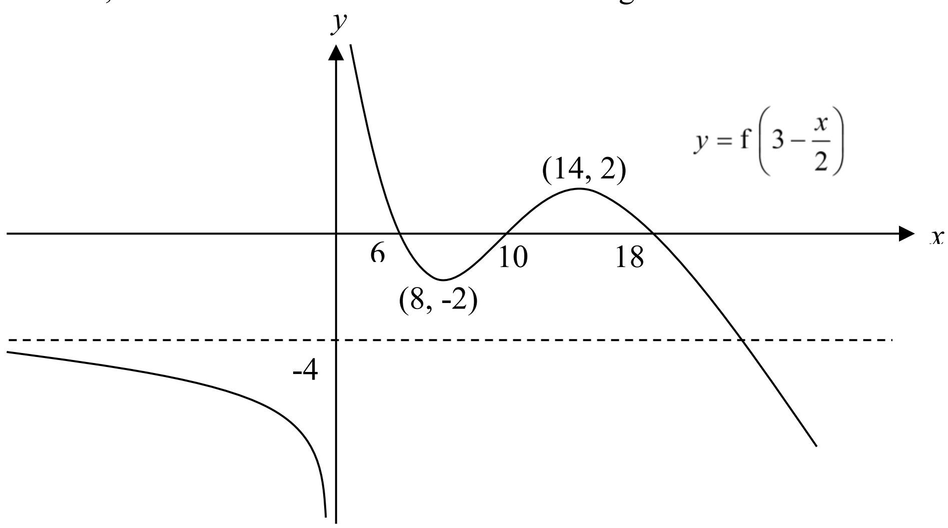
Transformations

1. **AJC/I/11**

The graph of $y = \mathrm{f}\left(3 - \frac{x}{2}\right)$ has 2 stationary points at (8, -2) and (14, 2) and intersects the x-axis at $x = 6$, $x = 10$ and $x = 18$ as shown in the diagram below.



Sketch, on separate clearly labeled diagrams, the graphs of

(a) $y = \dfrac{1}{\mathrm{f}\left(3 - \frac{x}{2}\right)}$; [3]

(b) $y = \mathrm{f}(x)$. [3]

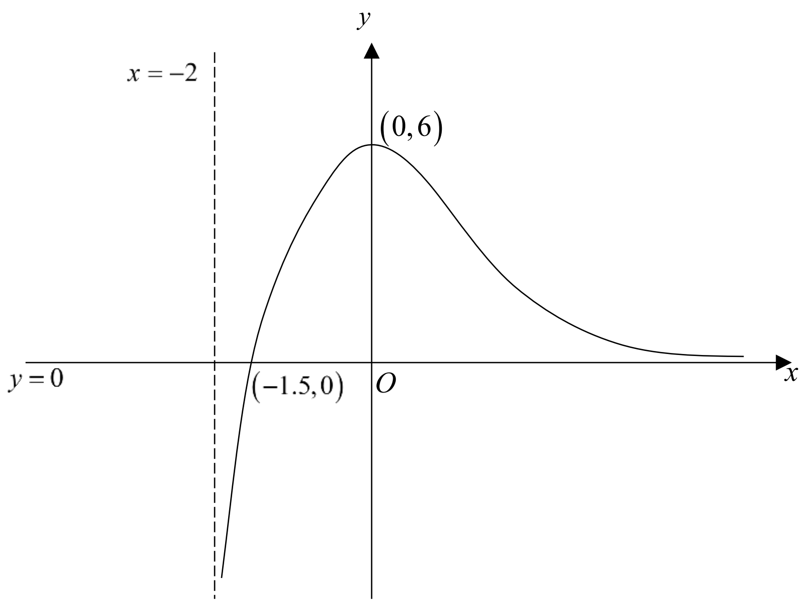
Show clearly the asymptotes, stationary points and points of intersection with the axes in your diagrams.

2. **VJC/I/6**

A curve C_1 has equation $x^2 - a^2y^2 = a^2$ where $a > 1$. Sketch C_1, indicating the axial intercepts, asymptotes and stationary points, if any. [2]

C_1 undergoes a single transformation to become C_2. Given that C_2 has a line of symmetry $y = 4$ and the point (4, 3) lies on C_2, find a. [3]

3. **EJC Prelim/2020/02/Q1**



The diagram above shows the curve $y=\mathrm{f}(x)$. The curve cuts the x-axis at the point $(-1.5,0)$ and has a maximum point at $(0,6)$. The curve also has vertical asymptote $x=-2$ and horizontal asymptote $y=0$. Stating the equations of the asymptotes and coordinates of any turning points and points where the curves cross the axes, if it is possible to do so, sketch on separate diagrams, the curves

(i) $y=\frac{1}{2}\mathrm{f}(1-x)$, [3]

(ii) $y=\frac{1}{\mathrm{f}(x)}$. [3]

4. **ACJC/2011/II/5**

The curves C_1 and C_2 have equations $9y^2=(x+k)^2-9$ and $\frac{x^2}{k^2}+y^2=1$ respectively, where k is a real constant such that $k>3$.

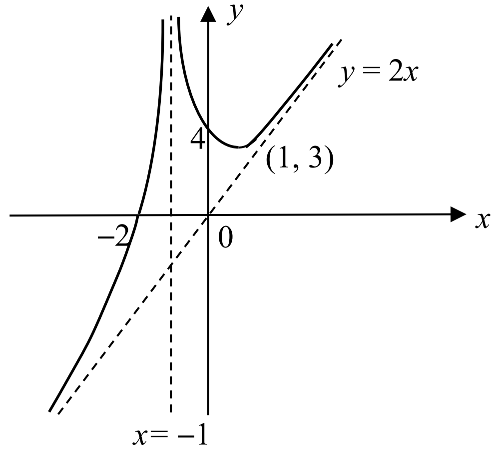
(a) Describe a sequence of transformations which transforms the graph of $x^2-y^2=1$ to the graph of C_1.

(b) (i) On the same diagram, sketch the graphs of C_1 and C_2, stating clearly the coordinates of any points of intersection with the axes and the equations of any asymptotes.

(ii) Find the range of values of the positive constant a, where $a>k$, such that the equation $\frac{x^2}{a^2}+\frac{(x+k)^2-9}{9}=1$ has two real roots.

5. **TJC/2013/I/7**

(a) The diagram below shows the graph of $y = \mathrm{f}(x)$. The graph has a minimum point at $(1,3)$ and intersects the axes at $x = -2$ and $y = 4$. The equations of the asymptotes are $y = 2x$ and $x = -1$.



On separate diagrams, sketch the graphs of

(i) $y = \mathrm{f}'(x)$, [2]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$, [2]

In each case, give if possible, the equations of the asymptotes, the coordinates of the turning points and the coordinates of the points where the graph crosses the x- and y-axes.

(b) A curve undergoes the transformations A, B and C in succession:

A: a translation of 2 units in the negative direction of the x-axis.
B: a scaling parallel to the x-axis by a factor 2.
C: a translation of 1 unit in the positive direction of the y-axis.

The equation of the resulting curve is $y = 2x + 9 - \dfrac{2}{x+4}$.

Find the equation of the curve before the three transformations were effected. [3]

6. **AJC/2011/I/10**

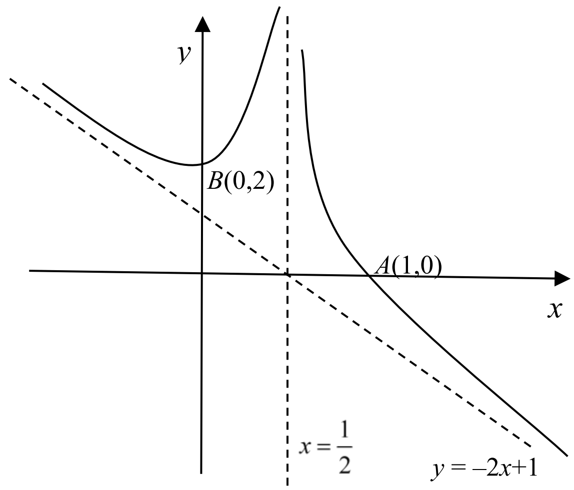
The sketch below shows the graph of $y = \mathrm{f}(x)$. The curve passes through the point A (1,0) and has a minimum point at B(0,2). The equation of the asymptotes are $y = -2x + 1$ and $x = \dfrac{1}{2}$.

On separate diagrams, sketch the following graphs indicating the points corresponding to A, B and asymptotes where necessary.

(i) $y = \mathrm{f}\left(\left|x + \dfrac{1}{2}\right|\right)$ [3]

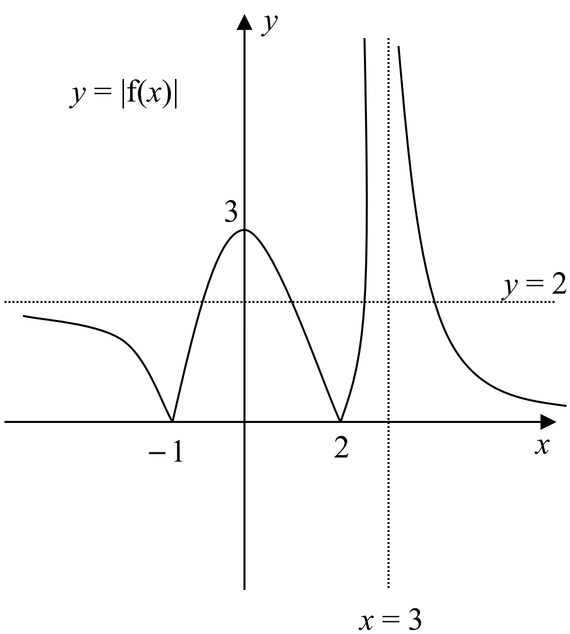
(ii) $y = \dfrac{1}{\mathrm{f}(x)}$ [3]

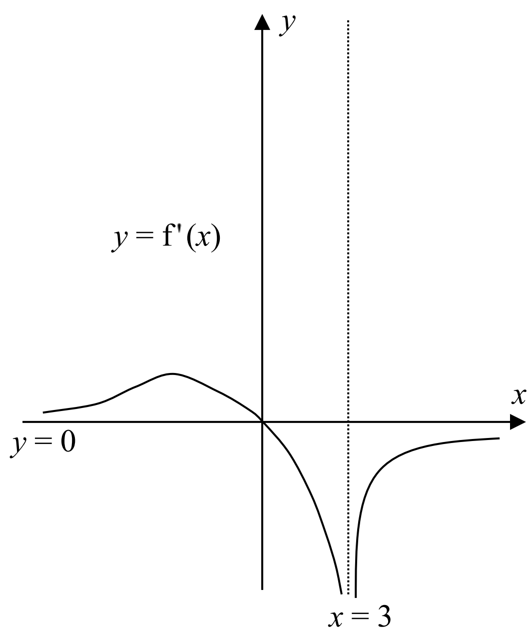
(iii) $y = \mathrm{f}'(2x)$ [3]



7. **NYJC/2011/II/4**

(a) The diagrams below show the graphs of $y = |\mathrm{f}(x)|$ and $y = \mathrm{f}'(x)$.

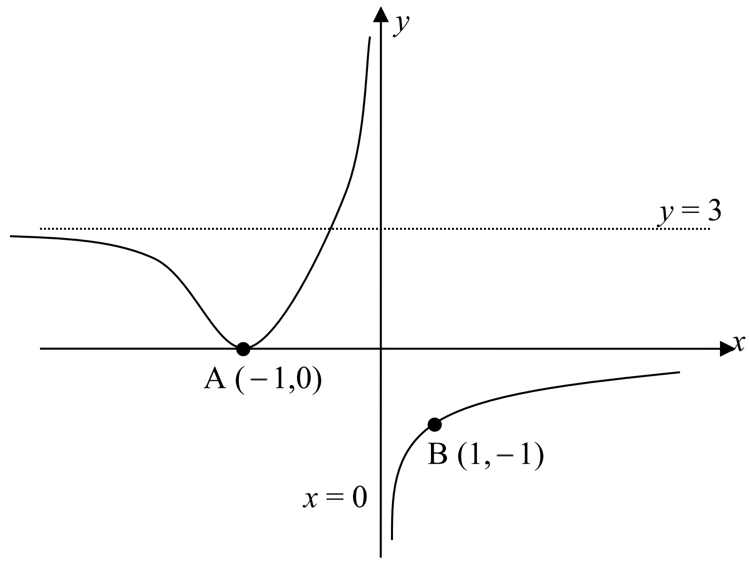




Sketch the graph of $y = \mathrm{f}(x)$, stating the equations of any asymptotes and the coordinates of any axial intercepts and turning points. [3]

Hence, find the range of values of k if there is exactly 1 real root to the equation $\mathrm{f}(x) - k = 0$. [2]

(b) The diagram below shows the graph of $y = \mathrm{f}(2x-1)$. The curve passes through the point A(-1, 0) and B(1, -1). The asymptotes are $x = 0$ and $y = 0$ and $y = 3$.



Sketch, on separate clearly labelled diagrams, the graphs of

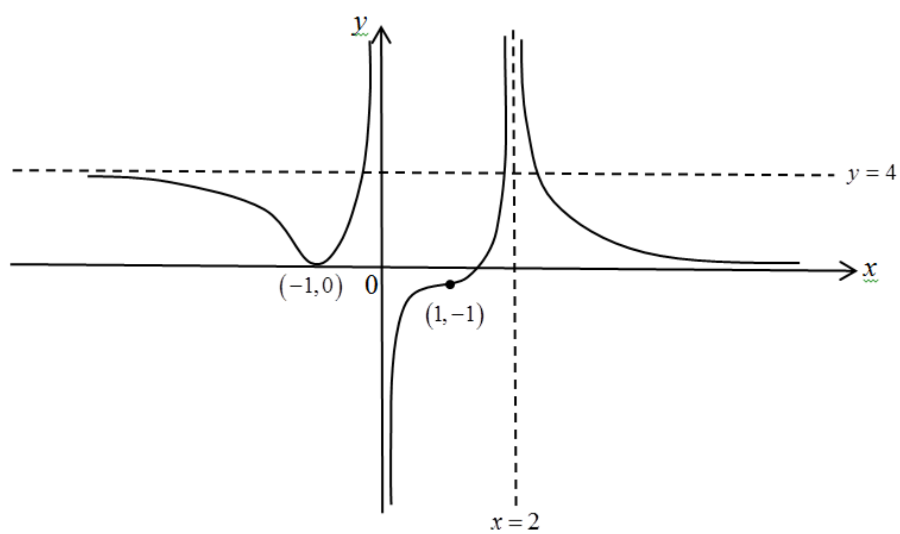
(i) $y = \mathrm{f}(-|2x-1|)$, [6*]

(ii) $y = \mathrm{f}(x)$, describing the sequence of transformations involved. [5]

Your sketch should show clearly the equations of any asymptotes and the coordinates of the points corresponding to A and B.

8. **AJC/2013/I/7**

The diagram shows the graph of $y = \mathrm{f}(x)$. The curve has a minimum point at (-1, 0) and a stationary point of inflexion at (1, -1). The asymptotes are $y = 4$, $y = 0$, $x = 0$ and $x = 2$.

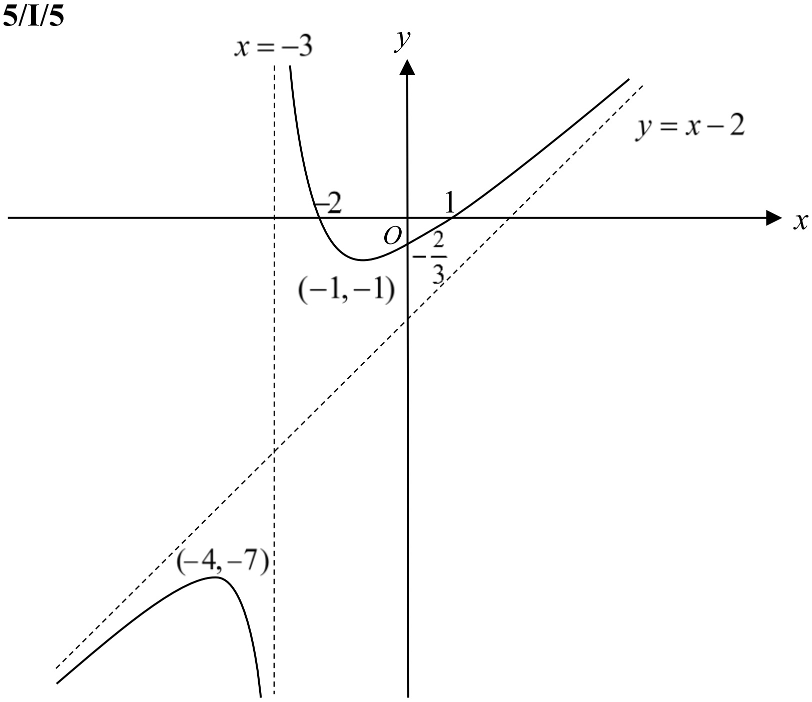


Sketch the graph of $y = \mathrm{f}'(x)$. [3]

In each case, indicate clearly the coordinates of any points where the curve crosses the axes, turning points and equations of asymptotes whenever possible.

Find the number of points of intersection between the curves $y = \mathrm{f}(1-x)$ and $y = \mathrm{f}'(x)$. [3]

9. **DHS/2015/I/5**



The diagram shows the graph of $y = \mathrm{f}(x)$. The graph has a minimum point at $(-1,-1)$ and a maximum point at $(-4,-7)$. It intersects the axes at $x = -2$, $x = 1$ and $y = -\frac{2}{3}$. The equations of the asymptotes are $y = x - 2$ and $x = -3$.

(i) Sketch the graph of $y = \dfrac{1}{\mathrm{f}(x)}$, giving the coordinates of any stationary points, points of intersection with the axes and the equations of any asymptotes. [3]

(ii) Solve the inequality $\mathrm{f}\left(\frac{1}{x}\right) < 0$. [3]

10. **SAJC/2012/II/3**

Find the values of the constants A and B such that $\frac{x^2-4x}{(x-2)^2} = A + \frac{B}{(x-2)^2}$ for all values of x except $x = 2$. Hence state a sequence of transformations by which the graph of $y = \frac{x^2-4x}{(x-2)^2}$ may be obtained from the graph of $y = \dfrac{1}{x^2}$. [4]

11. **CJC/2012/I/6**

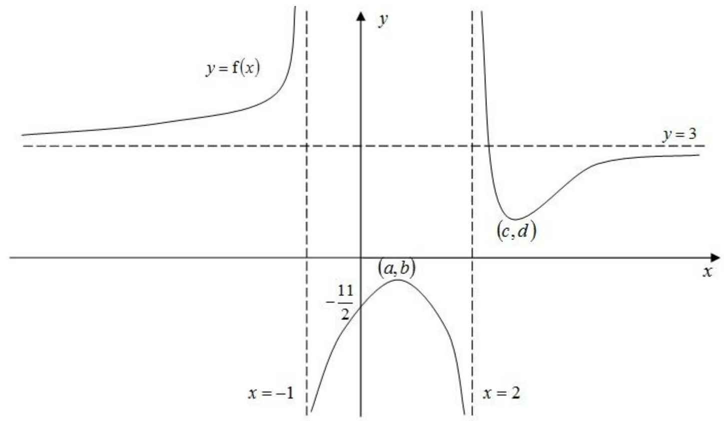
The sketch below shows the graph of $y = \mathrm{f}(x)$. The curve intersects the y-axis at the point $\left(0, -\dfrac{11}{2}\right)$, has a maximum point at (a, b) and a minimum point at (c, d). The equation of the asymptotes are $y = 3$, $x = -1$ and $x = 2$.

On separate diagrams, sketch the following graphs indicating the points corresponding to the stationary points, axial intercepts and asymptotes where necessary.

(i) $y = \mathrm{f}(|x|)$ [2]

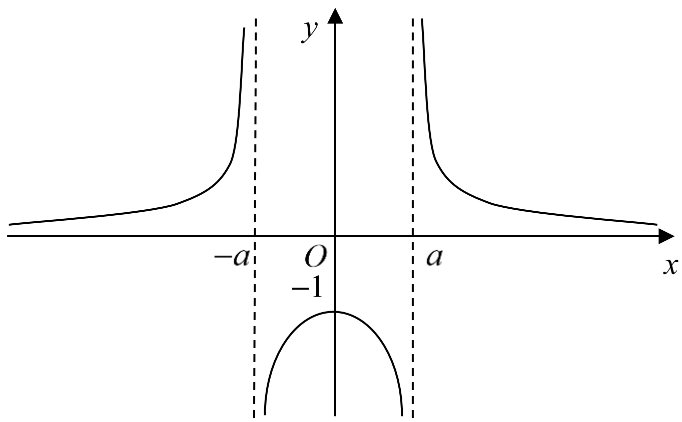
(ii) $y = \dfrac{1}{\mathrm{f}(x)}$ [3]

(iii) $y = \mathrm{f}'(x)$ [2]



12. **JJC/2015/I/8**

The diagram below shows the curve with equation $y = \mathrm{f}(x)$ which has vertical asymptotes $x = a$ and $x = -a$ where $a > 0$, horizontal asymptote $y = 0$ and a stationary point at $(0, -1)$.



Sketch, on separate diagrams, the graphs of

(a) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

(b) $y = \mathrm{f}'(x)$, [3]

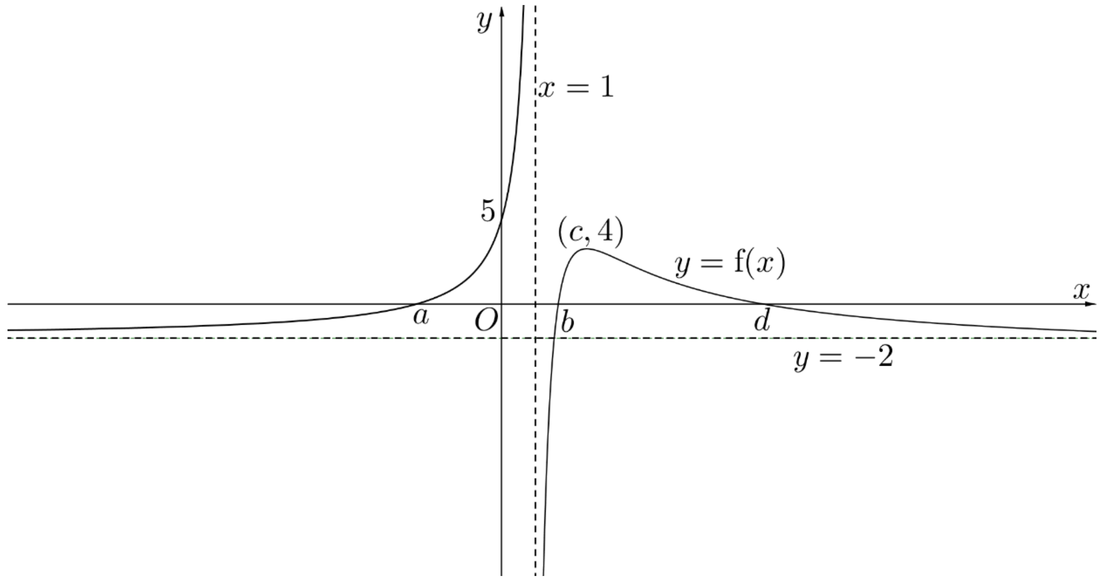
indicating clearly the asymptotes and the axial intercepts if any.

The graph of $y = \mathrm{f}(x)$ undergoes in succession, the following transformations:

A: A translation of 1 unit in the negative x-direction.
B: A reflection in the x-axis.
C: A scaling parallel to the y-axis by a factor of 2.

The equation of the resulting curve is given by $y = \dfrac{2}{1-(x+1)^2}$. Find the value of a. [3]

13. **NJC/2014/II/2**



The diagram shows the curve with equation $y = \mathrm{f}(x)$. The curve crosses the x-axis at $x = a$, $x = b$ and $x = d$, and has a turning point at (c, 4), where a, b, c and d are real constants. The lines $x = 1$ and $y = -2$ are asymptotes to the curve.

Sketch the graphs of

(a) $y = \dfrac{1}{\mathrm{f}(x)}$, and [4]

(b) $y = \mathrm{f}'(x)$ [3]

on separate diagrams. Label clearly any points where the graphs cross the axes, the coordinates of any turning points and the equations of any asymptotes.

14. **RI/2014/I/5**

The curve with equation $y = -\dfrac{1}{x}$ undergoes a translation of 1 unit in the positive x-direction, followed by a stretch with factor $\dfrac{1}{2}$ parallel to the x-axis and then a translation of 3 units in the positive y-direction.

Given that the equation of the new curve is $y = \mathrm{f}(x)$. Sketch the curve, stating its asymptotes and the coordinates of the points of intersection with the axes**.** [4]

By sketching the graphs of $y = |\mathrm{f}(x)|$ and $y = \mathrm{f}(|x|)$ on separate diagrams, solve the inequality $|\mathrm{f}(x)| > \mathrm{f}(|x|)$. [4]

15. **SAJC/2018/2/1**

A curve C with equation $y = a\left(x-1\right)+\dfrac{b}{x+c}$, where a, b and c are real constants, undergoes in succession, the following transformations:

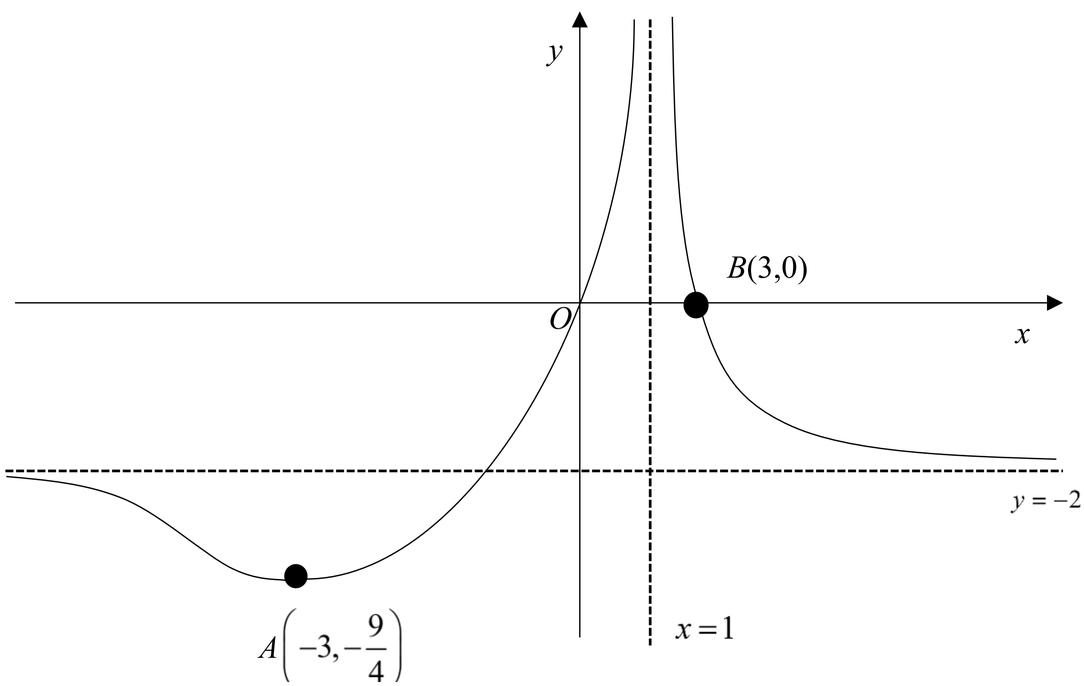
A: A reflection in the x-axis
B: A translation of 1 unit in the negative x-direction

The resulting curve with equation $y = \mathrm{f}\left(x\right)$ has the y-axis as one of its asymptotes.

Given that $\left(1, \dfrac{1}{6}\right)$ is a turning point of $y = \dfrac{1}{\mathrm{f}\left(x\right)}$, find the values of a, b and c.

16. **SAJC/2019/I/Q1**

The diagram below shows the graph of $y = 2\mathrm{f}(3-x)$. The graph passes through the origin O, and two other points $A\left(-3, -\dfrac{9}{4}\right)$ and $B(3,0)$. The equations of the vertical and horizontal asymptotes are $x = 1$ and $y = -2$ respectively.



(a) State the range of values of k such that the equation $\mathrm{f}(3-x)=k$ has exactly two negative roots. **[1]**

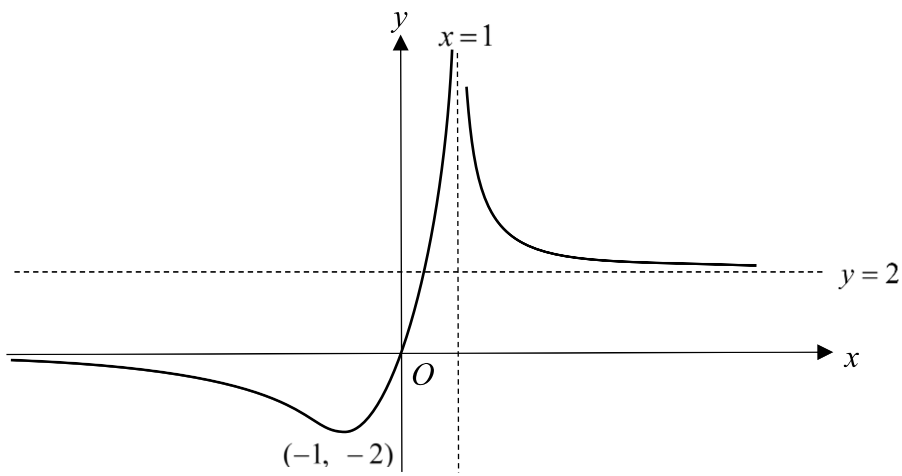
(b) By stating a sequence of two transformations which transforms the graph of $y = 2\mathrm{f}(3-x)$ onto $y = \mathrm{f}(3+x)$, find the coordinates of the minimum point on the graph of $y = \mathrm{f}(3+x)$. Also, write down the equations of the vertical asymptote(s) and horizontal asymptote(s) of $y = \mathrm{f}(3+x)$. **[5]**

17. **TJC/2021/I/Q1**

The curve with equation $y = p + \dfrac{1}{x+q}$, where p and q are constants, is transformed by a scaling of factor 4 parallel to the x-axis, followed by a translation of 3 units in the negative y-direction, followed by a reflection about the y-axis.
The resulting curve has asymptotes $x = 2$ and $y = 1$. Find the values of p and q. [5]

18. **EJC/2022/2/Q2**

The graph of $y = \mathrm{f}(x)$ is given below. It has one vertical asymptote at $x = 1$ and two horizontal asymptotes $y = 0$ and $y = 2$. The graph passes through the origin O and has a turning point at $(-1,\ -2)$.



(a) On separate diagrams, sketch the following graphs, indicating the coordinates of the points where the graphs cross the axes, the turning points, and the equations of any asymptotes if possible.

(i) $y = \dfrac{1}{\mathrm{f}(x)}$ [3]

(ii) $y = \mathrm{f}'(x)$ [3]

(iii) $y = \mathrm{f}(|x-1|)$ [2]

(b) State the range of values of a such that $\mathrm{f}(|x-1|) = a$ has only positive root(s). [1]

19. **EJC Prelim 9758/2023/01/Q6**

A curve R has equation $y = \mathrm{f}(x)$. The point P with coordinates $(7, -9)$ lies on R. The tangent to R at point P has gradient 10.

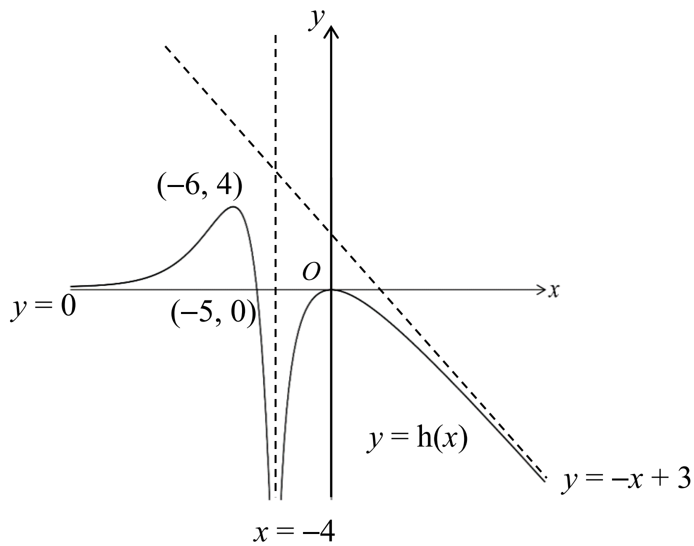
(a) The curve C is obtained from R by applying the following two transformations:

- A scaling parallel to the y-axis with scale factor 3, and
- A translation in the negative x-direction by 4 units.

Find the coordinates of the point on C corresponding to P, and state the gradient of C at this point. [2]

(b) The curve D has equation $y = \dfrac{1}{\mathrm{f}(x)}$. Find the coordinates of the point on D corresponding to P, and find the gradient of D at this point. [3]

20. **DHS Prelim 9758/2024/01/Q6**



(a) The diagram shows the curve $y = \mathrm{h}(x)$. The curve has maximum points at $(-6,\ 4)$ and the origin, and crosses the x-axis at $(-5,\ 0)$. The lines $y = 0$, $x = -4$ and $y = -x + 3$ are the horizontal, vertical and oblique asymptotes to the curve respectively.

(i) On the diagram given above, sketch the graph of $(x+6)^2 + (y-9)^2 = r^2$, where r is a positive constant. Find the range of values of r for the equation $(x+6)^2 + (\mathrm{h}(x)-9)^2 = r^2$ to have at least one real root. [3]

(ii) On a separate diagram, sketch the graph of $y = \dfrac{1}{\mathrm{h}(x)}$. [3]

(b) The graph of $y = 10 - |x+1|$ undergoes a sequence of transformations which transform its equation into $y = |x-1|$. Describe and write down the transformations. [3]

21. **HCI Prelim 9758/2024/01/Q1**

The curve $y = \mathrm{f}(x)$ cuts the axes at $\left(0, \dfrac{1-p}{p}\right)$ and $(1-p, 0)$, where p is a constant such that $0 < p < 1$. It is given that f^{-1} exists.

State, if it is possible to do so, the coordinates of the points where the following curves cut the axes.

(a) $y = \mathrm{f}(x) + 1$

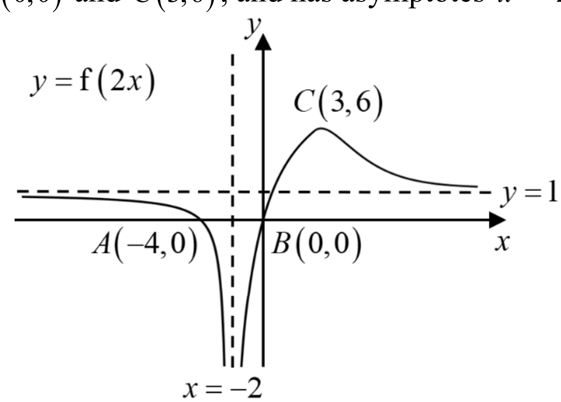
(b) $y = \mathrm{f}(x - p)$

(c) $y = \mathrm{f}(3x - p)$

(d) $y = \mathrm{f}^{-1}(x)$ [4]

22. **VJC Prelim 9758/2024/01/Q8**

(a) The diagram shows the graph with equation $y = \mathrm{f}(2x)$. The graph passes through the points $A(-4,0)$, $B(0,0)$ and $C(3,6)$, and has asymptotes $x = -2$ and $y = 1$.



On separate clearly labelled diagrams, deduce the graphs of

(i) $y = \mathrm{f}(2x - 2)$, [2]

(ii) $y = \mathrm{f}(x)$. [2]

(b) The curve C_1 undergoes the transformations in the order given below:

1. A translation of 2 units in the negative x direction.
2. A stretch parallel to the x axis, factor 2, y axis invariant.
3. A translation of 1 unit in the positive y direction.

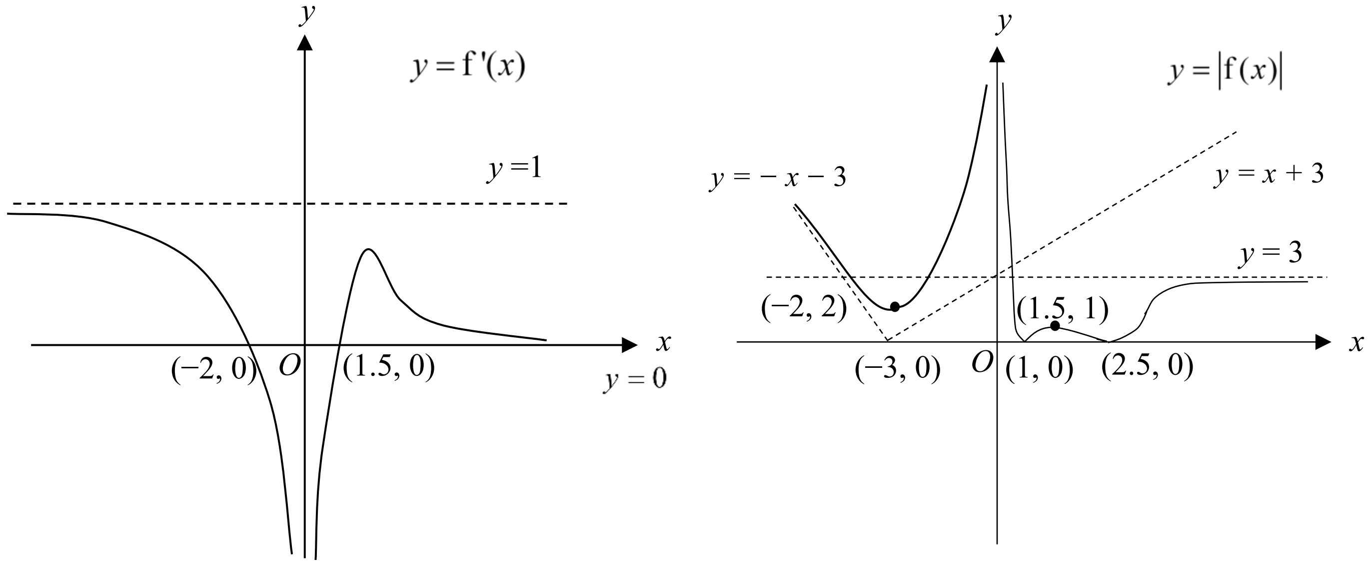
The resulting curve C_2 has equation

$$y = \frac{x^2 + 9x + 22}{x + 4}, \quad x \in \mathbb{R}, \quad x \neq -4.$$

Find, in the simplest form, the equation for C_1. [4]

23. **DHS Prelim 9758/2025/P1/Q5**

The graphs of $y = \mathrm{f}'(x)$ and $y = |\mathrm{f}(x)|$ are shown below.



(a) State the nature of all turning point(s) of the graph of $y = \mathrm{f}(x)$. [2]

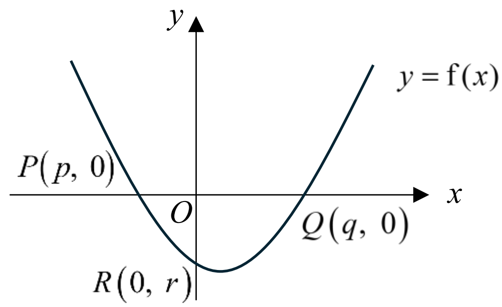
(b) State the range of values of x where f is decreasing. [2]

(c) Sketch the graph of $y = \mathrm{f}(x)$ indicating clearly the equations of the asymptotes, coordinates of the turning point(s) and the intersections with the axes. [3]

(d) On the copy of the graph of $y = |\mathrm{f}(x)|$ in the Printed Answer Booklet, sketch and label a line $y = kx + 3k$, where k is a constant. Hence state the range of values of k for which there is no real solution to the equation $|\mathrm{f}(x)| = kx + 3k$. [2]

24. **MI Prelim 9758/2025/P2/Q1**

The diagram shows the curve with equation $y = \mathrm{f}(x)$. The curve passes through the points $P\left(p,\ 0\right)$, $Q\left(q,\ 0\right)$ and $R\left(0,\ r\right)$.



(a) The curve $y = \mathrm{f}(x)$ is transformed onto the curve with equation $y = \mathrm{f}(2x-1)$. Find the coordinates of the points on the graph of $y = \mathrm{f}(2x-1)$ which correspond to the points P, Q and R on curve $y = \mathrm{f}(x)$. [3]

(b) It is given that $I = \int_p^q \mathrm{f}(x)\,\mathrm{d}x$. Find, in terms of I, the area of the finite region bounded by

(i) the curve with equation $y = -\mathrm{f}(x)$ and the x-axis, [1]

(ii) the curve with equation $y = \dfrac{1}{2}\mathrm{f}(x+3)$ and the x-axis. [1]

(c) **F**ind the value of $\int_0^q \mathrm{f}'(x)\,\mathrm{d}x$, justifying your answer. [1]

Answers

2. $a = 2\sqrt{2}$

3. Ellipse; (a) (1)Scale parallel to x-axis by factor a, then (2)Translate in the positive x-direction by 2 units or (1) Translate in the positive x-direction by $\dfrac{2}{a}$ units, then (2) Scale parallel to x-axis by factor a.

4. (a) Sequence of transformations:
EITHER (1) Scaling parallel to the x-axis by factor 3
(2) Translation in the negative x-axis direction by k units

OR (1) Translation in the negative x-axis direction by $\dfrac{k}{3}$ units
(2) Scaling parallel to the x-axis by factor 3

(bii) $a > k+3$

5. (b) $y = 4x - \dfrac{1}{x}$

8. (ii) $-\dfrac{1}{3} < x < 0$ or $x < -\dfrac{1}{2}$ or $x > 1$

9. 2 solutions

11. 1. Translate the graph of $y = \dfrac{1}{x^2}$ by 2 units in the positive x direction
2. Stretch the resulting graph parallel to the y-axis with a scale factor of -4.
3. Translate the resulting graph by 1 unit in the positive y direction.

12. $a = 1$

14. $x < -\dfrac{1}{2}$ or $\dfrac{1}{2} < x < \dfrac{2}{3}$

15. $a = -3,\ b = -3$ and $c = -1$

16. (a) $-\dfrac{9}{8} < k < -1$, (b) $\left(3, -\dfrac{9}{8}\right)$, $x = -1$, $y = -1$

17. $p = 4$, $q = \dfrac{1}{2}$

18. (b) $0 \le a \le 2$

19. (a) $(3, -27)$, 30, (b) $(7, -\frac{1}{9})$, $-\frac{10}{81}$

20. (ai) $r \ge 5$

21. (a) $\left(0, \dfrac{1}{p}\right)$ (b) $(1,0)$ (c) $\left(\dfrac{1}{3}, 0\right)$ (d) $\left(\dfrac{1-p}{p}, 0\right)$, $(0, 1-p)$

22. (b) $y = 2x + \dfrac{1}{x}$

24.(a) P becomes $P'\left(\dfrac{1}{2}p + \dfrac{1}{2},\ 0\right)$, Q becomes $Q'\left(\dfrac{1}{2}q + \dfrac{1}{2},\ 0\right)$ and R becomes $R'\left(\dfrac{1}{2},\ r\right)$.

(b)(i) $-I$ or $|I|$ (b)(ii) $-\dfrac{1}{2}I$ or $\dfrac{1}{2}|I|$ (c) $-r$