Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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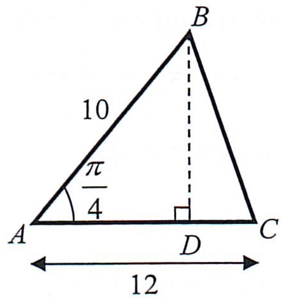
MA304.3.1 – Area of a Triangle

At the end of this unit, you should be able to

- **find the area of a triangle using $\frac{1}{2}ab\sin C$**

E1. The figure below shows a triangle with the length of two sides given and an angle between the two sides.



(a) Show that the perpendicular distance from B to AC is given by $BD = 10\sin\frac{\pi}{4}$.

$\sin\left(\frac{\pi}{4}\right) = \frac{BD}{10}$

$BD = 10\sin\left(\frac{\pi}{4}\right)$ (shown)

(b) Hence determine the area of the triangle using AC as the base and BD as the height.

Area of triangle $= \frac{1}{2} \times 12 \times 10\sin\left(\frac{\pi}{4}\right)$ units^2

$= 6 \times 10 \times \frac{1}{\sqrt{2}}$ units^2

$= 30\sqrt{2}$ units^2

E2. Given the triangle below, show that the area of the triangle is given by $\frac{1}{2}ab\sin C$.

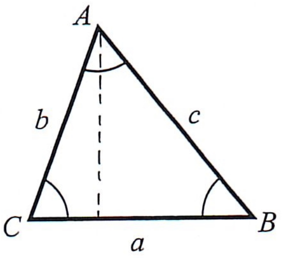
Let D be a line on CB such that AD is $\perp$ to CB.

$AD = b \sin C$

$\therefore$ Area of triangle $= \frac{1}{2} \cdot AD \cdot CB$

$= (\frac{1}{2} \cdot b\sin C \cdot a)$

$= \frac{1}{2} ab \sin C$ (shown)



Given any triangle such that two of the sides (a, b) and the angle (C) in between the two sides are given, the area of the triangle is given by

$$\textbf{Area of triangle} = \frac{1}{2}ab\sin C$$

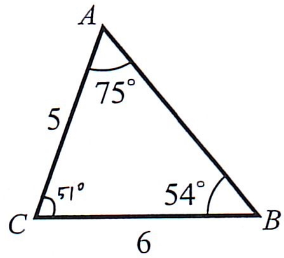
E3. Find the area of the triangle as shown on the right.

$\measuredangle ACB = 180° - 75° - 54°$

$= 51°$

Area of triangle $= \frac{1}{2} \times 5 \times 6 \times \sin(51°)$ units^2

$= 11.7$ units^2 (3 s.f.)

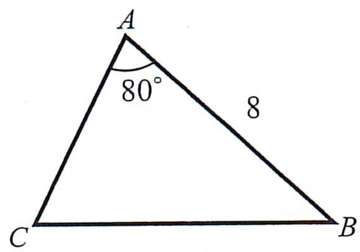


P1. Given that the area of the triangle below is 24 units^2, find the length of AC.

$24 = \frac{1}{2} \times 8 \times AC \times \sin(80°)$ units^2

$AC = 24 \div \frac{1}{2} \div 8 \div \sin(80°)$ units

$= 6.09$ units (3 s.f.)



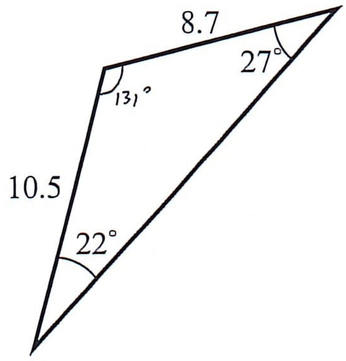
P2. Find the area of the triangle shown on the right.

other angle $= 180° - 22° - 27°$

$= 131°$

Area of triangle $= \frac{1}{2} \times 10.5 \times 8.7 \times \sin(131°)$ units²

$= 34.5$ units² (3s.f.)



P3. In the figure, ΔABC is similar to ΔDAC. Given that $AB = 10$ cm and $\measuredangle ADC = \frac{\pi}{6}$,

(a) Determine $\measuredangle ABC$.

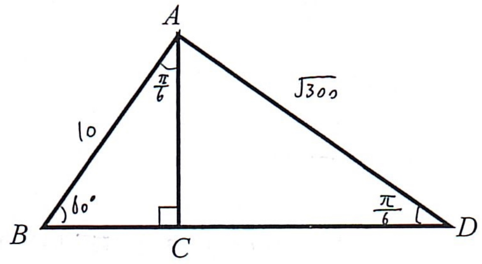
$\measuredangle BAC = \measuredangle ADC = \frac{\pi}{6}$

$\measuredangle ABC = 90° - \measuredangle BAC$

$= 90° - 30°$

$= 60°$

$= \frac{\pi}{3}$



(b) Find the length of *BD*.

$\cos(60°) = \frac{AB}{BD}$

$BD = \frac{10}{\cos(60°)}$ cm

$= 20$ cm

(c) Hence or otherwise find the area of ΔABD.

$AD = \sqrt{20^2 - 10^2}$ units

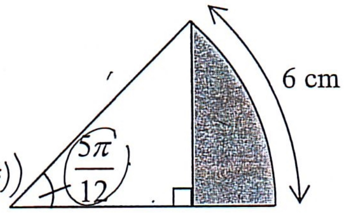
$= \sqrt{300}$ units

Area of $\Delta ABD = \frac{1}{2} \times 10 \times \sqrt{300}$ units²

$= 5\sqrt{300}$ units²

$= 50\sqrt{3}$ units²

P4. Find the area of the shaded region in the figure on the right.



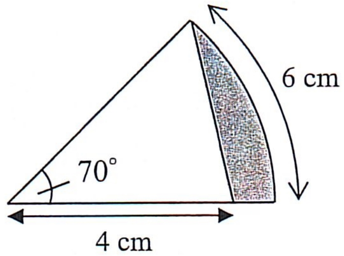
Radius $= 6 \div \frac{5\pi}{12}$

$= \frac{72}{5\pi}$ cm

Area $= \frac{1}{2}\left(\frac{72}{5\pi}\right)(6^2) - \frac{1}{2}\left(\frac{72}{5\pi}\cos\left(\frac{5\pi}{12}\right)\right)\left(\frac{72}{5\pi}\sin\left(\frac{5\pi}{12}\right)\right)$

$= 11.1$ cm^2 (3 s.f.)

P5. Find the area of the shaded region in the figure on the right.



$70^\circ = \frac{7\pi}{18}$

$\frac{6\text{ cm}}{x\text{ cm}} = \frac{7\pi}{18}$ cm

$x = \frac{108}{7\pi}$ cm

Area of triangle $= \frac{1}{2}(4)\left(\frac{108}{7\pi}\right)\sin(70^\circ)$ cm^2

$= \frac{216}{7\pi}\sin(70^\circ)$ cm^2

Area of sector $= \frac{1}{2} \times \frac{108}{7\pi} \times 6$ cm^2

$= \frac{324}{7\pi}$ cm^2

Area of shaded region $= \frac{324}{7\pi} - \frac{216}{7\pi}\sin(70^\circ)$ cm^2

$= 5.50$ cm^2 (3 s.f.)