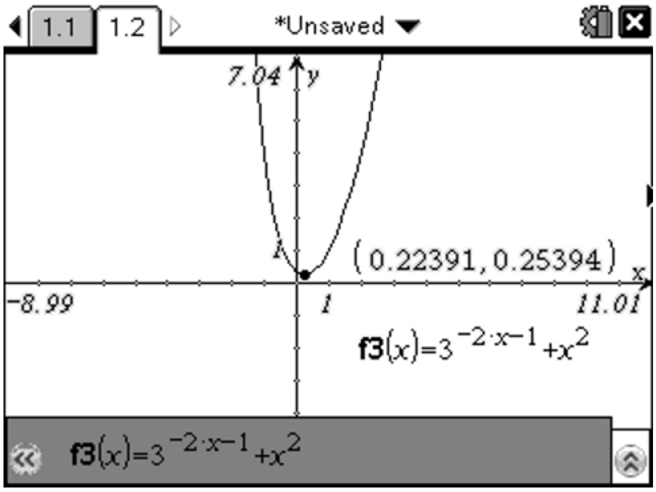
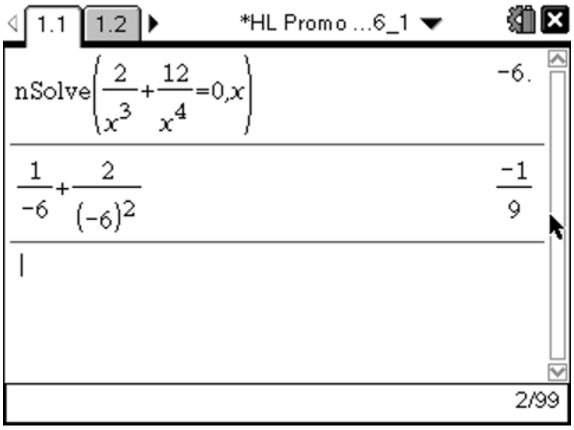
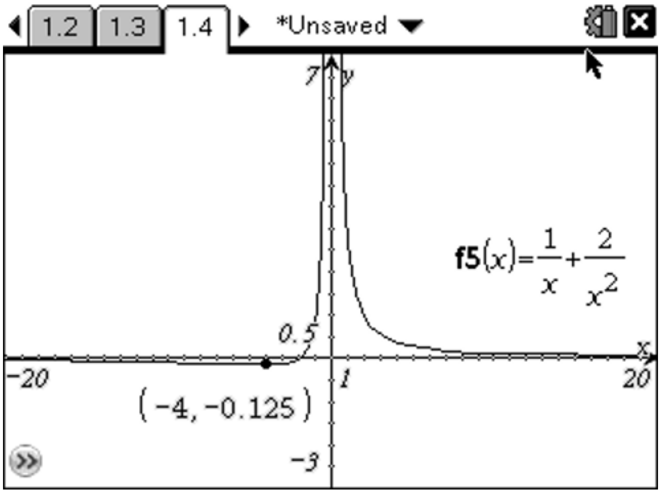
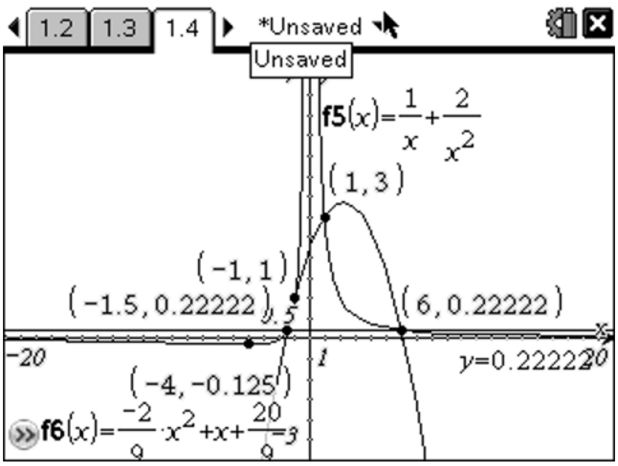
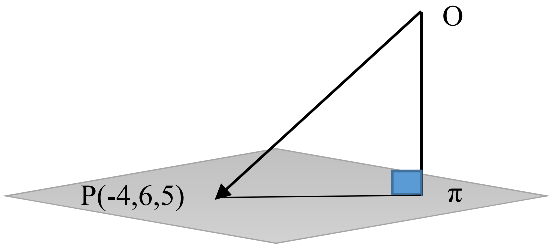
2016 HL Final Examination Paper 2

	Section A Questions/ Solutions	Comments
1.	A polynomial $x^4 + ax^2 + bx + 8$ has factor $x^2 - 3x + 2$. Find the values of a and b. [6 marks]	6
	Let $P(x) = x^4 + ax^2 + bx + 8$ Since $x^2 - 3x + 2 = (x-2)(x-1)$ $(x-2)$ and $(x-1)$ are linear factors. By Factor theorem, $P(2) = 2^4 + a2^2 + 2b + 8$ $= 4a + 2b + 24 = 0$ $P(1) = 1^4 + a + b + 8$ $= a + b + 9 = 0$ Solving (using GDC or otherwise), $a = -3$ and $b = -6$	Generally well attempted
2.	(a) If $\cos(x-\alpha) = k\cos(x+\alpha)$, express $\tan x$ in terms of k and $\tan\alpha$. (b) Hence find the value(s) of x, where $0° \le x < 360°$, when $k = \frac{1}{2}$ and $\alpha = 70°$. Give your answer(s) to the nearest degree.	8
	(a) $\cos(x-\alpha) = k\cos(x+\alpha)$ $\cos x\cos\alpha + \sin x\sin\alpha = k(\cos x\cos\alpha - \sin x\sin\alpha)$ $\cos x\cos\alpha - k\cos x\cos\alpha = -\sin x\sin\alpha + k(-\sin x\sin\alpha)$ $\frac{\sin x}{\cos x} = \frac{(1-k)\cos\alpha}{(-1-k)\sin\alpha}$ $\tan x = \frac{(k-1)}{(k+1)\tan\alpha}$ (b) $\tan x = \frac{1-0.5}{(-0.5-1)\times\tan 70°}$ $= -0.1213$ [A1] $x = 180° - tan^{-1}(0.1213),\ \ 360° - tan^{-1}(0.1213)$ $= 173°, 353°$ (to 3 sf)	Well attempted
3.	(a) Describe a sequence of **two** geometric transformations which maps the function $g(x) = 3^{2x+1}$ to $y = \frac{1}{g(x)}$. [4 marks] (b) If $\frac{1}{g(x)}$ intersects $y = -x^2 + k$ at only one point, find the value of k corrected to 3 decimal places. [4 marks]	8
	(a) 1. Reflect $f(x)$ in the x-axis. 2. Stretch by scale factor 4 parallel to the y-axis. **<u>Method 1:</u>** $\frac{1}{g(x)} = 3^{-2x-1} = 3^{[2(-x)+1]-2}$ $= \frac{1}{9} \times 3^{[2(-x)+1]} = \frac{1}{9}g(-x)$ 1. Reflect $g(x)$ about the y-axis. 2. Scale by factor $\frac{1}{9}$ parallel to the y-axis. **<u>Method 2:</u>** $\frac{1}{g(x)} = 3^{-2x-1} = 3^{[2(-x)+1]-2}$ $= \frac{1}{9} \times 3^{[2(-x)+1]} = \frac{1}{9}g(-x)$ 1. Scale by factor $\frac{1}{9}$ parallel to the y-axis. 2. Reflect about the y-axis. **<u>Method 3:</u>** $\frac{1}{g(x)} = 3^{-2x-1} = 3^{[2(-x-1)+1]}$ $= \frac{1}{9} \times 3^{[2(-x)+1]} = \frac{1}{9}g(-x)$	Many students did not show any working to show the sequence of transformation .

	1. Translate 1 unit parallel to the x-axis. 2. Reflect about the y-axis. **Method 4:** $\frac{1}{g(x)} = 3^{-2x-1} = 3^{[2(-(x+1))+1]}$ $= \frac{1}{9} \times 3^{[2(-x)+1]} = \frac{1}{9} g(-x)$ 1. Reflect about the y-axis. 2. Translate -1 unit parallel to the x-axis. (b) $3^{-2x-1} = -x^2 + k$ $3^{-2x-1} + x^2 = k$ [M1: attempt to formulate equation]  The minimum point is at *x* = 0.254 Hence, *k* = 0.254	Not well attempted. A lot of students do not think of finding the minimum point of the graph y= $3^{-2x-1} + x^2$
4.	Solve the simultaneous equations $iz + 2w = 0$ and $z - w^* = 3$, giving z and w in the form of $a + ib$, where a and b are real. [6 marks]	6
	$z = 3 + w^*$ $i(3 + w^*) + 2w = 0$ Let $w = a + ib$, $i(3 + a - ib) + 2a + 2ib = 0$ $(b + 2a) + i(3 + a + 2b) = 0$ Comparing real and imaginary parts, $b + 2a = 0$ and $3 + a + 2b = 0$ Solving (using GDC or otherwise), $a = 1, b = -2$ $w = 1 - 2i$ $z = 4 + 2i$	Quite a number of students do not fully understand the concept of *w** and mistakenly assume *w**= -*w* or *iw* or $\frac{1}{w}$
5.	A curve has equation $y = \frac{1}{x} + \frac{2}{x^2}, x \neq 0$. (a) Using differentiation or otherwise, find the coordinate of the non-stationary inflexion point of the curve. [3 marks] (b) Sketch the curve, stating the equations of any asymptotes and the coordinates of any turning points and points of intersections with the axes. [3 marks] (c) Hence find the solution set of the inequality $-\frac{2}{9}x^2 + x + \frac{20}{9} > \frac{1}{x} + \frac{2}{x^2}$. [2 marks]	8
	(a) $y = \frac{1}{x} + \frac{2}{x^2}$ $\frac{dy}{dx} = -\frac{1}{x^2} - \frac{4}{x^3}$	Quite a number of students find the turning

	$\frac{d^2y}{dx^2} = \frac{2}{x^3} + \frac{12}{x^4} = 0$ Using GDC,  OR solving $\frac{2}{x^3} + \frac{12}{x^4} = 0$ $x = -6 \;\; or \; 0(reject)$ $y = -\frac{1}{9}$ (b)  [Correct shape and asymptotes x = 0 and y = 0] [min. point (-4, -1/8), inflexion (-6, -1/9)] [x-intercept (-2, 0)] (c)  Reading off from the intersection points and vertical asymptotes $-1.5 < x < -1 \qquad or \quad 1 < x < 6$	point instead of the point of inflexion. Many students did not indicate the minimum point in their answer. Quite a number of students only show one set of solutions but left out the solution

		$-1.5 < x <$ -1
6.	Two boats departs from the same pier at right angles to each other. The faster boat travels at 8km/h and the slower boat travels at 6km/h. (a) Show that the rate at which the distance between the boats changes is independent of time. [3 marks] (b) A cruise ship departs from the same pier according to $x(t) = 3t - 1, y(t) = 2t + 3$, where the distance units are km and the time units are hours. Find the speed of the cruise ship. [2 marks]	5
	(a) Relative distance, $s = \sqrt{(f(t)^2 + s(t)^2)} = \sqrt{(8^2 + 6^2)}\, t$ $= 10t$ km Rate of change of distance, $\frac{ds}{dt} = 10\ km/h.$ (Hence rate is independent of time.) (b) Direction vector of the cruise ship $= \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ Speed of the cruise ship $= \sqrt{3^2 + 2^2} = \sqrt{13}$	Not well attempted. Many students could not formulate the equation S= $\sqrt{(8^2 + 6^2)}\, t$
7.	The plane π has equation $3x - y - 2z = -28$. P is a point on the plane with coordinates $(-4, 6, 5)$.  A line perpendicular to the plane passes through P, find the exact value of the shortest distance from the line to the origin.	5
	Method 1. $\lvert\overrightarrow{OP}\rvert = \left\lvert\begin{pmatrix} -4 \\ 6 \\ 5 \end{pmatrix}\right\rvert = \sqrt{4^2 + 6^2 + 5^2} = \sqrt{77}$ Perpendicular distance of plane to origin $= \frac{\lvert d\rvert}{\lvert\boldsymbol{n}\rvert} = \frac{28}{\sqrt{3^2+1^2+2^2}} = \frac{28}{\sqrt{14}} = 2\sqrt{14}$ By Pythagoras Theorem, taking $\overrightarrow{OP}$ as the hypotenuse or otherwise, shortest distance = $\sqrt{\sqrt{77}^2 - (2\sqrt{14})^2} = \sqrt{21}$ **Method 2.** Let $\overrightarrow{OF}$ be the vector perpendicular to the plane. $\overrightarrow{OF} = m\boldsymbol{n} = m\begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}$ Since $\overrightarrow{OF}$ lies on the plane, $\begin{pmatrix} 3m \\ -m \\ -2m \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} = -28$ $9m + m + 4m = -28$ $m = -2$ $\overrightarrow{OF} = -2\boldsymbol{n} = \begin{pmatrix} -6 \\ 2 \\ 4 \end{pmatrix}$ Shortest distance = $\lvert\overrightarrow{PF}\rvert = \left\lvert\begin{pmatrix} -6 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \\ 5 \end{pmatrix}\right\rvert = \left\lvert\begin{pmatrix} -2 \\ -4 \\ -1 \end{pmatrix}\right\rvert$ $= \sqrt{2^2 + 4^2 + 1^2} = \sqrt{21}$	Fairly well attempted. Recommend that students sketch to visualise the question. Common error to substitute the line equation into the plane. Ie. $\begin{pmatrix} -4 + 3m \\ 6 - m \\ 5 - 2m \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} = -28$ Note that the above will give $m = 0$ since the intersection is just P(-4, 6, 5).

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Method 3.

Equation of line passing through P, perpendicular to the plane

$$l: \boldsymbol{r} = \begin{pmatrix} -4 \\ 6 \\ 5 \end{pmatrix} + m\begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} -4+3m \\ 6-m \\ 5-2m \end{pmatrix}, m \in \boldsymbol{R}$$

The foot of the perpendicular, N is when line is perpendicular to the normal of the plane.

$$\begin{pmatrix} -4+3m \\ 6-m \\ 5-2m \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} = 0$$

$$m = -2$$

$$\overrightarrow{ON} = \begin{pmatrix} -4+3(2) \\ 6-2 \\ 5-2(2) \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}$$

Shortest distance = $\left|\overrightarrow{ON}\right| = \left|\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}\right| = \sqrt{2^2+4^2+1^2} = \sqrt{21}$

</td><td>Question requires the exact value and the answer 4.58 is penalised.</td></tr>
<tr><td>8.</td><td>

Given that $z_1 = 2$ and $z_2 = 1 + i\sqrt{3}$ are roots of the cubic equation $z^3 + az^2 + bz + c = 0$ where a, b, $c \in$ R,

(i) write down the third root, z_3, of the equation;

(ii) find the values of a, b and c.

(iii) Explain why $z^4 - 6z^3 + 16z^2 - 24z + 16 = 0$ has **three distinct** roots, z_1, z_2 and z_3.

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(i) $1 - i\sqrt{3}$

(ii) $\left(z - \left(1 + i\sqrt{3}\right)\right)\left(z - \left(1 - i\sqrt{3}\right)\right)$

$= z^2 - 2z + 4$

$(z-2)(z^2 - 2z + 4)$

$= z^3 - 4z^2 + 8z - 8$

(iii) $z^4 - 6z^3 + 16z^2 - 24z + 16 = (z-2)(z^3 - 4z^2 + 8z - 8)$

$= (z-2)^2\left(z - \left(1 + i\sqrt{3}\right)\right)\left(z - \left(1 - i\sqrt{3}\right)\right)$

Hence **three distinct** roots, z_1, z_2 and z_3.

</td><td>Well attempted. Most were able to observe the repeated root in 8c).</td></tr>
<tr><td>9.</td><td>

A group of 6 men and 6 women are assigned to sit in three distinct rows of four seats each.

(a) Find the number of ways in which the 12 people can be arranged if there is no restriction. [2 marks]

(b) Find the number of ways to arrange the 12 people if each row has at least one woman.
[4 marks]

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(a) No. of ways = 12! = 479001600

(b) Method 1.

No. of ways that 12 people are seated with no women in at least one of the rows

$= \binom{6}{4} \times 4! \times \binom{3}{1} \times 8!$ [select a row of all men $\binom{6}{4} \times 4!$, arrange the rest]

No. of ways $= 12! - \binom{6}{4} \times 4! \times \binom{3}{1} \times 8!$ [Complement set]

$= 435456000$

Method 2.

No. of ways $= 12! - \binom{8}{6} \times 6! \times \binom{3}{1}$

$= 435456000$

Method 3.

</td><td>9b) is poorly attempted.

Most students were not able to use the complement set to solve the question.</td></tr>
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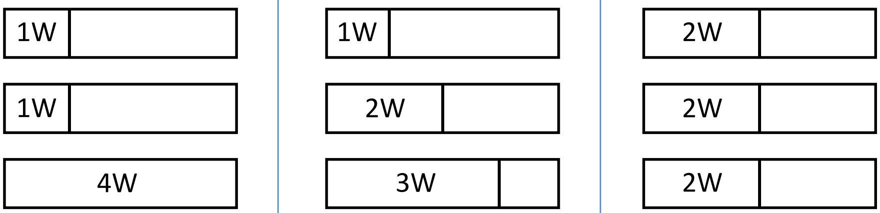
Case 1	Case 2	Case 3



Case 1	Case 2	Case 3
${}^{6}C_1 \times {}^{6}C_3 \times 4!$ $\times {}^{5}C_1 \times {}^{3}C_3 \times 4!$ $\times {}^{4}C_4 \times 4!$ $\times 3$ = 24,883,200	${}^{6}C_1 \times {}^{6}C_3 \times 4!$ $\times {}^{5}C_2 \times {}^{3}C_2 \times 4!$ $\times {}^{3}C_3 \times {}^{1}C_1 \times 4!$ $\times 3!$ = 298,598,400	${}^{6}C_2 \times {}^{6}C_2 \times 4!$ $\times {}^{4}C_2 \times {}^{4}C_2 \times 4!$ $\times {}^{2}C_2 \times {}^{2}C_2 \times 4!$ = 111,974,400

Total no. of ways = 24,883,200 + 298,598,400 + 111,974,400
= 435,456,000

Topic	Section B Questions/ Solutions	Marks
10.	(a) Find the values of h for which the following system of equations has i. no solution, ii. an infinite number of solutions. $\pi_1: x + 2y - 2z = -2$ $\pi_2: -x + 2y - z = 5$ $\pi_3: -4y + 3z = h$ [4 marks] (b) Given that the system of equations can be solved, find the line of intersection of the planes with direction vector $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$. . [5 marks] (c) Show that the line $\frac{x-4}{2} = z + 2, \ \ y = -5$ lies on the plane π_1. [5 marks] (d) The z-axis meets the plane π_2 at the point P. i. Find the coordinates of P. [2 marks] ii. Hence find the coordinates of F, the foot of the perpendicular from P to π_1. [5 marks]	21

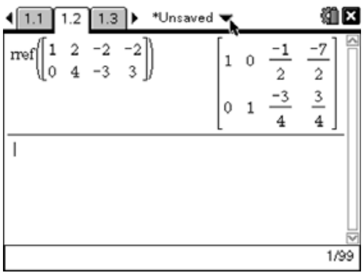
(a) $\begin{pmatrix} 1 & 2 & -2 & -2 \\ -1 & 2 & -1 & 5 \\ 0 & -4 & 3 & h \end{pmatrix} \xrightarrow{R2+R3} \begin{pmatrix} 1 & 2 & -2 & -2 \\ 0 & 4 & -3 & 3 \\ 0 & -4 & 3 & h \end{pmatrix}$

$\xrightarrow{R3+R2} \begin{pmatrix} 1 & 2 & -2 & -2 \\ 0 & 4 & -3 & 3 \\ 0 & 0 & 0 & h+3 \end{pmatrix}$

i. There is no solution when $h \neq -3$.
ii. There is an infinite number of solutions when $h = -3$.

10a) Students should be the proper presentation of the augmented matrix for their solution.

A number of students were not aware of the condition for no solution.

(b) 

[Substitute $h = -3$]

$rref \begin{pmatrix} 1 & 0 & -\frac{1}{2} & -\frac{7}{2} \\ 0 & 1 & -\frac{3}{4} & \frac{3}{4} \end{pmatrix}$

	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{7}{2} \\ \frac{3}{4} \\ 0 \end{pmatrix} + z\begin{pmatrix} \frac{1}{2} \\ \frac{3}{4} \\ 1 \end{pmatrix}$ $= \begin{pmatrix} -\frac{7}{2} \\ \frac{3}{4} \\ 0 \end{pmatrix} + m\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \ , m \in \boldsymbol{R}$ (c) $\frac{x-4}{2} = z + 2,\ \ y = -5$ $l{:}\,\boldsymbol{r} = \begin{pmatrix} 4 \\ -5 \\ -2 \end{pmatrix} + h\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, h \in \boldsymbol{R}$ $\begin{pmatrix} 4 \\ -5 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = -2$ Position vector is on π_1. $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = 0$ The direction vector is perpendicular to the normal vector of π_1. Hence line lies on π_1. (d) i) $\pi_2{:}\,\boldsymbol{r} \cdot \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = 5$ At the z-axis, $P = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}$. Since P lines on π_2, $\begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = 5$ $z = -5$ or $P = \begin{pmatrix} 0 \\ 0 \\ -5 \end{pmatrix}$. [A1] ii) Line passing through PF, $l{:}\,\boldsymbol{r} = \begin{pmatrix} 0 \\ 0 \\ -5 \end{pmatrix} + k\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} k \\ 2k \\ -5-2k \end{pmatrix}, k \in \boldsymbol{R}$ $\begin{pmatrix} k \\ 2k \\ -5-2k \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = -2$ $k + 4k + 10 + 4k = -2$ $k = -\frac{4}{3}$ $F = \begin{pmatrix} -\frac{4}{3} \\ -\frac{8}{3} \\ -2\frac{1}{3} \end{pmatrix}$	
Z 11.	A function is defined by $4y^2 + 8y + x^2 = 1$. (a) Show that $\frac{dy}{dx} = \frac{-x}{4y+4}$. [3 marks] (b) Find the equation of the normal to the curve at the point $\left(\frac{1}{4}, -\frac{1}{8}\right)$. [4 marks] (c) Find the distance between the two points on the curve where each tangent is parallel to the line $y = \frac{1}{4}x$. [7 marks] (d) By completing the square or otherwise, show that the curve can be expressed in the form $(y-h)^2 + \frac{x^2-a}{k} = 1$, where *a*, *h* and *k* are integers to be determined. [4 marks] (e) Sketch the curve, stating the coordinates of any stationary point(s) and point(s) of intersection(s) with the axes. [5 marks]	23
	(a) $4y^2 + 8y + x^2 = 1$ $8y\frac{dy}{dx} + 8\frac{dy}{dx} + 2x = 0$	

$8\frac{dy}{dx}(y+1)+2x=0$

$\frac{dy}{dx}=\frac{-x}{4y+4}$

(b) $\frac{dy}{dx}=\frac{-\frac{1}{4}}{4\left(-\frac{1}{8}\right)+4}=\frac{-1}{14}$

Gradient of normal = $\frac{1}{\frac{1}{14}}=14$

Equation of the normal $y+\frac{1}{8}=14\left(x-\frac{1}{4}\right)$

$y=14x-\frac{29}{8}$

(c) Equate $\frac{dy}{dx}=\frac{-x}{4y+4}=\frac{1}{4}$

$x=-y-1$

Substituting into original equation

$4y^2+8y+(-y-1)^2=1$

$5y^2+10y=0$

$y=0 \qquad or \qquad y=-2$

Points $(-1,0)$ and $(1,-2)$.

Distance = $\sqrt{((2)^2+(1+1)^2)}$

= $2\sqrt{2}$ =2.83 (to 3 s.f)

(d) $4y^2+8y+x^2=1$

$4(y^2+2y+1-1)+x^2-1=0$

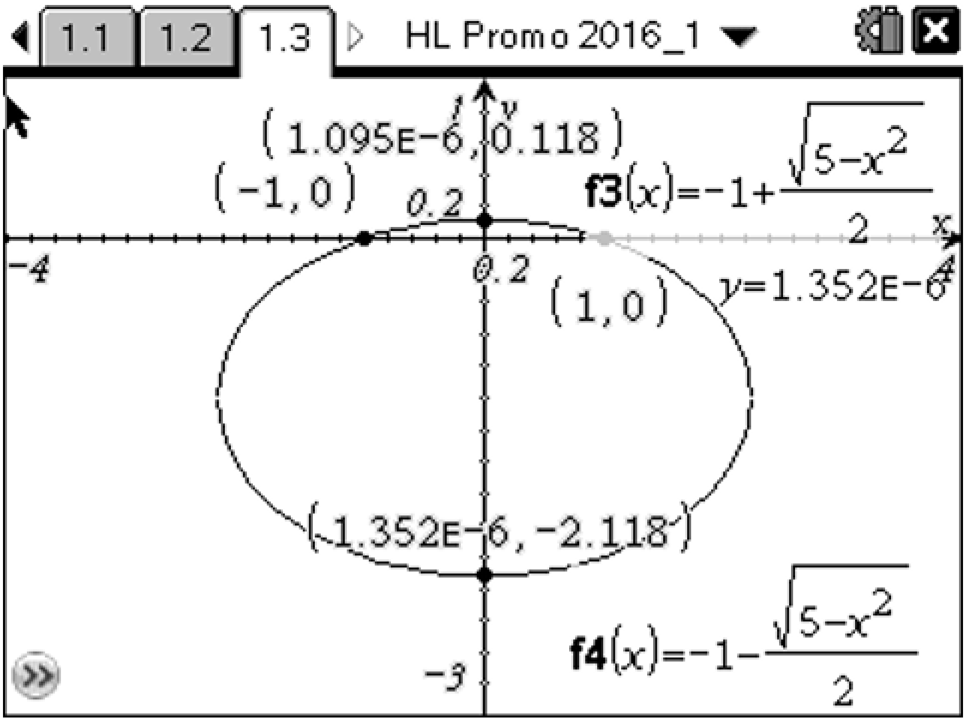
$(y+1)^2-1+\frac{x^2-1}{4}=0$

$(y+1)^2+\frac{x^2-1}{4}=1$

A number of students were not able to complete the square for 11d). An alternative method would be to use the quadratic formula.

(e) Rearranging the equation

$y=-1\pm\sqrt{\left(1-\frac{x^2}{4}\right)}$



11e). A number of students used the wrong equation to sketch the graph. An even greater number didn't realised that $\pm$ accompanies the square root in this case and sketch only the positive half of the function.

Minimum point (0, -2.12) to 3s.f

Maximum point (0, 0.118) to 3s.f

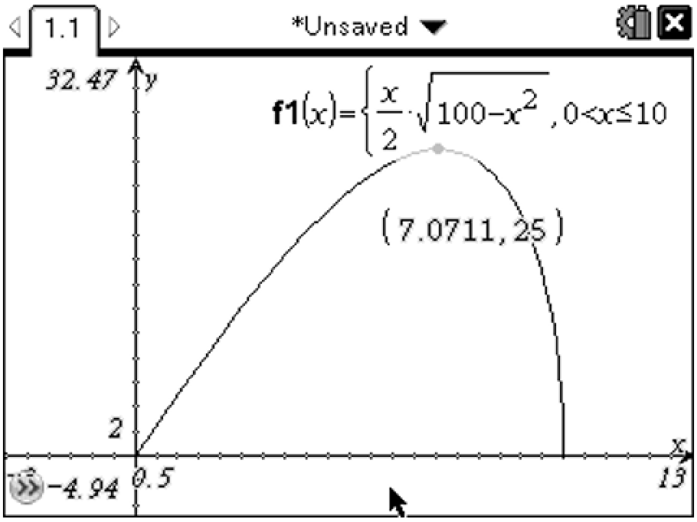
Axial intercepts: $(-1,0)$, $(1,0)$

12. (a) Differentiate the following with respect to *x*: — 16

i. $y=\arccos x+2\sqrt{1-x^2}$, $x\in[-1,1]$ [3 marks]

ii. $y=\sin^2\left(\frac{1}{x}\right)$ [4 marks]

	(b) A ladder is sliding down along a vertical wall. The ladder is 10 meters long and the top is slipping at the constant rate of 10 m/s. (i) How fast is the bottom of the ladder moving along the ground when the bottom is 6 meters from the wall? [4] (ii) If A is the area under the ladder formed by the ladder, wall and ground, how far is the bottom of the ladder away from the wall when A is maximum? [3] (iii) State the range of values the bottom of the ladder will be from the wall when A is decreasing. [2]	
	(a) (i) $y = \arccos x + 2\sqrt{1-x^2}$ Differentiating with respect to *x*, $\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}} - \frac{2x}{\sqrt{1-x^2}}$ $= -\frac{1+2x}{\sqrt{1-x^2}}$ (ii) $y = \sin^2\left(\frac{1}{x}\right)$ Differentiating with respect to *x*, $\frac{dy}{dx} = 2\sin\left(\frac{1}{x}\right) \times \cos\left(\frac{1}{x}\right) \times \frac{-1}{x^2} \quad = -\frac{\sin\left(\frac{2}{x}\right)}{x^2}$ (b) (i) Method 1 By Pythagorean Theorem, $x^2 + y^2 = 10^2$ When *x* = 6, $6^2 + y^2 = 10^2$ $y = 8$ $y^2 = 10^2 - x^2$ $2y\frac{dy}{dx} = -2x$ $\frac{dy}{dx} = -\frac{x}{y}$ $\frac{dx}{dt} = \frac{dy}{dt} \div \frac{dy}{dx}$ $= -10 \div \frac{-6}{8} = 13\frac{1}{3}ms^{-1}$ Method 2 By Pythagorean Theorem, $x^2 + y^2 = 10^2$ When *x* = 6, $6^2 + y^2 = 10^2$ $y = 8$ $x^2 + y^2 = 10^2$ Differentiate wrt *t* $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ Sub. $\frac{dy}{dt} = -10, x = 6, y = 8$ $2(8)(-10) + 2(6)\frac{dx}{dt} = 0$ $\frac{dx}{dt} = 13\frac{1}{3}ms^{-1}$ (ii) Since $x^2 + y^2 = 10^2$, $y = \sqrt{100 - x^2}$ $A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{100 - x^2}$ Method 1.	12ai). differentiation of $2\sqrt{1-x^2}$ was poorly attempted with many careless mistakes. 12aii). Overall poorly attempted. Most students could not express the solution in the simplest form by using the double-angle formula. 12bi). As the ladder slips faster when it moves away from the wall, $\frac{dx}{dt}$ is positive. Also note that The top of the ladder decreases in height as the ladder slips. Hence $\frac{dy}{dt} = -10$. 12bii). Students should be aware that due to the marks allocated, the solution involves that use of GDC



[Sketch to show max]

$x = 7.07m$ (to 3s.f)

Method 2.

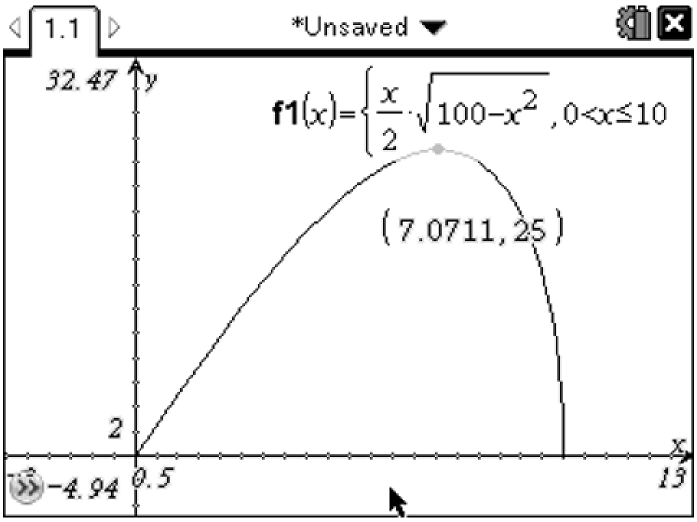
$\frac{dA}{dx} = \frac{1}{2}\left(\frac{-2x^2}{2\sqrt{100-x^2}} + \sqrt{100-x^2}\right) = 0$

$\frac{x^2}{\sqrt{100-x^2}} = \sqrt{100-x^2}$

$x^2 = 100 - x^2$

$2x^2 = 100$

$x = \sqrt{\frac{100}{2}} = 7.07m$ (to 3s.f)



(iii)

From graph, A is decreasing in $7.07m < x \leq 10m$.

rather than differentiation by formula.

GDC method should include evidence of method – graph.

12biii). Solution should involve the use of GDC to find the maximum point (x = 7.07m) as differentiation of the function is complicated.

Note that the function continues to decrease pass x = 10m.

~End~