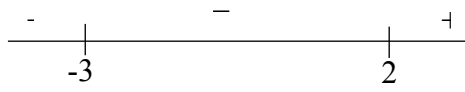


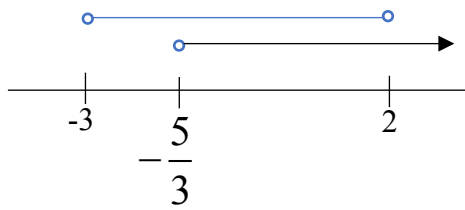
2025 ESSS 4NA Preliminary Examination Additional Mathematics Paper 1 (4051/01)
MARKING SCHEME

Qn	Suggested Solutions	Marks	Total Marks	Remarks
1a(i)	$\sin \theta = \frac{5}{13}$, then $\alpha = \frac{5}{13}$ is in first quadrant. By Pythagoras' theorem, $r^2 = 13^2 - 5^2$ $= 144$ $r = \sqrt{144} = 12$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\tan \theta = \frac{5}{12}$	B1	1	
1a(ii)	$\sin 2\theta$ $= 2 \sin \theta \cos \theta$ $= 2 \left(\frac{5}{13} \right) \left(\frac{12}{13} \right)$ $= \frac{120}{169}$	M1 A1	2	
1b	$\cos^{-1} \left(-\frac{\sqrt{2}}{2} \right)$ The principal value of $\cos^{-1} \left(-\frac{\sqrt{2}}{2} \right)$ is an angle in the 2 nd quadrant in the interval $0 \leq \cos^{-1} x \leq \pi$. $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ Since $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$, the principal value of $\cos^{-1} \left(-\frac{\sqrt{2}}{2} \right)$ is $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$.	B1	1	
2	$14x^3 + 112$ $= 14(x^3 + 8)$ $= 14(x^3 + 2^3)$ $= 14(x+2)(x^2 - 2x + 4)$	M1 A2	3	A1 Either $(x+2)$ or $(x^2 - 2x + 4)$ is correct.

4



Putting both cases together,



Answer:

M1

M1

A1

3

1 mark for either
correct solution

5	Therefore	M1 M1 M1 A1	4	Long division seen Factorisation seen Either correct values of A or B
6a	$ \begin{aligned} &2x^2 - 6x - 10 \\ &= 2(x^2 - 3x - 5) \\ &= 2\left[x^2 - 3x - 5 + \left(-\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] \\ &= 2\left[\left(x - \frac{3}{2}\right)^2 - 5 - \frac{9}{4}\right] \\ &= 2\left[\left(x - \frac{3}{2}\right)^2 - \frac{29}{4}\right] \\ &= 2\left(x - \frac{3}{2}\right)^2 - \frac{29}{2} \quad (\text{shown}) \end{aligned} $	M1 A1	2	

6b	Minimum value of $y = x^2 - 2\sqrt{2}x - 2$ = Minimum value of x when it occurs is Coordinates of minimum point	B1	1	
6c	The curve does not lie completely above or below the x-axis because the coefficient of x^2 is positive. So, the curve is U-shaped and the minimum value of $y < 0$ (minimum point below x-axis). So, the curve cuts the x-axis twice.	B1	1	
7	$\frac{x}{2} - \frac{1}{x} = \sqrt{2}$ $x^2 - 2 = 2\sqrt{2}x \quad (\text{multiply by } 2x)$ $x^2 - 2\sqrt{2}x - 2 = 0$ $a = 1, \quad b = -2\sqrt{2}, \quad c = -2$ $x = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$ $x = \frac{2\sqrt{2} \pm \sqrt{16}}{2}$ $x = \frac{2\sqrt{2} \pm 4}{2}$ $x = \sqrt{2} \pm 2$ $= 2 \pm \sqrt{2}$	M1 M1 M1 A1	4	-quadratic eqn -Use of formula -simplifying $\sqrt{16}$ to 4
8a	$a \sin \theta - b \cos \theta = R \sin(\theta - \alpha)$ Comparing $8 \sin \theta - 7 \cos \theta = R \sin(\theta - \alpha)$ $a = 8, \quad b = 7$ $R = \sqrt{a^2 + b^2}$ $= \sqrt{8^2 + 7^2}$ $= \sqrt{113}$ $\tan \alpha = \frac{b}{a}$ $= \frac{7}{8}$ $\alpha = 41.18592517$ $\therefore 8 \sin \theta - 7 \cos \theta = \sqrt{113} \sin(\theta - 41.2^\circ) \text{ (1d.p.)}$	M1 M1 A1	3	

9	Find V in terms of h: Subst into V To find rate of change in height, Height, (Subst. $h = 9$ cm)	M1 M1 M1 M1 M1 A1	6	
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10a		M1 A1	2	
10b	$\int_2^4 \frac{6x}{\sqrt{4x-5}} dx$ $= \frac{1}{2} \int_2^4 \frac{12x}{\sqrt{4x-5}} dx$ $= \frac{1}{2} \left[(2x+5)\sqrt{4x-5} \right]_2^4$ $= \frac{1}{2} \left[((2(4)+5)\sqrt{4(4)-5}) - ((2(2)+5)\sqrt{4(2)-5}) \right]$ $= \frac{1}{2} [13\sqrt{11} - 9\sqrt{3}]$ $= 13.7638325$ $= 13.76 (2 \text{ d.p.})$	M1 M1 M1 A1	4	
11	$\sec^2 x - 4 \tan x = 6$ $\tan^2 x + 1 - 4 \tan x = 6$ $\tan^2 x - 4 \tan x - 5 = 0$ $(\tan x + 1)(\tan x - 5) = 0$ $\tan x = -1 \quad \text{or} \quad \tan x = 5$ $\alpha = \tan^{-1}(5) \qquad \alpha = \tan^{-1}(1)$ $= 78.690^\circ \qquad = 45^\circ$ $x = \alpha, 180^\circ + \alpha \qquad x = 180^\circ - \alpha, 360^\circ - \alpha$ $x = 78.6900675^\circ, 258.6900675^\circ \quad x = 135^\circ, 315^\circ$ $\therefore x = 78.7^\circ, 135^\circ, 258.7^\circ, 315^\circ (1 \text{d.p.})$	M1 M1 M1 A1	5	

12	When (tangent is parallel to the x-axis). Subst $x = 0$ into $y, y = 0$ Subst , Coordinates of the points are	M1 M1 M1 M1 A1, A1	6	
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13		M1 M1 M1 M1 M1 M A1	7	
14(a)	$\tan \frac{\pi}{3} = \frac{y}{x}$ (equilateral triangle, use trigo ratio) $\sqrt{3} = \frac{y}{x}$ Note: $\tan \frac{\pi}{3} = \sqrt{3}$ $y = \sqrt{3}x$ Or Since triangle ASR is equilateral triangle, $AS = SR = PQ$ $= 6 - 2x$ cm. $BS = 6 - (6 - 2x) = 2x$. By Pythagoras' Theorem, $(2x)^2 = y^2 + x^2$ $4x^2 - x^2 = y^2$ $3x^2 = y^2$ $y = \sqrt{3}x$ ($x, y > 0$)	M1		

	Area of shaded region, A =	M1 A1	3	1 mark: either area seen
14(b)	$A = \sqrt{3}(9 - 6x + 2x^2).$ $\frac{dA}{dx} = \sqrt{3}(-6x + 2x(2))$ $= \sqrt{3}(-6 + 4x)$ For stationary values of A , $\frac{dA}{dx} = 0$ $\sqrt{3}(-6 + 4x) = 0$ $4x = 6$ $x = \frac{3}{2}$ When $x = \frac{3}{2}$, $A = \sqrt{3}\left[9 - 6\left(\frac{3}{2}\right) + 2\left(\frac{3}{2}\right)^2\right]$ $= \frac{9}{2}\sqrt{3}$ The stationary point of A is $\frac{9}{2}\sqrt{3}$,	M1 A1	3	
14c	$\frac{dA}{dx} = \sqrt{3}(-6 + 4x)$ $\frac{d^2A}{dx^2} = 4\sqrt{3} > 0$ Thus by second derivative test, this value of A is a minimum.	M1 A1	2	