2026 J2 H2 FM
Linear Algebra (Set 3 – Solutions)

1 In this question, V denotes the set of vectors of the form $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$, where a, b, c and d are real numbers. You may assume that V forms a linear space under the usual operations of vector addition and multiplication by a scalar.

(a) Show that the subset of V for which $a+b-d=c^2$ does not form a linear space. [1]

(b) Determine whether or not the subset of V for which both $a+4b=2c+d$ and $a-2b=c+d$ forms a linear space. [2]

(c) Determine whether or not the subset of V for which $a+4b=2c+d$ or $a-2b=c+d$ forms a linear space. [2]

(d) State the dimension of the subspace of V such that $3a+2b-c+d=0$ and provide a basis for this linear space. [3]

	Solution: 2020/HCI Promo/Q8	
(a)	Let V_2 be the given subset of V. $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \in V_2$ as $1+1-1=1^2$. However, $2\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \notin V_2$ as $2+2-2=2 \neq 2^2$. Since V_2 is not closed under scalar multiplication, V_2 is not a linear space.	
(b)	Let V_3 be the given subset of V. Clearly, $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \in V_3$ as $0+4(0)=2(0)+0$ and $0-2(0)=0+0$. Consider $\begin{pmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{pmatrix}, \begin{pmatrix} a_2 \\ b_2 \\ c_2 \\ d_2 \end{pmatrix} \in V_3$, and $k_1, k_2 \in \mathbb{R}$ Since $\begin{aligned}(k_1a_1+k_2a_2)+4(k_1b_1+k_2b_2) &= (k_1a_1+4k_1b_1)+(k_2a_2+4k_2b_2) \\ &= 2(k_1c_1+k_1d_1)+2(k_2c_2+k_2d_2) \\ &= 2(k_1c_1+k_2c_2)+(k_1d_1+k_2d_2)\end{aligned}$ and $\begin{aligned}(k_1a_1+k_2a_2)-2(k_1b_1+k_2b_2) &= (k_1a_1-2k_1b_1)+(k_2a_2-2k_2b_2) \\ &= (k_1c_1+k_1d_1)+(k_2c_2+k_2d_2) \\ &= (k_1c_1+k_2c_2)+(k_1d_1+k_2d_2)\end{aligned}$	Intersection of subspaces are subspaces.

	Thus $k_1\begin{pmatrix}a_1\\b_1\\c_1\\d_1\end{pmatrix}+k_2\begin{pmatrix}a_2\\b_2\\c_2\\d_2\end{pmatrix}\in V_3$ $\therefore$ V_3 is closed under addition and scalar multiplication	
(c)	Consider $\begin{pmatrix}1\\1\\2\\1\end{pmatrix}$ which satisfies $a+4b=2c+d$ but not $a-2b=c+d$. Consider $\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$ which satisfies $a-2b=c+d$ but not $a+4b=2c+d$. However $\begin{pmatrix}1\\1\\2\\1\end{pmatrix}+\begin{pmatrix}1\\0\\1\\0\end{pmatrix}=\begin{pmatrix}2\\1\\3\\1\end{pmatrix}$ does not satisfy both $a+4b=2c+d$ and $a-2b=c+d$ as $2+4(1)\neq 2(3)+1$ and $2-2(1)\neq(3)+1$. Thus subset is not a linear space as it is not closed under addition.	A union of subspaces may not be a subspace.
(d)	$3a+2b-c+d=0\Rightarrow c=3a+2b+d$ $\begin{pmatrix}a\\b\\c\\d\end{pmatrix}=a\begin{pmatrix}1\\0\\3\\0\end{pmatrix}+b\begin{pmatrix}0\\1\\2\\0\end{pmatrix}+d\begin{pmatrix}0\\0\\1\\1\end{pmatrix}$ $\therefore \dim(V_1)=3$ Basis $=\left\{\begin{pmatrix}1\\0\\3\\0\end{pmatrix},\begin{pmatrix}0\\1\\2\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\\1\end{pmatrix}\right\}$ since the 3 vectors are clearly linearly independent.	

2 2018/NJC/Prelim

An *orthogonal* matrix $\mathbf{Q}$ is one such that

$$\mathbf{Q}^{\mathrm{T}}\mathbf{Q}=\mathbf{I},$$

where $\mathbf{I}$ is the identity matrix.

(a) Find the possible determinants of an orthogonal matrix. [3]

(a)	$\mathbf{Q}^{\mathrm{T}}\mathbf{Q}=\mathbf{I}$ $\det\left(\mathbf{Q}^{\mathrm{T}}\mathbf{Q}\right)=\det(\mathbf{I})$ $\det\left(\mathbf{Q}^{\mathrm{T}}\right)\det(\mathbf{Q})=\det(\mathbf{I})$ $\det(\mathbf{Q})\det(\mathbf{Q})=\det(\mathbf{I})$ $\left(\det(\mathbf{Q})\right)^2=1$ $\det(\mathbf{Q})=\pm 1$

For any vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$, it can be shown that $\mathbf{u} \cdot \mathbf{v} = \mathbf{u}^{\mathrm{T}}\mathbf{v}$.

(b) Suppose $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ are the column vectors (in that order) of a 3×3 orthogonal matrix $\mathbf{Q}$ with real entries. Use the above result to show that $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ are unit vectors that are pairwise perpendicular. [4]

(b)

$$\mathbf{Q}^{\mathrm{T}}\mathbf{Q} = \begin{pmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \\ \mathbf{v}_3^T \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{pmatrix} = \begin{pmatrix} \mathbf{v}_1^T\mathbf{v}_1 & \mathbf{v}_1^T\mathbf{v}_2 & \mathbf{v}_1^T\mathbf{v}_3 \\ \mathbf{v}_2^T\mathbf{v}_1 & \mathbf{v}_2^T\mathbf{v}_2 & \mathbf{v}_2^T\mathbf{v}_3 \\ \mathbf{v}_3^T\mathbf{v}_1 & \mathbf{v}_3^T\mathbf{v}_2 & \mathbf{v}_3^T\mathbf{v}_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Consider $\mathbf{v}_i^T\mathbf{v}_j$, for $i, j = 1, 2, 3$.

If $i = j$,	If $i \neq j$,
$\mathbf{v}_i^T\mathbf{v}_i = 1$ $\mathbf{v}_i \cdot \mathbf{v}_i = 1$ $\lvert\mathbf{v}_i\rvert^2 = 1$ $\lvert\mathbf{v}_i\rvert = 1$	$\mathbf{v}_i^T\mathbf{v}_j = 0$ $\mathbf{v}_i \cdot \mathbf{v}_j = 0$ $\mathbf{v}_i \perp \mathbf{v}_j$

Alternatively:

Note that $\mathbf{Qe}_i = \mathbf{v}_i$ for $i = 1, 2, 3$, where $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ are the standard basis vectors for $\mathbb{R}^3$.

For $i, j = 1, 2, 3$, such that $i \neq j$,

$$\begin{aligned} \mathbf{v}_i \cdot \mathbf{v}_j &= \mathbf{v}_i^{\mathrm{T}}\mathbf{v}_j \\ &= (\mathbf{Qe}_i)^{\mathrm{T}}(\mathbf{Qe}_j) \\ &= \mathbf{e}_i^{\mathrm{T}}\mathbf{Q}^{\mathrm{T}}\mathbf{Qe}_j \\ &= \mathbf{e}_i^{\mathrm{T}}\mathbf{e}_j \text{ (since } \mathbf{Q}^{\mathrm{T}}\mathbf{Q} = \mathbf{I}) \\ &= \mathbf{e}_i \cdot \mathbf{e}_j = 0 \end{aligned}$$

For $i, j = 1, 2, 3$, such that $i = j$,

$$\begin{aligned} \lvert\mathbf{v}_i\rvert^2 = \mathbf{v}_i \cdot \mathbf{v}_i &= \mathbf{v}_i^{\mathrm{T}}\mathbf{v}_i \\ &= (\mathbf{Qe}_i)^{\mathrm{T}}(\mathbf{Qe}_i) \\ &= \mathbf{e}_i^{\mathrm{T}}\mathbf{Q}^{\mathrm{T}}\mathbf{Qe}_i \\ &= \mathbf{e}_i^{\mathrm{T}}\mathbf{e}_i \text{ (since } \mathbf{Q}^{\mathrm{T}}\mathbf{Q} = \mathbf{I}) \\ &= 1 \end{aligned}$$

Suppose $\mathbf{v}_1 = \frac{1}{3}\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$, $\mathbf{v}_2 = \frac{1}{3}\begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{v}_3 = \frac{1}{3}\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$, and the linear transformation T is defined as follows:

$$\mathrm{T}(\mathbf{u}) = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} \text{ if and only if } \mathbf{u} = \lambda_1\mathbf{v}_1 + \lambda_2\mathbf{v}_2 + \lambda_3\mathbf{v}_3,$$

where λ_1, λ_2 and λ_3 are real constants.

(c) The column vectors of an orthogonal matrix are called *orthonormal* vectors. Given that $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ as defined above are orthonormal vectors, find the matrix $\mathbf{A}$ representing the linear transformation T. [2]

(c)

$$\mathbf{u} = \lambda_1\mathbf{v}_1 + \lambda_2\mathbf{v}_2 + \lambda_3\mathbf{v}_3 = \begin{pmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{pmatrix}\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \mathbf{Q}\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$$

	Thus $\begin{pmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{pmatrix} = \mathbf{Q}^{-1}\mathbf{u} = \mathbf{Q}^{\mathrm{T}}\mathbf{u} = \frac{1}{3}\begin{pmatrix}1 & 2 & 2\\ -2 & -1 & 2\\ 2 & -2 & 1\end{pmatrix}\mathbf{u}$ i.e. $\mathrm{T}(\mathbf{u}) = \frac{1}{3}\begin{pmatrix}1 & 2 & 2\\ -2 & -1 & 2\\ 2 & -2 & 1\end{pmatrix}\mathbf{u}$. Hence, $\mathbf{A} = \frac{1}{3}\begin{pmatrix}1 & 2 & 2\\ -2 & -1 & 2\\ 2 & -2 & 1\end{pmatrix}$ Alternatively: Since $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are **orthonormal**, if $$\mathbf{u} = \lambda_1\mathbf{v}_1 + \lambda_2\mathbf{v}_2 + \lambda_3\mathbf{v}_3,$$ taking the dot product with each $\mathbf{v}_i$ gives $$\lambda_1 = \mathbf{v}_1 \cdot \mathbf{u}, \qquad \lambda_2 = \mathbf{v}_2 \cdot \mathbf{u}, \qquad \lambda_3 = \mathbf{v}_3 \cdot \mathbf{u}.$$ Using $\mathbf{u} \cdot \mathbf{v} = \mathbf{u}^T\mathbf{v}$, $$T(\mathbf{u}) = \begin{pmatrix}\mathbf{v}_1^T\\ \mathbf{v}_2^T\\ \mathbf{v}_3^T\end{pmatrix}\mathbf{u}.$$ Hence $$A = \begin{pmatrix}\mathbf{v}_1^T\\ \mathbf{v}_2^T\\ \mathbf{v}_3^T\end{pmatrix}.$$ Substituting the given vectors, $$\boxed{A = \frac{1}{3}\begin{pmatrix}1 & 2 & 2\\ -2 & -1 & 2\\ 2 & -2 & 1\end{pmatrix}}$$

(d) Hence find the subspace of $\mathbb{R}^3$ whose range space under T is given by $\left\{\begin{pmatrix}p\\ q\\ p+q\end{pmatrix} : p, q \in \mathbb{R}\right\}$. [3]

(d)	Let $\mathrm{T}(\mathbf{u}) = \begin{pmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{pmatrix} = \begin{pmatrix}p\\ q\\ p+q\end{pmatrix}$	

	$\mathbf{u} = \lambda_1\mathbf{v}_1 + \lambda_2\mathbf{v}_2 + \lambda_3\mathbf{v}_3$ $= p\mathbf{v}_1 + q\mathbf{v}_2 + (p+q)\mathbf{v}_3$ $= p(\mathbf{v}_1 + \mathbf{v}_3) + q(\mathbf{v}_2 + \mathbf{v}_3)$ $= p\left(\frac{1}{3}\begin{pmatrix}1\\2\\2\end{pmatrix} + \frac{1}{3}\begin{pmatrix}2\\-2\\1\end{pmatrix}\right) + q\left(\frac{1}{3}\begin{pmatrix}-2\\-1\\2\end{pmatrix} + \frac{1}{3}\begin{pmatrix}2\\-2\\1\end{pmatrix}\right)$ $= p\begin{pmatrix}1\\0\\1\end{pmatrix} + q\begin{pmatrix}0\\-1\\1\end{pmatrix}$ Required subspace $= \left\{\begin{pmatrix}p\\-q\\p+q\end{pmatrix} : p, q \in \mathbb{R}\right\}$	

3 Let V be the vector space of all real polynomials of degree at most n, and let $D : V \to V$ be the linear transformation representing differentiation, that is, $D(p) = \frac{\mathrm{d}p}{\mathrm{d}x}$.

(a) Explain why D is a linear transformation. [1]

(b) State bases for the kernel and range of D. [2]

(c) Give a reason why the set of real polynomials of degree n is not a vector space. [1]

	ACJC JC2 Prelim 9649/2019/02/Q1	
(a)	For any two polynomials p and q in V and any two real numbers α and β, $D(\alpha p + \beta q) = \frac{\mathrm{d}}{\mathrm{d}x}(\alpha p + \beta q)$ $= \alpha\frac{\mathrm{d}p}{\mathrm{d}x} + \beta\frac{\mathrm{d}q}{\mathrm{d}x}$ $= \alpha D(p) + \beta D(q)$ Hence D is a linear transformation.	
(b)	Need $D(p) = \frac{\mathrm{d}p}{\mathrm{d}x} = 0$. Thus $p = k \in \mathbb{R}$. A basis for the kernel of D is $\{1\}$. Let $p = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ $\frac{\mathrm{d}p}{\mathrm{d}x} = na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1$ A basis for the range of D is $\{x^{n-1}, x^{n-2}, ..., x, 1\}$	
(c)	The set of real polynomials of degree n is not a vector space as it does not contain a zero element. (It must have ax^n term, $a \neq 0$). OR If p and q are polynomials with the same term ax^n as the term in the highest power, then $p-q$ would be a polynomial of degree $n-1$ and does not belong to the set. Hence it is not closed under addition.	

4	**Do not use a calculator in answering this question.** The linear transformation T, from $\mathbb{R}^2 \to \mathbb{R}^2$, defined by $\mathrm{T}:\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}$ where $\mathbf{A}=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where a,b,c and d are constants. It is given that the transformation under T has a line L_1 of invariant points. **(a)** Explain why $\mathbf{A}$ has an eigenvalue equal to 1. [1] **(b)** Hence show that $ad+1=a+d+bc$. [2] It is given that $\mathbf{A}=\frac{1}{5}\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$. **(c)** Find an eigenvector $\mathbf{e_1}$ that corresponds to eigenvalue 1 of $\mathbf{A}$ and the Cartesian equation of L_1. [2] It is given that the transformation under T has an invariant line L_2. **(d)** By finding the other eigenvalue and its corresponding eigenvector $\mathbf{e_2}$ of $\mathbf{A}$, give the Cartesian equation of L_2. [3] **(e)** Show that L_1 and L_2 are perpendicular. [1] **(f)** By considering your answer to part **(d)** and **(e),** and the image of $\alpha\mathbf{e_1}+\beta\mathbf{e_2}$ when it is transformed under T, where $\alpha,\beta\in\mathbb{R}$, describe the geometrical transformation T explaining your answer. [3]

Solution: [2022/MYE/RI/JC2/6]

(a)	Let (x, y) be a point on the line of invariant points. Thus $\mathrm{T}\begin{pmatrix} x \\ y \end{pmatrix}=\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}=1\begin{pmatrix} x \\ y \end{pmatrix}$, so $\mathbf{A}$ has eigenvalue of 1.	
(b)	Since $\lambda=1$ is an eigenvalue: $\det(\mathbf{A}-1\mathbf{I})=0$ $\det\begin{pmatrix} a-1 & b \\ c & d-1 \end{pmatrix}=0$ $(a-1)(d-1)=bc$ $ad+1=a+d+bc$ (shown)	
(c)	$\begin{pmatrix} \frac{3}{5}-1 & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5}-1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ From 1st row, the equation of L_1 is $x-2y=0$ or $y=\frac{1}{2}x$. $\Rightarrow\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ \frac{x}{2} \end{pmatrix}=\frac{1}{2}x\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ Hence $\mathbf{e_1}=\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.	

(d)

$$\det\begin{pmatrix} \frac{3}{5}-\lambda & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5}-\lambda \end{pmatrix}=0$$

$$\left(\frac{3}{5}-\lambda\right)\left(-\frac{3}{5}-\lambda\right)-\frac{16}{5}=0$$

$$\lambda = 1 \text{ or } -1$$

The other eigenvalue is -1.

$$\begin{pmatrix} \frac{8}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{2}{5} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The equation of L_2 is $2x+y=0$ or $y=-2x$.

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix}=x\begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad \text{Hence } \mathbf{e_2}=\begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

(e)

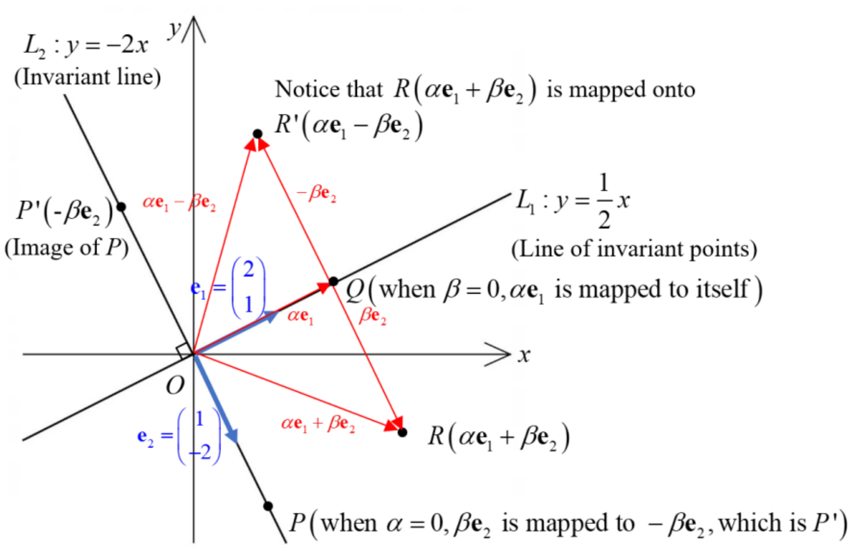
$$L_1:\ y=\frac{1}{2}x \qquad L_2: y=-2x$$

Since product of gradients $=\left(\frac{1}{2}\right)(-2)=-1$, L_1 and L_2 are perpendicular.

(f)

Image of $\alpha\mathbf{e_1}+\beta\mathbf{e_2}$ under T is $\mathbf{A}(\alpha\mathbf{e_1}+\beta\mathbf{e_2})=\alpha\mathbf{Ae_1}+\beta\mathbf{Ae_2}=\alpha\mathbf{e_1}-\beta\mathbf{e_2}$

Since $\mathbf{e_1}$ and $\mathbf{e_2}$ spans $\mathbb{R}^2$ and are linearly independent, $\mathbf{e_1}$ and $\mathbf{e_2}$ are perpendicular, the transformation T is a reflection about L_1.



5 **(a)** State with clear justifications if the following statements are true or false.

(i) $(A+B)^2 = A^2 + 2AB + B^2$, where A and B are square matrices of same order. [1]

(ii) If A is a square matrix, then $A^2 = 0 \Rightarrow A = 0$. [1]

(iii) If A is a singular square matrix, then there exists a square matrix B such that $AB = I$, where I is the identity matrix. [1]

(b) Let $\mathbf{A}$ be a nonzero square matrix such that $\mathbf{A}^2 = \mathbf{A}$.

(i) Determine all possible values of $\det(\mathbf{A})$. [2]

(ii) Determine if the following statements are true. Justify your answer.

(A) $\mathbf{I} - \mathbf{A}$ is always invertible. [1]

(B) $\mathbf{I} + \mathbf{A}$ is always invertible. [2]

(c) Let $\mathbf{B} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$. Given that $\mathbf{B}$ is the inverse of a matrix $\mathbf{C}$, and $\mathbf{D}$ is the matrix obtained from $\mathbf{C}$ by adding to the second row of $\mathbf{C}$ twice the first row of $\mathbf{C}$, find $\mathbf{D}^{-1}$ in a similar form to $\mathbf{B}$. [3]

	HCI Promo 9649/2021/Q1 & NJC JC2 Prelim 9649/2019/02/Q2	
(a)(i)	False. Take $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $(A+B)^2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $A^2 + 2AB + B^2 = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$	
(ii)	False. Take $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $A^2 = 0$ but $A \neq 0$	
(iii)	False. Assume it is true, then $\det(AB) = \det I$ $\det I = \det(AB) = \det A \times \det B = 0 \times \det B = 0$ Contradiction. Hence false.	
(b)(i)	$\det(\mathbf{A}^2) = \det(\mathbf{A})$ $[\det(\mathbf{A})]^2 = \det(\mathbf{A})$ $\det(\mathbf{A})[\det(\mathbf{A}) - 1] = 0$ Hence, $\det(\mathbf{A}) = 0$ or 1.	

(b)(ii) (A)	No. Counterexample: $\mathbf{A}=\mathbf{I}$. Then $\mathbf{I}-\mathbf{A}=\mathbf{0}$.	
(B)	True. Suppose not, then there exists a nonzero $\mathbf{x}$ such that $$\begin{aligned}(\mathbf{I}+\mathbf{A})\mathbf{x}&=\mathbf{0}\\ \mathbf{x}+\mathbf{A}\mathbf{x}&=\mathbf{0} \qquad ---(1)\\ \mathbf{A}(\mathbf{x}+\mathbf{A}\mathbf{x})&=\mathbf{A}\mathbf{0}\\ \mathbf{A}\mathbf{x}+\mathbf{A}^2\mathbf{x}&=\mathbf{0}\\ \mathbf{A}\mathbf{x}+\mathbf{A}\mathbf{x}&=\mathbf{0}\\ \mathbf{A}\mathbf{x}&=\mathbf{0}\end{aligned}$$ Sub $\mathbf{A}\mathbf{x}=\mathbf{0}$ into (1), we get $\mathbf{x}=\mathbf{0}$, a contradiction. Alternatively, Consider $\frac{1}{2}(2\mathbf{I}-\mathbf{A})$. Then we have $$\begin{aligned}\frac{1}{2}(2\mathbf{I}-\mathbf{A})(\mathbf{I}+\mathbf{A})&=\frac{1}{2}\left(2\mathbf{I}^2+2\mathbf{A}-\mathbf{A}-\mathbf{A}^2\right)\\ &=\frac{1}{2}(2\mathbf{I}+\mathbf{A}-\mathbf{A})\\ &=\frac{1}{2}(2\mathbf{I})\\ &=\mathbf{I}\end{aligned}$$	
(c)	$\mathbf{C}=\mathbf{B}^{-1}$ Let $\mathbf{E}=\begin{pmatrix}1&0&0\\2&1&0\\0&0&1\end{pmatrix}$, which defines the row operation. $$\begin{aligned}\mathbf{D}&=\mathbf{E}\mathbf{C}\\ &=\mathbf{E}\mathbf{B}^{-1}\\ \mathbf{D}^{-1}&=\mathbf{B}\mathbf{E}^{-1}\\ &=\begin{pmatrix}a&b&c\\d&e&f\\g&h&i\end{pmatrix}\begin{pmatrix}1&0&0\\-2&1&0\\0&0&1\end{pmatrix}\\ &=\begin{pmatrix}a-2b&b&c\\d-2e&e&f\\g-2h&h&i\end{pmatrix}\end{aligned}$$	