Anglo-Chinese School

(Independent)

FINAL EXAMINATION 2020

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 2

FRIDAY 25 SEP 2020

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics Analysis and Approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**
- Questions with an asterisk (*) are common to both HL and SL papers

Section A (55 Marks)		Section B (55 Marks)	
Question	**Marks**	**Question**	**Marks**
1		**10**	
2			
3			
4		**11**	
5			
6			
7		**12**	
8			
9			
Subtotal		**Subtotal**	
TOTAL	**/ 110**		

This question paper consists of 14 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Solve the simultaneous equations

$$x^3 y = 8$$
$$\log_2 x + \log_4 y = 3$$

2. *[Maximum mark: 3]

Prove that the product of two odd numbers is odd.

3. ***[Maximum mark: 9]

It is speculated that there is an association between the height of a student, *x* centimetres and the time taken, *y* seconds, to do a 100 m sprint. The following paired data are collected..

Height (x cm)	160	171	157	172	183	161	182	156	166	191
Time (y s)	16.0	15.2	14.0	15.1	12.0	12.4	11.5	13.7	13.1	11.2

(a) Find the Pearson's product moment correlation coefficient between the height and the time taken. Comment on your answer. [3]

(b) Find the equation of the regression line of y on x, where x is the height of the students and y is the time taken to do a 100 m sprint. [2]

(c) (i) Find the expected time taken if the height of the student is 168 cm. [1]

(ii) Find the expected height of the student if he takes 11.8s to run 100m. [3]

4. [Maximum mark: 5]

Eight women and six men form a queue. Find the number of ways this can be done if

(a) a particular man and woman have to stand next to each other. [2]

(b) no two men are to stand next to each other [3]

5. [Maximum mark: 5]

During recess, Ryan will eat either chicken rice or chicken noodles but not both. If he eats chicken rice, the probability of him feeling full is 0.6, while if he eats chicken noodles, the probability of him feeling full is 0.7. The probability that Ryan feels full on any given day is 0.64.

(a) Find the probability that Ryan eats chicken rice. [3]

(b) Find the probability that Ryan eats chicken rice given that does not feel full. [2]

6. [Maximum mark: 7]

The probability that an archer hits the bullseye is 0.68. For a training set, the archer will shoot six arrows at the target. On a training day, the archer will go through eight training sets.

(a) Find the probability that the archer will hit the target more than thrice in a training set.[2]

(b) Find the expected number of training sets in a day where the archer will hit the target more than thrice in the set. [3]

(c) The archer now decides to shoot n arrows in a training set. Find the minimum value of n such that the probability of hitting the target in a training set more than thrice is more than 0.90. [2]

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7. *[Maximum mark: 7]

X is the random variable that denotes the weights of 17 year old male students in a school and it is normally distributed with mean μ and standard deviation σ. It is known that $P(X > 61) = 0.25$ and $P(X < 41) = 0.21$.

(a) Find the value of μ and σ. [5]

(b) A male student is considered severely overweight if his weight lies within the top 1% of the population. Find the minimum value of X for a student to be considered severely overweight. [2]

8. [Maximum mark: 7]

Let the complex number $z = \alpha + i\beta$ where α, β are non-zero real numbers.

(a) The complex number $\frac{z^*}{z^2}$ is purely imaginary. Find z in terms of α only. [4]

(b) If $\arg(z) > 0$, find the possible values of $\arg(z)$. [3]

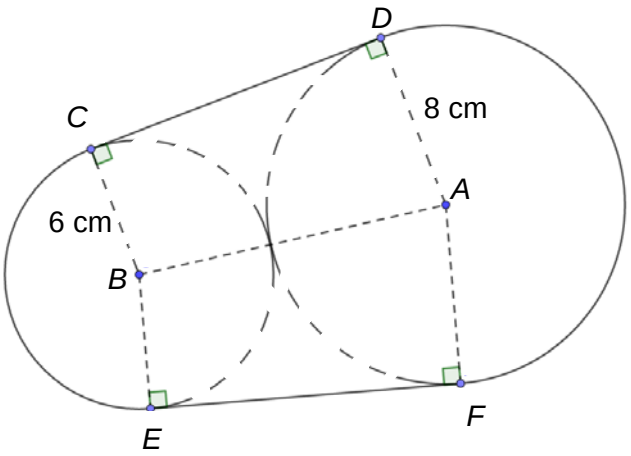
9. [Maximum mark: 8]

A circle with radius 8 cm and another with radius 6 cm are located such that they touch at one point. A piece of string is to be wrapped around the perimeter of the diagram below.

(a) Show that $CD = \sqrt{192}$ cm. [2]

(b) Find the obtuse angle $\angle CBE$ in radians. [3]

(c) Hence, or otherwise, determine the length of string needed to go around the perimeter of the diagram, such that it is just sufficiently tight. [3]



Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

10. [Maximum Marks: 14]

Consider a function $f(x) = (x-5)^2 - 2|x-5| - 9$ for $0 \le x \le 10$.

(a) Sketch the graph of $y = f(x)$. Label the coordinates of axial intercepts and the minimum and maximum points clearly. [5]

(b) Hence, or otherwise, solve the inequality $(x-5)^2 - 2|x-5| - 9 \le 0$ [2]

(c) State the range of function f. [1]

(d) State the values of k where the equation $(x-5)^2 - 2|x-5| - 9 = k$ has 4 distinct roots. [3]

(e) Given a function $g(x) = \ln(2x+1)$ for $x > -\frac{1}{2}$, find the roots of the equation $f(x) = g(x)$. [3]

11. [Maximum Marks: 23]

The overall score obtained by a figure skater in a competition is based on the technical score X and the score for style, Y.

Assuming that X is a **continuous** random variable with probability density function

$$f(x) = \begin{cases} \frac{x}{50} & 0 \le x \le 10 ; \\ 0 & \text{elsewhere,} \end{cases}$$

find

(a) the probability that a random figure skater scores at least 5 for his technical score and; [3]

(b) the value of $E(X)$ and $Var(X)$. [5]

It is also given that Y is a **discrete** random variable with probability function

$$f(y) = \begin{cases} ay & y = 1, 2, 3, 4, 5 \\ 0 & \text{elsewhere.} \end{cases}$$

(c) Find the value of the constant a. [3]

(d) Hence or otherwise, find

(i) $P(Y < 3)$ and;

(ii) the expected score for style. [5]

(e) Given that $Var(Y) = \frac{14}{9}$, find the value of $E(2Y + 5)$ and $Var(2Y + 5)$. [4]

(f) Given the probability for a figure skater to score at least 5 for his technical score and less than 3 for his style is $\frac{1}{10}$, state with reason, whether the variables X and Y are independent. [3]

12. [Maximum Marks: 18]

In the following Argand diagram, ΔABC is an equilateral triangle inscribed in a circle centred at the origin with radius 2 units. Point B represents a complex number w.

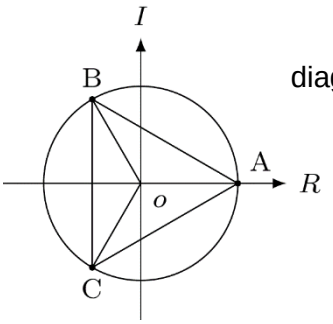


diagram not to scale

(a) Calculate the size of $A\hat{O}B$ in **radians**. [2]

(b) Hence, express w in the form $re^{i\theta}$, where r and θ are real numbers. [2]

(c) Show that

(i) $w^3 = 8$ and;

(ii) $w^2 + 2w + 4 = 0.$ [4]

(d) (i) Using results from part (c), show that $f(z) = (z-1)(z-\frac{w}{2})(z-\frac{w^2}{4})$ can be simplified to $f(z) = z^3 - \frac{w^3}{8}$, where $z \in \mathbb{C}$.

(ii) Hence, solve $f(z) = 26$ giving your answers in terms of π. [10]

END OF PAPER