

1 QUANTITIES & MEASUREMENT

H2 Physics 9478



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Learning Outcomes

Candidates should be able to:

- recall and use the following SI base quantities and their units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol)
- recall and use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T)
- express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate
- use SI base units to check the homogeneity of physical equations
- make reasonable estimates of physical quantities included within the syllabus.
- show an understanding of the distinction between random and systematic errors (including zero error), which limit precision and accuracy
- assess the uncertainty in derived quantities by adding absolute or relative (i.e. fractional or percentage) uncertainties or by numerical substitution (rigorous statistical treatment not required).
- distinguish between scalar and vector quantities, and give examples of each
- add and subtract coplanar vectors
- represent a vector as two perpendicular components

1.1 Introduction

Physics is an experimental science. Precise and accurate measurements enable the collection of useful experimental data that can be tested against theoretical predictions to refine the development of physical theories. Experimental evidence is the ultimate authority in discriminating between competing physical theories. Scientific knowledge continues to evolve as data from new or improved measurements helps us to better understand and explain physical phenomena.

Measurements are subject to uncertainties, and it is important to estimate these to understand the reliability of the measurements. Error analysis involves estimating the uncertainties in measurements and finding ways to reduce them if necessary. In an experiment, the record of measurements made should include the estimated uncertainties and an analysis of the possible sources of errors with a discussion of steps taken to reduce the uncertainties should be documented. Doing this enables better conclusions to be drawn from the experimental data.

The act of measurement affects the object being measured due to the interaction between the measuring device and the object. Common examples of this include measurements made using a thermometer, voltmeter or ammeter. Thus, improving the accuracy of measurements often requires the use of better instruments and enhanced experimental techniques.

Physicists are very serious about measurements, and the other sciences and society as a whole have benefitted from the spill-over effects of the invention of many amazing measuring devices and techniques. Modern engineering also depends heavily on accurate measurements in areas such as design, construction, optimization and communication. Precise measurements have made many advanced technological applications possible; examples include the study and manipulation of materials, and breakthroughs in fields as diverse as geophysics and biology. Measurements using sophisticated devices like magnetic resonance imaging (MRI) scanners are important in the medical industry as it provides a wealth of data that aids in clinical diagnosis and influences decisions with regards to treatment.

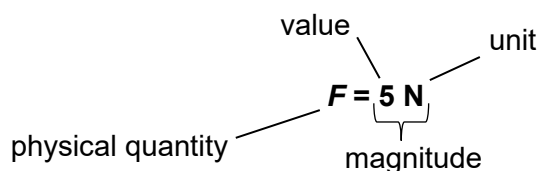
1.2 Quantities and Units

❖ Physical Quantities

The laws of Physics are commonly expressed as mathematical relationships among physical quantities and are verified by conducting experiments which involved the measurements of these quantities.

The magnitude of a physical quantity is expressed as a value together with a unit of measurement.

All physical quantities consist of a numerical value and a unit. For “a force of five newtons”, force ' F ' is the physical quantity, '5 N' is the magnitude with newtons 'N' being the unit.



❖ S.I. Base Quantities and Units

Base quantities are physical quantities that are the most fundamental and they are used to define other physical quantities. Scientists, in the interest of simplicity, chose seven base quantities that give a full description of the physical world.

The *Système Internationale d'Unités* (International System of Units or SI units) is based on the seven base quantities, and their corresponding base units are listed below. In 2019, some of SI base units were redefined. (Use the Internet to find out what they are.)

Base quantity	Base unit	Symbol
time	second	s
length	metre	m
mass	kilogram	kg
current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
luminous intensity (not in syllabus)	candela	cd



[International System of units](#)



[The True Measure of a Kilogram](#)



[The International Atomic Time](#)

❖ S.I. Derived Quantities and Units

Derived quantities are physical quantities formed by combining base quantities according to algebraic relations involving products and/or quotients. Hence, **derived units** are defined in terms of base units and are expressed as products and/or quotients of base units.

Derived units are obtained from the base units according to a **defining equation** that relates the physical quantities.

For example, the defining equation for speed v is given by $v = \frac{s}{t}$, where s is distance and t is time. Hence, the unit of speed is metre per second, i.e. m s^{-1} .

There is a space between the m and the s^{-1} . Otherwise, it may be misread as per millisecond (ms^{-1}). Units should always be expressed in indices form, e.g. m s^{-2} instead of m/s^2 .

Other examples of derived units are given below.

Derived quantity	Defining equation	Base units	Derived unit	Symbol of derived unit
force	force = mass $\times$ acceleration	$(\text{kg}) \times (\text{m} \div \text{s} \div \text{s})$ = kg m s^{-2}	newton	N
work done	work done = force $\times$ displacement	$(\text{kg m s}^{-2}) \times (\text{m})$ = $\text{kg m}^2 \text{s}^{-2}$	joule	J



[NASA Mars Orbiter Unit Conversion Mistake](#)

❖ Homogeneity of Equations

For a physical equation to be operational or meaningful, each term in the equation must have the **same base units** (or dimensions). Only quantities with the same base units can be added, subtracted or equated. In other words, each term separated by “+”, “-” or “=” sign must have the same base units.

When each of the terms in a physical equation has the same base units, the equation is said to be **homogeneous** or **dimensionally consistent**.

Example 1

Analyse whether the equation $v = u + at$ is homogeneous.

Example 2

Analyse whether the equation $v^2 = u^2 + 2as^2$ is homogeneous.

Checking the homogeneity of an equation using base units is a powerful way of establishing if the physical equation is **plausible**. See Example 4.

Example 3

Analyse whether the equation $s = ut + at^2$ is homogeneous.

However, we know that the correct equation that describes how the displacement of an object moving with constant acceleration varies with time is $s = ut + \frac{1}{2}at^2$. Hence, the equation $s = ut + at^2$ is homogenous but is **physically wrong!**

Note: An equation which is found to be homogenous **need not be physically correct** due to either (i) wrong coefficients/signs, or (ii) missing/additional terms.

Example 4

The period T of a simple pendulum is thought to depend on its length L , its mass m and the acceleration due to gravity g according to the equation:

$$T = k L^x m^y g^z$$

where k is a **dimensionless constant** (i.e. a constant with no unit).

Deduce the relation between T , L , m and g by finding the indices x , y and z .

Note: It is not possible to deduce the value of any constant such as k through this analysis. Constants can only be determined through more rigorous derivations or experimentations.

1.3 Prefixes, Standard Form and Significant Figures

❖ Decimal, Sub-multiples or Multiples

The following prefixes and their symbols can be used to indicate decimal sub-multiples or multiples of both base and derived units:

Prefix	pico	nano	micro	milli	centi	deci	kilo	mega	giga	tera
Symbol	p	n	μ	m	c	d	k	M	G	T
Multiple	10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{-1}	10^3	10^6	10^9	10^{12}

E.g. $0.00000123 \text{ J} = 1.23 \times 10^{-6} \text{ J} = 1.23 \mu\text{J}$
 $4850000 \text{ V} = 4.85 \times 10^6 \text{ V} = 4.85 \text{ MV}$

❖ Standard Form

The standard form expresses a number as $N \times 10^n$, where n is an integer, either positive or negative, and N is any number such that $1 \leq N < 10$.

E.g. $0.00000123 \text{ J} = 1.23 \times 10^{-6} \text{ J}$
 $4850000 \text{ V} = 4.85 \times 10^6 \text{ V}$

❖ Significant Figures

Zeroes on the left of the first non-zero digit are not significant. Zeroes on the right of the last non-zero digit are only significant after a decimal point.

E.g. 0.0600 g has 3 significant figures
 $6.080 \times 10^4 \text{ J}$ has 4 significant figures
 $6.08 \times 10^4 \text{ J}$ has 3 significant figures

Note:

A speed of 100 m s^{-1} can be considered as 3 or 2 or even 1 s.f.. This is dependent on the resolution of the instruments used to measure the speed directly or the measurements that lead to the calculation of this value. It also depends on the s.f. of the other quantities appearing within a question. **If in doubt, assume 3 s.f..**

1.4 Estimation

❖ Order of Magnitude

Sometimes, due to incomplete information of available data, coupled with the need to quickly get an idea of the magnitude of a physical quantity, we compute an estimated value of the quantity. The estimated value can also serve as a check on a calculation to ensure that no blunders were made. In making an approximation, it is usually necessary to make estimates of values.

We refer to an **order of magnitude** estimate of a physical quantity as the **power of ten** of the number that describes it. Attaching the prefix "kilo-" to a unit increases the size of the unit by three orders of magnitude, for example, in "kg".

If two numbers differ by one order of magnitude, one is about ten times larger than the other. Two numbers have the same order of magnitude if the larger value is less than ten times the smaller value. When bacteria in a flask have multiplied from some hundreds (10^2) to some millions (10^6), the population of the bacteria has increased by four orders of magnitude (or 10^4 times).

Physical quantity	Estimate	Order of Magnitude
mass of a car	1300 kg $\sim 10^3$ kg	3
power of a domestic bulb	100 W $\sim 10^2$ W	2
speed of a car along an expressway	25 m s ⁻¹ $\sim 10^1$ m s ⁻¹	1

Example 5

Estimate the volume of a wooden half-metre rule found in the school laboratory.

1.5 Measuring Instruments and Methods of Measurements

❖ Resolution

The resolution of an instrument is the smallest graduation on the scale. As the resolution of an instrument increases, the number of significant figures of a measurement increases. For example, the diameter of a coin measured with a micrometer screw gauge (e.g. 2.500 cm) has more s.f. than if it is measured with a metre rule (e.g. 2.5 cm).

For the same instrument, the d.p. of a measurement depends on the unit used. For example, the diameter of the above coin measured with a metre rule can be stated as 25 mm (0 d.p.), 2.5 cm (1 d.p.) or 0.025 m (3 d.p.).

Physical quantity	Measuring instrument	Resolution
Length	Metre rule	0.1 cm [1mm]
	Vernier calipers	0.01 cm [0.1 mm]
	Micrometer screw gauge	0.001 cm [0.01 mm]
Time	^Δ Digital stopwatch	0.01 s
Mass	^Δ Electronic balance	0.01 g
Voltage or current	^Δ Digital multimeter	Depends on setting used

^Δ For digital/electronic meters, measurements are recorded as displayed on the meters.

❖ Uncertainty

In experiments, uncertainty of a measurement is likely to be larger than the resolution of the instrument used. Uncertainty of a measurement takes into account of factors such as limitations of the methods of measurements and imperfections of objects to be measured.

1.6 Systematic and Random Errors

In the process of taking measurements, we may encounter errors that give rise to incorrect readings. Errors are generally classified as either systematic or random.

❖ Systematic Errors

Systematic Errors

Systematic errors result in all readings or measurements being either always above or always below the **true** value by a fixed amount.

A systematic error can be **eliminated** if the source of the error is known and accounted for. It cannot be eliminated by repeating the measurements and averaging them.

Examples of systematic errors:

- Not accounting for zero-error in a measurement.
An instrument has a zero error if the scale reading is non-zero before a reading is taken. Instruments should be checked for zero error and the zero error must be accounted for in the measurement.
- Not accounting for background radiation
Background radiation is constantly present in the environment and is emitted by a variety of natural and artificial sources. When measuring the activity of a radioactive source, the value of background radiation must be subtracted from the count rate recorded by the Geiger Muller counter.

❖ **Random Errors**

Random Errors

Random errors result in readings or measurements being scattered about the **mean** value. These errors have equal probability of being positive or negative.

Random errors may be **reduced** by:

- repeating the measurements and finding the average value.
- plotting a graph and drawing a line of best fit for the plotted points.

Examples of random errors:

- variation in diameter of a piece of wire along the length of a piece of wire
- fluctuations in the count-rate of a radioactive decay

Example 6

Complete the table below.

Systematic or Random Error?	
The initial reading of the scale was not zero before taking measurement.	
Five different persons measuring the length of the same rod using the same measuring tape.	

Note:

Errors in timings occur due to a variety of reasons. It is important to establish how the error occurs to determine which type of error it is and whether it can be eliminated.

1.7 Accuracy and Precision

❖ Accuracy vs Precision

Accuracy

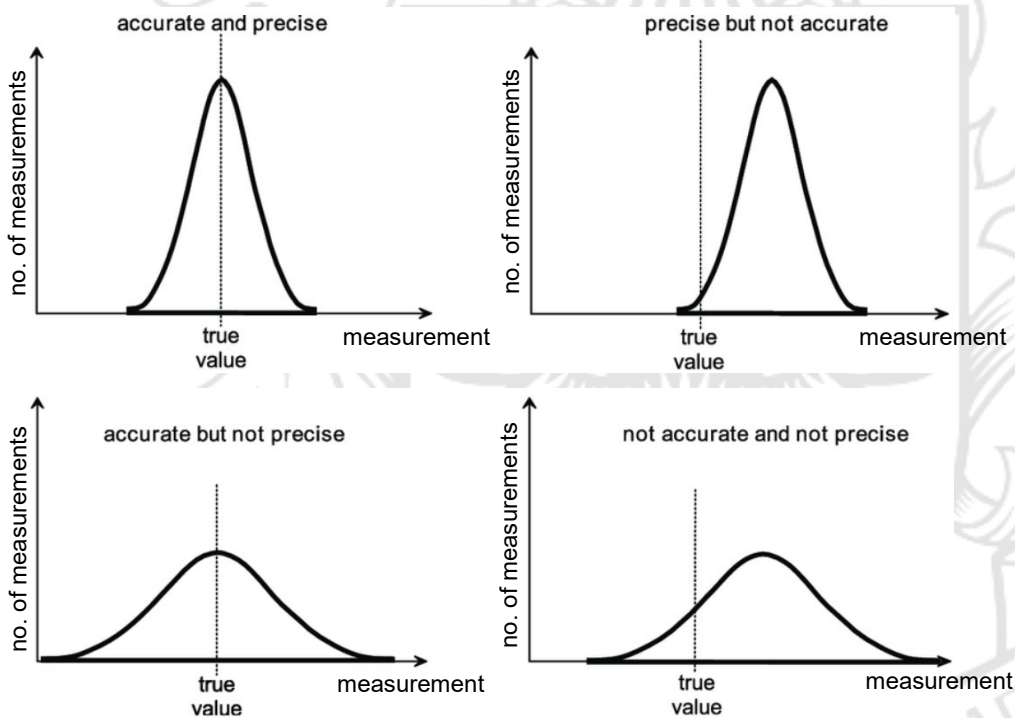
Accuracy is the degree of closeness of the mean value of the measurements to the true value. It is affected by **systematic** error.

Precision

Precision is the degree of agreement between repeated measurements of the same quantity. It is affected by **random** error.

Note:

- “Accuracy” and “Precision” are comparative terms, i.e. we need to compare at least two sets of measurements to decide which is more accurate or precise.
- When a large number of repeated measurements are plotted in a distribution graph, a bell-shaped graph is obtained. Accuracy and precision are graphically illustrated below.



❖ **Related Terminologies**

1. **Responsiveness** of an instrument is the ability of the instrument to detect rapid variations in the quantity being measured.
2. **Sensitivity** of an instrument is the ability of the instrument to detect small changes in the quantity being measured.
3. **Reliability** of a measurement is a measure of the confidence that can be placed in a set of measurements, i.e. whether they are both accurate and precise.
4. **Reproducibility** of a measurement is the agreement between repeated measurements of the same physical quantity.

1.8 Calculations of Uncertainties of Derived Quantities

❖ **Uncertainties of Derived Quantities**

We can combine two or more quantities that we have measured to produce a derived quantity for which the equation to determine it involves product and/or quotient of the base quantities. For example:

- dividing distance by time to get speed,
- adding two lengths to get a total length.

We need to learn how these uncertainties propagate to the uncertainty of the derived quantity. We assume that the two measured quantities are x and y with **absolute (or actual) uncertainties Δx and Δy** , respectively.

Note:

- The measurements x and y must be independent of each other.
- Absolute uncertainties are quoted as positive values to only 1 s.f.

❖ **Uncertainty Formulae**

The formulae used to calculate the uncertainties of derived quantities are summarised in the table below. (They are derived using calculus.)

1. Addition	$Q = aX + bY$	Absolute uncertainty: (1 sf)	$\Delta Q = a \Delta X + b \Delta Y$
2. Subtraction	$Q = aX - bY$	Absolute uncertainty: (1 sf)	$\Delta Q = a \Delta X + b \Delta Y$
3. Product	$Q = aX \times Y$	Fractional uncertainty: (2 sf)	$\frac{\Delta Q}{Q} = \frac{\Delta X}{X} + \frac{\Delta Y}{Y}$
4. Division	$Q = a \frac{X}{Y}$	Fractional uncertainty: (2 sf)	$\frac{\Delta Q}{Q} = \frac{\Delta X}{X} + \frac{\Delta Y}{Y}$
5. Product with powers	$Q = aX^m \times Y^n$	Fractional uncertainty: (2 sf)	$\frac{\Delta Q}{Q} = m \frac{\Delta X}{X} + n \frac{\Delta Y}{Y}$
6. Quotient with powers	$Q = a \frac{X^m}{Y^n}$	Fractional uncertainty: (2 sf)	$\frac{\Delta Q}{Q} = m \frac{\Delta X}{X} + n \frac{\Delta Y}{Y}$

Note:

- a, b, m, n are numerical constants.
- **Always re-arrange a given equation such that only the quantity of interest appears as the subject on the left-hand-side of the equation.**
- Uncertainties do not cancel one another. They always add up to maximise the uncertainty because we are estimating the largest possible uncertainty.
- The **absolute uncertainty** of a quantity is expressed as a positive value to **1 s.f.** The **quantity** is then expressed to the **same d.p. and unit** as the absolute uncertainty.
- The **fractional uncertainty** is the value of the absolute uncertainty divided by the value of the quantity i.e. $\Delta x/x$. The fractional uncertainty multiplied by 100 is the percentage uncertainty,

$$\text{i.e. \% uncertainty of } x = \frac{\Delta x}{x} \times 100\%$$



[Introduction to Measurements & Error Analysis](#)

Example 7(i) Using Uncertainty Formulae

A rectangle has length $L = (34.3 \pm 0.1)$ cm and breadth $B = (21.8 \pm 0.1)$ cm. Calculate the perimeter P and area A of the rectangle and express their values with their absolute uncertainties.

Example 8

A metal cube of side L has mass M . Its density ρ can be calculated from $M = \rho L^3$. If $M = (0.065 \pm 0.001)$ kg and $L = (0.200 \pm 0.001)$ m, express ρ with its associated uncertainty.

❖ The Upper-Lower Bound Method of Uncertainty Propagation

The simplest way to determine the uncertainty of derived quantity is to use the **upper-lower bound method** of uncertainty propagation.

This method uses the uncertainty ranges of each measurement to calculate the **maximum** $Q_{\max}$ and **minimum** $Q_{\min}$ values of the derived quantity Q , and then using them to determine the **absolute uncertainty** ΔQ of the derived quantity:

$$\Delta Q = \frac{Q_{\max} - Q_{\min}}{2}$$

Worked Example:

An angle θ is measured using a protractor, where $\theta = (25 \pm 1)^\circ$, and you need to calculate $Q = \cos \theta$ and its absolute uncertainty ΔQ . Hence,

$$Q = \cos 25^\circ = 0.90631$$

$$Q_{\min} = \cos(25 + 1)^\circ = 0.89879$$

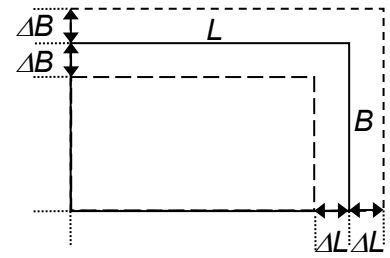
$$Q_{\max} = \cos(25 - 1)^\circ = 0.91355$$

$$\Delta Q = \frac{Q_{\max} - Q_{\min}}{2} = \frac{0.91355 - 0.89879}{2} = 0.007 \text{ (1 s.f.)}$$

$$\therefore Q = 0.906 \pm 0.007 \quad (\text{both numbers must have the same d.p.})$$

Note: This method is especially useful when the relationship between the variables is complex.

Example 7(ii) Using Upper-Lower Bound Method



Example 9

The equation connecting object distance u , image distance v and focal length f for a lens is

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

A student measures values of u and v , with their associated uncertainties, which are

$$u = 50 \text{ mm} \pm 3 \text{ mm},$$
$$v = 200 \text{ mm} \pm 5 \text{ mm}.$$

Determine the value f and its associated uncertainty Δf .



1.9 Scalars and Vectors

All physical quantities can be divided into *scalar* and *vector* quantities.

A **scalar** quantity has a magnitude only.

A **vector** quantity has both a magnitude and a direction.

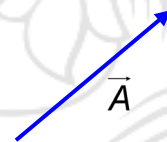
Scalar Quantity	Vector Quantity
distance	displacement
speed	velocity
energy	moment
mass	weight
time	acceleration
temperature	momentum

❖ Vector Representation

A vector is denoted by $\vec{A}$, $\vec{A}$ or $\mathbf{A}$.

A vector is represented by an arrow:

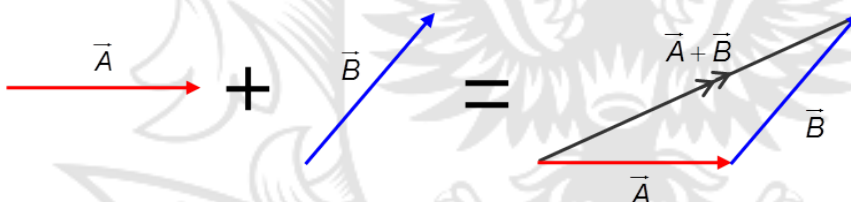
- the length of the arrow represents the magnitude of the vector.
- the direction of the arrow represents the direction of the vector.



❖ Vector Addition - Polygon Law (for any number of vectors)

Two or more vectors may be added to form a resultant vector.

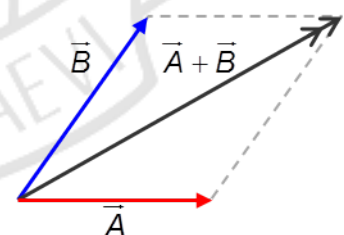
Consider the vector addition of two vectors $\vec{A}$ and $\vec{B}$ by joining the head of one vector to the tail of another. The resultant vector $\vec{A} + \vec{B}$ joins the tail of the first vector to the head of the last vector.



❖ Vector Addition - Parallelogram Law (for only two vectors)

If vectors $\vec{A}$ and $\vec{B}$ form the sides of a parallelogram by starting or ending at the same point, then the resultant vector $\vec{A} + \vec{B}$ is the diagonal of the parallelogram.

This is called the parallelogram law of vector addition, and it is equivalent to the polygon law. Note that **vector addition is commutative**.



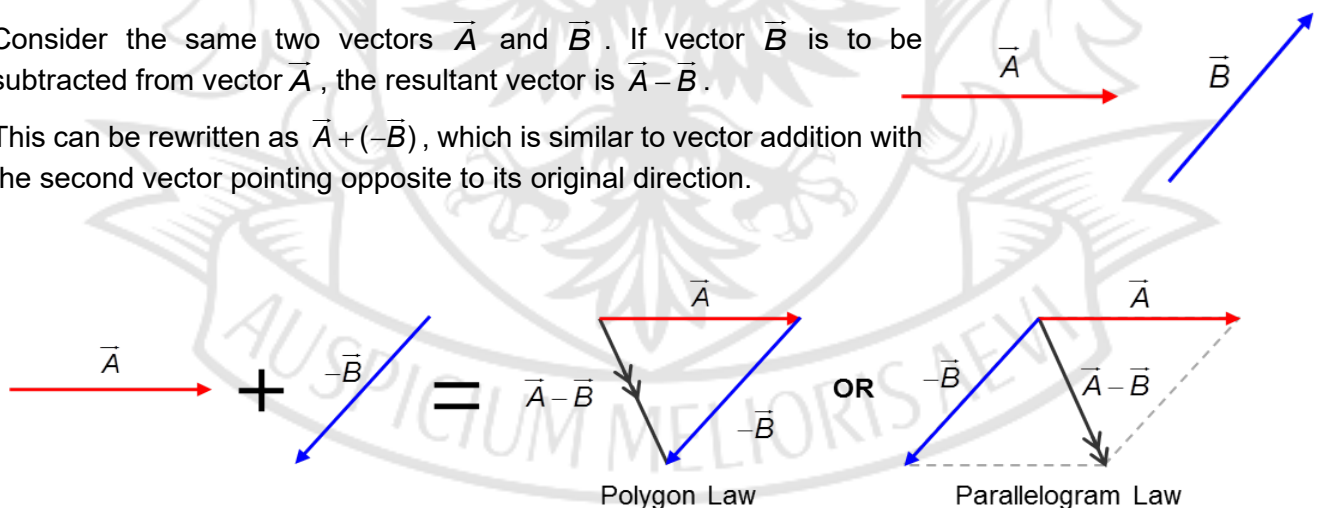
Example 10

A boat is travelling due north with a velocity v_{BW} of 4.00 m s^{-1} relative to the water. The current in the water is flowing at a velocity v_{WS} of 2.00 m s^{-1} in an easterly direction relative to the shore. Determine the velocity V_{BS} of the boat relative to the shore.

❖ **Vector Subtraction**

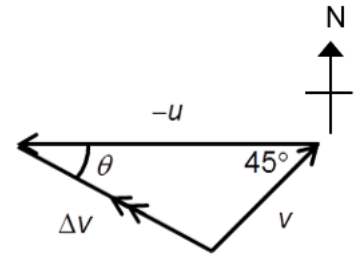
Consider the same two vectors $\vec{A}$ and $\vec{B}$. If vector $\vec{B}$ is to be subtracted from vector $\vec{A}$, the resultant vector is $\vec{A} - \vec{B}$.

This can be rewritten as $\vec{A} + (-\vec{B})$, which is similar to vector addition with the second vector pointing opposite to its original direction.



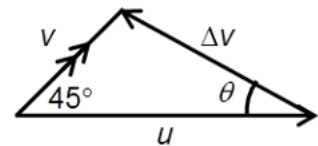
Example 11

An object was initially moving with a constant speed of 20 m s^{-1} towards the east. It changed direction and moved with a constant speed of 10 m s^{-1} in the north-easterly direction. Calculate the change in velocity.



$$v - u = \Delta v$$

OR



$$u + \Delta v = v$$

❖ Resolution of Vectors

Any vector can be resolved into two components that are perpendicular to each other.

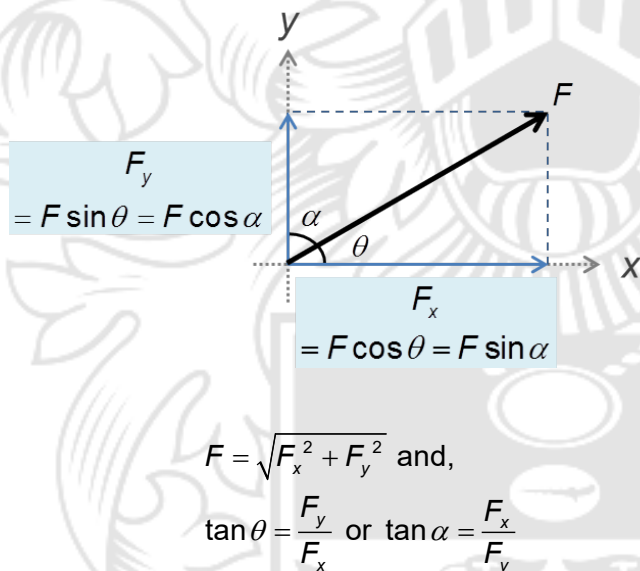
The orientation of the axes are usually

- (i) *vertical and horizontal*, or
- (ii) *parallel and perpendicular to a slope*.

The magnitude of a component is smaller than that of the vector (unless the vector lies along an axis).

The original vector forms the hypotenuse of the right-angle triangle.

Just as a vector can be “broken” into two components, these components can be “recombined” to give the magnitude and direction of the vector.

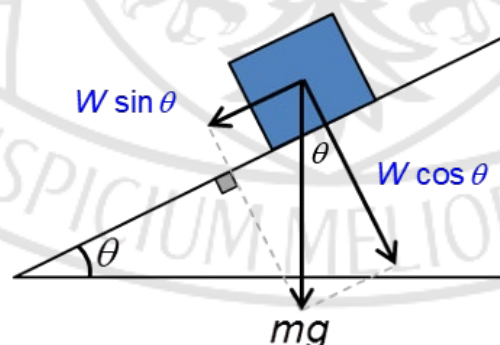


Example 12

Indicate the component of the weight W of the object acting

- (a) along the slope and
- (b) perpendicular to the slope.

[Solution]



❖ Strategies For Adding Multiple Vectors

When two or more vectors are added, the following procedures can be applied to determine the resultant vector:

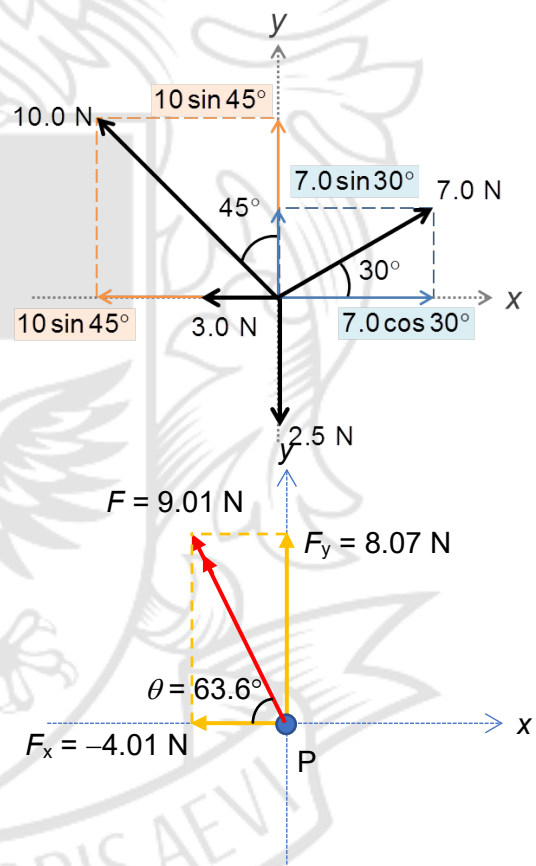
1. Choose a convenient orientation of the x-y coordinate system.
2. Draw all vectors starting from the origin with the appropriate lengths and directions.
3. Determine the x- and y-components of all vectors.
4. Determine the resultant components along the x- and y-axis.
5. Apply Pythagoras theorem to find the magnitude of the resultant vector.
6. Apply trigonometry to find the angle that the resultant vector makes with the x- or y-axis, or any other reference line.

Note:

The resultant vector is independent of the orientation of the x-y axes.

Example 13

The figure below shows four forces acting at a point P. Calculate the resultant force on P by resolving each force in two perpendicular directions.



The resultant force is 9.01 N, acting at an angle 63.6° clockwise from the 3.0 N force.

1.10 Summary

❖ S.I. Units

The seven **base units** are: metre (m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol) and candela (cd).

Derived units are defined in terms of base units. They are expressed as products or quotients of base units.

An equation is said to be **homogeneous** or **dimensionally consistent** if every term on both sides of the equation has the same base units.

A homogeneous equation may not be physically correct.

❖ Errors and Uncertainties

Systematic errors result in all readings or measurements being either always above or always below the true value by a fixed amount.

Random errors result in readings or measurements being scattered about a mean value. These errors have equal probability of being positive or negative.

Accuracy is the degree of closeness of the mean value of the measurements to the true value. It is affected by systematic error.

Precision is the degree of agreement between repeated measurements of the same quantity. It is affected by random error.

❖ Calculating Uncertainties of Derived Quantities

$$\text{If } Q = aX \pm bY \quad \text{then} \quad \Delta Q = |a|\Delta X + |b|\Delta Y$$

$$\left. \begin{array}{l} \text{If } Q = (aX^m)(bY^n) \\ \text{OR } Q = \frac{aX^m}{bY^n} \end{array} \right\} \quad \text{then} \quad \frac{\Delta Q}{Q} = |m|\frac{\Delta X}{X} + |n|\frac{\Delta Y}{Y}$$

$$\text{Percentage uncertainty} = \frac{\Delta Q}{Q} \times 100\%$$

The absolute uncertainty is expressed to 1 s.f., and the quantity is expressed to the same d.p. as that of the absolute uncertainty.

Fractional and percentage uncertainty can be expressed to 2 s.f.

❖ Scalars and Vectors

A **scalar** quantity has a magnitude only.

A **vector** quantity has both a magnitude and a direction.

❖ Reference Links

NASA Mars Orbiter unit conversions mistake

<https://www.youtube.com/watch?v=IWHTyibmB7U>

List of physical quantities

https://en.wikipedia.org/wiki/List_of_physical_quantities International

Atomic Time

<https://www.bipm.org/en/-/2021-12-21-record-tai>

True Measure of a Kilogram

<https://www.asiaone.com/singapore/true-measure-kilogram>

Scale of Objects

<https://htwins.net/scale2/>

Introduction to Measurements & Error Analysis

<https://users.physics.unc.edu/~deardorf/uncertainty/UNCguide.html>