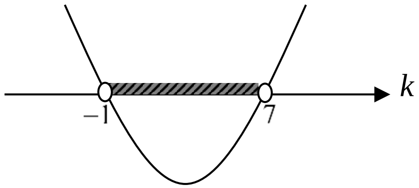
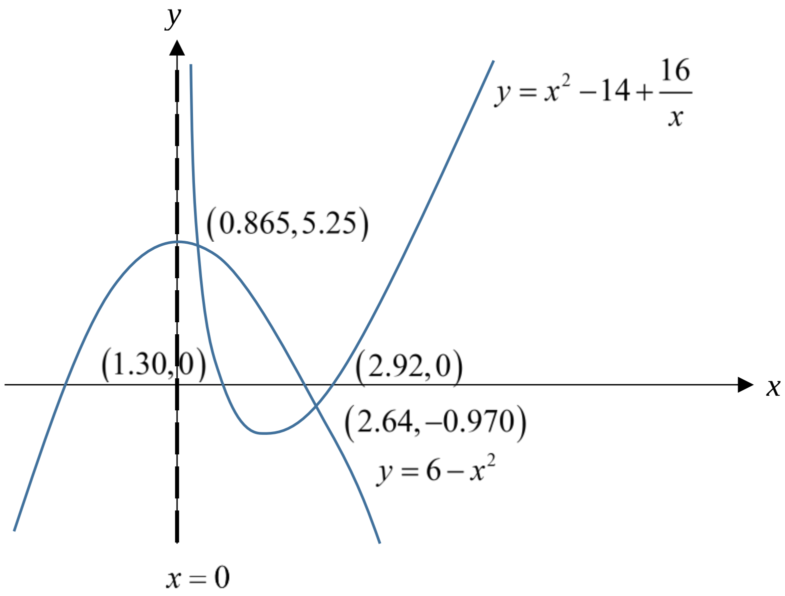
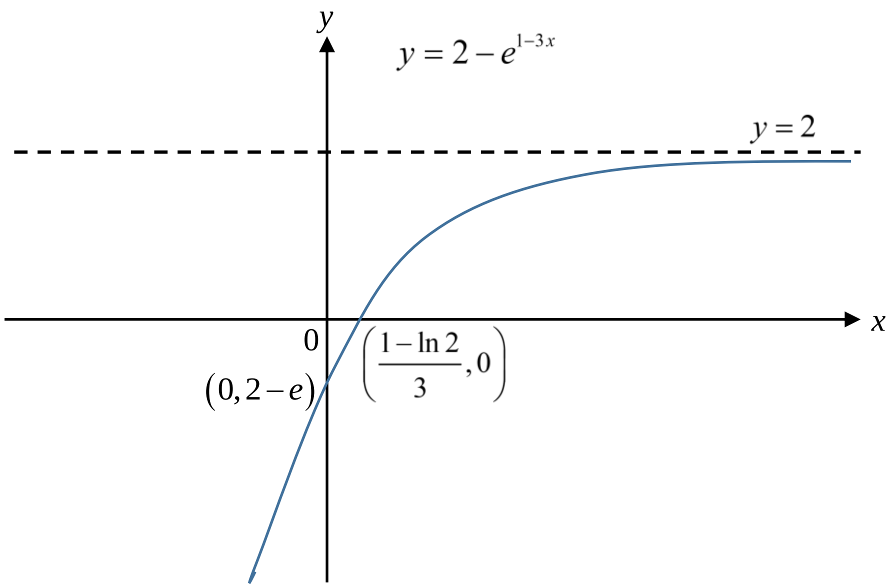
Appendix 2

by:

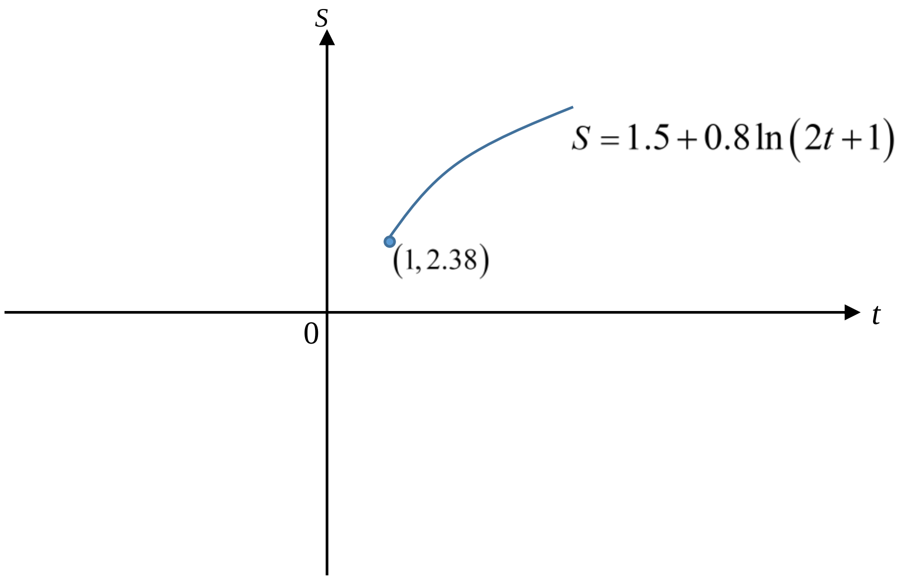
Question	**Suggested Solutions**
1	Since $x^2+(k-1)x+k+2>0$, Discriminant < 0 $(k-1)^2-4(1)(k+2)<0$ $k^2-2k+1-4k-8<0$ $k^2-6k-7<0$ $(k-7)(k+1)<0$  $-1<k<7$ Therefore the set of values of k is $\{k\in\mathbb{R}\mid -1<k<7\}$

2(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(2x^3-5x^{\frac{1}{3}}\right)^2=2\left(2x^3-5x^{\frac{1}{3}}\right)\left(6x^2-\frac{5}{3}x^{-\frac{2}{3}}\right)$ $=2\left(2x^3-5x^{\frac{1}{3}}\right)\left(6x^2-\frac{5}{3x^{\frac{2}{3}}}\right)$
2(b)	$\int\frac{1}{\sqrt{5-4x}}\,\mathrm{d}x=\int(5-4x)^{-\frac{1}{2}}\,\mathrm{d}x$ $=\frac{(5-4x)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)(-4)}+C$ $=-\frac{1}{2}(5-4x)^{\frac{1}{2}}+C,$ where C is an arbitrary constant
3(a)	$y=x^2-14+\frac{16}{x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2x-\frac{16}{x^2}$ At stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. $2x-\frac{16}{x^2}=0$ $2x=\frac{16}{x^2}$ $x^3=8$ $x=2$

	When $x=2$, $y=2^2-14+\dfrac{16}{2}=-2$ Therefore stationary point is $(2,-2)$.
(b)	
(c)	$2x^2-20+\dfrac{16}{x}=0$ $x^2-14+\dfrac{16}{x}+x^2-6=0$ $x^2-14+\dfrac{16}{x}+x^2=6-x^2$ The equation of the curve is $y=6-x^2$.
(d)	From GC, $x=0.865$ or $x=2.64$.

4(a)	 When $x = 0$, $y = 2 - e$ $e^{1-3x} = 2$ When $y = 0$, $1 - 3x = \ln 2$ $x = \dfrac{1-\ln 2}{3}$
4(b)	$y = 2 - e^{1-3x}$ $\dfrac{dy}{dx} = 3e^{1-3x}$ When $x = 1, y = 2 - e^{1-3} = 2 - \dfrac{1}{e^2}$, $\dfrac{dy}{dx} = 3e^{-2} = \dfrac{3}{e^2}$ Equation of tangent when $x = 1$,

	$y-\left(2-\dfrac{1}{e^2}\right)=\dfrac{3}{e^2}(x-1)$ $y=\dfrac{3}{e^2}(x-1)+2-\dfrac{1}{e^2}$ $y=\dfrac{3}{e^2}x-\dfrac{4}{e^2}+2$
4(c)	$A=\int_{\frac{1}{3}}^{1} 2-e^{1-3x}\ \mathrm{d}x$ $=\left[2x-\dfrac{e^{1-3x}}{-3}\right]_{\frac{1}{3}}^{1}=\left[2x+\dfrac{e^{1-3x}}{3}\right]_{\frac{1}{3}}^{1}$ $=\left[2(1)+\dfrac{1}{3}e^{1-3}\right]-\left[\dfrac{2}{3}+\dfrac{1}{3}e^{0}\right]$ $=1+\dfrac{1}{3e^2}\ \ =p+\dfrac{q}{e^2}$ Comparing $p=1,\ q=\dfrac{1}{3}$
5(a)	Let \$$x$, \$$y$ and \$$z$ be the amount spent on internet advertising, television advertising and posters advertising respectively.

	$x+y+z=220000-(1)$ $y=2x \Rightarrow 2x-y=0-(2)$ $y-11000=3z \Rightarrow y-3z=11000-(3)$ Solving (1), (2) and (3) using GC, $x = \$61000$ $y = \$122000$ $z = \$37000$
(b)	When $t=4$, $S=1.5+0.8\ln\left[2(4)+1\right]$ $=1.5+0.8\ln 9$ $=3.26$ 3.26 represents the annual sales in millions of dollars in the 4th year.
(c)	

(d)	$S = 1.5 + 0.8\ln(2t+1)$ $\dfrac{\mathrm{d}S}{\mathrm{d}t} = 0.8\left(\dfrac{2}{2t+1}\right)$ $= \dfrac{8}{5(2t+1)}$
(e)	For $t \geq 1$, $\dfrac{\mathrm{d}S}{\mathrm{d}t} > 0$. Since $\dfrac{\mathrm{d}S}{\mathrm{d}t} > 0$, S increase when t increases. Therefore the sales in the future will increase. As t increases, $\dfrac{\mathrm{d}S}{\mathrm{d}t}$ decreases, so the sales in the future will increase at a decreasing rate in the future.
6	Let X be the random variable that denotes the height of a sunflower plant $X \sim \mathrm{N}(\mu, \sigma^2)$ Given that $\mathrm{P}(X > 1.72) = 0.3$ and $\mathrm{P}(X < 1.52) = 0.1$,

	$\mathrm{P}(X > 1.72) = 0.3$ $\mathrm{P}\left(\frac{X-\mu}{\sigma} > \frac{1.72-\mu}{\sigma}\right) = 0.3$ $\mathrm{P}\left(Z > \frac{1.72-\mu}{\sigma}\right) = 0.3, \quad Z \sim (0,1)$ $\frac{1.72-\mu}{\sigma} = 0.52440$ $1.72-\mu = 0.52440\sigma$ $\mu + 0.52440\sigma = 1.72 ...(1)$	$\mathrm{P}(X < 1.52) = 0.1$ $\mathrm{P}\left(\frac{X-\mu}{\sigma} < \frac{1.52-\mu}{\sigma}\right) = 0.1$ $\mathrm{P}\left(Z < \frac{1.52-\mu}{\sigma}\right) = 0.1, \quad Z \sim (0,1)$ $\frac{1.52-\mu}{\sigma} = -1.2816$ $1.52-\mu = -1.2816\sigma$ $\mu - 1.2816\sigma = 1.52 ...(2)$
	Solving equations (1) and (2), $\mu = 1.66$ and $\sigma = 0.111$ (use GC)	
7	Number of arrangements $= {}^{7}C_{5} \times 5! \times 2! \times 3!$ $= 30240$	
8(a)	$\mathrm{P}(A \mid B) = 0.8$ $\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = 0.8$ $\mathrm{P}(A \cap B) = 0.8\mathrm{P}(B)$ $\mathrm{P}(B) = \frac{\mathrm{P}(A \cap B)}{0.8}$ $= \frac{0.2}{0.8}$ $= \frac{1}{4}$	

	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $0.9 = \mathrm{P}(A) + \frac{1}{4} - 0.2$ $\mathrm{P}(A) = 0.85$
(b)	$\mathrm{P}(A' \cap B') = \mathrm{P}(A \cup B)'$ $= 1 - \mathrm{P}(A \cup B)$ $= 1 - 0.9$ $= 0.1$ $\mathrm{P}(A' \cap B')$ is the probability that both events *A* and *B* do not occur
(c)	<u>Method 1</u> Since $\mathrm{P}(A \mid B) = 0.8$ and $\mathrm{P}(A) = 0.85$ $\mathrm{P}(A \mid B) \neq \mathrm{P}(A)$ $\therefore$ Events *A* and *B* are not independent. <u>Method 2</u> $\mathrm{P}(A \cap B) = 0.2$ $\mathrm{P}(A) \times \mathrm{P}(B) = 0.85 \times 0.25$ $= 0.2125$ $\neq 0.2$ $\therefore$ Events *A* and *B* are not independent.

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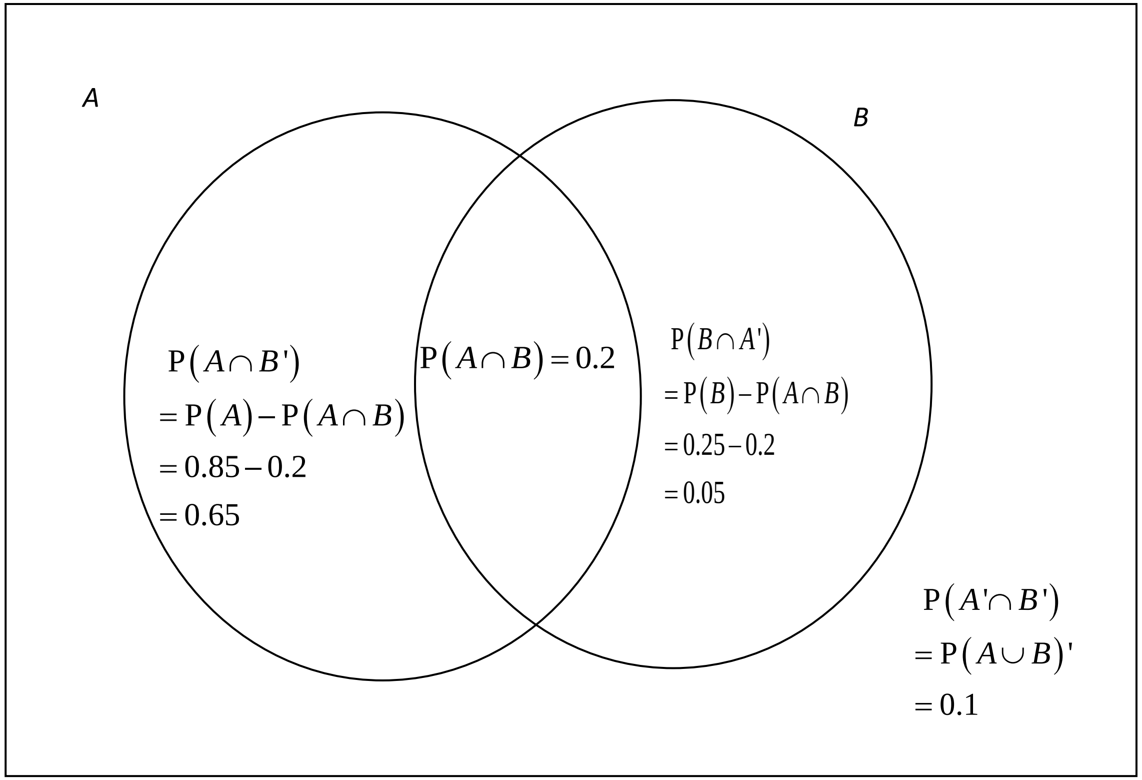
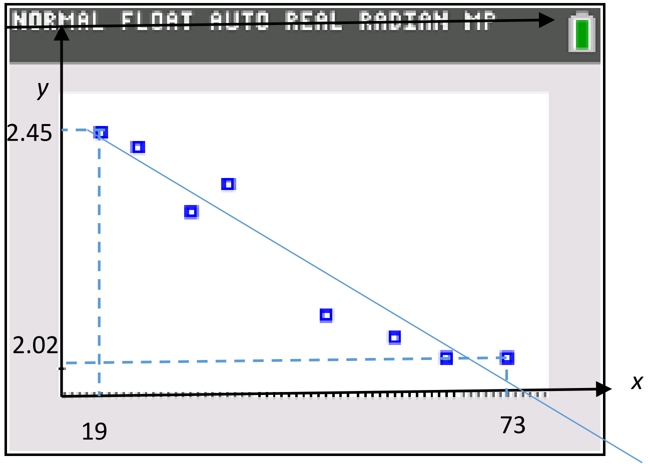
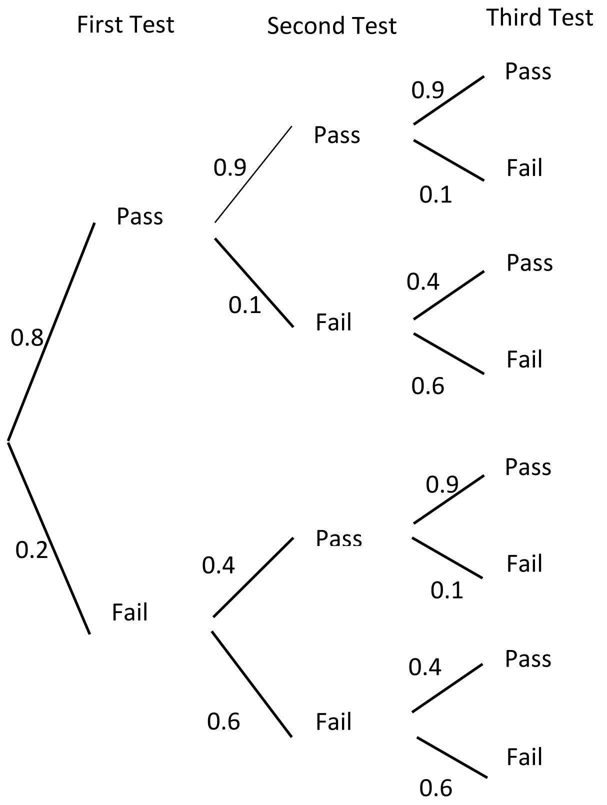
(d)


A
B
$P(A \cap B')$
$= P(A) - P(A \cap B)$
$= 0.85 - 0.2$
$= 0.65$
$P(A \cap B) = 0.2$
$P(B \cap A')$
$= P(B) - P(A \cap B)$
$= 0.25 - 0.2$
$= 0.05$
$P(A' \cap B')$
$= P(A \cup B)'$
$= 0.1$

9(a)	 $y = -0.00893x + 2.61$
(b)	Using GC, $r = -0.966$(to 3 s.f.) It shows that there is a strong negative linear correlation between x (the ages of the adults) and y (maximum walking speed).
(c)	$y = -0.00893x + 2.61$ [*For the sketch of the regression line on the scatter diagram, please refer to the scatter diagram in (i)]*
(d)	When x = 55, y = 2.12 m/s
(e)	

	Since $x = 55$ is within the range of x $\left(19 \le x \le 73\right)$ given in the table, the estimate is an interpolation thus I would expect the estimate to be reliable.
10(a)	
(b)	Let P: pass the test Required probability = P(PPP) + P(PP’P) + P(P’PP) + P(PPP’) = (0.8)(0.9)(0.9) + (0.8)(0.1)(0.4) + (0.2)(0.4)(0.9) + (0.8)(0.9)(0.1) = 0.824

(c)	Required Probability = P (chosen applicant is not accepted | applicant passed the first test) $=\dfrac{\mathrm{P}(PFF)}{\mathrm{P}(P)}$ $=\dfrac{0.8\times0.1\times0.6}{0.8}$ $=0.06$
(d)	Required Probability $={}^{3}C_{2}\left(0.824\right)^{2}(1-0.824)$ = 0.358
11(ai)	Let Y be the random variable denoting the number of orders that the company delivers within 24 hours $Y\sim\mathrm{B}(10,0.75)$ $\mathrm{P}(Y=4\quad\text{or}\quad Y=5)$ $=\mathrm{P}(Y=4)+\mathrm{P}(Y=5)$ $=0.0746$ (to 3 s.f.)
(aii)	$\mathrm{P}(Y>6)$ $=1-\mathrm{P}(Y\le6)$ $=0.776$ (to 3 s.f.)
(b)	Unbiased estimate of population mean = $\overline{x}=\dfrac{708.6}{60}=11.81$ (3 s.f.) Unbiased estimate of population variance, $s^{2}=\dfrac{1}{59}\left[8408.5-\dfrac{708.6^{2}}{60}\right]=0.67685$ (3 s.f.) Let μ be the population mean delivery time.

	Test $H_0: \mu = 12$ vs. $H_1: \mu < 12$ at the 5% level of significance. Under H_0, since $n = 60$ is large, $\overline{X} \sim N(12, \frac{0.67685}{60})$ approximately by Central Limit Theorem. $Z = \frac{\overline{X} - 12}{\sqrt{0.67685/60}} \sim N(0,1)$ Using a one-tailed *z*-test, $\overline{x} = 11.81$ gives $Z_{cal} = -1.79$ and *p*-value = 0.0368 < 0.05 $\therefore$ We reject H_0 and conclude that at the 5% level of significance, there is sufficient evidence that the mean delivery timer of an order is less than 12 hours and the manager's claim is supported by data.
(c)	Since the sample size (60) is large, Central Limit Theorem can be used to approximate the sample mean delivery time to a normal distribution. Hence not necessary to assume that the delivery times are distributed normally for the test to be valid
12(a)	Let *X* and *Y* be the random variable denoting the duration of a randomly chosen song by band *A* and band *B* respectively. $X \sim N\left(3.6, 0.5^2\right)$ $Y \sim N\left(4.2, 0.8^2\right)$

	$\mathrm{E}(X-Y)=\mathrm{E}(X)-\mathrm{E}(Y)=3.6-4.2=-0.6$ $\mathrm{Var}(X-Y)=\mathrm{Var}(X)+\mathrm{Var}(Y)=0.5^2+0.8^2=0.89$ $X-Y \sim \mathrm{N}(-0.6, 0.89)$ $\mathrm{P}(X-Y>0)=0.262$
12(b)	Let $T=X_1+X_2+Y_1+Y_2+Y_3$ $\mathrm{E}(X_1+X_2+Y_1+Y_2+Y_3)=2\mathrm{E}(X)+3\mathrm{E}(Y)=2(3.6)+3(4.2)=19.8$ $\mathrm{Var}(X_1+X_2+Y_1+Y_2+Y_3)=2\mathrm{Var}(X)+3\mathrm{Var}(Y)=2(0.5)^2+3(0.8)^2=2.42$ $T \sim \mathrm{N}(19.8, 2.42)$ $\mathrm{P}(18<T<22)=0.798$
12(c)	Let I be random variable denoting the duration of an interval (in mins) $I \sim \mathrm{N}(1.5, 0.4^2)$ Let $S=X_1+X_2+X_3+X_4+Y_1+Y_2+Y_3+Y_4+I_1+I_2+......+I_7$ $\mathrm{E}(S)=4\mathrm{E}(X)+4\mathrm{E}(Y)+7\mathrm{E}(I)=4(3.6)+4(4.2)+7(1.5)=41.7$ $\mathrm{Var}(S)=4\mathrm{Var}(X)+4\mathrm{Var}(Y)+7\mathrm{Var}(I)=4(0.5)^2+4(0.8)^2+7(0.4)^2=4.68$ $S \sim \mathrm{N}(41.7, 4.68)$ $\mathrm{P}(S>40)=0.784$
12(d)	Let $Q=X_1+X_2+X_3+X_4+Y_1+Y_2+Y_3+Y_4+I_1+I_2+......+I_7$ $Q \sim \mathrm{N}(41.7, 4.68)$ $\mathrm{P}(S>40)\mathrm{P}(Q>40)=0.615$
12(e)	The events found in (d) is a subset of the events found in that of "the two halves of the concert last for a total of more than 80 minutes.

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	$S + Q > 80$ can have cases such as ($S > 30$ and $Q > 50$) and ($S > 20$ and $Q > 60$). Therefore ($S > 40$ and $Q > 40$) is a subset of $S + Q > 80$.