



RAFFLES INSTITUTION
2026 YEAR 6 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS

9758/02

Paper 2

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [] at the end of each question or part question.

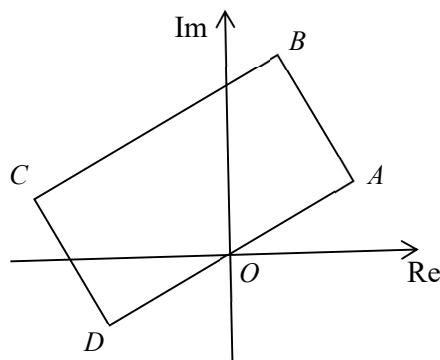
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RAFFLES INSTITUTION

Mathematics Department

Section A: Pure Mathematics [40 marks]

- 1** **(a)** The complex number w is such that $w^2 + ww^* = 6 + 2\sqrt{3}i$. Find exactly the possible values of w , giving your answer in the form $x + iy$ where x and y are real numbers. [3]
- (b)** The complex numbers found in part **(a)** are represented by points A and D in an Argand diagram as shown below. Points B and C represent complex numbers b and c respectively.



Given that $ABCD$ is a rectangle with $AD = 2AB$, find complex numbers b and c , giving your answers in the form $u + iv$, where u and v are real numbers. [3]

- (c)** By finding $\arg(b)$, find the exact value of $\tan \frac{5\pi}{12}$. [2]
- 2** **(a)** Find $\int \frac{2x+3}{x^2+2x-3} dx$. [3]
- (b)** Using the substitution $u = \tan \frac{x}{2}$, show that $\int \frac{1}{2+\cos x} dx = \int \frac{2}{3+u^2} du$. Hence find the exact value of $\int_0^{\frac{\pi}{2}} \frac{1}{2+\cos x} dx$. [6]

3 The function f is given by

$$f : x \mapsto a - e^{|x-a|}, \quad x \in \mathbb{R},$$

where a is a positive constant.

It is given that the range of f is $(-\infty, k]$, where $k > 0$.

- (a) Find the range of values of a . [2]
- (b) Show that f^{-1} does not exist. [2]
- (c) If the domain of f is restricted to $(-\infty, m]$, state the largest value of m in terms of a for which f^{-1} exists. [1]
- (d) Using your answer found in part (c), find f^{-1} in similar form. [3]

The function g is given by

$$g : x \mapsto \ln\left(1 - \frac{x}{a}\right), \quad x < a.$$

- (e) Show that the composite function gf exists and find the range of gf in terms of a . [3]

4 Do not use a calculator in answering this question.

- (a)** The curve $y = f(x)$ passes through the origin and has gradient given by

$$\frac{dy}{dx} = \frac{x+y}{1+xy}.$$

(i) Show that $(1+xy)\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + (y-1)\frac{dy}{dx} = 1.$ [2]

- (ii)** By further differentiation of the result in part **(a)(i)**, show that

$$(1+xy)\frac{d^3y}{dx^3} + p(x)\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) + q(y)\frac{d^2y}{dx^2} + r\left(\frac{dy}{dx}\right)^2 = 0,$$

where the functions $p(x)$, $q(y)$ and the constant r are to be determined.

[3]

- (iii)** Hence find the Maclaurin series for y up to and including the term in x^3 .

[2]

- (b)** The equation

$$2\sqrt{2}\cos\left(x - \frac{\pi}{4}\right) + 3\sin 2x - 3 = 0$$

has a root θ , which is close to zero.

- (i)** Show that $\theta^2 + a\theta + b \approx 0$, where a and b are integers to be determined.

[3]

- (ii)** Hence find an approximation for θ in surd form.

[2]

Section B: Probability and Statistics [60 marks]

- 5** A random variable X has probability distribution given by

$$P(X = x) = \begin{cases} kx^2 & \text{for } x = 1, 2, 3, \\ k(7 - x) & \text{for } x = 4, 5, 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(a) Show that $k = \frac{1}{20}$. [2]

(b) Find the values of $E(X)$ and $\text{Var}(X)$. [3]

Another random variable Y is defined by $Y = 3X_1 - X_2$, where X_1 and X_2 are independent observations of X .

(c) Find $\text{Var}(Y)$. [2]

- 6** **(a)** For a special opening act in a talent show, 8 performers (3 singers, 3 dancers and 2 musicians) are to stand in a circle on stage. Find the number of possible arrangements if

(i) there are no restrictions, [1]

(ii) all performers of the same role must stand together. [2]

- (b)** For the main performance, 12 performers (3 singers, 5 dancers and 4 musicians) are arranged in a line on stage such that no two musicians are adjacent. Find the number of possible arrangements. [2]

- (c)** 7 performers are selected from 15 performers (6 singers, 5 dancers and 4 musicians) for the closing act. Find the number of possible selections if there must be at least 2 performers from each role. [3]

7 In this question, you should state the parameters of any distributions you use.

In a running competition, the time taken by members from the Sports Recreational Club to complete a 20 km race is normally distributed with a mean of 105 minutes and a standard deviation of 12 minutes.

- (a) Find the probability that a randomly chosen Sports Recreational Club member finishes the 20 km race in less than 100 minutes. [1]
- (b) Find the time that is exceeded by 10% of the Sports Recreational Club members. [1]

Members from the Swift Runners Club competed in the same 20 km race. After analysing their results, it was found that 36% of them completed the race in less than 95 minutes and 8% of them completed the race in more than 125 minutes. You may assume that the Swift Runners Club members' running times can be modelled using a normal distribution with mean μ and standard deviation σ .

- (c) Determine the values of μ and σ . [4]
- (d) Using the values of the parameters found in part (c), find the probability that the total running time of 2 randomly chosen Swift Runners Club members is less than 1.8 times of the running time of a randomly chosen Sports Recreational Club member.

State an assumption you have made in your calculation. [4]

- 8** A light source is placed at varying distances from a sensor. The intensity of light measured by the sensor is recorded. The results are as shown in the table below; the distance, x , is in cm and the intensity of light, y , is in lux.

x	10	15	20	30	40	50	70	100
y	112.2	60.1	42.5	28.8	24.2	22.0	20.1	19.2

- (a)** Sketch the scatter diagram for the data. Hence, explain why a linear model $y = a + bx$ is likely to be inappropriate. [3]

- (b)** The following two models are proposed, where c , d , e and f are constants:

$$y = c + \frac{d}{x} \qquad y = e + f \ln x$$

Calculate the product moment correlation coefficient for both models. State, with a reason, why the model $y = c + \frac{d}{x}$ is the more appropriate model. [3]

- (c)** Find the regression equation for the model $y = c + \frac{d}{x}$, giving your constants c and d correct to 2 decimal places. [1]

- (d)** Use your equation found in part **(c)** to estimate the light intensity when $x = 120$. Comment on the reliability of this estimate. [2]

- (e)** In this context, what does the value of c in your regression line in part **(c)** represent? [1]

- (f)** Suggest a physical interpretation for the constant c . [1]

- 9** A quality control inspector at an electronics factory tests components to see if any are defective. The inspector randomly selects 30 components for testing in a day. Historical data shows that 0.8% of the components are defective.

- (a) State, in context, two assumptions needed for the number of defective components found each day to be well-modelled by a binomial distribution. [2]

Assume now that the number of defective components found each day has a binomial distribution.

- (b) Find the probability that at most 1 component is found to be defective. [1]
- (c) Find the probability that the first defective component is the fourth component tested and a total of 3 defective components are found in a day. [2]
- (d) Given that the inspector finds at least 1 defective component in a day, find the probability that she finds less than 3 defective components. [2]
- (e) Find the probability that, in a randomly chosen 5-day working week, at most 1 component is found to be defective on more than 3 days. [2]

The owner of the factory decides to expand his business by producing memory chips. On average, 5% of the memory chips are defective. Another quality control inspector was tasked to randomly select 100 memory chips for testing in a day. You may assume that the number of defective memory chips found each day has a binomial distribution.

- (f) The inspector records down the number of defective memory chips she found each day. Estimate the probability that the average number of defective memory chips found on 50 randomly chosen days is between 4.5 and 5.5. [3]

- 10 (a)** The waiting time for a bus during peak hours in a city is X minutes and it is known that the standard deviation of X is 1.2 minutes. The Metropolitan Transport Board (MTB) which runs a large fleet of buses claims that the mean waiting time for buses during peak hours in the city is 6.5 minutes.

With various changes to the infrastructure in the city, MTB wants to review whether the mean waiting time has changed since the claim was made. MTB records a random sample of 20 waiting times and found that the mean waiting time was $\bar{x}$ minutes.

- (i)** Given that $\bar{x} = 6.9$, test at the 5% level of significance, whether the mean waiting time differs from 6.5 minutes.

State an assumption for the test to be valid. [4]

- (ii)** Explain what you understand by the phrase ‘at the 5% level of significance’ in the context of this question. [1]

- (iii)** Find the range of values of $\bar{x}$ for which there is sufficient evidence at 1% significance level that the mean waiting time differs from 6.5 minutes. [2]

- (b)** MTB operates the train system in the city as well and has received public feedback that the mean waiting time for trains has been getting worse. It is known that the waiting times for trains is normally distributed. MTB wishes to test, at the 5% level of significance, if the mean waiting time for trains is more than 3.7 minutes. A random sample of n observations, where $n \geq 30$, was taken and a mean of 3.95 minutes and standard deviation of 0.9 minutes were obtained.

Given that the null hypothesis is not rejected, obtain an inequality involving n , and hence find the set of values that n can take. [5]

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