NANYANG JUNIOR COLLEGE
JC2 Timed Practice 2026
General Certificate of Education Advanced Level
Higher 3

CANDIDATE NAME

CIVICS GROUP | | | - | | |

REGISTRATION NUMBER | | |

PHYSICS 9814/01

Paper 1

09 July 2026

1 hour 30 min

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and registration number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

The use of an approved scientific calculator is expected where appropriate.

Answer **all** questions.

For Examiner's Use	
1	**10**
2	**12**
3	**10**
4	**20**
Total	**52**

This document consists of **14** printed pages

Data

speed of light in free space,	c	=	3.00×10^{8} m s^{-1}
permeability of free space,	μ_o	=	$4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space,	ε_o	=	8.85×10^{-12} F m^{-1}
			$\left(\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^{9}\,\text{m F}^{-1}\right)$
elementary charge,	e	=	1.60×10^{-19} C
the Planck constant,	h	=	6.63×10^{-34} J s
unified atomic mass constant,	u	=	1.66×10^{-27} kg
rest mass of electron,	m_e	=	9.11×10^{-31} kg
rest mass of proton,	m_p	=	1.67×10^{-27} kg
molar gas constant,	R	=	8.31 J K^{-1} mol^{-1}
the Avogadro constant,	N_A	=	6.02×10^{23} mol^{-1}
the Boltzmann constant,	k	=	1.38×10^{-23} J K^{-1}
gravitational constant,	G	=	6.67×10^{-11} N m^{2} kg^{-2}
acceleration of free fall,	g	=	9.81 m s^{-2}

Formulae

uniformly accelerated motion	s	=	$ut + \frac{1}{2}at^2$
	v^2	=	$u^2 + 2as$
moment of inertia of rod through one end	I	=	$\frac{1}{3}ML^2$
moment of inertia of hollow cylinder through axis	I	=	$\frac{1}{2}M(r_1^2 + r_2^2)$
moment of inertia of solid sphere through centre	I	=	$\frac{2}{5}MR^2$
moment of inertia of hollow sphere through centre	I	=	$\frac{2}{3}MR^2$
work done on/by a gas	W	=	$p\,\Delta V$
Pressure	p	=	$\frac{F}{A}$
gravitational potential	ϕ	=	$-\frac{Gm}{r}$
temperature	T/K	=	T / °C + 273.15
pressure of an ideal gas	p	=	$\frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle$
mean translational kinetic energy of an ideal gas particle	E	=	$\frac{3}{2}kT$

displacement of particle in s.h.m.	x	=	$x_o \sin \omega t$	alternating current/voltage,	x	=	$x_o \sin \omega t$
velocity of particle in s.h.m.	v	=	$v_o \cos \omega t$	magnetic flux density due to a long straight wire	B	=	$\dfrac{\mu_o I}{2\pi d}$
		=	$\pm \omega\sqrt{(x_o{}^2 - x^2)}$	magnetic flux density due to a flat circular coil	B	=	$\dfrac{\mu_o NI}{2r}$
electric current,	I	=	$Anvq$	magnetic flux density due to a long solenoid	B	=	$\mu_o nI$
resistors in series,	R	=	$R_1 + R_2 + \ldots$	energy in an inductor	U	=	$\dfrac{1}{2}LI^2$
resistors in parallel,	$\dfrac{1}{R}$	=	$\dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots$	RL series circuits	τ	=	$\dfrac{L}{R}$
capacitors in series	$\dfrac{1}{C}$	=	$\dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$	RLC series circuits (underdamped)	ω	=	$\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$
capacitors in parallel	C	=	$C_1 + C_2 + \ldots$	energy states for quantum particle in a box	E_n	=	$\dfrac{h^2}{8mL^2}n^2$
energy in a capacitor	U	=	$\dfrac{1}{2}QV$	radioactive decay	x_0	=	$x_0 e^{-\lambda t}$
		=	$\dfrac{1}{2}\dfrac{Q^2}{C}$	radioactive decay constant	λ	=	$\dfrac{\ln 2}{t_{1/2}}$
		=	$\dfrac{1}{2}CV^2$	Lorentz factor	γ	=	$\dfrac{1}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}$
charging a capacitor	Q	=	$Q_0\left[1 - e^{-\frac{t}{\tau}}\right]$	length contraction	L	=	$\dfrac{L_0}{\gamma}$
discharging a capacitor	Q	=	$Q_0 e^{-\frac{t}{\tau}}$	time dilation	T	=	γT_0
RC time constant	τ	=	RC	Lorentz transformation equations (1 dimension)	x'	=	$\gamma(x - vt)$
electric potential,	V	=	$\dfrac{Q}{4\pi\varepsilon_o r}$		t'	=	$\gamma\left(t - \dfrac{vx}{c^2}\right)$
electric field strength due to a long straight wire	E	=	$\dfrac{\lambda}{2\pi\varepsilon_o r}$	velocity addition	u'	=	$\dfrac{(u - v)}{\left(1 - \dfrac{uv}{c^2}\right)}$
electric field strength due to a large sheet	E	=	$\dfrac{\sigma}{2\varepsilon_o}$	mass-energy equivalence	E^2	=	$(pc)^2 + (mc^2)^2$

Answer **all** questions.

1 Two perfectly elastic balls, A and B, are held above the ground at a height h, such that h is much greater than the diameter of each ball. The balls are almost touching, with ball A directly above ball B.

The balls are released simultaneously. Ball A rebounds to a maximum height H above the ground.

(a) Sketch a labelled diagram to show the velocities of the balls relative to the ground just before they collide with each other.

Hence show that when the balls collide with each other, they do so with relative velocity

$$v_{rel} = \sqrt{8gh}$$

By conservation of energy,
Gain in KE = Loss in GPE

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh} \quad [B1]$$

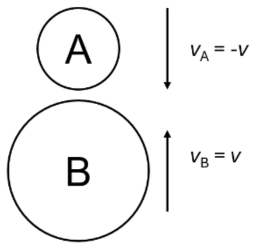

Diagram showing balls moving in opposite directions with same speed [B1]

Taking upwards as positive.
Since the balls have the same speed just before collision,
AND the ball collides elastically with the ground and changes direction,

$$v_{rel} = \sqrt{2gh} - (-\sqrt{2gh}) = 2\sqrt{2gh} = \sqrt{8gh} \text{ (shown)} \quad [B1]$$

It's a "show" question, clear explanation of the steps used is important to score. Don't just show the maths!

(b) The mass of the lower ball m_B is greater than or equal to the mass of the upper ball m_A so that:

$$\frac{m_B}{m_A} = n \quad \text{and} \quad n \geq 1$$

Determine an expression for the velocity of the centre of mass frame v_{CoM} relative to the ground immediately before the two balls collide.

$$(m_A + m_B)v_{CM} = m_A(-v) + m_B(v) \quad [C1]$$

$$v_{CM} = \frac{-m_A v + m_B v}{(m_A + m_B)}$$

$$v_{CM} = \frac{\left(-\frac{m_A}{m_A} + \frac{m_B}{m_A}\right)}{\left(\frac{m_A}{m_A} + \frac{m_B}{m_A}\right)}v = \left(\frac{-1+n}{1+n}\right)v = \left(\frac{n-1}{n+1}\right)\sqrt{2gh} \text{ (upwards)} \quad [A1]$$

[2]

(c) **(i)** Determine an expression for the ratio of the rebound height H to the drop height h.

Using $\vec{v} - \vec{v}_{CM} = \vec{v}'$ (where $\vec{v}'$ = velocity of body relative to CM)

$$\vec{u}_A - \vec{u}_{CM} = \vec{u}_A'$$

$$-\sqrt{2gh} - \left(\frac{n-1}{n+1}\right)\sqrt{2gh} = \vec{u}_A'$$

$$-\sqrt{2gh}\left[1+\left(\frac{n-1}{n+1}\right)\right] = \vec{u}_A' \qquad \text{[B1]}$$

In CM frame, to conserve momentum, $\vec{v}_A' = -\vec{u}_A'$ [B1]

$$\vec{v}_A' = \sqrt{2gh}\left[1+\left(\frac{n-1}{n+1}\right)\right]$$

$$\vec{v}_A = \vec{v}_{CM} + \vec{v}_A'$$

$$= \left(\frac{n-1}{n+1}\right)\sqrt{2gh} + \sqrt{2gh}\left[1+\left(\frac{n-1}{n+1}\right)\right]$$

$$= \sqrt{2gh}\left[1+2\left(\frac{n-1}{n+1}\right)\right]$$

$$= \sqrt{2gh}\left(\frac{3n-1}{n+1}\right) = \sqrt{2gh}\left(3-\frac{4}{n+1}\right)$$

since $h \propto v^2$

$$\frac{H}{h} = \frac{\left[\sqrt{2gh}\left(3-\frac{4}{n+1}\right)\right]^2}{\left(\sqrt{2gh}\right)^2} = \left[3-\frac{4}{(n+1)}\right]^2 \qquad \text{[A1]}$$

[3]

(ii) Deduce the maximum value of $\frac{H}{h}$ for $n \geq 1$.

Rearrange answer in 1(c)(i) to give $\frac{H}{h} = \left[3-\frac{4}{n+1}\right]^2$ [M1]

As n tends to infinity, the expression tends to 9 [A1]

maximum value of $\frac{H}{h}$ =...................................... [2]

[Total: 10]

2 A consequence of special relativity is that observers in different inertial reference frames disagree on the lengths of objects and the times between events.

(a) State what is meant by an inertial observer.

an observer, moving at constant velocity / not accelerating [B1] ……….

…… [1]

(b) An inertial reference frame (x, y, z, t) has Earth as its origin.

Ignore the rotational motion of Earth as negligible on the length scale of interest.

In this reference frame, the star Proxima Centauri is a distance of 4.25 light years (ly) along the positive x-axis, as shown in Figure 2.1.

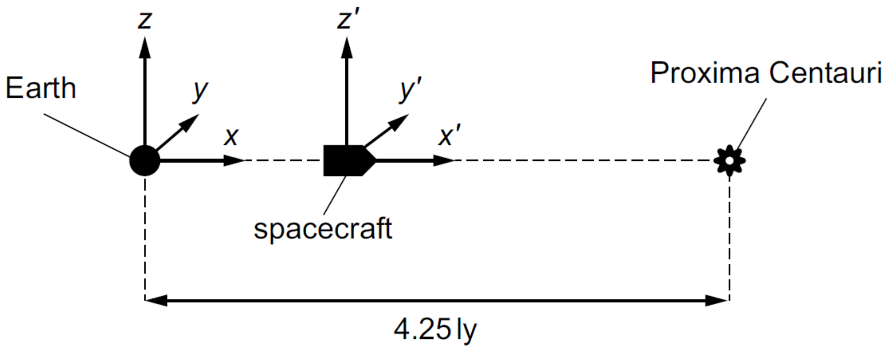

Figure. 2.1

A spacecraft leaves Earth at $t = 0$ and travels directly to Proxima Centauri at a speed of $\frac{c}{2}$.

Assume that there is negligible relative motion between Earth and Proxima Centauri.

The origin of an inertial reference frame (x', y', z', t') moving with the spacecraft coincides with the origin of the Earth's reference frame at the moment the spacecraft leaves the Earth.

(i) Write down the x and t coordinates of the spacecraft in Earth's reference frame at the moment of departure from Earth and at the moment of arrival at Proxima Centauri.

Use units of light years (ly) for x and years (yr) for t. In these units, $c = 1$.

departure: $x = 0$ AND $t = 0$ [A1]

arrival: $x = 4.25$ (ly) [A1]

$t = 8.50$ (yr) [A1]

x at departure = ... ly

t at departure = ... yr

x at arrival = ... ly

t at arrival = ... yr

[3]

(ii) Use the Lorentz transformation equations to write down the x' and t' coordinates of the spacecraft at the moment of departure from Earth and at the moment of arrival at Proxima Centauri.

Use units of light years (ly) for x' and years (yr) for t'. In these units, $c = 1$.

departure: $x' = 0$ and $t' = 0$ [B1]

arrival: $x' = 0$ [B1]

use of Lorentz transformation equation: $t' = \gamma\left(t - \frac{vx}{c^2}\right)$ [M1]

$$t' = \frac{1}{\sqrt{1 - \frac{\left(\frac{c}{2}\right)^2}{c^2}}}\left(8.50 - \frac{\left(\frac{c}{2}\right)(4.25c)}{c^2}\right) = 7.36\ \text{(yr)}$$ [A1]

x' at departure = ... ly

t' at departure = ... yr

x' at arrival = ... ly

t' at arrival = ... yr

[4]

(iii) Using your answer in **2(b)(ii)** or otherwise, calculate the distance from Earth to Proxima Centauri in the (x', y', z', t') frame.

Distance = $vt' = \dfrac{7.36}{2} = 3.68$ ly [A1]

distance = ... ly [1]

(iv) The spacecraft returns immediately to Earth at the same speed of $\dfrac{c}{2}$.

Explain why, for an inertial observer onboard the spacecraft, this journey could be considered a form of time travel.

Support your explanation with calculations.

time passed on Earth = 17.0 yr [B1]

time passed for observer on spacecraft = 14.7(2) yr [B1]

fewer years pass for observer on spacecraft than on Earth so observer has travelled into Earth's future / idea of age difference as a form of time travel, e.g. person on Earth has aged more (twin paradox) [B1]

....

....

....

[3]

[Total: 12]

3 Fig. 3.1 shows a side view and top view of a helicopter hovering stationary in mid-air.

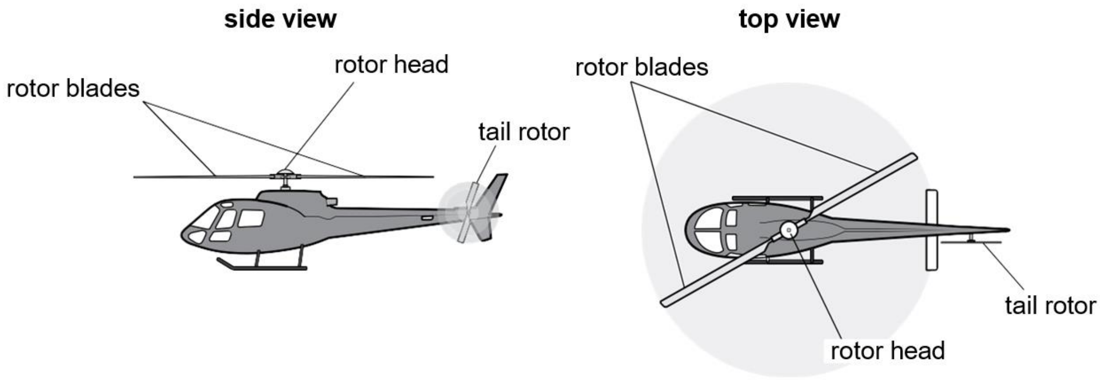

Fig. 3.1

The two rotor blades of the helicopter can be modelled as uniform thin rectangular plates of length R and width c, as shown in Fig. 3.2.

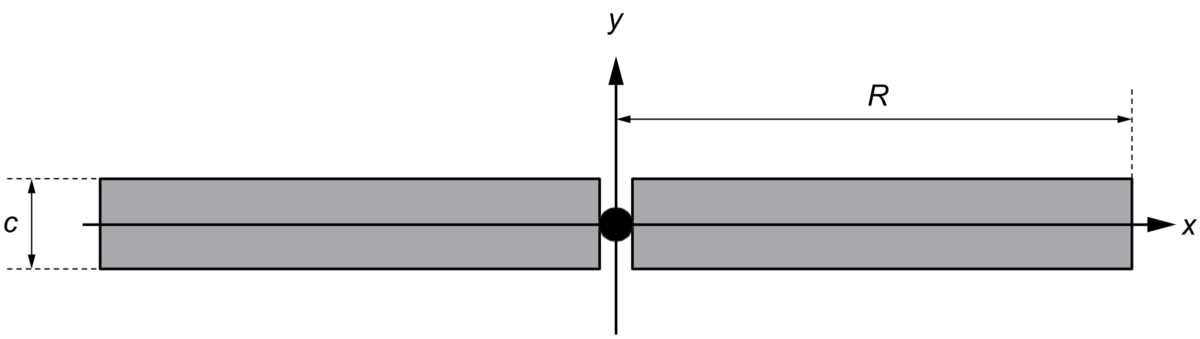

Fig. 3.2 (not to scale)

The gap between the two rotor blades is negligible when compared with the total length of the blades.

(a) Show, from first principles, that the moment of inertia, I_x, of the rotor blades about the x-axis in Fig. 3.2 is given by

$$I_x = \frac{1}{12}Mc^2$$

where M is the **total** mass of the rotor blades.

$\int dI = \int r^2 dm = \sigma \int y^2 dA$(where $\sigma = \frac{dm}{dA}$) C1

$I_x = 2\sigma R \int_{-c/2}^{c/2} y^2 dy$, dA=2Rdy M1

$= \frac{\sigma R c^3}{6}$ M1

$= \frac{Mc^2}{12}$ since $\sigma = \frac{M}{2Rc}$ A0

[3]

(b) The total lift force on the helicopter can be calculated by summing the lift on infinitesimal elements of the blade.

Fig. 3.3 shows an element of the rotor blade with width *c* and length *dr* at a distance *r* from the axis of rotation.

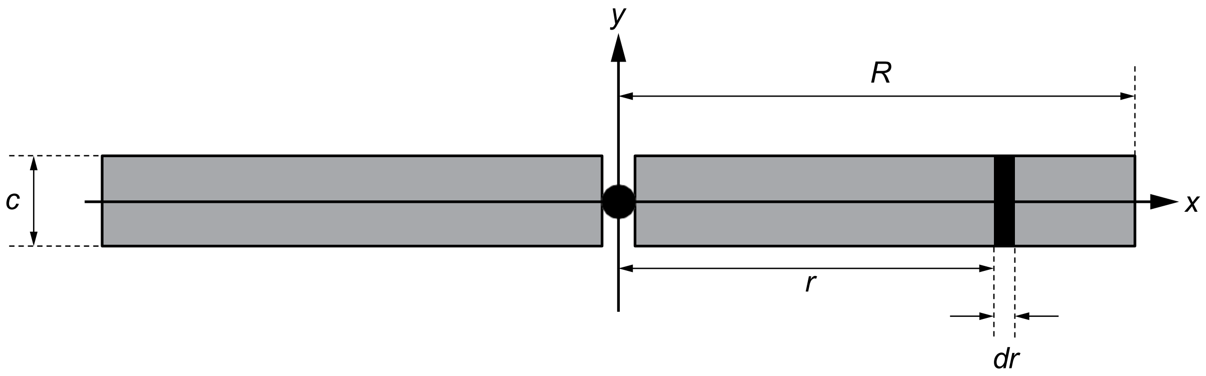

Fig. 3.3 (not to scale)

The lift *dL* on the blade element is given by the equation:

$$dL = \frac{1}{2}C_L\rho(r\omega)^2 c\,dr$$

where:

C_L is the coefficient of lift

ρ is the density of air and can be taken to be 1.25 kg m^{-3}

ω is the angular velocity of the rotor blades.

(i) Show that the coefficient of lift C_L is dimensionless.

[N] = [kg m s^{-2}] = [C_L] × [kg m^{-3}] × [m s^{-1}]^2 × [m] × [m] M1

C_L is dimensionless A0

[1]

(ii) Show that the total lift force *L* generated by the two rotor blades is given by:

$$L = \frac{1}{3}C_L\rho\omega^2 cR^3$$

integrating for a single blade: $\int_0^L dL = \frac{1}{2}C_L\rho\omega^2 c\int_0^R r^2 dr$ C1

$L = \frac{1}{6}C_L\rho c\omega^2 R^3$ C1

doubling this answer for the second blade will give the shown result M1

$L = \frac{1}{3}C_L\rho c\omega^2 R^3$ A0

[3]

(c) Coning is an effect where the rotor blades of the helicopter tilt upwards from the horizontal, as shown in Fig. 3.4.

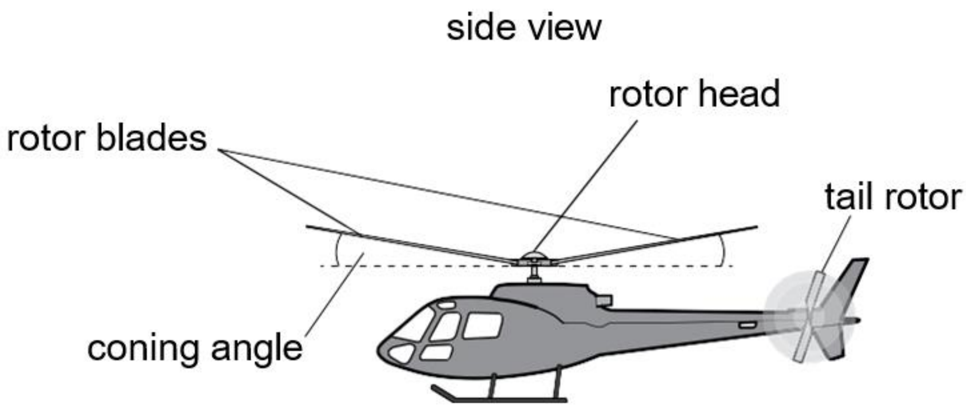

Fig. 3.4

(i) Explain why coning occurs.

by Newton's third law there is a reaction force from the air pushing the rotor blades upwards (the upwards force is larger than the weight of the rotor blades)
B1
OR
the lift generated becomes larger with distance along the blades from the rotor head
(B1)
OR
the weight of the helicopter acts through the rotor head and lift acts on the blades, creating moments B1

.... [1]

(ii) The total mass of the helicopter, including blades, is 2500 kg.

Calculate the angular velocity ω of the rotor blades, in rpm, required for the helicopter to hover at a constant height above the ground with a coning angle of 10.0°.

Use a typical value for C_L of 0.400, R = 3.84 m and width c = 20 cm.

$mg = L \cos\theta \rightarrow mg = \frac{1}{3} C_L \rho c \omega^2 R^3 \cos\theta$ M1

ω = 114.9 rad s^{-1} ≈ 1100 rpm A1

ω = ... rpm [2]

[Total: 10]

4 A uniformly charged thin disc of radius R lies in the x–y plane as shown in Fig. 4.1.

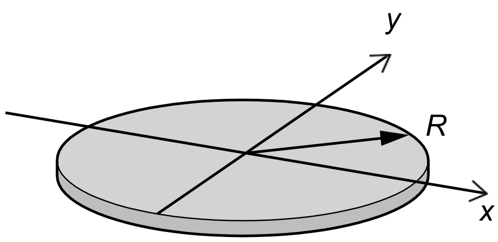

Fig. 4.1

The total amount of charge on the disc is Q.

(a) (i) State an expression for the surface charge density σ in terms of Q and R.

$$\boxed{\sigma = \frac{Q}{\pi R^2}}$$

[1]

(ii) Use your answer in **4(a)(i)** and apply Gauss's law with an appropriately chosen Gaussian surface to show that an approximation for the electric field at the position (0,0,z), where $z \ll R$, is given by:

$$E_z = 2\pi k\sigma$$

where k is a constant you need to determine.

You may wish to draw a diagram to help your answer.

$\oiint_S E.dA = \frac{Q_{enc}}{\varepsilon_0}$ B1

pillbox shaped surface chosen AND field lines taken normal to the surface B1

$\oiint_S E.dA = \oiint_S E.dA\cos 0 = E\oiint_S dA = E(2A) = E \times 2\pi r^2$ C1

$Q_{enc} = \pi r^2\sigma = \frac{Qr^2}{R^2}$ C1

$E = \frac{Q}{2\pi\varepsilon_o R^2}$ M1

$= \frac{\sigma}{2\varepsilon_o} = 2\pi k\sigma,$ $k = \frac{1}{4\pi\varepsilon_o}$ B1

[6]

(iii) Determine an expression for the electric potential at the point (0,0,z) relative to the origin.

$$E_z = -\frac{dV}{dz} \quad -> \quad V = -\int_0^z E_z dz \quad \text{M1}$$

$$V = -\int_0^z 2\pi k\sigma dz = -2\pi k\sigma z (= -\frac{\sigma z}{2\varepsilon_o}) \qquad \text{A1}$$

[2]

(b) The electric field can be determined more accurately than determined in **4(a)** by superimposing the point charge fields of infinitesimal charge elements. This can be done by summing the fields of charged rings of width dr, as shown in Fig. 4.2.

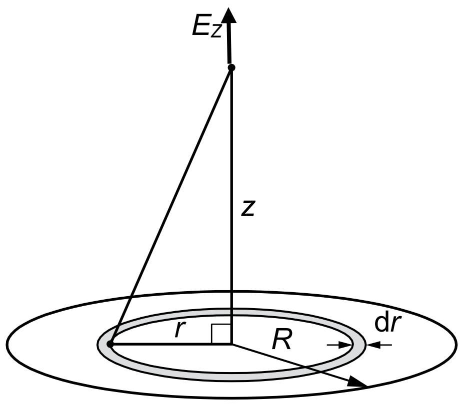

Fig. 4.2

i. Show that the electric field at position (0,0,z) is given by:

$$E_Z = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

Coulomb's Law: E= $\dfrac{q}{4\pi\varepsilon_o d^2}$ B1

Pythagoras gives $h^2 = z^2 + r^2$ B1

$$\cos\theta = \frac{z}{\sqrt{z^2 + r^2}} \qquad \text{C1}$$

$$E = \frac{\sigma}{4\pi\varepsilon_o}\int\int_0^R \frac{z}{(z^2 + r^2)^{3/2}} \times 2\pi r dr \qquad \text{C1}$$

$$= \frac{\sigma z}{2\varepsilon_o}\left(\frac{1}{z} - \frac{1}{\sqrt{z^2 + R^2}}\right) \qquad \text{M1}$$

$$E_Z = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right] \qquad \text{A0}$$

[5]

ii. Determine an expression for E_z when $z \ll R$.

$$\frac{z}{\sqrt{z^2+R^2}} \text{ -> } \frac{z}{R} -> 0 \qquad \text{M1}$$

$$Hence\ ,\ \text{E} = \frac{\sigma}{2\varepsilon_o} \qquad \text{A1}$$

[2]

iii. Determine a non-zero expression for E_z when $z \gg R$.
You may wish to use the approximation shown when x is small.
$(1 + x)^n \approx 1 + nx$

$$\frac{z}{\sqrt{z^2+R^2}} \text{ -> } \frac{z}{z} -> 1 \text{ (will give a result of 0 and thus unphysical)}$$

Use of binomial,

$$\frac{z}{\sqrt{z^2+R^2}} = [1+\frac{R^2}{z^2}]^{-1/2} \approx 1-\frac{R^2}{2z^2} \qquad \text{M1}$$

$$Hence\ ,\ \text{E} = \frac{\sigma R^2}{4\varepsilon_o z^2} \qquad \text{A1}$$

[2]

iv. Comment on the form of the expressions in **4(b)(ii)** and **4(b)(iii)**.

first agrees with the Gaussian pillbox approach in earlier part: an (infinitely) large slab / plate / sheet B1

second agrees with Coulomb's law for a point charge of charge Q B1

....[2]

[Total: 20]

END OF PAPER