Photoelectric Effect

1. Estimate the rate of emission of photons emitted by a 4.00 mW red laser (632 nm).

$$P = \frac{Nhf}{t} = \frac{Nhc}{t\lambda}$$

$$\frac{N}{t} = \frac{P\lambda}{hc} = \frac{4.00\times10^{-3}\times632\times10^{-9}}{6.63\times10^{-34}\times3.00\times10^{8}} = 1.27\times10^{16}\ \text{s}^{-1}$$

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2. Light of wavelength 3.82×10^{-7} m is incident on a substance and electrons are emitted with a maximum speed of 6.87×10^{5} m s^{-1}. Calculate the work function[1] energy of the substance. [N07 H2 P3 Q7c]

Using Einstein's photoelectric equation,

$$hf = \Phi + KE_{\max}$$

$$\Phi = \frac{hc}{\lambda} - KE_{\max}$$

$$= \frac{6.63\times10^{-34}\times3.00\times10^{8}}{3.82\times10^{-7}} - \frac{1}{2}\left(9.11\times10^{-31}\right)\left(6.87\times10^{5}\right)^2$$

$$= 3.06\times10^{-19}\ \text{J}$$

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3 When light of wavelength 350 nm falls on a potassium surface, electrons are emitted that have a maximum kinetic energy of 1.31 e V. Determine
(a) the work function of potassium,
(b) the cut-off wavelength,
(c) the frequency corresponding to the cut-off wavelength.

(a) Using Einstein's photoelectric equation,

$$\frac{hc}{\lambda} = \Phi + KE_{\max} \Rightarrow \Phi = \frac{hc}{\lambda} - KE_{\max} = \left(\frac{6.63\times10^{-34}\times3.00\times10^{8}}{350\times10^{-9}}\right) - \left(1.31\times1.60\times10^{-19}\right) = 3.59\times10^{-19}\ \text{J}$$

(b) At cut-off wavelength λ_c, $KE_{\max} = 0$

$$\frac{hc}{\lambda_c} = \Phi \Rightarrow \lambda_c = \frac{hc}{\Phi} = \frac{6.63\times10^{-34}\times3.00\times10^{8}}{3.59\times10^{-19}} = 554\times10^{-9}\ \text{m} = 554\ \text{nm}$$

(c) Threshold frequency

$$f_c = \frac{c}{\lambda_c} = \frac{3.00\times10^{8}}{554\times10^{-9}} = 5.42\times10^{14}\ \text{Hz}$$

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[1] There are conduction electrons in a metal which are not attached to a particular atom. Removing these electrons from a metal surface using a photon involves a different amount of energy from that for ionizing a single metal atom. Thus, the work function energy of a metal differs from the energy required to remove the outer electron from an isolated atom. There is a simple connection between the work function energy Φ of a solid metal and the first ionization energy $(E_i)_1$ of its corresponding gaseous atoms: $\Phi \sim ½(E_i)_1$.

4 In an experiment on photoelectric effect, a beam of light is used and the voltage across the electrodes required to stop the photocurrent is 1.5 V.

(a) Determine the maximum speed of the photoelectrons.

(b) Sketch a graph to illustrate how the photocurrent varies with the voltage across the electrodes and label it **(b)**.

(c) With reference to photoelectrons, explain the significance of the sloping section of your graph for negative values of potential difference.

(d) The metal is replaced by another with a greater work function. Add a new line to the graph above to indicate how the photocurrent now varies with the voltage and label it **(d)**.

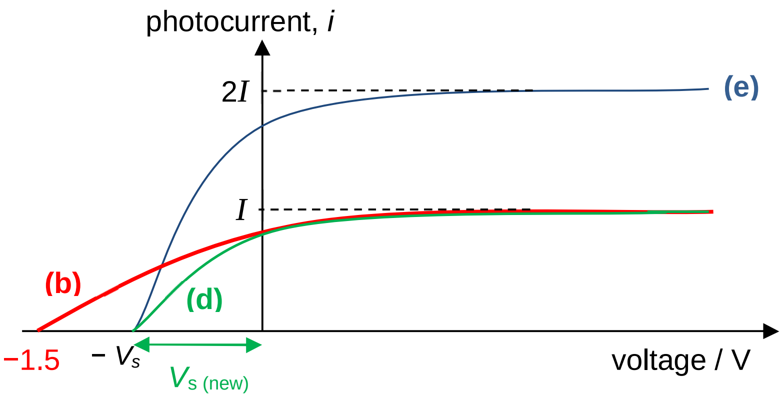
(e) With this new metal, the intensity of the light is now doubled. Add another line to the graph to indicate how the photocurrent now varies with the voltage and label it **(e)**.

(a)

$$eV_s = \frac{1}{2}mv_{max}^2$$

$$v_{max} = \sqrt{\frac{2eV_s}{m}} = \sqrt{\frac{2(1.60\times10^{-19})(1.5)}{9.11\times10^{-31}}} = 7.26\times10^5\ \text{m s}^{-1}$$

(b) (d), (e)



(c) It shows that electrons are emitted with a range[2] of kinetic energy.

5 In a photoelectric emission experiment, ultraviolet radiation of wavelength 254 nm and intensity 210 Wm^{-2}, was incident on a silver surface in an evacuated tube, so that an area of 12 mm^2 was illuminated. A photocurrent of 4.8×10^{-10} A was collected at an adjacent electrode.

(a) What was the rate of incidence of photons on the silver surface?

(b) What was the rate of emission of electrons?

(c) The photoelectric quantum yield is defined as the ratio of

$$\frac{\text{number of photoelectrons emitted per second}}{\text{number of photons incident per second}}$$

(i) Find the quantum yield of this silver surface at wavelength of 254 nm.

[2] This shows that while some photoelectrons are able to overcome the negative potential difference applied across the two plates to contribute to the current, less energetic electrons (electrons with less kinetic energy) are stopped by the negative potential difference. As the potential difference becomes more negative, more electrons are prevented from reaching the collector, resulting in less photocurrent.

(ii) Give two reasons why this value might be expected to be much less than one.

(d) When the experiment was repeated with the radiation of wavelength 313 nm, no photoelectron was emitted. Explain this observation.

[J83 P1 Q16 part] [3.22×10^{15}, 3×10^{9}, 9.3×10^{-7}]

(a) Intensity $I = \dfrac{N_p hf}{At} = \dfrac{N_p hc}{\lambda At}$

$$\frac{N_p}{t} = \frac{I\lambda A}{hc}$$

$$= \frac{(210)(254\times 10^{-9})(12\times 10^{-6})}{(6.63\times 10^{-34})(3.00\times 10^{8})} = 3.22\times 10^{15} \text{ photons per second}$$

(b) Using the photocurrent,

$$i = \frac{Q}{t} = \frac{N_e e}{t}$$

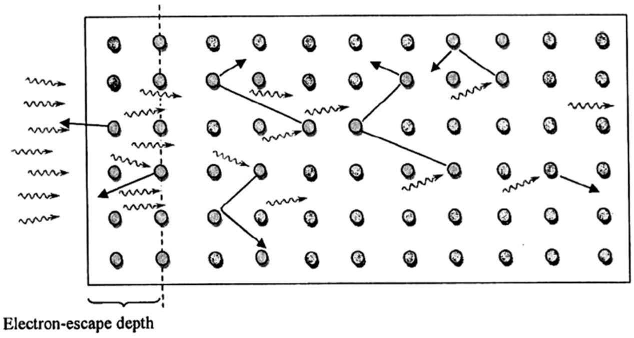
$$\frac{N_e}{t} = \frac{i}{e}$$

$$= \frac{(4.8\times 10^{-10})}{(1.60\times 10^{-19})} = 3.0\times 10^{9} \text{ electrons per second}$$

(c)(i) Quantum Yield $= \dfrac{(3.0\times 10^{9})}{(3.22\times 10^{15})} = 9.3\times 10^{-7}$

(ii) 1. Most of the photons are reflected from the metal surface.

2. Atoms are made up of mostly empty space and thus the probability of a photon hitting a surface electron is very small. Instead, photons may hit electrons which are deeper below the surface, and these electrons may lose all its kinetic energy on its way up to the surface (due to collisions with other free electrons) before they are emitted.



3. Electrons emitted may travel at an angle away from the normal and may not be collected at the anode.

(d) With a longer wavelength, the energy of the incoming photons is lower. If it is lower than the work function energy of the silver surface, then no photoemission occurs.

Energy distribution of photoelectron energy

Photo-emission from a metal surface is a multistep process. When the photon hits the metal surface it excites a photoelectron with a very high quantum yield, but that photoelectron is travelling in the same direction as the photon i.e. down into the metal. For an electron to be emitted from the surface the initial photoelectron has either to ricochet back out of the metal, or more likely to transfer energy to other electrons so one of the other electrons has enough energy to leave the surface. This process is essentially random and consequently has a very low quantum yield, so the overall quantum yield for photoemission is around 10^{-5} to 10^{-6}.

The scattering of the initial photoelectron occurs via inelastic collisions, so whether it's the original electron scattered backwards, or other electrons scattered by the initial photoelectron, the energy of the electron leaving the surface is less that the energy of the photon. The balance of the energy goes into lattice vibrations of the metal i.e. internal energy.

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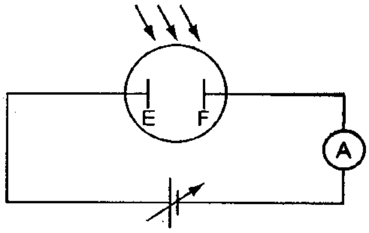
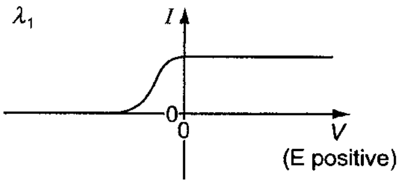
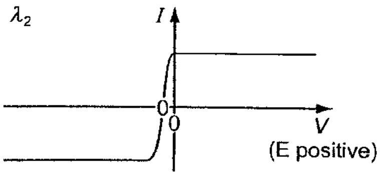
6 The diagram shows a circuit used for photoelectric emission experiments.



The two electrodes E and F are made of different metals. The work function of electrode E is ϕ_E and the work function of electrode F is ϕ_F.
Current-voltage (I-V) characteristics are obtained when both electrodes are illuminated with monochromatic light.
When the wavelength of the light is λ_1, the I-V characteristic is as shown.



When the wavelength of the light is λ_2 the I-V characteristic is as shown.



Explain whether

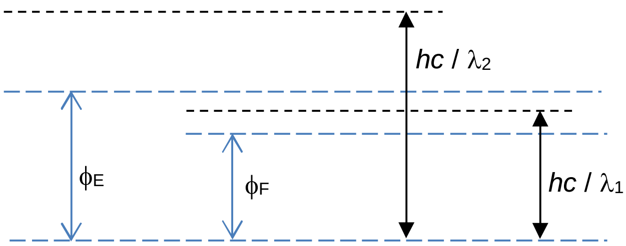
(a) λ_2 is greater or less than λ_1.

(b) ϕ_E is greater or less than ϕ_F.

[N07 H2 P1 Q36 mod]

(a) **λ_2 is less than λ_1.** This is because photons with wavelength of λ_2 have sufficient energy to overcome the work functions of **both** metals to cause electron emission, whereas photons with wavelength of λ_1 could only cause emission in plate F.

(b) **ϕ_E is greater than ϕ_F.** This is because photons with wavelength of λ_1 could cause electron emission only for metal F, but not E.



7

(a) In a photoelectric effect demonstration, photoelectrons are being produced at a rate of 2.7×10^{13} per second. Each electron has energy of 1.2×10^{-19} J.

(i) Calculate the current giving this rate of production of photoelectrons.

(ii) Determine the e.m.f.[3] generated.

[Answer: 4.32×10^{-6} A, 0.75 V]

(b) State, with a reason, what modifications to the apparatus of the demonstration would be required, if separately, to increase

(i) the energy of the photoelectron,

(ii) the rate of production of photoelectrons.

[N92/III/6 mod]

(a) **(i)** $i = \dfrac{Q}{t} = \dfrac{N_e e}{t} = \left(2.7\times10^{13}\ \text{s}^{-1}\right)\left(1.60\times10^{-19}\ \text{C}\right) = 4.32\times10^{-6}\,\text{A}$

(ii) The photoelectrons gain kinetic energy due to energy transfer from the photons. When they move across a retarding potential difference *V*, they do work against an electric field. So, in photelectric experiments, e.m.f. is effectively "measured" by the retarding potential needed to prevent photoelectrons from reaching the other electrode. This retarding potential plays the role of the e.m.f. in terms of how much energy per unit charge is imparted to the photoelectrons: $\text{e.m.f. (in this context)} = V = \dfrac{KE}{e}$

e.m.f. generated = energy per unit charge provided by photons
= (1.2×10^{-19} J) / (1.60×10^{-19} C) = 0.75 V

(b) **(i)** Increase the frequency of the electromagnetic radiation, so that the energy of the incoming photons, and hence the kinetic energy of the photoelectrons, increases.

OR Change the metal to one with a smaller work function energy, so that an photoelectron ejected will get more energy from a photon.

(ii) Increase the intensity of the electromagnetic radiation, so that the number of incoming photons per unit time increases. With more photons hitting the surface per unit time, the rate of photoelectrons produced will increase.

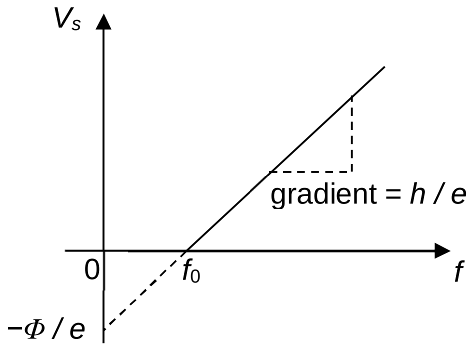
OR Use a cleaner surface for the metal. The state of contamination of the metal surface by adsorbed gas is one important factor which determine the photoelectric yield.

[3] In an electrical circuit, there must be a device somewhere in the loop that acts like the water pump in a water fountain. The influence in this device that makes a charge travels “uphill” from lower to higher potential energy (even though the electrostatic force is trying to push it from higher to lower potential energy) is called electromotive force (e.m.f.) and a circuit device that provides e.m.f. is called a source of e.m.f. Note that “electromotive force” is a poor term because e.m.f. is not a force but an energy-per-unit-charge quantity, like potential.

8 (a) With reference to the photoelectric effect experiment, sketch a graph to show the variation of the stopping potential with the frequency of radiation.

(b) Explain the change in the graph if
(i) the metal surface is replaced by one with a greater work function,
(ii) the intensity of light increases (frequency is kept constant).

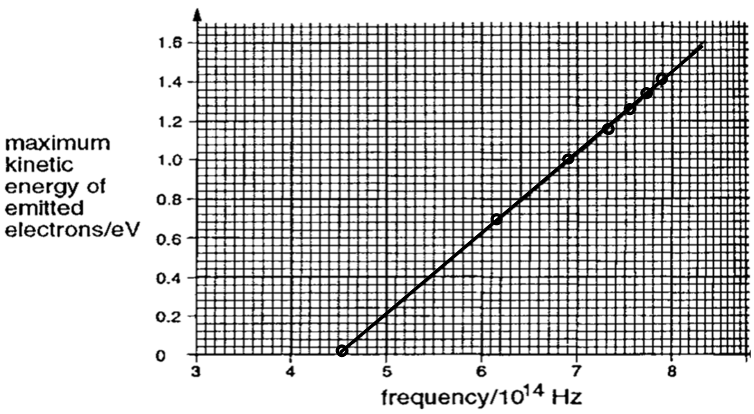
(a)



(b) **(i)** Using $V_S = \frac{h}{e}f - \frac{\Phi}{e}$, the vertical intercept will be more negative, but the gradient of the line remains unchanged.

(ii) If the intensity of light increases and the frequency of light remains the same, there will be no change in the graph. This is because although the number of photons incident on the metal surface per unit time increases, the energy of each incident photon or the energy absorbed by each electron does not change. Hence, V_s is independent of the intensity of the light.

9 (a) The graph below shows the maximum kinetic energies, in electron volts, of electrons emitted from a sodium surface by light of different frequencies from a hydrogen light source.



In calculations on electron emission, an equation which is often used is

photon energy = work function + max K.E. of the emitted electrons

With the help of the graph, determine the numerical values for these three terms in joules when the incident radiation from the hydrogen lamp has a frequency of 6.2×10^{14} Hz. [Ans: 4.11×10^{-19}, 2.96×10^{-19} J, 1.15×10^{-19} J]

(b) The frequencies of light from the lamp, shown by the small circles on the graph are the frequencies obtained in this range. Explain how this shows the existence of discrete energy levels in hydrogen atoms. Without giving numerical values, sketch the pattern of these levels. [N94/III/6 mod]

(a) $KE_{\max} = hf - \Phi$

From the graph, when $f = 6.2\times10^{14}$ Hz,

$KE_{\max} = 0.72$ eV $= 1.15\times10^{-19}$ J

Photon energy= $hf = (6.63\times10^{-34})(6.2\times10^{14}) = 4.11\times10^{-19}$ J

Work function $\Phi = hf - KE_{\max} = 4.11\times10^{-19}\text{ J} - 1.15\times10^{-19}\text{ J} = 2.96\times10^{-19}$ J

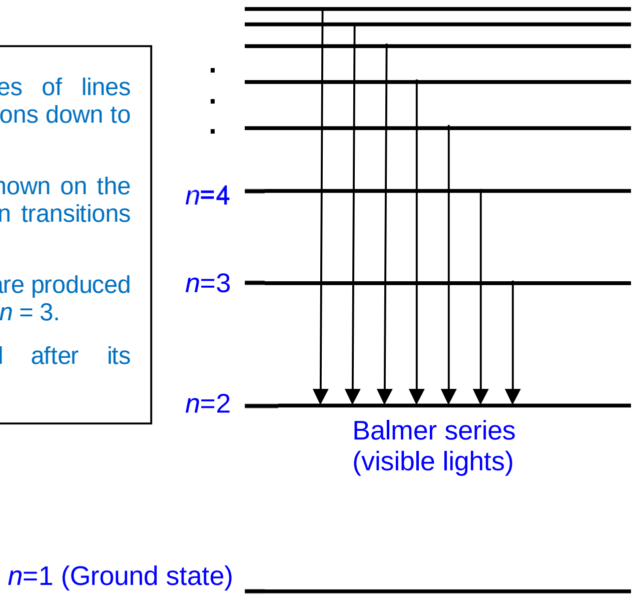
(b) The discrete number of frequencies obtained in the range shows that the photons emitted from the hydrogen lamp has discrete energy values. Energy of photon = *hf*. Photon is emitted when an electron transits from a higher to a lower energy level. Energy of emitted photon is equal to the difference in energy between two energy levels. Discrete energy changes implies the existence of discrete energy levels in hydrogen atoms.

The Lyman (ultraviolet) series of lines corresponds to electron transitions down to the level $n = 1$.

The Balmer (visible) series (shown on the right) are produced by electron transitions down to $n = 2$.

The Paschen (infrared) series are produced by electron transitions down to $n = 3$.

Each series being named after its discoverer.



Wave-particle duality • Energy levels in atoms • Line spectra

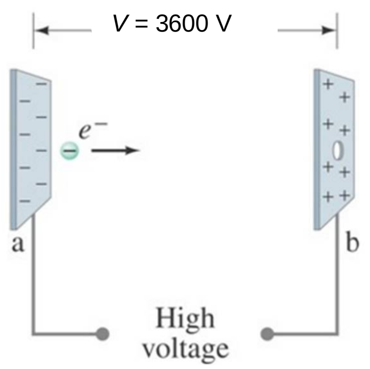
10 (a) Express the de Broglie wavelength of an electron in terms of its kinetic energy, E, the electron mass m, and the Planck constant h.

(b) Estimate the wavelength of a beam of electrons which was accelerated through a p.d of 3600 V.

[Answer: 2.05×10^{-11} m]

(a) $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE_k}$

The de Broglie's wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}$



(b) increase in kinetic energy of electrons
= decrease in electric potential energy

assume the electrons start from rest,

final kinetic energy of electrons = electrical energy supplied

$$= eV$$
$$= (1.60\times10^{-19}\ \text{C})(3600\ \text{V})$$
$$= 5.76\times10^{-16}\ \text{J}$$

The de Broglie's wavelength, $\lambda = \frac{h}{\sqrt{2mE_k}} = \frac{6.63\times10^{-34}}{\sqrt{2(9.11\times10^{-31})(5.76\times10^{-16})}} = 2.05\times10^{-11}$ m

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11 The measured wavelength, λ_m, of selected lines in the hydrogen spectrum are given by empirically by

$$\frac{1}{\lambda_m} = R\left(\frac{1}{4} - \frac{1}{m^2}\right)$$

where R is a constant and has the value 1.097×10^7 m^{-1} and m is an integer taking the values of 3, 4, 5, …, etc.

(a) Calculate the value of the wavelength when m = 4.

(b) Calculate the minimum wavelength given by this equation.

(c) Draw a diagram showing the approximate position of the line on a horizontal axis of wavelength. Mark the value calculated in (a) and the red and violet ends of the spectrum.

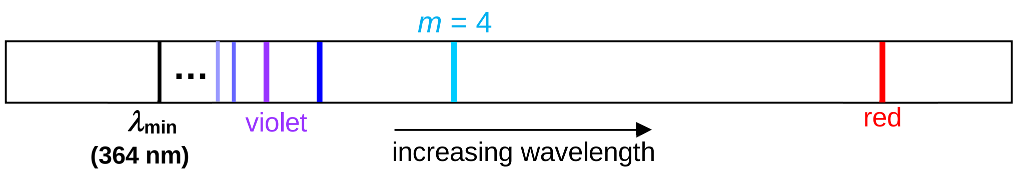
(d) Explain why it is that although there is an infinite number of lines in this spectrum, the spectrum is nevertheless seen as a line spectrum.

[J88/II/12 (c) part] [Answer: 3.64×10^{-7} m]

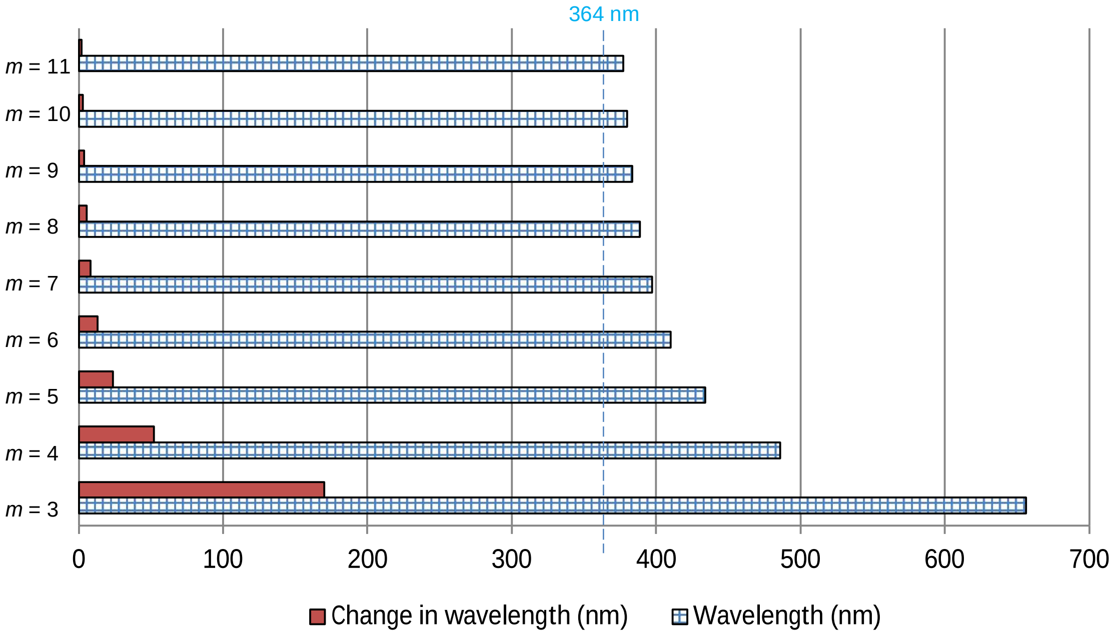
(a) when m = 4, $\lambda = 4.86 \times 10^{-7}$ m (colour bluegreen or cyan)

(b) λ_m is minimum when $\frac{1}{\lambda_m}$ or $\left(\frac{1}{4} - \frac{1}{m^2}\right)$ is maximum, so substitute $m = \infty$ into the given equation $\frac{1}{\lambda_m} = R\left(\frac{1}{4} - \frac{1}{m^2}\right)$ to get $\lambda_{min} = 3.64\times10^{-7}$ m.

(c)



(d) At low integer values of *m* (3 to 8), the calculated wavelengths form distinct lines in the visible range for each *m* value. As *m* increases towards large values, the difference between adjacent spectral lines decreases, and the wavelength of these spectral lines converges to the minimum wavelength value of 364 nm. The infinite continuum for wavelengths below about 380 nm is in the ultraviolet region that they can't be easily seen with naked eyes as it approaches a limit of 364 nm. Thus, the spectrum is nevertheless seen as a line spectrum.



12 re ie

(a) Suggest why using neutrons may be preferable to using electrons when investigating matter.

(b) Calculate the speed of a neutron having a de Broglie wavelength of 2.6×10^{-10} m. The mass of a neutron is 1.7×10^{-27} kg. [Answer: 1.5×10^{3} m s^{-1}]

- Neutrons are electrically neutral particles, while electrons carry a negative charge. The absence of an electric charge in neutrons reduces their interactions with the electron cloud surrounding atoms. This property makes neutrons particularly useful for studying materials that are sensitive to electrical charge, such as certain magnetic materials.
- Neutrons have a much larger mass than electrons. This means that neutrons have a greater penetration power compared to electrons. Neutrons can easily penetrate deep into materials without being significantly deflected or absorbed, allowing us to study the internal structure of thick samples or materials that are difficult to investigate using electrons.

(b) Applying the de Broglie's equation, $\lambda = h / (m\, v)$

$$2.6\times10^{-10} = 6.63\times10^{-34} / (1.7\times10^{-27} \times v)$$

$$v = 1.5 \times 10^{3} \text{ m s}^{-1}$$

13 An atom is assumed to have zero energy in the ground state; and its energy in the first, second and third excited states are: 1.635×10^{-18} J, 1.936×10^{-18} J and 2.019×10^{-18} J respectively.

(a) Determine the wavelength of the photon which would excite the atom from the first excited state to the second excited state.

(b) A blue line of wavelength 5.18×10^{-7} m is observed in the spectrum of this atom. Determine the transition between energy levels that will give rise to this spectral line.

(c) Suggest whether light incident on a metal surface exerts a pressure on the surface.

(d) The atom in the ground state is bombarded with

(i) electrons of energy 1.800×10^{-18} J

(ii) photons of energy 1.800×10^{-18} J

State whether any transition takes place in each case.

[Answer: 661 nm, 3.84×10^{-19} J]

(a) Difference in energy, $E_2 - E_1 = h\,c\,/\,\lambda$

$$(1.936\times10^{-18} - 1.635\times10^{-18}) = (6.63\times10^{-34})\,(3\times10^{8})\,/\,\lambda$$

$$\lambda = 661 \text{ nm}$$

(b) Energy of photon causing the blue line $= h\,c\,/\,\lambda = (6.63\times10^{-34})\,(3\times10^{8})\,/\,(5.18\times10^{-7})$

$$= 3.84\times10^{-19} \text{ J}$$

Thus, the blue line is formed by a photon emitted when an electron transits from the third excited state to the first excited state.

(c) Since light possesses momentum $p = h\,/\,\lambda$, when light is incident on the surface of a piece of metal, its momentum changes. According to Newton's laws, light exerts a force on the metal that is proportional to the rate of change of momentum. This force acting on unit area of the metal exerts a pressure on the metal.

(d) **(i)** Yes. The atom may collide with an electron and absorb 1.635×10^{-18} J which is the amount needed to put the atom in the first excited state. The remaining kinetic energy of the colliding electron is (1.800×10^{-18} J – 1.635×10^{-18} J) = 0.165×10^{-18} J. The atom in the first excited state then spontaneously emits a photon and drops from first excited state to the ground state.

(ii) No. No collisional excitation would occur. An atom in the ground state may absorb a photon of energy *hf* exactly equal to the energy difference ΔE between the ground state and the excited states. Photons of energy 1.800×10^{-18} J does not match any of ΔE values.

14 (a) Describe how line spectra can be explained using the idea of discrete electron energy levels in isolated atoms.

(b) State the conditions under which atoms are sufficiently isolated so that line spectra can be observed.

(c) Explain the difference between how the emission line spectrum and the absorption line spectrum are produced.

(d) Explain what can be deduced from observing the line spectrum.

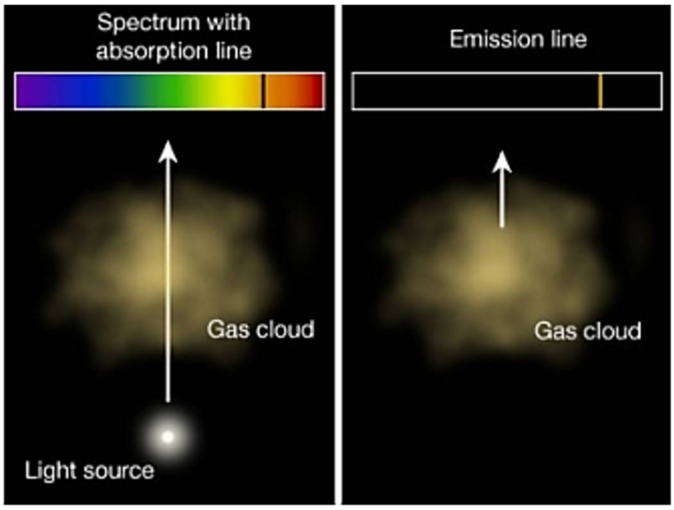
(e) Suggest a practical application of observing the line spectrum.

(a) [Emission spectra] Energy of the photon is proportional to its frequency $E = hf$. Each spectral line corresponds to a specific photon energy. When electron transits from one higher level to another lower energy level, a photon is released. Energy of emitted photon is equal to the difference in energy between the two energy levels. Discrete energy changes implies discrete electron energy levels in isolated atoms.

(b) Atoms must be in gaseous form/vaporized.

A low [4] density/pressure, hot gas seen against a cooler background emits a BRIGHT LINE or EMISSION LINE spectrum. (Right figure.)

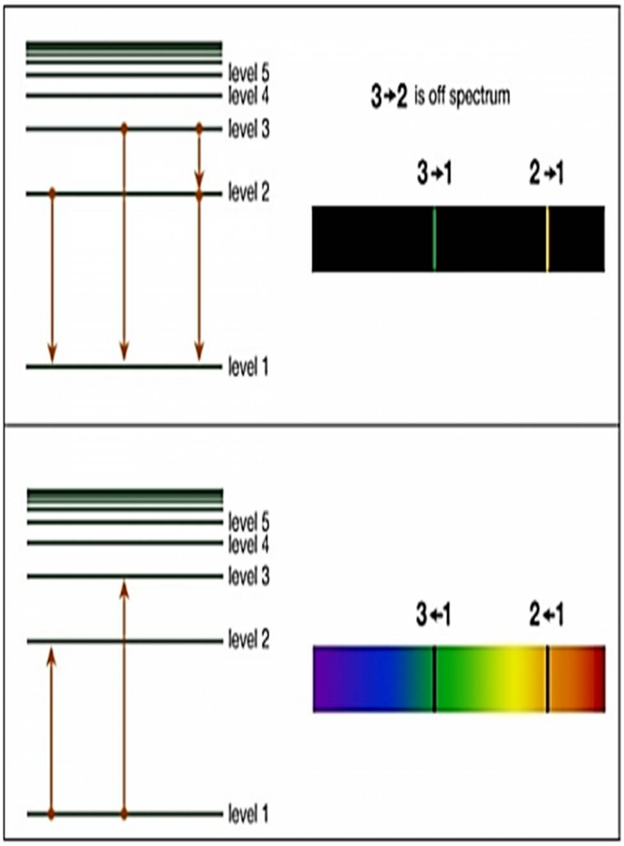
A low density/pressure, cool gas in front of a hotter source of a continuous spectrum creates a DARK LINE or ABSORPTION LINE spectrum. (Left figure.)



[A cloud of gas in space, such as an interstellar cloud, is made up of atoms, which are the smallest components of an element that retain all the properties of that element. A typical cloud of gas in space is likely to contain a lot of hydrogen and helium (due to their abundance in the universe), along with trace amounts of heavier elements, like oxygen, nitrogen, carbon, and perhaps iron.]

[4] Having low pressure to ensure the gas atoms are far apart and do not interact with one another.

(c) The emission line spectrum is produced by electron transitions within individual gas atom as the electrons move from higher to lower energy levels after the gas atoms are excited by some processes (such as laser excitation, or collisions with high-energy particles), or applying high electric field through the gas (electrical discharge). Since electrons exist only at specific energy levels in the atom, only photons of specific frequencies are observed when viewed with a spectrometer.



The absorption line spectrum is produced by passing a white (continuous spectrum) light through a cloud of cool gas. As white light passes through the gas, certain frequencies of light are absorbed by the gas atoms and re-emitted in random directions, thus a series of black lines corresponding to the wavelengths that have been absorbed are observed when the transmitted light is viewed with a spectrometer.

(d) The line spectrum shows that only photons of specific frequencies are emitted or absorbed by gas. This implies that only specific energy transitions can take place in a gas atom. Thus, it could be deduced that electrons exist at discrete energy levels in the atom.

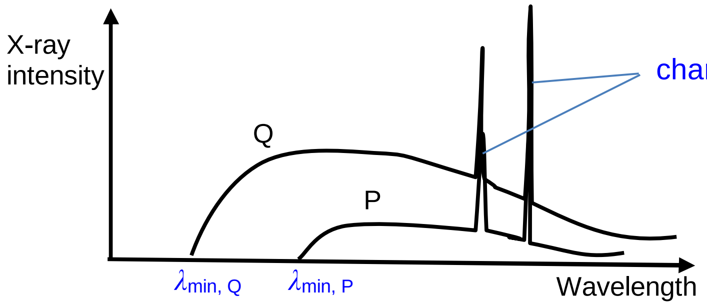
(e) The line spectrum could be used to identify gases as each gas produces a unique line spectrum.

The energy levels of the electrons in an atom are like fingerprints—no two elements have the same set of energy levels. What this means is that if we observe absorption[5] lines caused by a cloud of gas, we can tell what elements make up that cloud by the wavelengths or frequencies of the absorption lines. Tables exist that list all the known wavelengths of the lines from a particular element as measured in the lab.

[5] A star will create an absorption line spectrum because the continuous spectrum emitted by the dense, extremely hot, opaque gas that makes up core of the star (where nuclear fusion reactions take place) passes through the cooler, transparent atmosphere of the star, where specific wavelengths of light are absorbed by atoms and ions present in the gas, leading to the formation of dark absorption lines in the spectrum.

X-ray Spectrum

15 X-ray spectra are taken from two X-ray tubes P and Q. The intensity of the X-rays is plotted against the wavelength in both cases and is shown below.



(a) Explain whether or not the target material in both tubes is the same.
(b) Explain whether X-ray tube Q has the higher or lower voltage applied to it compared with X-ray tube P

(a) The target material in both tubes are the same as their characteristic x-rays produced have the same wavelengths, which shows that the energy levels of the target atoms in both tubes are the same.

(b) X-ray tube Q has a higher voltage applied because the minimum wavelength of x-rays produced in tube Q is lower than that in tube P. This shows that the bombarding electrons produced by tube Q have higher initial kinetic energy, hence tube Q has a higher voltage applied to it.

Note: The minimum wavelength occurs when all of the electron's initial kinetic energy is given to a *single* photon as a result of an interaction with *one* atom of the target: $E_k = eV = hf_{max} = h\left(c/\lambda_{min}\right)$.

16 The potential difference between the target and cathode of an X-ray tube is 50 kV and the current in the tube is 20 mA. Only 1% of the total energy supplied is emitted as X-ray radiation.
(a) Calculate the maximum frequency of emitted radiation.
(b) Determine the rate at which heat must be removed from the target in order to keep its temperature steady.

(a) By equating the energy of the photon to the energy supplied,

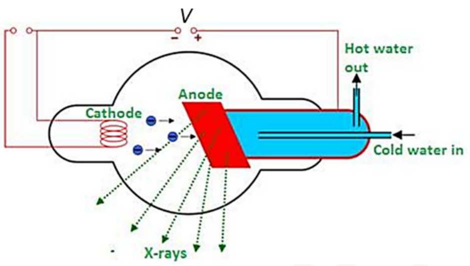
$$eV = hf$$

$$(1.60\times10^{-19})\ (50\ 000) = (6.63\times10^{-34}) \times f$$

$$f = 1.2\times10^{19}\text{ Hz} \qquad \text{[max frequency]}$$

(b) Since 1% of energy supplied becomes x-ray, the remaining 99% becomes heat.

rate of heat removal = 0.99 × power supplied
= 0.99 × (tube current i × applied p.d. V)
= 0.99 × ($20\times10^{-3} \times 50\ 000$)
= 990 W



The tube current i is measured by number of electrons per second striking the target.

Heisenberg Uncertainty Principle

17 Suppose that the x-component of the velocity of a 2×10^{-4} kg mass is measured to an accuracy of $\pm10^{-6}$ m s^{-1}. Determine the limit of the accuracy with which we can locate the particle along the *x*-axis.

$$\Delta x\Delta p \geq h \Rightarrow \Delta x \geq \frac{h}{\Delta p}$$

Since $\Delta p = m\Delta v$,

$$\Delta x \geq \frac{h}{m\Delta v} = \frac{6.63\times10^{-34}}{(2\times10^{-4})(10^{-6})} = 3\times10^{-24} \text{ m}$$

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18 A measurement establishes the position of a proton with an accuracy of $\pm1.50\times10^{-11}$ m. Find the minimum uncertainty in the proton's position 1.00 s later. [Answer: 26500 m]

Using the position-momentum uncertainty principle,

Let Δx_0 to be the uncertainty in the proton's position at t = 0.

The uncertainty in its momentum at this time is, from $\Delta x\Delta p \geq h$,

$$\Delta p \geq \frac{h}{\Delta x_0}$$

The uncertainty in the proton's velocity is

$$\Delta v = \frac{\Delta p}{m} \geq \frac{h}{m\Delta x_0}$$

The distance x the proton covers in the time t cannot be known more accurately than

$$\Delta x \approx t\Delta v \geq \frac{ht}{m\Delta x_0}$$

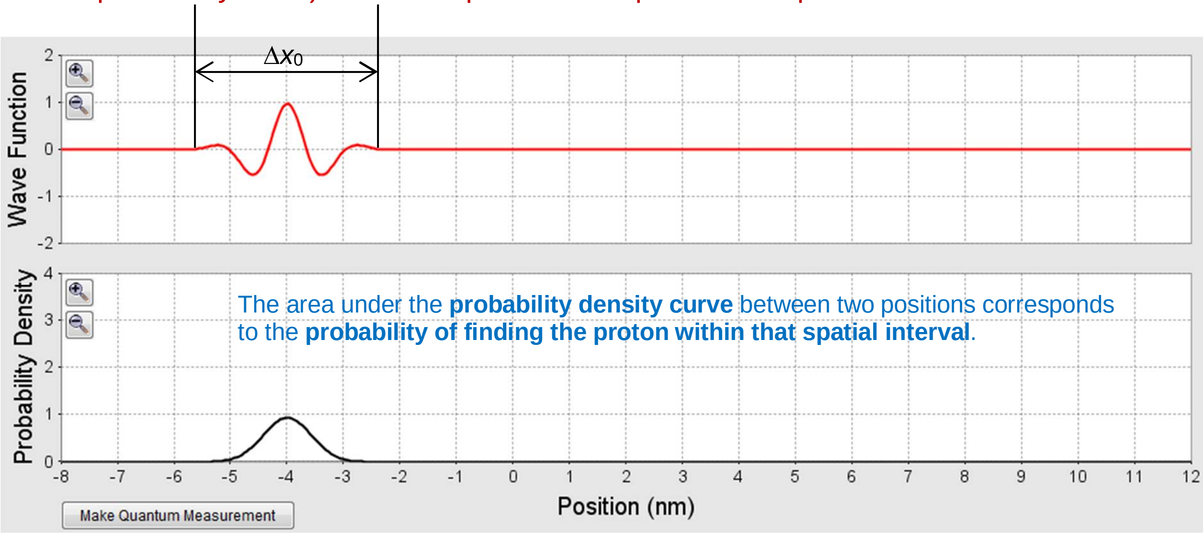
Hence Δx is inversely proportional to Δx_0 : the more we know about the proton's position at t = 0, the less we know about its later position at t > 0.

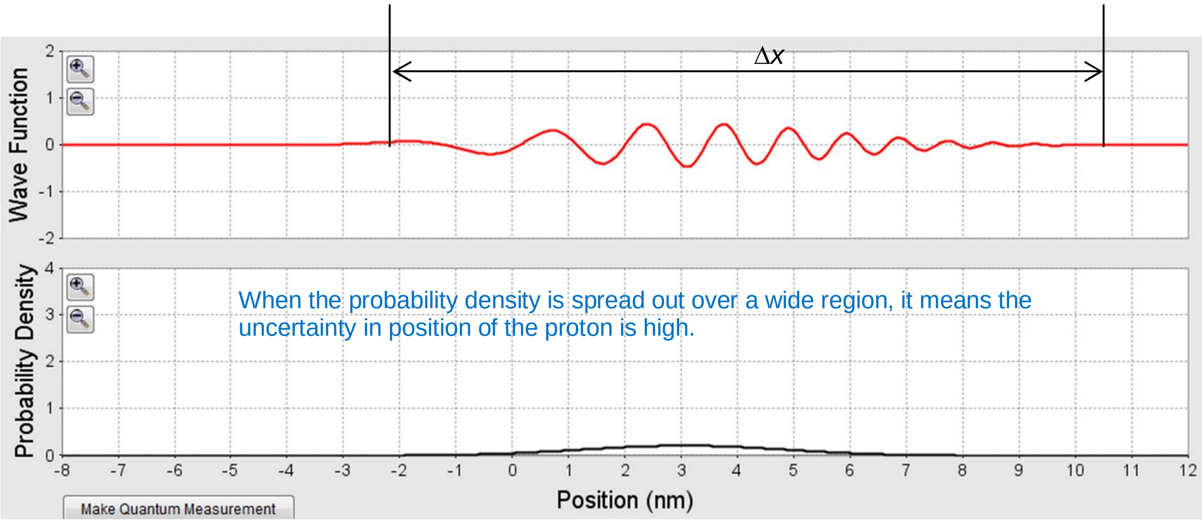
At t = 1.00 s,

$$\Delta x \geq \frac{(6.63\times10^{-34})(1.00\text{ s})}{(1.67\times10^{-27})(1.50\times10^{-11})} = 26500 \text{ m} \quad \text{(to 3 s.f.)}$$

Hence, the uncertainty in the position after 1.00 s is **26500 m**.

What has happened is that the original de Broglie wave packet[6] (removed from the present syllabus) that corresponds to the proton has spread out to a much wider one:





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19 What is the uncertainty in the location of a photon of wavelength 300 nm if this wavelength is known to an accuracy of one part in a million?
[Answer: 300 mm]

$$\Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{1}{10^6} \Rightarrow \Delta\lambda = \frac{\lambda}{10^6} = \frac{300\times10^{-9}}{10^6} = 3.00\times10^{-13}\text{ m}$$

Using de Broglie relation,

$$p = \frac{h}{\lambda}$$

$$\frac{dp}{d\lambda} = -\frac{h}{\lambda^2} \Rightarrow \Delta p = -\frac{h}{\lambda^2}\Delta\lambda$$

$$\boxed{\text{or } p = \frac{h}{\lambda} \Rightarrow \frac{\Delta p}{p} = \frac{\Delta\lambda}{\lambda} \Rightarrow \Delta p = \frac{\Delta\lambda}{\lambda}p = \frac{\Delta\lambda}{\lambda}\left(\frac{h}{\lambda}\right) = \frac{h}{\lambda^2}\Delta\lambda}$$

$$\Delta x \geq \frac{h}{\Delta p} = \frac{h}{\left(-\frac{h}{\lambda^2}\Delta\lambda\right)} = \frac{-\lambda^2}{(\Delta\lambda)} = \frac{-(300\times10^{-9})^2}{(3.00\times10^{-13})} = -0.300\text{ m}$$

Hence, the uncertainty in the location is **300 mm**

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[6] A wave packet represents a quantum particle in space. A wave packet allows us to describe a particle that behaves both like a wave (it spreads and interferes) and a particle (it is localized and can be detected at a point).

20 A pulse of radio wave lasts for 1.0×10^{-5} s. A photon of the radio wave may be considered to be at a point anywhere within this pulse, although the location of the point is not known. Calculate

(i) the length of the pulse,

(ii) the uncertainty in the position of the photon,

(iii) the uncertainty in the momentum of the photon.

[N07 H2 P3 Q7 (d) & (e) mod]

(i) The pulse length refers to the entire spatial extent of the pulse of wave.

Length of the pulse, x = speed of radio wave $\times$ duration of the pulse

$= (3.00 \times 10^{8})\ (1.0 \times 10^{-5}) = 3000$ m

(ii) Uncertainty in the position of the photon, $\Delta x = 3000$ m (here we take $\Delta x \approx x$)

(iii) Uncertainty in the momentum of the photon,

$$\Delta p = \frac{h}{\Delta x} = \frac{h}{(3000)} = 2 \times 10^{-37} \text{ kg m s}^{-1}$$

End of Tutorial