Answer **all** the questions.

1 Differentiate each function with respect to x.

(a) $y = 4x^3 - 3x^2 + x - 1$ [1]

$\frac{dy}{dx} = 12x^2 - 6x + 1$ — (B1)

(b) $y = \frac{(2x^2-1)^2}{\sqrt{2x^2-1}}$, leaving your answer in the form $kx\sqrt{2x^2-1}$, where k is a constant. [2]

$y = (2x^2-1)^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{2}(2x^2-1)^{\frac{1}{2}}(4x)$ — (M1)

$= 6x\sqrt{2x^2-1}$ — (A1)

If students apply quotient rule

$u = (2x^2-1)^2$ $\quad v = (2x^2-1)^{\frac{1}{2}}$

$\frac{du}{dx} = 2(2x^2-1)(4x)$ $\quad \frac{dv}{dx} = \frac{1}{2}(2x^2-1)^{-\frac{1}{2}}(4x)$

$= 8x(2x^2-1)$ $\quad = \frac{2x}{\sqrt{2x^2-1}}$

$\frac{dy}{dx} = \frac{\left[8x(2x^2-1)\cdot\sqrt{2x^2-1} - \frac{2x}{\sqrt{2x^2-1}}(2x^2-1)^2\right]}{(2x^2-1)}$ — (M1)

$= \frac{8x(2x^2-1)^{\frac{3}{2}} - 2x(2x^2-1)^{\frac{3}{2}}}{2x^2-1}$

$= \frac{(2x^2-1)^{\frac{3}{2}}(8x-2x)}{2x^2-1}$

$= 6x\sqrt{2x^2-1}$ — (A1)

2 A curve has the equation $y = (px-5)^2$. The gradient of the curve at $(2, q)$ is -4. Find the value(s) of p and q given that $p > q$. [4]

$\frac{dy}{dx} = 2(px-5)(p)$

$= 2p(px-5)$

at $x = 2$, $\frac{dy}{dx} = -4$.

$\therefore 2p(2p-5) = -4$ — (M1)

$4p^2 - 10p + 4 = 0$

$2p^2 - 5p + 2 = 0$

$(p-2)(2p-1) = 0$

$\therefore p = 2$ or $p = \frac{1}{2}$ — (M1)

at $p = 2$,

$y = (2x-5)^2$

Sub $(2, q)$ into $y = (2x-5)^2$

$q = (4-5)^2$

$= 1$.

$\therefore p = 2, q = 1$

at $p = \frac{1}{2}$,

$y = (\frac{1}{2}x - 5)^2$

Sub $(2, q)$ into $y = (\frac{1}{2}x - 5)^2$

$\therefore q = (-4)^2 = 16$

$\therefore p = \frac{1}{2}, q = 16$ [rejected].

A1 for either

$\therefore \underbrace{p = 2, q = 1}$ #

Award A1 only if both sets of p & q values are shown.

3 **(a)** Find the first three terms, in ascending powers of x, the expansion of $(2-px)^8$. [2]

$(2-px)^8 = 2^8 + \binom{8}{1}(2)^7(-px) + \binom{8}{2}(2)^6(-px)^2 + \dots$ — (M1)

$= 256 - 1024px + 1792p^2x^2 + \dots$ — (A1)

(b) When $(1-2x)^2(2-px)^8$ is expanded as far as the term in x^2, the result is $q+rx^2$. Find the value of p, q and r. [4]

$(1-2x)^2(256 - 1024px + 1792p^2x^2 + \dots)$

$= (1-4x+4x^2)(256 - 1024px + 1792p^2x^2 + \dots)$

$= 256 - 1024px + 1792p^2x^2 - 1024x + 4096px^2 + 1024x^2 + \dots$

$= 256 + (-1024p - 1024)x + (1792p^2 + 4096p + 1024)x^2 + \dots$

$\therefore -1024p - 1024 = 0$

$-1024p = 1024$

$p = -1$. — (A1)

$r = 1792p^2 + 4096p + 1024$

$= 1792(-1)^2 + 4096(-1) + 1024$

$= -1280$ — (A1)

M1 for either (Comparing coefficient for x or x^2)

$q = 256$. — (B1)

4 **(a)** Write down the general term in the binomial expansion of $\left(3x+\frac{1}{x^3}\right)^{14}$. [1]

$$T_{r+1} = \binom{14}{r}(3x)^{14-r}\left(\frac{1}{x^3}\right)^r$$

$$= \binom{14}{r} \cdot 3^{14-r} \cdot x^{14-r} \cdot x^{-3r}$$

$$= \binom{14}{r} 3^{14-r} x^{14-4r}$$

B1 for either
(No B1 if students apply formula correctly, but simplify wrongly)

(b) Write down the power of x in this general term. [1]

$14-4r$ — (B1)

(c) Explain why the term in $\frac{1}{x^4}$ does not exist in the expansion of $\left(3x+\frac{1}{x^3}\right)^{14}$. [2]

If $\frac{1}{x^4}$ exists, $14-4r=-4$

$$4r = 18$$

$$r = \frac{9}{2}$$

(M1)

Since $r=\frac{9}{2}$ is not an integer, $\frac{1}{x^4}$ term does not exist. (A1)

(d) Find the middle term in the expansion of $\left(3x+\frac{1}{x^3}\right)^{14}$. [2]

middle term: $\frac{15+1}{2} = 8$.

$T_8 = T_{7+1}$

$= \binom{14}{7} 3^{14-7} x^{14-4(7)}$ — (M1)

$= 7505784\, x^{-14}$

$= \frac{7505784}{x^{14}}$ } A1 for either.

5 The equation of a circle, C_1, with centre A, is $x^2+y^2-6x+4y=27$.

(a) Show that coordinates of A is $(3,-2)$ and that radius of C_1 is $\sqrt{40}$ units. [3]

$x^2-6x+y^2+4y=27$

$x^2-6x+3^2+y^2+4y+2^2=27+3^2+2^2$ — (M1)

$(x-3)^2+(y+2)^2=40$. — (M1)

$\therefore$ centre: $(3,-2)$

radius: $\sqrt{40}$ } (A1)

Alternative ②

$2gx=-6x$ $\quad 2fy=4y$

$g=-3$ $\quad f=2$ } (M1)

$\therefore C(-3,2)$

$\sqrt{2^2+(-3)^2-(-27)}$ — (M1)

$=\sqrt{40}$ — (A1)

Alternative ①: let centre be (a,b), radius be r.

$\therefore (x-a)^2+(y-b)^2=r^2$

$x^2-2ax+a^2+y^2-2by+b^2=r^2$

$x^2+y^2-2ax-2by=r^2-a^2-b^2$ — (M1)

M1: $[-2a=-6$ | $-2b=4]$ | $r^2-(-3)^2-2^2=27$

$a=3$ | $b=-2$ | $r^2=40$

$r=\sqrt{40}$

$\therefore$ Centre $(-3,2)$

radius: $\sqrt{40}$ } (A1)

(b) The highest point on circle, C_1, is P.
Find the equation of the tangent to circle, C_1, at P. [1]

$y=-2+\sqrt{40}$ — (B1)

(c) Explain why the point $Q(-3,8)$ lies outside the circle, C_1. [2]

$C(3,-2)$

$Q(-3,8)$

length of CQ: $\sqrt{[3-(-3)]^2+(-2-8)^2}$ — (M1)

$=\sqrt{136}$ $(>\sqrt{40})$

Since distance between Q and centre of C_1 is greater than length of radius of C_1, Q lies outside the circle. } (A1)

(d) A second circle, C_2, passes through the points A and Q.
Given that the line $3y-2x=7$ passes through the centre of C_2, find the equation of C_2 [5]

$A(3,-2)$ $Q(-3,8)$

$m_{AQ}: \frac{8-(-2)}{-3-3} = \frac{10}{-6}$

$= -\frac{5}{3}$.

$m_{\perp \text{bisector of } AQ}: (-1) \div (-\frac{5}{3}) = \frac{3}{5}$ — (M1)

Midpt of AQ: $\left(\frac{3+(-3)}{2}, \frac{-2+8}{2}\right)$

$= (0, 3)$

$\therefore$ Eqn of $\perp$ bisector of AQ: $y-3 = \frac{3}{5}(x-0)$ — (M1)

$y = \frac{3}{5}x + 3$ — ①

$3y - 2x = 7$ — ②

Sub ① into ②: $3\left(\frac{3}{5}x+3\right) - 2x = 7$ — (M1)

$\frac{9x}{5} + 9 - 2x = 7$

$\frac{9x}{5} - 2x = -2$

$9x - 10x = -10$

$-x = -10$

$x = 10$

Sub $x=10$ into ①: $y = \frac{3}{5}(10) + 3$ (A1)

$y = 9$

$\therefore$ Centre of C_2 $(10, 9)$

Radius of C_2: $\sqrt{(10-3)^2 + [9-(-2)]^2} = \sqrt{170}$

$\therefore$ equation of C_2: $(x-10)^2 + (y-9)^2 = 170$ — (A1)

[also accept]
$x^2 - 20x + y^2 - 18y + 11$
(general form)

END OF PAPER