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Class: 3Z1

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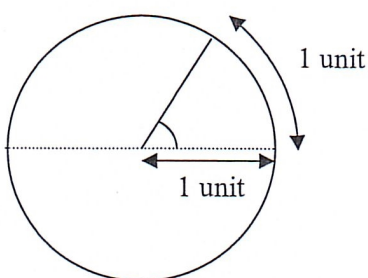
MA304.1.1 – The Radian

At the end of this unit, you should be able to

- understand and convert between degrees and radians
- find the arc length given the angle subtended by the arc at the centre and the radius

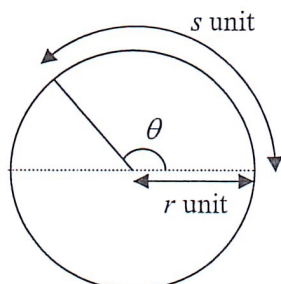
While angles have been commonly measured in degrees ($^{\circ}$) it is actually not the scientific unit for measuring angles and cannot be used to simplify certain mathematical and physical formulas. An alternative method of measuring angles is derived from the unit circle.

A radian is defined by the angle made by an arc whose length is equal to the radius.



For the unit circle, it means when the arc has a length of one unit.

In general, the angle made by an arc is defined as the ratio of the arc length to the radius of the circle.



$$\theta = \frac{s \text{ (arc length)}}{r \text{ (radius)}}$$

E1. A goat tied to a pole walks around with the rope taut. If it walks a complete round, it would have completed a 360° revolution. The (arc) distance it would have walked will be the circumference of the circle, i.e. $2\pi r$, where r is the length of the rope, which is also the radius of the circle. Hence, the angle is equivalent to

$$\frac{2\pi r}{r} = 2\pi \text{ radians}$$

In other words, we have the following equivalent relationship

$360^\circ = 2\pi \text{ radians}$

E2. Convert the following angles into its corresponding equivalent degree/radian.

(a) 90°

$$90 \times \frac{\pi}{180} = \frac{\pi}{2}$$

(c) $\frac{3\pi}{4}$

$$\frac{3\pi}{4} \times \frac{180}{\pi} = 135^\circ$$

(b) 210°

$$210 \times \frac{\pi}{180} = \frac{7\pi}{6}$$

(d) $\frac{5\pi}{3}$

$$\frac{5\pi}{3} \times \frac{180}{\pi} = 300^\circ$$

P1. Convert the following angles into its corresponding equivalent degree/radian.

(a) -30°

$$-30 \times \frac{\pi}{180} = -\frac{\pi}{6}$$

(c) $-\frac{\pi}{3}$

$$-\frac{\pi}{3} \times \frac{180}{\pi} = -60^\circ$$

(b) -135°

$$-135 \times \frac{\pi}{180} = -\frac{3\pi}{4}$$

(d) $-\frac{7\pi}{12}$

$$-\frac{7\pi}{12} \times \frac{180}{\pi} = -105^\circ$$

E3. The radius of a circle is 40 cm and a twine, 1 metre in length, is to go around the circumference. Determine the angle made by the arc and also the percentage of the circumference of which the twine can go around it.

$$\text{Angle} = \frac{100}{40}$$

$$= 2.5$$

$$\text{Percentage} = \frac{2.5}{2\pi} \times 100\%$$

$$= 39.8\%$$