

- 1 The gradient function of a curve is $\frac{1}{x^2} + \frac{1}{4}x$ and the curve passes through the point $\left(4, -\frac{1}{4}\right)$. Find the equation of the curve. [3]

2 (i) Show that $\log_x \sqrt{x} - \ln x$ can be expressed as a single logarithm $\ln \left(\frac{e^{\frac{1}{2}}}{x} \right)$. [2]

(ii) Hence, solve the equation $\log_x \sqrt{x} - \ln x = \lg \sqrt[3]{10}$, leaving your answer in terms of e. [2]

3 **(i)** The equation of a curve is $y = \frac{x^2}{3x-1}$. Find $\frac{dy}{dx}$, simplifying your answer. [2]

(ii) The tangent to the curve at the point where $x=1$ makes an angle of θ ($0^\circ < \theta < 90^\circ$) with x -axis. Find θ . [2]

4 The equation of a curve is $y = (a - x)(x - b)$, where a and b are real numbers.

(i) Write down the equation of the line of symmetry of the curve in terms of a and b . [1]

(ii) Hence, or otherwise, find an expression in terms of a and b , for the maximum value of y , leaving your answer in the simplest form. [2]

- 4 (iii) It is given that the equation $(a-x)(x-b)=k$, where k is a real number, has no real roots. State the range of values of k in terms of a and b . [1]

5 **(a)** Find $\int \frac{e^{4x} - 1}{e^{2x+1} + e} dx$. [2]

- 5** **(b)** Evaluate $\int_{-4}^{-1} \sqrt{\frac{3}{1-2x}} \, dx$, leaving your answer in surd form. [3]

6 The equation of a curve is given by $y = \frac{ax^3 + b}{x}$, where a and b are constants.

(i) Show that $x^2 \frac{d^2y}{dx^2} = 2y$. [3]

- 6 **(ii)** Given that the gradient of the curve at the point $(1, 2)$ is 3, find the values of a and b . [3]

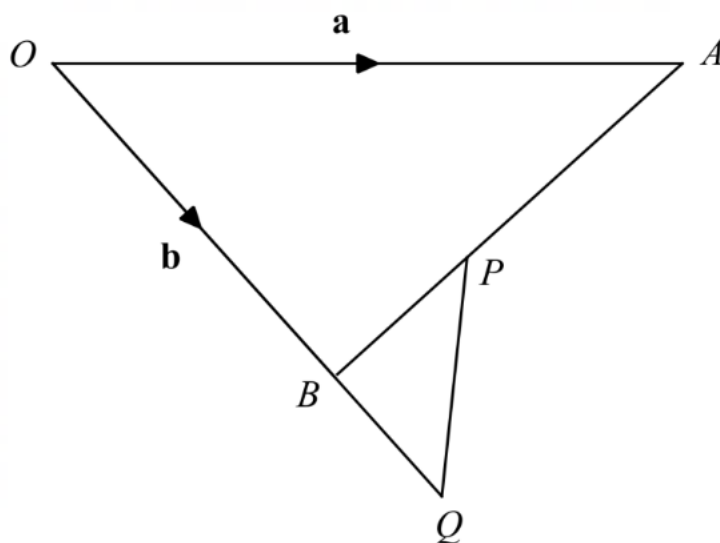
- 7 (a) The equation of a curve is $y = \ln(3x+1)\ln(3x-1)$. Find the gradient of the curve at the point where the curve cuts the line $x=1$, giving your answer to 3 significant figures. [3]

7 **(b)** The equation of a curve is $y = \sqrt{e^{16x}} \left(e^{-\frac{3}{x}} \right)$, where $x \neq 0$.

(i) Find $\frac{dy}{dx}$, simplifying your answer. [2]

(ii) Showing your explanation clearly, determine if there are any tangents to the curve that are parallel to the x -axis. [1]

- 8 In the diagram (not drawn to scale), $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. P is the point on AB such that $AB = 3PB$. The point Q lies on OB produced such that $OQ = 4BQ$.



- (i) Express, as simply as possible, $\vec{OP}$ in terms of $\mathbf{a}$ and/or $\mathbf{b}$. [2]

- (ii) Show that $\vec{PQ} = -\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$. [2]

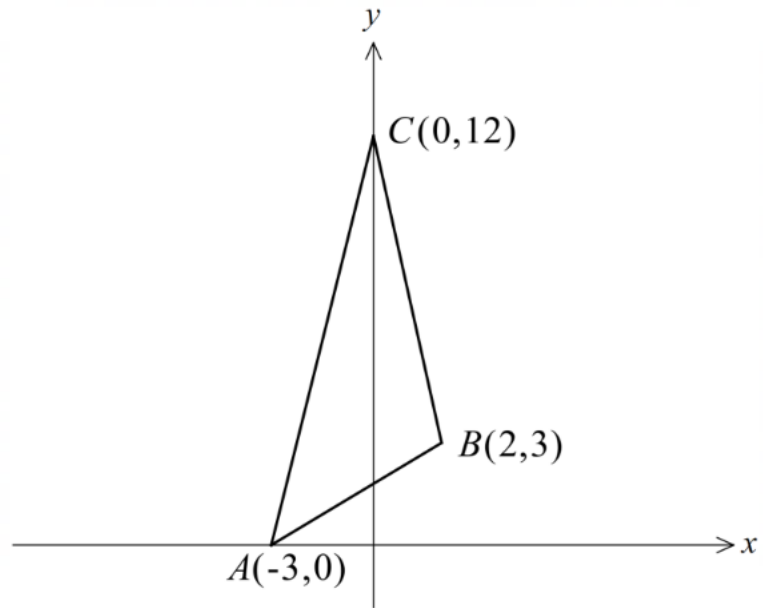
8 R is a point on OA such that $\overrightarrow{OR} = \frac{2}{3}\mathbf{a}$.

(iii) Using a vector method, determine whether P , Q and R are collinear. [3]

- 9** **(i)** Factorise $2x^3 - x^2 + 12x - 6$ completely. [2]

- 9 (ii) Hence, express $\frac{x^2 - x + 9}{2x^3 - x^2 + 12x - 6}$ as partial fractions. [5]

10



The diagram (not drawn to scale) shows triangle ABC with coordinates of A , B and C as $(-3, 0)$, $(2, 3)$ and $(0, 12)$ respectively.

- (i) Find the equation of the line perpendicular to AC and passes through B . [2]

10 **(ii)** Find the coordinates of the point D such that $ABCD$ is a kite. [4]

(iii) Find the area of the kite $ABCD$. [2]

- 11** Positive Auto, a car dealer, records the number of cars sold in the first quarter of some models and their unit selling prices in the following table.

Model	January	February	March	Unit selling price (thousand dollars)
HRV-3	8	10	7	120
N Note	15	12	10	80
MB C-Class	2	1	0	350
RZ 500h	3	1	2	200

The information given above can be represented by

$$\mathbf{G} = \begin{pmatrix} 8 & 10 & 7 \\ 15 & 12 & 10 \\ 2 & 1 & 0 \\ 3 & 1 & 2 \end{pmatrix} \text{ and } \mathbf{P} = (120 \ 80 \ 350 \ 200).$$

- (i) Write down a matrix $\mathbf{M}$ such that the elements in the matrix product $\mathbf{MG}$ gives the total number of cars that were sold in each month. Evaluate $\mathbf{MG}$. [2]
- (ii) Write down a matrix $\mathbf{D}$ such that the elements in the matrix product $\mathbf{GD}$ gives the total number of each model of cars that were sold during the 3-month period. [1]

- 11 (iii) Evaluate the matrix product $\mathbf{PGD}$ and explain what the element(s) in the product represent. [2]

- (iv) During the month of June, Positive Auto decided to hold a Father's Day promotion by giving a 5% discount on the selling price of HRV-3 and N Note, and a 8% discount on the selling price of MB C-Class and RZ 500h. Write down a 4×4 matrix $\mathbf{R}$ such that the matrix product $\mathbf{PR}$ will give the new discounted price of each model of car. Evaluate $\mathbf{PR}$. [2]

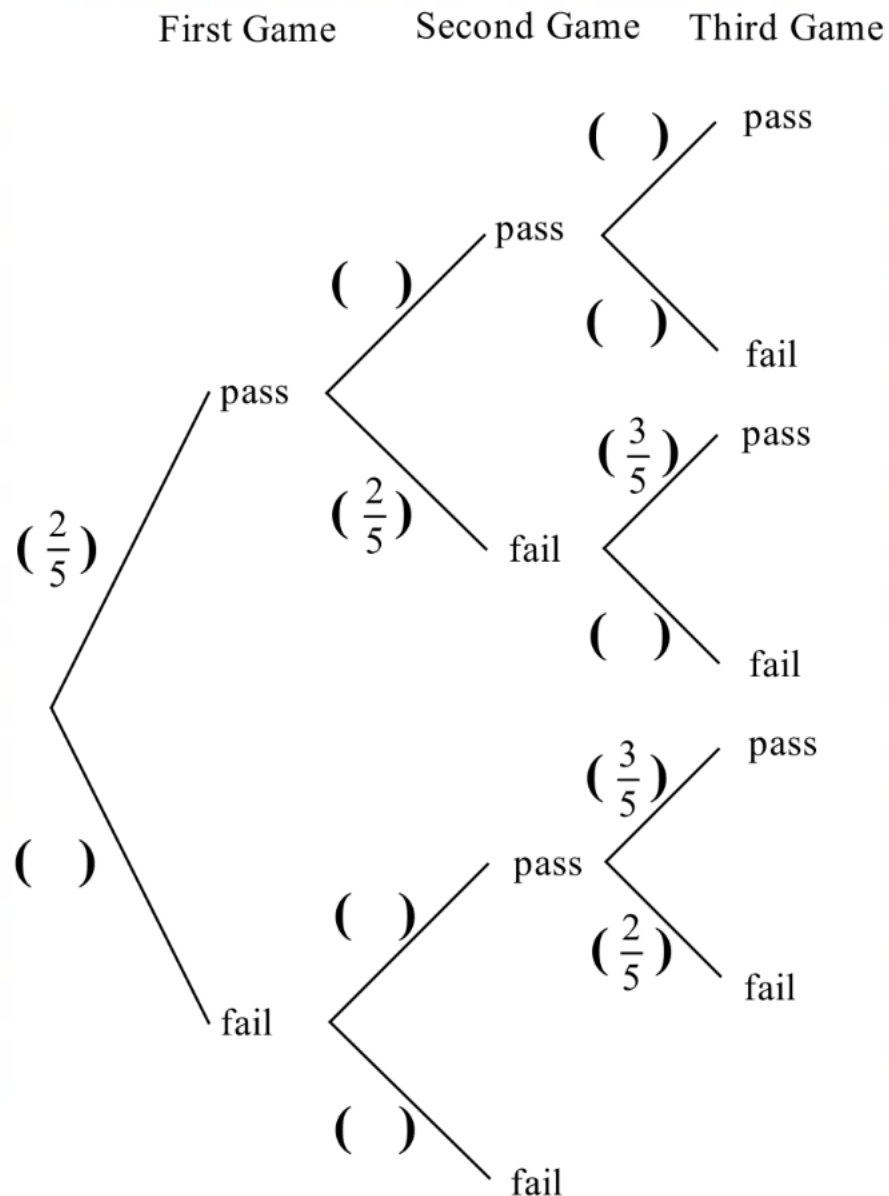
- 12 Old Rafflesian Hockey Club invites new players to take part in a series of three games. At the end of each game, the performance of each player is assessed as pass or fail. Players who fail in the first two games will not play the third game.

The probability of any player passing the first game is $\frac{2}{5}$.

For each of the player's second and third game,

- if he has passed in the preceding game, then the probability of passing is 1.5 times the probability of passing that preceding game,
- if he has failed in the preceding game, then the probability of passing is the same as the probability of passing that preceding game.

- (i) Two of the branches in the tree diagram are filled up. Complete the tree diagram. [2]



- 12** Players who achieve a pass in all three games are invited to join the first team squad.
Players who achieve a pass in exactly two games are invited to join the second team squad.
Players who fail in two games will not be able to join any team squad.

(ii) Find the probability that a randomly selected player is selected to join the second team squad. [2]

(iii) Find the probability that a randomly selected player is not able to join any team squad. [2]

(iv) Daniel and Rohit take part in the games at the same time. Given that their performances are independent, find the probability that they pass a total of 4 games exactly. [3]

- 13** When the polynomial $P(x) = 2x^3 + ax^2 + bx - 20$, where a and b are constants, is divided by $x^2 - 2x + 3$, it leaves a remainder of $x + 13$.

(i) Show that $a = -15$ and $b = 29$. [4]

13 **(ii)** Evaluate $P(5)$ and hence solve $P(x) = 0$. [4]

(iii) Hence, solve the equation $2y - a\sqrt[3]{y^2} + b\sqrt[3]{y} + 20 = 0$. [2]

END OF PAPER

