \underset{\sim}{u}$

$= \underset{\sim}{v} + (-\underset{\sim}{u})$

$= 37 - (-25)$

$= 62 \text{ m s}^{-1}$

Hence, change of velocity is 62 m s^{-1} to the left

11 $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

$\Delta\vec{v} = \vec{v}_f + \left(-\vec{v}_i\right)$

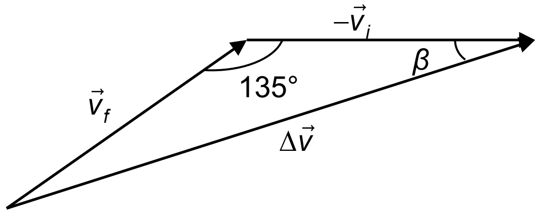
$\Delta v^2 = v_f{}^2 + v_i{}^2 - 2v_f v_i \cos 135°$

$\Delta v^2 = 160^2 + 120^2 - \left(2 \times 160 \times 120 \cos 135°\right)$

$\Delta v = 260 \text{ m s}^{-1}$

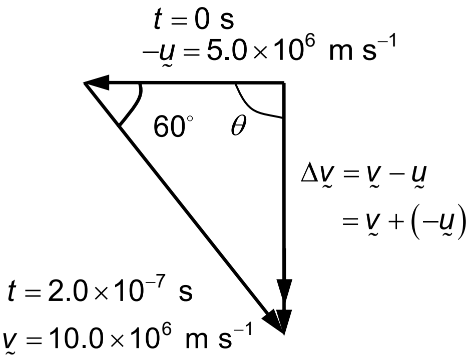
$\dfrac{\sin 135°}{\Delta v} = \dfrac{\sin\beta}{160}$

$\beta = 26°$



The change in velocity is 260 m s^{-1} at an angle of 26° North of East, as shown in the diagram.

12 **(a)**



Change in velocity,

$|\Delta\underset{\sim}{v}| = \sqrt{(5.0\times10^6)^2 + (10\times10^6) - 2(5.0\times10^6)(10\times10^6)\cos 60^0}$

$= 8.6603 \times 10^6$

$\approx 8.66 \times 10^6 \text{ m s}^{-1}$

(b) Acceleration is rate of change of velocity.

$a = \dfrac{\Delta v}{\Delta t} = \dfrac{8.6603 \times 10^6}{2.0 \times 10^{-7}}$

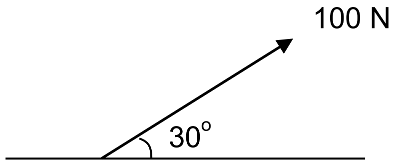
$= 4.3302 \times 10^{13}$

$\approx 4.33 \times 10^{13} \text{ m s}^{-2}$

$\dfrac{\sin\theta}{\sin 60^0} = \dfrac{10 \times 10^6}{8.6603 \times 10^6} \Rightarrow \theta = 90^0$

The average acceleration is $4.33 \times 10^{13} \text{ m s}^{-2}$ in the downwards direction, as shown in the diagram.

13



Effective force along the ground

$= 100\cos 30^\circ$

$= 87$ N

Effective force normal to the ground

$= 100\sin 30^\circ$

$= 50$ N

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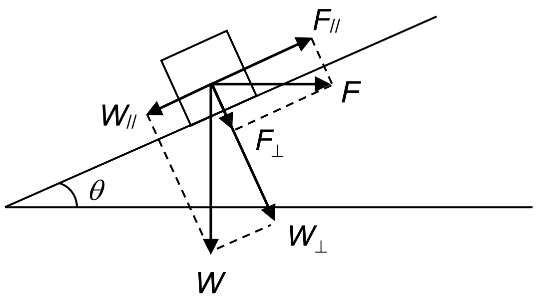

Resolving F into its components,

$F_{//} = F\cos\theta$

$F_{\perp} = F\sin\theta$

Resolving W into its components,

$W_{//} = W\sin\theta$

$W_{\perp} = W\cos\theta$

15 **(a)** Consider the mercury in the bore as having the same shape as a cylinder.

Density of mercury, $\rho = \dfrac{M}{V} = \dfrac{M}{\pi\left(\dfrac{d}{2}\right)^2 L}$

Diameter of bore, $d^2 = \dfrac{4M}{\pi\rho L} = \dfrac{4(6.884 - 6.610)}{\pi(13.6)(10.926 - 4.758)} = 4.16\times10^{-3}\ \text{cm}^2$

$d = 0.06449$ cm

$$2\frac{\Delta d}{d} = \frac{\Delta M}{M} + \frac{\Delta\rho}{\rho} + \frac{\Delta L}{L}$$

$$2\frac{\Delta d}{d} = \frac{0.004}{6.884 - 6.610} + 0.001 + \frac{0.004}{10.926 - 4.758} = 0.0162$$

$$\frac{\Delta d}{d} = 0.00812$$

$\Delta d = 0.00812 \times 0.06449 = 0.0005$ cm

$d = (0.0645 \pm 0.0005)$ cm

(b) The uncertainty of d (0.0005 cm) is much smaller as compared to the uncertainty if d is measured directly (0.004 cm).