

Solutions:		
1	$\int \frac{3(1-x)^3 - 2}{1-x} dx = \int \left[\frac{3(1-x)^3}{1-x} - \frac{2}{1-x} \right] dx$ $= \int \left[3(1-x)^2 - \frac{2}{1-x} \right] dx$ $= \frac{3(1-x)^3}{(3)(-1)} + 2\ln(1-x) + c$ $= -(1-x)^3 + 2\ln(1-x) + c$	
2	$p + q\sqrt{2} = \frac{3+2\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$ $= \frac{3+3\sqrt{2}+2\sqrt{2}+2(2)}{1-2}$ $= \frac{7+5\sqrt{2}}{-1}$ $= -7-5\sqrt{2}$ $\therefore p = -7, q = -5$	
3	$V = \frac{4}{3}\pi r^3$ $\frac{dV}{dr} = 4\pi r^2$ $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ $20 - 4 = 4\pi r^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{4}{\pi r^2}$ When $r = 11$, $\frac{dr}{dt} = \frac{4}{\pi(11)^2}$ $\approx 0.0105 \text{ cm/s (correct to 3 s.f.)}$	
4	$2x^2 + 5xy - y^2 + 48 = 0 \quad \text{--- (1)}$	

Solutions:		
	$y + 8 = 2x \quad \text{--- (2)}$ From (2): $y = 2x - 8 \quad \text{--- (3)}$ Put (3) into (1): $2x^2 + 5x(2x - 8) - (2x - 8)^2 + 48 = 0$ $2x^2 + 10x^2 - 40x - (4x^2 - 32x + 64) + 48 = 0$ $2x^2 + 10x^2 - 40x - 4x^2 + 32x - 64 + 48 = 0$ $8x^2 - 8x - 16 = 0$ $x^2 - x - 2 = 0$ $(x + 1)(x - 2) = 0$ $x = -1 \quad \text{or} \quad x = 2$ Put $x = -1$ into (3): $y = 2(-1) - 8$ $y = -10$ Put $x = 2$ into (3): $y = 2(2) - 8$ $y = -4$ $\therefore$ Coordinates of the points of intersection are $(-1, -10)$ and $(2, -4)$.	
5	Let $\frac{8x^2 + 3x + 1}{x(2x + 1)^2} = \frac{A}{x} + \frac{B}{2x + 1} + \frac{C}{(2x + 1)^2}$ Multiplying throughout by the denominator, $8x^2 + 3x + 1 = A(2x + 1)^2 + Bx(2x + 1) + Cx$ When $x = 0$, $A = 1$ When $x = -\frac{1}{2}$, $8\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 1 = -\frac{1}{2}C$ $\frac{3}{2} = -\frac{1}{2}C$ $C = -3$ When $x = 1$, $8(1)^2 + 3(1) + 1 = (1)(3)^2 + 3B - 3(1)$ $12 = 9 + 3B - 3$ $B = 2$	

Solutions:		
	$\therefore \frac{8x^2 + 3x + 1}{x(2x+1)^2} = \frac{1}{x} + \frac{2}{2x+1} - \frac{3}{(2x+1)^2}$	
6(a)	$4^{x-1} = 3 + 2^{x+2}$ $\frac{(2^x)^2}{4} = 3 + 2^2(2^x)$ $(2^x)^2 - 16(2^x) - 12 = 0$	
(b)	Let $u = 2^x$. $u^2 - 16u - 12 = 0$ $u = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(-12)}}{2(1)}$ $u = \frac{16 \pm \sqrt{304}}{2}$ $u = -0.7177978871 \quad \text{or} \quad u = 16.71779789$ $2^x = -0.7177978871 \quad \text{or} \quad 2^x = 16.71779789$ (no solutions) * $x = \frac{\ln 16.71779789}{\ln 2}$ ≈ 4.06 ** (correct to 2 d.p.)	
7(a)	$C(x) = 40x^2 - 480x + 1442$ $= 40(x^2 - 12x) + 1442$ $= 40 \left[x^2 - 12x + \left(\frac{-12}{2} \right)^2 - \left(\frac{-12}{2} \right)^2 \right] + 1442$ $= 40[(x-6)^2 - 36] + 1442$ $= 40(x-6)^2 + 2$	
(b)	Minimum value of $C(x) = 2$ Corresponding value of $x = 6$	

Solutions:		
(c)	Minimum average cost per chair = \$2 Number of chairs produced = 6000	
8(a)	$\angle ABC = \angle CAF$ (Alternate Segment Theorem) $= \angle DEF$ ($\angle$s in the same segment) Since $\angle ABC = \angle DEF$, by the Alternate Angles Property, AB is parallel to ED. (proven)	
(b)	In $\triangle ACF$ and $\triangle BAF$, $\angle AFC = \angle BFA$ (common $\angle$) $\angle CAF = \angle ABF$ (Alternate Segment Theorem) $\therefore$ Triangle ACF is similar to triangle BAF. (proven)	
(c)	$\frac{AF}{BF} = \frac{CF}{AF}$ $AF^2 = BF \times CF$ (shown)	
9(a)	L.H.S. $= \frac{\cos 2\theta + 1}{3 + 3\sin \theta}$ $= \frac{1 - 2\sin^2 \theta + 1}{3(1 + \sin \theta)}$ $= \frac{2 - 2\sin^2 \theta}{3(1 + \sin \theta)}$ $= \frac{2(1 - \sin^2 \theta)}{3(1 + \sin \theta)}$ $= \frac{2(1 - \sin \theta)(1 + \sin \theta)}{3(1 + \sin \theta)}$ $= \frac{2(1 - \sin \theta)}{3}$ $= \text{R.H.S. (proven)}$	
(b)	$\frac{\cos 2\theta + 1}{3 + 3\sin \theta} = \frac{1}{2}$	

Solutions:		
	$\frac{2(1 - \sin \theta)}{3} = \frac{1}{2}$ $1 - \sin \theta = \frac{3}{4}$ $\sin \theta = \frac{1}{4}$ Basic angle, $\alpha = \sin^{-1} \frac{1}{4}$ $= 14.47751219^\circ$ $\theta = 14.47751219^\circ,$ $180^\circ - 14.47751219^\circ$ $\approx 14.5^\circ, 165.5^\circ$ (correct to 1 d.p.)	
10(a)	$\frac{d}{dx} \ln(\sin x) = \frac{\cos x}{\sin x}$ $= \cot x$	
(b)	$y = \cot x \quad \text{--- (1)}$ $y = \sqrt{2} \quad \text{--- (2)}$ (1) = (2): $\cot x = \sqrt{2}$ $\tan x = \frac{1}{\sqrt{2}}$ $x = \tan^{-1} \frac{1}{\sqrt{2}}$ Area of shaded region $= \int_{\tan^{-1} \frac{1}{\sqrt{2}}}^{\frac{\pi}{2}} \cot x \, dx$ $= \left[\ln(\sin x) \right]_{\tan^{-1} \frac{1}{\sqrt{2}}}^{\frac{\pi}{2}}$ $= \ln \left(\sin \frac{\pi}{2} \right) - \ln \left[\sin \left(\tan^{-1} \frac{1}{\sqrt{2}} \right) \right]$ $= 0 - (-0.5493061443)$	

Solutions:		
	$\approx 0.549 \text{ units}^2$ (correct to 3 s.f.)	
11(a)	$f(-2) = a(-2)^3 + 8(-2)^2 + b(-2) + 18$ $0 = -8a + 32 - 2b + 18$ $8a = 50 - 2b$ $a = \frac{25}{4} - \frac{1}{4}b \quad \text{--- (1)}$ $f(-1) = a(-1)^3 + 8(-1)^2 + b(-1) + 18$ $4 = -a + 8 - b + 18$ $a = 22 - b \quad \text{--- (2)}$ $(1) = (2): \quad \frac{25}{4} - \frac{1}{4}b = 22 - b$ $b = 21$ Put $b = 21$ into (2): $a = 22 - 21$ $= 1$	
(b)	$f(x) = x^3 + 8x^2 + 21x + 18$ By long division, $ \begin{array}{r} x^2 + 6x + 9 \\ x + 2 \overline{) x^3 + 8x^2 + 21x + 18} \\ \underline{x^3 + 2x^2} \\ 6x^2 + 21x + 18 \\ \underline{6x^2 + 12x} \\ 9x + 18 \\ \underline{9x + 18} \\ 0 \end{array} $ $f(x) = x^3 + 8x^2 + 21x + 18$ $= (x + 2)(x^2 + 6x + 9)$ $= (x + 2)(x + 3)^2$	
12(a)	$x = 10 - 3y \quad \text{--- (1)}$ $y = x - 2 \quad \text{--- (2)}$ Put (1) into (2): $y = 10 - 3y - 2$	

Solutions:		
	$y = 2$ Put $y = 2$ into (1): $x = 10 - 3(2)$ $= 4$ Coordinates of A are $(4, 2)$. Let the coordinates of C be (x, y). $(7.5, 3.5) = \left(\frac{4+x}{2}, \frac{2+y}{2} \right)$ $7.5 = \frac{4+x}{2} \quad \text{and} \quad 3.5 = \frac{2+y}{2}$ $x = 11 \qquad y = 5$ $\therefore$ Coordinates of C are $(11, 5)$.	
(b)	Gradient of $AC = \frac{2-5}{4-11}$ $= \frac{3}{7}$ Equation of AC is $y - 2 = \frac{3}{7}(x - 4)$ $y = \frac{3}{7}x + \frac{2}{7} \quad (\text{or } 7y = 3x + 2)$	
(c)	Area of triangle ABC $= \frac{1}{2} \begin{vmatrix} 4 & 7 & 11 & 4 \\ 2 & 1 & 5 & 2 \end{vmatrix}$ $= \frac{1}{2} [(4 + 35 + 22) - (14 + 11 + 20)]$ $= \frac{1}{2} (61 - 45)$ $= 8 \text{ units}^2$	
13(a)	Since $\text{period} = \frac{2\pi}{\frac{1}{b}}$,	

Solutions:

$$4\pi = \frac{2\pi}{\frac{1}{b}}$$

$$b = 2 \text{ (shown)}$$

(b) At $(\pi, 2)$, $2 = a \sin\left(\frac{\pi}{2}\right) + c$

$$2 = a + c$$

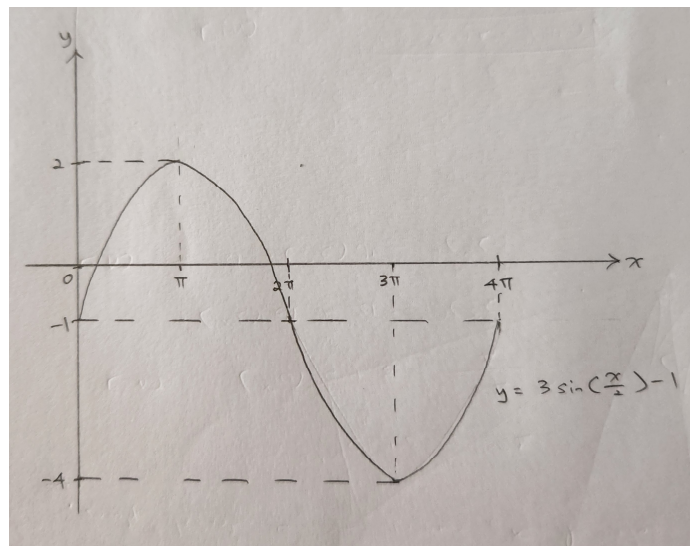
$$a = 2 - c \text{ --- (1)}$$

At $(4\pi, -1)$, $-1 = a \sin\left(\frac{4\pi}{2}\right) + c$

$$c = -1$$

Put $c = -1$ into (1): $a = 2 - (-1)$
 $= 3$

(c)



(d) $\sin\left(\frac{x}{2}\right) = \frac{x}{12\pi} - \frac{1}{3}$

$$3\sin\left(\frac{x}{2}\right) = \frac{x}{4\pi} - 1$$

$$3\sin\left(\frac{x}{2}\right) - 1 = \frac{x}{4\pi} - 2$$

Solutions:		
	Draw the line of $y = \frac{x}{4\pi} - 2$. The x-coordinates of the points of intersection of the graphs of $y = f(x)$ and $y = \frac{x}{4\pi} - 2$ give the solutions to the equation $\sin\left(\frac{x}{2}\right) = \frac{x}{12\pi} - \frac{1}{3}$.	
14(a)	$r\theta + 2r = 150$ $r\theta = 150 - 2r$ $\theta = \frac{150}{r} - 2$ Area of sector OAB, $A = \frac{1}{2}r^2\theta$ $= \frac{1}{2}r^2\left(\frac{150}{r} - 2\right)$ $= 75r - r^2 \text{ cm}^2 \text{ (shown)}$	
(b)	$\frac{dA}{dr} = 75 - 2r$ For stationary value, $\frac{dA}{dr} = 0$ $75 - 2r = 0$ $r = 37.5$ When $r = 37.5$, $A = 75(37.5) - (37.5)^2$ $= 1406.25$ $\frac{d^2A}{dr^2} = -2$ Since $\frac{d^2A}{dr^2} < 0$, $A = 1406.25$ is maximum when $r = 37.5$.	