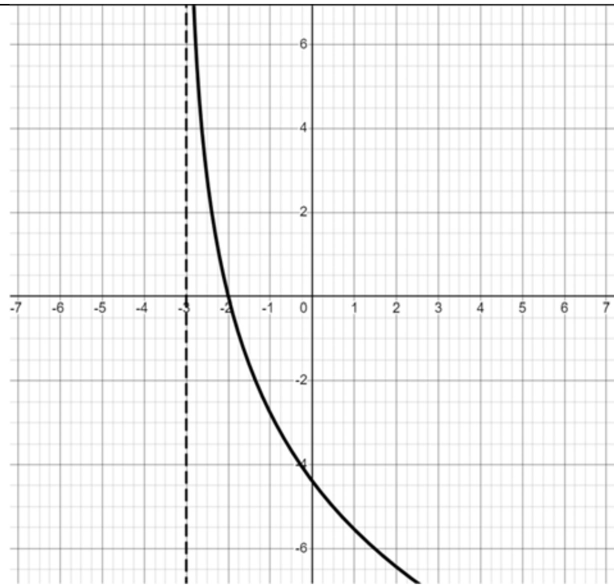
2024 Sec 3 EMath Practice Paper B Solution

Qn	Solutions
1(a)	$3x-6xy+4y^2-4y+1$ $=3x(1-2y)+(1-2y)^2$ $=(1-2y)(3x+1-2y)$ or $(2y-1)(2y-3x-1)$
1(b)	$\left(x-\frac{15}{x}-2\right)\div\left(x+\frac{12}{x}+7\right)$ $=\left(\frac{x^2-2x-15}{x}\right)\div\left(\frac{x^2+7x+12}{x}\right)$ $=\frac{(x-5)(x+3)}{(x+3)(x+4)}$ $=\frac{(x-5)}{(x+4)}$
1(c)	$x=\sqrt{\frac{a^2-z^2}{z^2}}+y$ $(x-y)^2=\frac{a^2-z^2}{z^2}$ $(x-y)^2z^2+z^2=a^2$ $a=\pm\sqrt{(x-y)^2z^2+z^2}$ or $a=\pm z\sqrt{(x-y)^2+1}$
2(a)	$\frac{1}{x}-\frac{3}{2x+1}=2$ $2x+1-3x=2x(2x+1)$ $4x^2+3x-1=0$ $(x+1)(4x-1)=0$ $\therefore x=-1$ or $x=\frac{1}{4}$
2(b)	$x^2-y=6x$ $x+y=6 \Rightarrow y=6-x$ sub into first eqn $x^2-(6-x)=6x$ $x^2-5x-6=0$ $(x-6)(x+1)=0$ $\therefore x=6$ or $x=-1$ $\therefore y=0$ or $y=7$

3(a)	$\mathbf{P} = \begin{pmatrix} 8 & 5 & 0.7 \\ 3 & 5 & 0.4 \end{pmatrix}$
3(b)(i)	$\mathbf{C} = \begin{pmatrix} 7.00 \\ 1.20 \\ 0.60 \end{pmatrix}$
3(b)(ii)	$\mathbf{PC} = \begin{pmatrix} 8 & 5 & 0.7 \\ 3 & 5 & 0.4 \end{pmatrix}\begin{pmatrix} 7 \\ 1.2 \\ 0.6 \end{pmatrix}$ $= \begin{pmatrix} 62.42 \\ 27.24 \end{pmatrix}$
3(b)(iii)	The top element represents the total cost for handmade toys while the bottom element represents the total cost for machine-made toys.
4(i)	$f(x) = -2x^2 + 8x + 15$ $= -2\left(x^2 - 4x + 2^2 - 2^2\right) + 15$ $= -2\left(x-2\right)^2 + 23$
4(ii)	$-2\left(x-2\right)^2 + 23 = 0$ $(x-2)^2 = \frac{23}{2}$ $(x-2) = \pm\sqrt{\frac{23}{2}}$ $x = -\sqrt{\frac{23}{2}} + 2$ or $\sqrt{\frac{23}{2}} + 2$
4(iii)	$x = 2$
5(i)	Gradient $= \frac{75-(-13)}{-10-1} = -8$ $Y = -8X + c$; at $(1, -13)$, $-13 = -8(1) + c$ $\therefore c = -5$ $\frac{y}{x^3} = -8x - 5$ $\therefore y = -8x^4 - 5x^3$
5(ii)	$Y = -8X - 5$ $-215 \neq -8(5) - 5$ She's incorrect.

6(i)	$A\hat{O}B = 2\times\cos^{-1}\frac{10}{30} \approx 2.46$
6(ii)	Major Arc Length APB $= 30\times\left(2\pi-2.4619\right)$ $= 114.6 \approx 115$ cm
6(iii)	Percentage of area below water level $= \dfrac{\frac{1}{2}\left(30^2\right)\left(2.4619\right)-\frac{1}{2}\left(30^2\right)\sin 2.4619}{\pi\left(30^2\right)}\times 100\%$ $\approx 29.2\%$
7(a)(i)	$y=\left(x+1\right)\left(x-k\right)$ At D, $21=\left(20+1\right)\left(20-k\right)$ $\therefore k=19$
7(a)(ii)	A(−1, 0), B(19, 0), C(0, −19) Area of ΔABC $= \frac{1}{2}\times 20\times 19$ $= 190$ units^2
7(a)(iii)	Gradient of CD $= \dfrac{21-\left(-19\right)}{20-0}=2$ Equation of CD through A: $y=2x+2$
7(b)	$x^2+y^2=2x+4y-4$ $x^2-2x+y^2-4y+4=0$ $(x-1)^2-1+(y-2)^2-4-4=0$ $(x-1)^2+(y-2)^2=3^2$ $\text{centre}:\left(1,2\right)$ $radius=3$ units Gradient of tangent $= \dfrac{-1}{\left(\dfrac{2-2\frac{4}{5}}{1-\frac{2}{5}}\right)}$ $=\dfrac{3}{4}$ Equation of tangent: $y-\dfrac{14}{5}=\dfrac{3}{4}\left(x-\dfrac{2}{5}\right)$ $y=\dfrac{3}{4}x+\dfrac{5}{2}$

8 (i)	$y^2+6x^2-7xy=0---(1)$ $Po\text{int}:(1,4)$ $P:\left(\frac{1}{2},3\right)$ $m=\frac{1}{\frac{1}{2}}$ $m=2$ Equation of line : $y-4=2(x-1)$ $y=2x+2---(2)$ $(2x+2)^2+6x^2-7x(2x+2)=0$ $-4x^2-6x+4=0$ $2x^2+3x-2=0$ $(2x-1)(x+2)=0$ $x=0.5$ or $x=-2$ $Q:(-2,-2)$
8(ii)	$P:\left(\frac{1}{2},3\right)$, $Q:(-2,-2)$ $m_{\text{perpendicular bisector}}$ $=\frac{-1}{2}$ midpoint of PQ : $\left(\frac{-3}{4},\frac{1}{2}\right)$ Equation of perpendicular bisector : $y-\frac{1}{2}=\left(\frac{-1}{2}\right)\left(x+\frac{3}{4}\right)$ $y=\frac{-1}{2}x+\frac{1}{8}$
9(i)	$I: y=\ln x--->y_1=\ln(x+3)$ Translate 3 units in negative x-direction $II: y_1=\ln(x+3)--->y_2=4\ln(x+3)$ Scale with scale factor 4 parallel to y-axis $III: y_2=4\ln(x+3)--->y_3=-4\ln(x+3)$ Reflect about x-axis

9(ii)	
10(i)	$\cos \angle DBC = \dfrac{6^2 + 14.8^2 - 10^2}{2(6)(14.8)}$ $\Rightarrow \angle DBC = \cos^{-1}\left(\dfrac{6^2 + 14.8^2 - 10^2}{2(6)(14.8)}\right) = 29.2°$ (1dp)
10(ii)	$\dfrac{\sin \angle DCB}{BD} = \dfrac{\sin \angle DBC}{CD}$ $\Rightarrow \angle DCB = \sin^{-1}\left(\dfrac{6 \sin 29.194°}{10}\right) = 17.0°$ (1dp) Bearing of B from C = $32° + 17.0° = 049.0°$
10(iii)	Shortest distance from D to BC $= BD \sin \angle DBC$ $= 6 \sin(29.194°)$ $= 2.93$ km (3sf)
10(iv)	TD $= 6 \tan 47.5°$ ≈ 6.548 km ≈ 6548 m
10(v)	$\tan \theta = \dfrac{6.548}{2.927}$ $\therefore \theta = 65.9°$ (1dp)

11(i)	$S:(a,b)$ $\left(\frac{a+9}{2},\frac{b+13}{2}\right)=(-1,9)$ $\frac{a+9}{2}=-1$ $a=-11$ $\frac{b+13}{2}=9$ $b=5$ $S:(-11,5)$ $R:(c,d)$ $\left(\frac{c-3}{2},\frac{d+14}{2}\right)=(-1,9)$ $c=1,d=4$ $R:(1,4)$
11(ii)	Area of PQRS $=\frac{1}{2}\begin{vmatrix}-11 & 1 & 9 & -3 & -11\\ 5 & 4 & 13 & 14 & 5\end{vmatrix}$ $=\frac{1}{2}\left[(-44+13+126-15)-(5+36-39-154)\right]$ $=116\text{ units}^2$
12(a)	$y=x(x+p)(x-q)$ $y=x(x+1)(x-2)$ $p=1,\ q=2$
12(b)	$4y=1-x---(1)$ $y=x(x+1)(x-2)---(2)\times 4$ $4y=4x(x^2-x-2)$ $4y=4x^3-4x^2-8x$ $1-x=4x^3-4x^2-8x$ $4x^3-4x^2-7x-1=0$