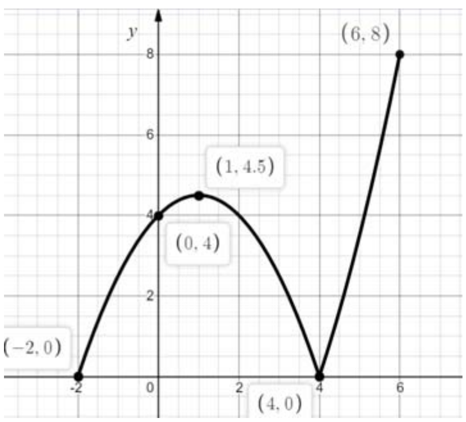
Paper G Sec 3 Solutions for students

Qn	Solutions
1(i)	$\dfrac{3-2x}{x^2-4}-\dfrac{2}{2-x}$ $=\dfrac{3-2x}{(x-2)(x+2)}-\dfrac{2}{2-x}$ $=\dfrac{3-2x}{(x-2)(x+2)}+\dfrac{2}{x-2}$ $=\dfrac{3-2x+2(x+2)}{(x-2)(x+2)}$ $=\dfrac{7}{(x-2)(x+2)}$
1(ii)	$\dfrac{3-2x}{x^2-4}-\dfrac{2}{2-x}=\dfrac{x^2-2x+4}{x^3+8}$ $\dfrac{7}{(x-2)(x+2)}=\dfrac{(x^2-2x+4)}{(x+2)(x^2-2x+4)}$ $\dfrac{7}{(x-2)(x+2)}=\dfrac{1}{(x+2)}$ $7=x-2$ $x=9$
2(a)	$2^{x+1}=5^{x-1}$ $2^{x+1}=5^{x-1}$ $2^x(2)=\dfrac{5^x}{5}$ $\dfrac{2^x}{5^x}=\dfrac{1}{10}$ $\left(\dfrac{2}{5}\right)^x=\dfrac{1}{10}$ $x\lg\left(\dfrac{2}{5}\right)=\lg\left(\dfrac{1}{10}\right)$ $x=\dfrac{-1}{\lg\left(\dfrac{2}{5}\right)}$ or $x=\log_{\frac{2}{5}}\left(\dfrac{1}{10}\right)$ $x=\dfrac{-1}{\lg 2-\lg 5}$ or $x=\dfrac{1}{\lg 5-\lg 2}$

2(b)	$x+\sqrt{x+1}=5$ $\sqrt{x+1}=(5-x)$ $x+1=(5-x)^2$ $x+1=x^2-10x+25$ $x^2-11x+24=0$ $(x-8)(x-3)=0$ $x=8$ (n.a.) or $x=3$
2 (c)	Area of rectangle =length (breadth) $8+2\sqrt{3}=(5-\sqrt{12})$(breadth) breadth $=\left(\dfrac{8+2\sqrt{3}}{5-\sqrt{12}}\right)$ breadth $=\left(\dfrac{8+2\sqrt{3}}{5-\sqrt{12}}\right)\left(\dfrac{5+\sqrt{12}}{5+\sqrt{12}}\right)$ $=\dfrac{40+8\sqrt{12}+10\sqrt{3}+2(6)}{25-12}$ $=\dfrac{52+16\sqrt{3}+10\sqrt{3}}{13}$ $=\left(4+2\sqrt{3}\right)$ $\therefore$ breadth is $\left(4+2\sqrt{3}\right)$ cm .
3(a)	$\boldsymbol{x}^2-3\boldsymbol{kx}+\boldsymbol{k}\left(1-\boldsymbol{k}\right)=0$ Two non-zero real roots, Discriminant, $b^2-4ac\geq 0$ $\left(-3k\right)^2-4\left(1\right)\left(k-k^2\right)\geq 0$ $13k^2-4k\geq 0$ $k\left(13k-4\right)\geq 0$ $k<0\ \ or\ k\geq\dfrac{4}{13}\ \ or\ \ k\neq 1$

3(b)(i)	$g(x)=3+\frac{4}{x+2}$ *Let* $g(x)=y$ $\Leftrightarrow g^{-1}(y)=x$ $3+\frac{4}{x+2}=y$ $\frac{4}{x+2}=y-3$ $(y-3)(x+2)=4$ $xy-3x+2y-6=4$ $x(y-3)=10-2y$ $x=\frac{10-2y}{(y-3)}$ $\therefore\ g^{-1}:\to\frac{10-2x}{x-3},\ x\neq 3$ Or $\therefore\ g^{-1}(x)=\frac{4}{x-3}-2,\ x\neq 3$
3(b)(ii)	Domain of $\boldsymbol{g}^{-1}$: $\boldsymbol{x}\neq 3$. Range of $\boldsymbol{g}^{-1}$: $\boldsymbol{g}^{-1}(\boldsymbol{x})\neq -2$.
4(a) (i)	Let $\mathrm{P}(y)=3y^3+2y^2-7y+2$ $\mathrm{P}(1)=3(1)^3+2(1)^2-7+2$ $\quad =0$ $\therefore (y-1)$ is a factor of $\mathrm{P}(y)$.
4(a) (ii)	$3y^3+2y^2-7y+2=(y-1)(3y^2+by-2)$ $\qquad =3y^3+by^2-2y-3y^2-by+2$ $\qquad =3y+(b-3)y^2+(-2-b)y+2$ Comparing coefficient of y^2 $(b-3)=2$ $\quad \therefore b=5$ $3y^3+2y^2-7y+2=(y-1)(3y^2+5y-2)$ $\qquad =(y-1)(3y-1)(y+2)$ **<u>Note :</u>** Long Division working acceptable

4 (a) (iii)	$24y^6+8y^4=14y^2-2$ $24y^6+8y^4-14y^2+2=0$ $3(8y^6)+2(4y^4)-7(2y^2)+2=0$ Replace "y" by $2y^2$ $3(2y^2)^3+2(2y^2)^2-7(2y^2)+2=0$ $(2y^2-1)(6y^2-1)(2y^2+1)=0$ $y=\pm\frac{1}{\sqrt{2}}$ or $y=\pm\frac{1}{\sqrt{6}}$
5 (i)	$\log_x(2x-1)\times\log_6 x$ $=\frac{\log_6(2x-1)}{\log_6 x}\times\log_6 x$ $=\log_6(2x-1)$
5 (ii)	$\log_6 x+\log_x(2x-1)\times\log_6 x=1$ $\log_6 x+\log_6(2x-1)=1$ $\log_6 x(2x-1)=1$ $x(2x-1)=6$ $2x^2-x-6=0$ $(2x+3)(x-2)=0$ $x=\frac{-3}{2}$(n.a.), $x=2$
6(i)	$y=\frac{1}{2}x^2-x-4$ $=\frac{1}{2}(x^2-2x-8)$ $=\frac{1}{2}\left[(x-1)^2-9\right]$ min. point : $\left(1\ ,\ -\frac{9}{2}\right)$ or $\left(1\ ,\ -4\frac{1}{2}\right)$

6(ii)	$y=\frac{1}{2}x^2-x-4$ $=\frac{1}{2}\left(x^2-2x-8\right)$ When $y=0$, $\frac{1}{2}\left(x^2-2x-8\right)=0$ $\frac{1}{2}(x-4)(x+2)=0$ $x=4$ or $x=-2$ Points : $\left(4\ ,\ 0\right)$ or $\left(-2\ ,\ 0\right)$
6 (iii)	
6 (iv)	For $\left\|\frac{1}{2}x^2-x-4\right\|=k$ to have 3 solutions, $0<k<4\frac{1}{2}$. Or $k=4.5$
7 (a)	0: $y=e^{-x}$ — Original graph I : $y=2e^{-x}$ — Stretch graph of $y=e^{-x}$ parallel to y-axis, scale factor 2. II: $y=-2e^{-x}$ — Reflect graph of $y=2e^{-x}$ along x-axis.

0:	$y=e^{-x}$	Original graph
I :	$y=2e^{-x}$	Stretch graph of $y=e^{-x}$ parallel to y-axis, scale factor 2.
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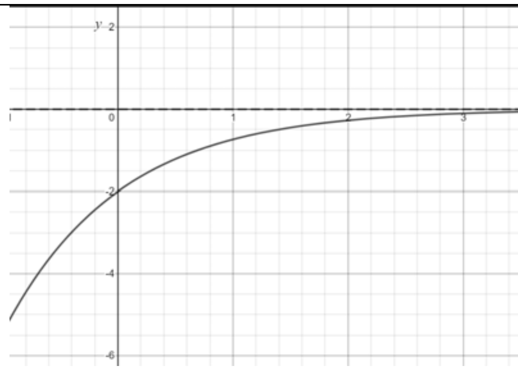
7(b)	
8 (i)	$(x-3)^2+(y+1)^2=7^2$ $x^2+y^2-6x+2y+10-49=0$ $x^2+y^2-6x+2y-39=0$ $x^2+y^2+2ax+2by+c=0$ $a=-3$ $b=1$ $c=-39$
8 (ii)	$B:(3\ ,\ 6)$
8 (iii)	Equation of line BC : $\mathrm{m}_{OB}=\mathrm{m}_{BC}$ $\mathrm{m}_{BC}=\frac{6-0}{3-0}=2$ $y=2x---(1)$ $x^2+y^2-6x+2y-39=0---(2)$ Sub. (1) into (2) : $x^2+4x^2-6x+4x-39=0$ $5x^2-2x-39=0$ $(5x+13)(x-3)=0$ $x=\frac{-13}{5}$ or $x=3$ (x-coordinate of point B) $y=\frac{-26}{5}$ $C:\left(\frac{-13}{5}\ ,\ \frac{-26}{5}\right)$ or $\left(-2\frac{3}{5},-5\frac{1}{5}\right)$

9 (a)	$2\boldsymbol{C} = 2\boldsymbol{B} - 3\boldsymbol{A}$ $= 2\begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix} - 3\begin{pmatrix} 8 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 6 & -4 \end{pmatrix} - \begin{pmatrix} 24 & 6 \\ 3 & -3 \end{pmatrix}$ $= \begin{pmatrix} -20 & -4 \\ 3 & -1 \end{pmatrix}$ $\boldsymbol{C} = \frac{1}{2}\begin{pmatrix} -20 & -4 \\ 3 & -1 \end{pmatrix}$ $= \begin{pmatrix} -10 & -2 \\ \frac{3}{2} & \frac{-1}{2} \end{pmatrix}$
9(b) (i)	$\boldsymbol{Q} = \begin{pmatrix} 400 & 250 & 50 \\ 450 & 200 & 140 \end{pmatrix}$ Or $\boldsymbol{Q} = \begin{pmatrix} 450 & 200 & 140 \\ 400 & 250 & 50 \end{pmatrix}$
9(b) (ii)	$\boldsymbol{P} = \begin{pmatrix} 3.00 \\ 1.80 \\ 2.00 \end{pmatrix}$
9(b) (iii)	$\boldsymbol{V} = \boldsymbol{QP}$ $= \begin{pmatrix} 400 & 250 & 50 \\ 450 & 200 & 140 \end{pmatrix}\begin{pmatrix} 3.00 \\ 1.80 \\ 2.00 \end{pmatrix} = \begin{pmatrix} 1200+450+100 \\ 1350+360+280 \end{pmatrix}$ $= \begin{pmatrix} 1750 \\ 1990 \end{pmatrix}$
9(b) (iv)	Total Amount of profit $= \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1750 \\ 1990 \end{pmatrix}$ $= (3740)$ OR Total Amount of profit $= \begin{pmatrix} 1750 & 1990 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $= (3740)$ Therefore, total amount of profit made was \$3740.

10	Plotting x^2y against x . $\text{gradient} = \frac{5-(-1)}{2-6}$ $\text{gradient} = -\frac{3}{2}$ $x^2y-5 = -\frac{3}{2}(x-2)$ $2x^2y-10 = -3x+6$ $y = \frac{-3x+16}{2x^2}$ OR $y = \frac{-3}{2x}+\frac{8}{x^2}$
11(i)	Eqn *AD* : $2\boldsymbol{y}-\boldsymbol{x} = -16$ $\boldsymbol{y} = \frac{1}{2}\boldsymbol{x}-8$ Gradient *AD* = gradient $BC = \frac{1}{2}$ Equation of BC: $\frac{y-18}{x-22} = \frac{1}{2}$ $y-18 = \frac{1}{2}x-11$ $y = \frac{1}{2}x+7$ ---(3) Equation of AB: $y = -2x+3$ ---(1) $\frac{1}{2}\boldsymbol{x}+7 = -2\boldsymbol{x}+3$ $x+14 = -4x+6$ $x = \frac{-8}{5}$ $= -1\frac{3}{5}$ $y = -2\left(\frac{-8}{5}\right)+3$ $= 6\frac{1}{5}$ $B:\left(-1\frac{3}{5}\ ,\ 6\frac{1}{5}\right)$

11 (ii)	Gradient of AB = -2 Gradient of BC = $\frac{1}{2}$ $m_{AB} \times m_{BC}$ $= (-2)\left(\frac{1}{2}\right)$ $= -1$ Hence line AB is $\perp$ to line BC. Thus $\angle ABC = 90^{\circ}$.
11 (iii)	Let $H : (a\ ,\ b)$ midpoint of AH = midpoint of BD $\left(\frac{4.4+a}{2}, \frac{-5.8+b}{2}\right) = \left(\frac{-1.6+12}{2}, \frac{6.2-2}{2}\right)$ $\frac{4.4+a}{2} = 5.2$ $a = 6$ $\frac{-5.8+b}{2} = 2.1$ $b = 10$ $H : (6\ ,\ 10)$
11 (iv)	Triangle AEC and Triangle ABC share common height, $\frac{\text{area of } \triangle AEC}{\text{area of } \triangle ABC} = \frac{\lvert AE\rvert}{\lvert AB\rvert}$ $\lvert \boldsymbol{AE} \rvert = \sqrt{\left(4\frac{2}{5}+16\right)^2 + \left(-5\frac{4}{5}-35\right)^2}$ $= \sqrt{\frac{10404}{5}}$ ***units*** $\lvert \boldsymbol{AB} \rvert = \sqrt{\left(4\frac{2}{5}+1\frac{3}{5}\right)^2 + \left(-5\frac{4}{5}-6\frac{1}{5}\right)^2}$ $= \sqrt{180}$ ***units***

	$\dfrac{\text{area of } \triangle AEC}{\text{area of } \triangle ABC} = \dfrac{\lvert AE \rvert}{\lvert AB \rvert}$ $\dfrac{\text{area of } \triangle AEC}{\text{area of } \triangle ABC} = \dfrac{\sqrt{\dfrac{10404}{5}}}{\sqrt{180}}$ $\dfrac{\text{area of } \triangle AEC}{\text{area of } \triangle ABC} = \dfrac{17}{5}$ OR Using shoelace method Area of triangle AEC $= \dfrac{1}{2}\begin{vmatrix} 4.4 & 22 & -16 & 44 \\ -5.8 & 18 & 35 & -5.8 \end{vmatrix}$ $= \dfrac{1}{2}(1203.6) = 601.8 \text{ units}^2$ Area of triangle ABC $= \dfrac{1}{2}\begin{vmatrix} 4.4 & 22 & -1.6 & 44 \\ -5.8 & 18 & 6.2 & -5.8 \end{vmatrix}$ $= \dfrac{1}{2}(354) = 177 \text{ units}^2$ $\dfrac{\text{area of } \triangle AEC}{\text{area of } \triangle ABC} = \dfrac{601.8}{177}$ $\dfrac{\text{area of } \triangle AEC}{\text{area of } \triangle ABC} = \dfrac{17}{5}$
12 (i)	$\angle PQR = 200^{\circ} - 115^{\circ} = 85^{\circ}$ $(RP)^2 = 26^2 + 30^2 - 2(26)(30)\cos 85^{\circ}$ $=1440.037041$ $\therefore$ RP $= 37.94781998$ $= 37.9 km \quad (3 s.f.)$
12 (ii)	$\angle PQ(\text{North}) = 360^{\circ} - 200^{\circ} = 160^{\circ}$ $\angle QP(North_2) = 180^{\circ} - 160^{\circ} = 20^{\circ}$ $\dfrac{\sin \angle QPR}{30} = \dfrac{\sin 85^{\circ}}{PR}$ $\sin \angle QPR = \dfrac{30 \sin 85^{\circ}}{PR} = 0.78755$ $\therefore \angle QPR = 51.957^{\circ} = 52.0^{\circ}(1dp)$ $\therefore \boldsymbol{bearing\ of\ R\ from\ P} = 20^{\circ} + 51.957^{\circ}$ $= 072.0^{\circ}(1\boldsymbol{dp})$

12 (iiii)	*Either* Let the shortest distance from island *R* to boat be *h* km. $\sin 85° = \frac{h}{30}$ $h = 30\sin 85° = 29.886$ $h = 29.9$ $\therefore$ the shortest distance of the boat from island R is 29.9 km. *Or* Let *h* km be the shortest distance of the boat from island *R*. Area of triangle PQR = $\frac{1}{2}(26)(h)$ $\frac{1}{2}(26)(30)\sin 85^o = \frac{1}{2}(26)(h)$ $h = (30)\sin 85^o = 29.886 = 29.9$ $\therefore$ the shortest distance of the boat from island R is 29.9 km.
13 (i)	Draw a $\perp$ line fr C to OA, intersecting at T $OT = 8 - r$ cm $(8-r)^2 + 12^2 = (8+r)^2$ $64 - 16r + r^2 + 144 = 64 + 16r + r^2$ $32r = 144$ $r = \frac{144}{32}$ $= \frac{36}{8}$ $= 4\frac{4}{8}$ $= 4.5$ (shown)
13 (ii)	$OC = 8 + 4.5$ $OC = 12.5$ cm $\sin \angle AOC = \frac{12}{12.5}$ $\therefore \angle AOC = 1.2870 = 1.29 \text{ rad(3sf)}$ $\therefore \angle OCB = 2\pi - \pi - \angle AOC$ $\angle OCB = 1.8545 = 1.85 \text{ rad (3sf)}$
13 (iii)	Area of shaded region = area of trapezium − area of sector 1 − area of sector 2

	$=\dfrac{12(8+4.5)}{2}-\dfrac{1}{2}(4.5)^2(1.8546)-\dfrac{1}{2}(8)^2(1.2870)$ $=15.0\text{ cm}^2\ (3\text{sf})$