Practice Paper C Sec 3

1 **(a)** Find the product of

$$(x^{\frac{1}{3}} - y^{\frac{1}{3}})(x^{\frac{1}{3}} + y^{\frac{1}{3}}) \text{ and } (x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})(x^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})$$

and simplify the answer. [2]

(b) If $y = \dfrac{\sqrt{x+a} - \sqrt{x-a}}{\sqrt{x+a} + \sqrt{x-a}}$, simplify $y + \dfrac{1}{y}$. [3]

(c) Solve the equation $\log_5 x + 1 = 6\log_x 5$. [3]

2 Angles A and B are in different quadrants such that $\tan A = \dfrac{3}{4}$ and $\sin B = -\dfrac{4}{5}$. Find, without using calculator, the possible values of $(2\sin A - \tan B)$ in the exact form. [6]

3 **(i)** Find $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$, the inverses of the matrices

$$\mathbf{A} = \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}.$$ [2]

(ii) Use $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$ from **(i),** to find the matrix $\mathbf{M}$

$$\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} \mathbf{M} \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}.$$ [3]

4 **(a)** Find the range of values of h , $h \neq 0$, for which the line $y = x - h$ meets the curve $y = hx^2 + 7x$. [3]

(b) Find the range of values of k for which $y = x^2 + (k-4)x + 1$ is always positive for all real values of x. [3]

5 Given that x^2+x-6 is a factor of $2x^4 + x^3 - ax^2 + bx + a + b = 1$, find the value of a and of b. [4]

6 **(a)** Simplify $\dfrac{a-b}{(b+c)(a+c)} + \dfrac{b-c}{(a+c)(a+b)} + \dfrac{c-a}{(a+b)(b+c)}$. [3]

(b) Simplify $\dfrac{2x^2-6x+4}{x^2-6x+5} \times \dfrac{x^2-x-20}{3x^2+6x-24}$. [3]

7 Sketch the graph $y = 1 - 4\cos 2x$ for $0° \le x \le 360°$. [3]

8 Solve the equations for $0° \le x \le 180°$.

(a) $\sin 3x = \cos 2x$, [3]

(b) $\tan(2x - 40^\circ) = -2.718$. [3]

9 Prove the identities.

(a) $\sin^4 x + \sin^2 x \cos^2 x + \cos^2 x \equiv 1$, [2]

(b) $\dfrac{2-\operatorname{cosec}^2 x}{\operatorname{cosec}^2 x + 2\cot x} \equiv \dfrac{\sin x - \cos x}{\sin x + \cos x}$. [4]

10 It is given that z varies jointly as the square of x and inversely as the square root of y, for all positive values of x, y and z. When $x = 2$ and $y = 4$, $z = 6$.

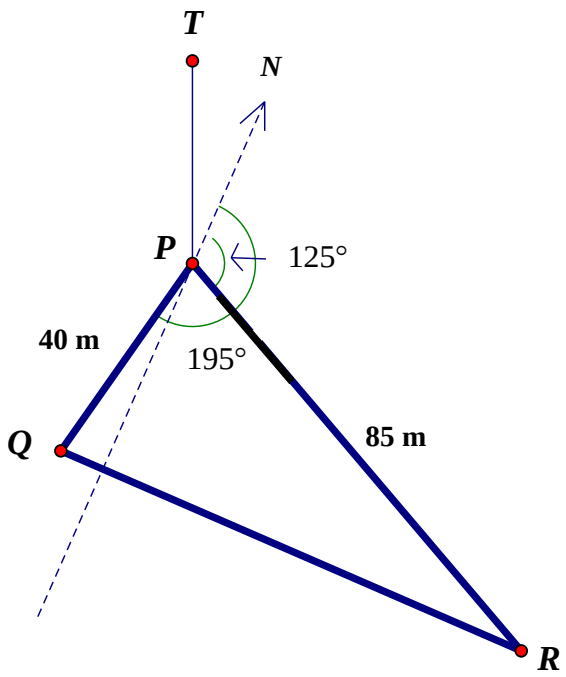
(a) Find the value of x when $y = 9$ and $z = 25$. [4]

(b) Find the percentage change in z when x is increased to 200% and y is reduced to 25%. [2]

11 Solve the equation $\left| x^2 - 3 \right| = 2x$ [4]

12 In the diagram, PQR represents a horizontal triangular field such that $PR = 85$ m and $PQ = 40$ m.

The bearing of R from P and the bearing of Q from P is $125°$ and $195°$ respectively.



(a) Calculate the distance from Q to R. [2]

(b) Calculate the area of the triangular field PQR. [2]

(c) Calculate the shortest distance from P to the road QR. [2]

TP represents a building at the corner P. Given that the angle of depression of the top of the building T to the corner R is $20°$,

(d) calculate the height of the building TP. [2]

A man walks along the road from Q to R,

(e) calculate the greatest angle of elevation of the top of the building. [2]

13 **(i)** The equation of a circle is given by $x^2 + y^2 - 10x + 2y + 1 = 0$.
Find the centre and the radius of the circle. [2]

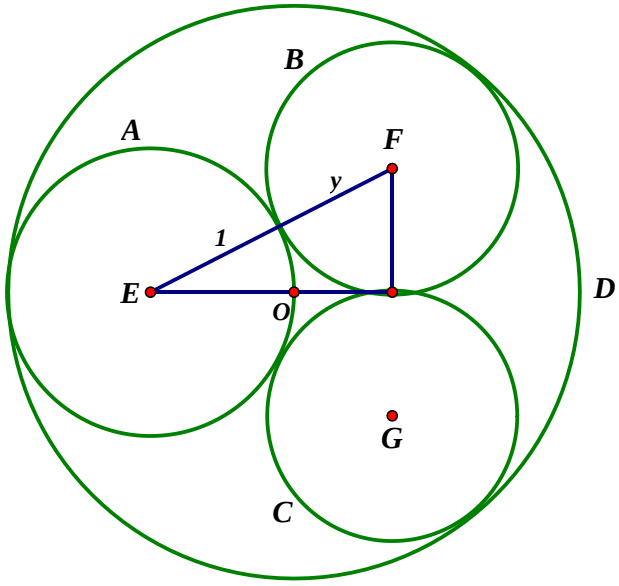
(ii) The tangent to the circle at a point A is parallel to the straight line $x - y + 2 = 0$.
Find the equation of the diameter of the circle through A. [2]

(iii) The circle is then reflected on the line $y = x$.
State the equation of the new circle formed. [1]

14 Circles A, B and C are externally tangent to each other and internally tangent to circle D. Circles B and C are congruent. E, F and G are the centres of circles A, B and C respectively**.**

Circle A has radius 1 and passes through O, the centre of circle D. Both circles B and C have radii y.

By considering <u>two right-angled triangles</u>, find the radius y of the circle B . [6]



15 **(a)** The table shows the experimental values of two variables x and y.

x	1	2	3	4	5
y	3.8	11.2	20.5	33.2	50.6
$\frac{y}{x}$	3.8	5.6		8.3	

It is known that x and y are related by the equation $y = ax + bx^2$

Plot $\frac{y}{x}$ against x and draw a suitable straight line.

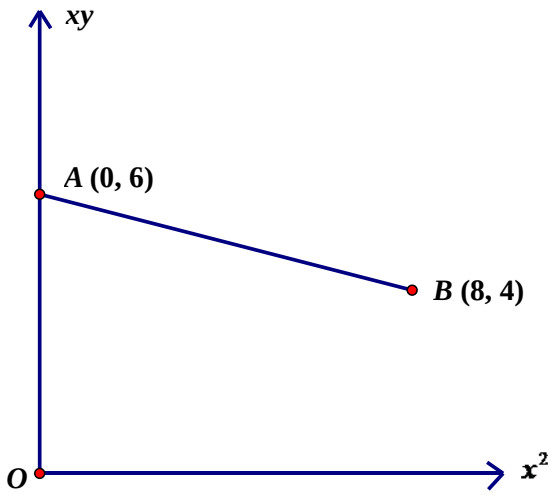
Use the graph to estimate

(i) the value of a and of b, [4]

(ii) the value of x when $y = 8x$. [1]

(b) The diagram shows the straight line graph of xy against x^2 passing through the points $A(0, 6)$ and $B(8, 4)$.

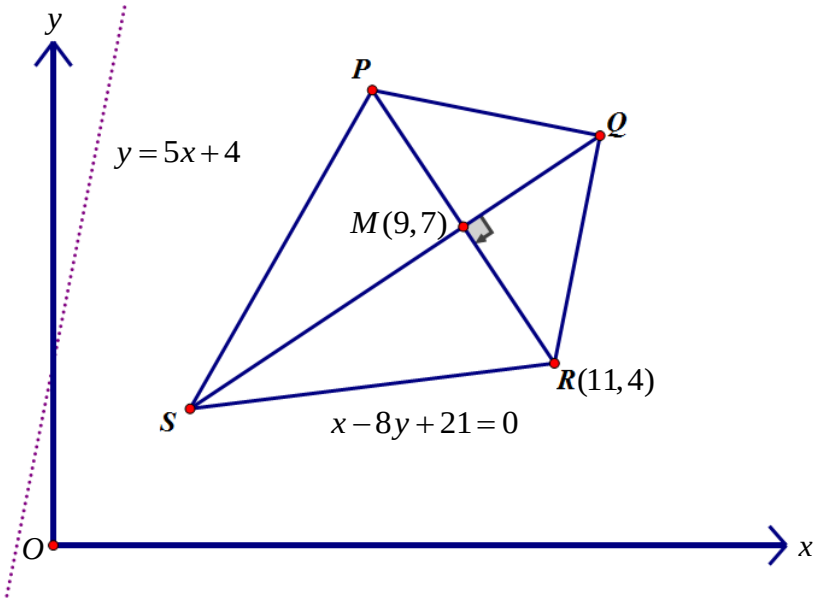
Express y in terms of x. [3]



16 **Solutions to this question by accurate drawing will not be accepted**

The diagram shows a quadrilateral $PQRS$ in which QS is the perpendicular bisector of PR. The mid-point of PR is M.

The coordinates of R and M are $(11, 4)$ and $(9, 7)$ respectively. RQ is parallel to the line $y = 5x + 4$ and the equation of SR is $x - 8y + 21 = 0$.



Find

(i) the coordinates of P, [2]

(ii) the equations of QR and QS, [4]

(iii) the coordinates of Q and of S, [3]

(iv) the area of $PQRS$. [2]

End Of Paper