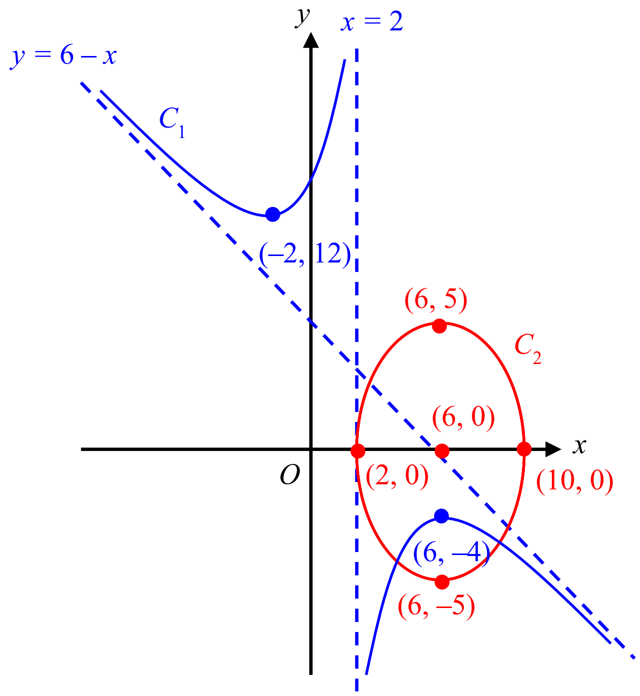
	Solution
1(a)	Differentiating C w.r.t. x, $3x^2+3y^2\dfrac{\mathrm{d}y}{\mathrm{d}x}-3x\dfrac{\mathrm{d}y}{\mathrm{d}x}-3y=0$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3y-3x^2}{3y^2-3x}=\dfrac{y-x^2}{y^2-x}$ When $y=0$, $x^3=k$, $x=k^{\frac{1}{3}}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=x=k^{\frac{1}{3}}$ $y=k^{\frac{1}{3}}\left(x-k^{\frac{1}{3}}\right)=k^{\frac{1}{3}}x-k^{\frac{2}{3}}$
(b)	For tangent to be parallel to y-axis, $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ must be undefined. Hence, $y^2-x=0$ $x=y^2$ Substituting into C and $k=3$, $y^6+y^3-3y^3-k=0$ $y^6-2y^3-3=0$ Factorising, $\left(y^3-3\right)\left(y^3+1\right)=0$ $y^3=3$ or $y^3=-1$ $y=3^{\frac{1}{3}}$ or $y=-1$

	Solution
2(a)	Let L be the limit of the sequence $u_{n+1} = 2u_n(1-u_n)$. $L = 2L(1-L)$ $2L^2 - L = 0$ $L(2L-1) = 0$ $L = 0$ or $L = \frac{1}{2}$
(b)	When $u_1 = 0$, $u_2 = 0$, $u_3 = 0$, sequence is convergent. When $u_1 = 0.01$, $u_2 = 0.0198$ (exact), $u_3 = 0.0388$ (3 s.f.), sequence is convergent. When $u_1 = -0.01$, $u_2 = -0.0202$ (exact), $u_3 = -0.0412$ (3 s.f.), sequence is not convergent.

<table>
<tr><td></td><td>Solution</td></tr>
<tr><td>3(a)</td><td>
</td></tr>
<tr><td>(b)</td><td>

x-coordinate of points of intersection are 4.6391, 7.7172 to 5 sf

Equation of ellipse: $y^2 = 25\left(1 - \dfrac{(x-6)^2}{16}\right)$

Required volume

$= \pi \displaystyle\int_{4.6391}^{7.7172} 25\left(1 - \frac{(x-6)^2}{16}\right) - \left(\frac{x^2 - 8x + 28}{2 - x}\right)^2 \, dx$

$= 58.7 \text{ units}^3$

</td></tr>
</table>

	Solution
4(a)	Normal vector of p $=\begin{pmatrix}0\\5\\2\end{pmatrix}\times\begin{pmatrix}5\\0\\t\end{pmatrix}=\begin{pmatrix}5t\\10\\-25\end{pmatrix}=5\begin{pmatrix}t\\2\\-5\end{pmatrix}$ plane p: $\mathbf{r}\bullet\begin{pmatrix}t\\2\\-5\end{pmatrix}=0$ Since l_1 and p are parallel, $\begin{pmatrix}t\\3-t^2\\1\end{pmatrix}\bullet\begin{pmatrix}t\\2\\-5\end{pmatrix}=0$ $t^2+6-2t^2-5=0$ $t^2=1$ $t=1$ or $t=-1$ Since point A is not on p, $\begin{pmatrix}1\\2\\1\end{pmatrix}\bullet\begin{pmatrix}t\\2\\-5\end{pmatrix}\neq 0$ $t+4-5\neq 0$ $t\neq 1$ $\therefore t=-1$ (Shown)
(b)	Let θ be the acute angle between l_1 and l_2 $\theta=\cos^{-1}\left\|\frac{1}{\sqrt{6}}\begin{pmatrix}-1\\2\\1\end{pmatrix}\bullet\frac{1}{\sqrt{6}}\begin{pmatrix}1\\2\\1\end{pmatrix}\right\|$ $=\cos^{-1}\left\|\frac{4}{6}\right\|$ $=0.841$ or 48.2°
(c)	$\|\mathbf{a}\cdot\mathbf{n}\|$ is the perpendicular distance from A to p $\|\mathbf{a}\cdot\mathbf{n}\|=\left\|\begin{pmatrix}1\\2\\1\end{pmatrix}\bullet\frac{1}{\sqrt{30}}\begin{pmatrix}-1\\2\\-5\end{pmatrix}\right\|$ $=\left\|\frac{-2}{\sqrt{30}}\right\|$ $=\frac{2}{\sqrt{30}}$

(d) Let N be the foot of perpendicular from A to p

$$\overrightarrow{ON} = \begin{pmatrix}1\\2\\1\end{pmatrix} - s\begin{pmatrix}-1\\2\\-5\end{pmatrix} = \begin{pmatrix}1+s\\2-2s\\1+5s\end{pmatrix}$$

Since N is on p,

$$\begin{pmatrix}1+s\\2-2s\\1+5s\end{pmatrix} \bullet \begin{pmatrix}-1\\2\\-5\end{pmatrix} = 0$$

$$-1 - s + 4 - 4s - 5 - 25s = 0$$

$$s = -\frac{1}{15}$$

$$\therefore \overrightarrow{ON} = \frac{1}{15}\begin{pmatrix}14\\32\\10\end{pmatrix}$$

Let B be the point of reflection of A in p

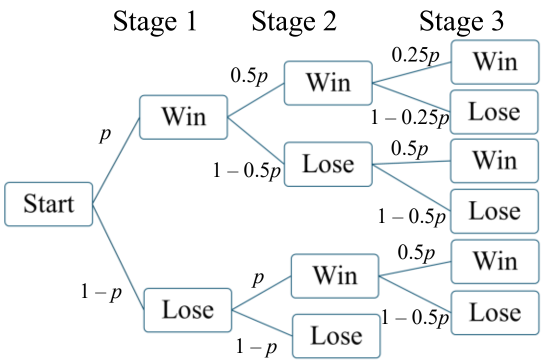
$$\overrightarrow{OB} = 2\overrightarrow{ON} - \overrightarrow{OA}$$

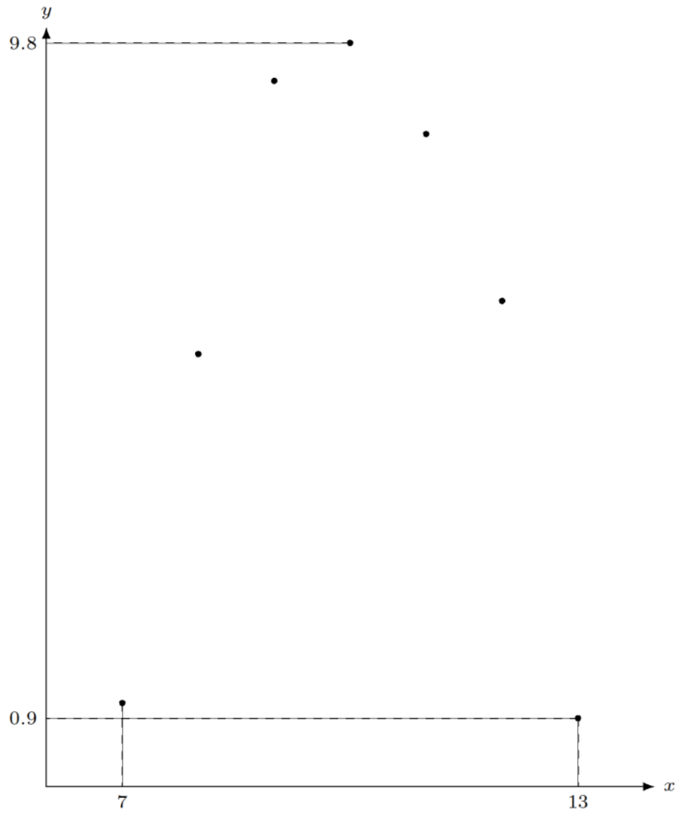
$$= \frac{2}{15}\begin{pmatrix}14\\32\\10\end{pmatrix} - \begin{pmatrix}1\\2\\1\end{pmatrix}$$

$$= \frac{1}{15}\begin{pmatrix}13\\34\\5\end{pmatrix}$$

line of reflection of l_2 in p: $\mathbf{r} = k\begin{pmatrix}13\\34\\5\end{pmatrix}, k \in \mathbb{R}$

	Solution
5(a)	Number of ways to choose the 6 glasses = total number of ways – number of ways with 1 missing colour $= \binom{8}{6} - \binom{4}{1} = 24$ Required number of ways $= 24\times(6-1)! = 2880$
(b)	Number of ways = $4\times 4 = 16$
(c)	Let *A* be the event that the blue glasses are adjacent. Let *B* be the event that the yellow glasses are adjacent. Then $\mathrm{n}(A') = 4!\times\binom{5}{2}\times 2! = 480$ [Explanation: arrange the four non-blue glasses (4!), then slot in the blue glasses (${}^{5}C_{2}\times 2!$)] Also $\mathrm{n}(A'\cap B) = 3!\times 2!\times {}^{4}C_{2}\times 2! = 144$ [Explanation: group the two yellow glasses (2!), arrange this group with the red and green glass (3!), then slot in the blue glasses (${}^{4}C_{2}\times 2!$)] So the required number of ways is $\mathrm{n}(A') - \mathrm{n}(A'\cap B) = 336$.

	Solution
6(a)	
(b)	$p(0.5p)(0.5^2 p) > 0.1$ $p^3 > 0.8$ $p > 0.928$
(c)	$\mathrm{P}(\text{wins exactly 2 stages out of 3} \mid \text{proceeds to Stage 3})$ $= \dfrac{\mathrm{P}(\text{wins exactly 2 stages out of 3} \cap \text{proceeds to Stage 3})}{\mathrm{P}(\text{proceeds to Stage 3})}$ $= \dfrac{0.9(0.45)(1-0.225)+0.9(0.55)(0.45)+0.1(0.9)(0.45)}{1-(0.1)(0.1)}$ $= \dfrac{0.577125}{0.99} = 0.583$

	Solution
7(a)	$r = 0.00208$
(b)	 There is a non-linear (curved) relationship between *x* and *y*.
(c)	$r = -0.997$. This means there is a (very) strong negative linear correlation between y and $(x-10)^2$.
(d)	Equation is $y = -0.987(x-10)^2 + 9.92$ When $x = 17$, $y = -38.4$ This estimate is not valid because $x = 17$ lies outside the valid range of $7 \le x \le 13$

	Solution

8(a)

$\mathrm{E}(X-3)=\mathrm{E}(X)-3=4-3=1$

$\mathrm{Var}(X-3)=\mathrm{E}\left((X-3)^2\right)-\left[\mathrm{E}(X-3)\right]^2=40-1=39$

$\mathrm{Var}(X)=\mathrm{Var}(X-3)=39$

Alternatively,

$\mathrm{E}\left((X-3)^2\right)=\mathrm{E}\left(X^2-6X+9\right)=40$

$\mathrm{E}\left(X^2\right)-6\mathrm{E}(X)+9=40$

$\mathrm{E}\left(X^2\right)=40+6\times4-9=55$

$\mathrm{Var}(X)=\mathrm{E}\left(X^2\right)-\left[\mathrm{E}(X)\right]^2=55-16=39$

(b)

y	0	1	2	3
$\mathrm{f}(y)$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
$\mathrm{P}(Y=y)$	$\left(\frac{2}{3}\right)^3$	${}^3C_1\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)$	${}^3C_2\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2$	$\left(\frac{1}{3}\right)^3$
$\mathrm{f}(y)\times\mathrm{P}(Y=y)$	$\frac{8}{27}$	$\frac{2}{9}$	$\frac{2}{27}$	$\frac{1}{108}$

$\mathrm{E}\left[\mathrm{f}(Y)\right]=\sum_y\left[\mathrm{f}(y)\times\mathrm{P}(Y=y)\right]$

$=\frac{8}{27}+\frac{2}{9}+\frac{2}{27}+\frac{1}{108}$

$=\frac{65}{108}$ (or 0.602 to 3 s.f.)

(c)

Since sum of probabilities is 1,

$p+q+0.15+r=1$

$p+q+r=0.85$ (1)

From $\mathrm{E}(W)=-0.2$,

$-3p-q+0.15+2r=-0.2$

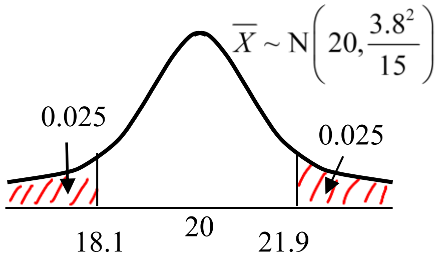
$-3p-q+2r=-0.35$ (2)

From $\mathrm{E}\left(W^3\right)=-3.2$,

$-27p-q+0.15+8r=-3.2$

$-27p-q+8r=-3.35$ (3)

From GC, solving simultaneous linear equations, $p=0.2, q=0.35, r=0.3$

	Solution
9(a)	The probability that any student is selected is the same, and the selection of any student is independent of the selection of other students.
(b)	Let X be the queue time at the canteen during peak hours of a randomly chosen EJC student and let μ be the population mean. $\mathrm{H}_0: \mu = 20, \quad \mathrm{H}_1: \mu \neq 20$ Assume that the queue times are normally distributed. Under H_0, $\overline{X} \sim \mathrm{N}\left(20, \frac{3.8^2}{15}\right)$ From GC,  Given that H_0 is not rejected at 5% level of significance, $18.1 < \bar{x} < 21.9$
(c)	$\bar{t} = \frac{\sum(t-20)}{50} + 20 = 19.3$ $s^2 = \frac{1}{n-1}\left(\sum(t-20)^2 - \frac{\left(\sum(t-20)\right)^2}{n}\right)$ $= \frac{1}{49}\left(489 - \frac{(-35)^2}{50}\right)$ $= \frac{929}{98}$ or $9.4796 \approx 9.48$ (3 s.f.)
(d)	Let μ be the population mean (of T). $\mathrm{H}_0: \mu = 20, \quad \mathrm{H}_1: \mu < 20$ Under H_0, since sample size n=50 is large, by Central Limit Theorem, $\overline{T} \sim \mathrm{N}\left(20, \frac{9.4796}{50}\right)$ approximately. At the 10% level of significance, the critical region is $\bar{t} < 19.442$. From data, $\bar{t} = 19.3 < 19.442$ Hence, we reject H_0 and conclude at 10% level of significance that there is sufficient evidence that EuOrder has reduced the queue time at the canteen.

	Solution
10(a)	Let X be the mass of a randomly chosen apple, in grams. Then $X \sim \mathrm{N}\left(100, 6^2\right)$. $\mathrm{P}(X \le 105) = 0.79767 = 0.798 \quad (3 \text{ s.f.})$
(b)	Let Y be the mass of a prepared apple, in grams. $\mathrm{E}(Y) = 0.7 \times 100 = 70$ $\mathrm{Var}(Y) = 0.7^2 \times 6^2 = 17.64$ Then, $Y \sim \mathrm{N}(70, 17.64)$. $\mathrm{P}(Y > 78) = 0.028405 = 0.0284 \quad (3 \text{ s.f.})$
(c)	Let T be the total mass of 2 prepared apples, 1 peeled banana, 12 blackcurrants and milk, in grams. $\mathrm{E}(T) = 2(70) + 130 + 12(2.5) + 50 = 350$ $\mathrm{Var}(T) = 2(17.64) + 8^2 + 12\left(0.6^2\right) = 103.6$ $T \sim \mathrm{N}(350, 103.6)$ $\mathrm{P}(T \ge m) = 0.9$ Using GC, $m = 336.96 = 337 \quad (\text{nearest gram})$
(d)	$\mathrm{P}(330 < T < 360) = 0.81236$ Let R be the number of Signature smoothies, out of 25, with masses between 330 and 360 grams. Then, $R \sim \mathrm{B}(25, 0.81236)$ $\mathrm{P}(R \ge 20) = 1 - \mathrm{P}(R \le 19) = 0.677 \quad (3 \text{ s.f.})$
(e)	Let W be the total mass of n blackcurrants and milk, in grams. $\mathrm{E}(W) = n(2.5) + 175 = 2.5n + 175$ $\mathrm{Var}(W) = n\left(0.6^2\right) = 0.36n$ $W \sim \mathrm{N}(2.5n + 175, 0.36n)$ $\mathrm{P}(W > 300) \ge 0.95$ Using GC,

n	$\mathrm{P}(W > 300)$
52	0.8761
53	0.957

Hence, the least value of n = 53.