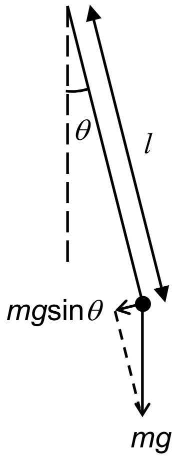
1(a) (i) torque $= I\ddot{\theta} = -mgl\sin\theta \approx -mgl\theta$, where moment of inertia $I = ml^2$

$$\Rightarrow \ddot{\theta} = -\frac{mgl}{ml^2}\theta = -\frac{g}{l}\theta \equiv -\omega^2\theta, \text{ where angular frequency } \omega = \sqrt{\frac{g}{l}}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$



Note 1: The negative sign in the first line is because, while the positive direction for the angle θ (as defined in the diagram) is anticlockwise, the torque is negative (clockwise).

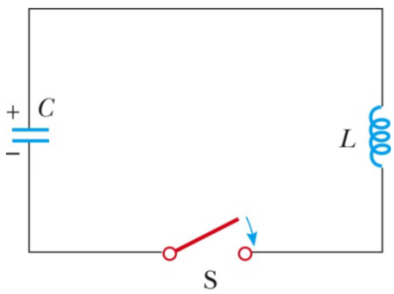
Note 2: One can also consider the oscillation of the displacement of the particle to arrive at the same conclusion.

1(a)(ii) $$f_{\text{pen, max}} = \frac{1}{T_{\text{min}}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{30.0\times10^{-2}}} = 0.910 \text{ Hz}$$

1(b)(i) Let the charge on the capacitor be q,

p.d. across the capacitor $= \dfrac{q}{C}$

emf induced in the inductor $= L\dfrac{dI}{dt} = L\dfrac{d^2q}{dt^2}$



By Kirchoff's law,

$$\frac{q}{C} + L\frac{d^2q}{dt^2} = 0$$

$$\frac{d^2q}{dt^2} = -\frac{q}{LC} \equiv -\omega^2 q, \quad \omega = \frac{1}{\sqrt{LC}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

1(b)(ii) $$f_{LC} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} = \frac{1}{2\pi}\frac{1}{\sqrt{82\times10^{-3}\times39000\times10^{-6}}} = 2.81 \text{ Hz}$$

$$\frac{f_{LC}}{f_{\text{pen, max}}} = \frac{2.81}{0.910} = 3.08 \approx 3$$

1(b)(iii) The frequency of the LC circuit needs to be lowered, hence a larger capacitance (by connecting a few capacitors in parallel) is needed.
Assume the length of the pendulum remains (the minimum value) 30.0 cm.
Let the number of capacitors needed to match the frequencies be n.

$$\sqrt{\frac{g}{l}} = \frac{1}{\sqrt{LC}}, \text{ or } l = gLC$$

$$30.0\times10^{-2} = 9.81\times82\times10^{-3}\times n\times39000\times10^{-6}$$

$$n = 9.6$$

Thus at least 10 capacitors are needed in parallel. To match frequencies, the length of the pendulum must be

$$l = gLC = 9.81\times82\times10^{-3}\times10\times39000\times10^{-6} = 0.3137 \text{ m} = 31.4 \text{ cm}$$

2(a) The magnitude of the dipole moment is the product of the magnitude of either charge and the separation between the charges.

2(b) Let the net charge be q.

$q \times 95.8 \times 10^{-12} \times 0.784 = 1.5 \times 3.34 \times 10^{-30}$

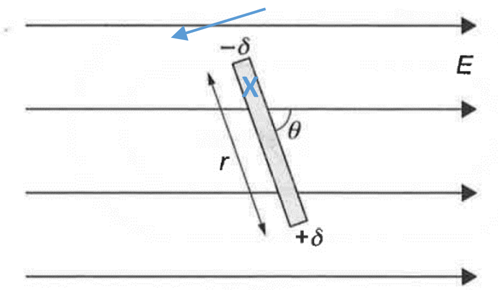
$q = 0.667 \times 10^{-19} = \dfrac{0.667}{1.6} e = 0.42e$

2(c) net electric dipole moment $= 2 \times 1.50\text{D} \times \cos\dfrac{104^\circ}{2} = 1.847\text{D} \approx 1.85\text{D}$

Note 1: Electric dipoles are vectors. We are adding up two vectors of equal length (1.50D), with an angle 104° between them.

Note 2: As a show question, it is important to show intermediate value (1.847D) before rounding to the value that we are trying to show.

2(d)(i)



Note: Oxygen atomic mass = 16, hydrogen atomic mass = 1. With two hydrogen atoms, total atomic mass = 2. Hence, the centre of mass is much closer to the oxygen atom (negative charge). Distance ratio should be roughly 1:8.

2(d)(ii) The two electric forces form a couple.

torque $= Fd = \delta E \times r \sin\theta = pE \sin\theta$

2(d)(iii) Maximum rotational kinetic energy is attained at $\theta = 0°$.

Method 1:

max KE = potential energy at 63° – potential energy at 0°

$= -pE\cos 63° - (-pE\cos 0°)$

$= pE(\cos 0° - \cos 63°)$

$= 1.85 \times 3.34 \times 10^{-30} \times 9500 \times (1 - \cos 63°)$

$= 3.21 \times 10^{-26}$ J

Method 2:

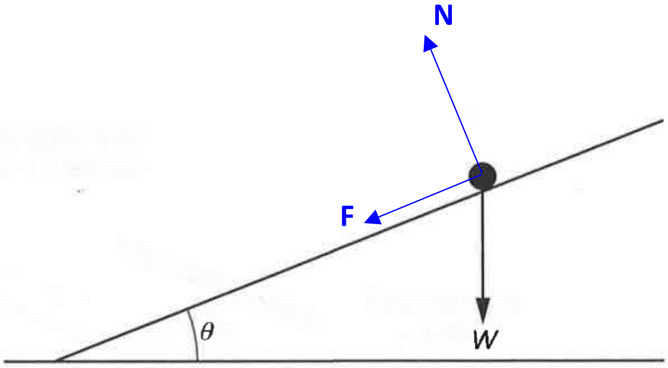
max KE $= -\int_{63^\circ}^{0^\circ} pE \sin\theta \, d\theta = pE(\cos 0^\circ - \cos 63^\circ)$

$= 1.85 \times 3.34 \times 10^{-30} \times 9500 \times (1 - \cos 63°)$

$= 3.21 \times 10^{-26}$ J

3(a) **Note 1**: Since the object is moving **at a high speed**, a large centripetal force is required. Hence, the frictional force must have a leftward component to contribute to the centripetal force.

Note 2: Normal force is normal to the slope and upwards. Its vertical component is equal in length to weight.



3(b) The centripetal force is the sum of the horizontal components of **F** and **N**:

$$F_c = F\cos\theta + N\sin\theta$$

The maximum frictional force is μN, hence the maximum centripetal force is

$$F_{c,\max} = \mu N\cos\theta + N\sin\theta = m\frac{v_{\max}^{2}}{r}$$

Consider the vertical direction:

$$-\mu N\sin\theta + N\cos\theta = mg$$

Taking ratio between the above two equations and simplify,

$$v_{\max} = \sqrt{\frac{rg(\mu+\tan\theta)}{1-\mu\tan\theta}}.$$

3(c)(i) Using the expression for $v_{\max}$ above, for $12° \le \theta \le 45°$:

Since $\tan\theta$ increases monotonically with θ, $\sqrt{\dfrac{\mu+\tan\theta}{1-\mu\tan\theta}}$ increases monotonically with θ, since the numerator increases, and denominator decreases, as θ increases.
Hence, the optimal value for θ is 45°.

Since $v_{\max} \propto \sqrt{r}$, the optimal value for r is the maximum value:

$$r_{\text{optimal}} = 20 + (0.20 + 0.70)\cos 45° = 20.6 \text{ m}.$$

Note: The above arguments are a little sketchy, since r is also a function of θ:

$$r = 20 + (0.20 + 0.70)\cos\theta$$

$\sqrt{\dfrac{\mu+\tan\theta}{1-\mu\tan\theta}}$ increases with θ, but $r = 20 + (0.20 + 0.70)\cos\theta$ decreases with θ. Ideally, one needs to replace r by the above function of θ in the expression for $v_{\max}$, differentiate the resulting expression w.r.t. θ and set the derivative to 0, to solve for the optimal value for θ at the stationary point. But the resulting equation is too tedious to solve.
Alternatively, one can plot the resulting function using a graphing calculator, and discover that it monotonically increases with θ. In other words, our conclusion in our solution is correct.

3(c)(ii) $$v_{\max} = \sqrt{\frac{rg(\mu+\tan\theta)}{1-\mu\tan\theta}} = \sqrt{\frac{20.6\times 9.81\times(0.70+\tan 45°)}{1-0.70\times\tan 45°}}$$

$$= \sqrt{\frac{20.6\times 9.81\times(0.70+1)}{1-0.70}} = 33.8 \text{ m s}^{-1}$$

4(a) By conservation of momentum,

$mv_0 = mv_1 + 2mv_2\cos\theta$, θ being the angle between v_2 and horizontal

$$= mv_1 + 2mv_2\frac{x}{\sqrt{\left(\frac{L}{4}\right)^2 + x^2}}.$$

Hence, $v_0 = v_1 + f(x)v_2$, where $f(x) = 2\dfrac{x}{\sqrt{\left(\frac{L}{4}\right)^2 + x^2}}$.

4(b) time taken for the two balls to reach middle pockets
= time taken for the cue ball to reach the right side:

$$\frac{\sqrt{x^2+\left(\frac{L}{4}\right)^2}}{v_2}=\frac{\frac{L}{2}+x}{v_1}\Rightarrow v_1=\frac{\frac{L}{2}+x}{\sqrt{x^2+\left(\frac{L}{4}\right)^2}}v_2$$

$$v_0=v_1+v_2\frac{2x}{\sqrt{\left(\frac{L}{4}\right)^2+x^2}}=v_1+v_1\times\frac{\sqrt{x^2+\left(\frac{L}{4}\right)^2}}{\frac{L}{2}+x}\times\frac{2x}{\sqrt{\left(\frac{L}{4}\right)^2+x^2}}$$

$$v_0=v_1\left(1+\frac{2x}{\frac{L}{2}+x}\right)=v_1\left(\frac{L+6x}{L+2x}\right).$$

4(c) Kinetic energy remains constant before and after an elastic collision:

$$\frac{1}{2}mv_0{}^2=\frac{1}{2}mv_1{}^2+2\times\frac{1}{2}mv_2{}^2 \text{ or } v_0{}^2=v_1{}^2+2v_2{}^2$$

Using equations in parts (a) and (b) to express v_0 and v_2 in terms of v_1, simplify and rearrange:

$$24x^2+8xL-\frac{L^2}{2}=0$$

Solving the quadratic equation for x,

$$x=\frac{L}{12}(\sqrt{7}-2).$$

Hence, b = 7.

4(d) Yes. The frictions on the balls are of equal magnitude. Since the cue ball moves a longer distance, friction does more negative work on it. The cue ball slows down more than the other two balls, hence will hit the right-hand cushion after the two balls have been potted.

5(a)(i) $M=8360\times3.0\times10^{-3}\times(12.0\times10^{-2})^2=0.361\,\text{kg}$

Using parallel axis theorem,

$$\text{moment of inertia}=\frac{Ma^2}{12}+Ml^2=M\left(\frac{a^2}{12}+l^2\right)$$

$$=0.361\times\left(\frac{(12.0\times10^{-2})^2}{12}+(25.0\times10^{-2})^2\right)=0.02299=0.023\text{ kg m}^2$$

5(a)(ii) mass of one cylindrical rod = $8360\times\pi\left(\frac{6.0\times10^{-3}}{2}\right)^2\times25.0\times10^{-2}=0.0591\,\text{kg}$

moment of inertia due to a single cylindrical rod about one end

$$=\frac{1}{3}ML^2=\frac{1}{3}\times0.0591\times\left(25.0\times10^{-2}\right)^2=0.00123\text{ kg m}^2$$

total moment of inertia = (0.023 + 0.00123) × 8 = 0.193 = 0.19 kg m^2.

5(a)(iii) By conservation of energy,
loss in GPE = gain in rotational kinetic energy

$$2mgh = \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\frac{4mgh}{I}} = \sqrt{\frac{4\times 1.81\times 9.81\times 11.0}{0.19}} = 64.1\text{ rad s}^{-1}$$

5(a)(iv) Assume the angular acceleration α of the paddles to be constant.
The angular acceleration α of the paddles and the linear acceleration a of the masses are related by $a = r\alpha$, r being the radius of the drum.
Let the time for the mass to move 11.0 m be t.

$$\alpha t = 64.1$$

$$\frac{1}{2}at^2 = \frac{1}{2}r\alpha t^2 = \frac{1}{2}\times\frac{8.6\times 10^{-2}}{2}\times 64.1\times t = 11.0 \Rightarrow t = 7.98 = 8.0\text{ s.}$$

5(a)(v) no. of rounds one paddle turns $= \frac{11.0}{\pi d} = \frac{11.0}{\pi\times 8.6\times 10^{-2}} = 40.7$
volume of air displaced by one paddle
$= \text{no. of rounds}\times 2\pi l\times a^2 = 40.7\times 2\pi\times 25.0\times 10^{-2}\times(12.0\times 10^{-2})^2 = 0.92\text{ m}^3$

5(b)(i) mass of water $m_{\text{water}} = 999\times 93.2\times 10^{-3} = 93.1\text{ kg}$
Assuming all GPE lost is converted into internal energy of water, and there is no heat loss.

$$2mgh = cm_{\text{water}}\Delta T$$

$$\Delta T = \frac{2mgh}{cm_{\text{water}}} = \frac{2\times 1.81\times 9.81\times 11.0}{4190\times 93.1} = 0.001\text{ K}$$

5(b)(ii)

$$\frac{2\times\frac{1}{2}mv^2}{2mgh} = \frac{v^2}{2gh} = \frac{0.305^2}{2\times 9.81\times 11.0} = 0.00043$$

$$\frac{1}{13} = 0.076$$

Since the heat capacity of the apparatus is a larger fraction of the total heat capacity than the kinetic energy of the mass is of the total GPE loss, the former will have a larger effect.

5(b)(iii) Measures (such as the insulation of the copper cylinder containing the water) were in place to prevent other forms of energy input. Hence, one can conclude that the temperature increase was due to the mechanical energy transfer by the paddles.

6(a) The object undergoes circular motion along the surface of the sphere, hence

$$F_c = mg\cos\theta - N = m\frac{v^2}{r}$$

By conservation of energy (ignoring the rotational KE),

$$\frac{1}{2}mv^2 = mgr(1-\cos\theta) \Rightarrow mv^2 = 2mgr(1-\cos\theta) \qquad (*)$$

Hence, $mg\cos\theta - N = \frac{2mgr(1-\cos\theta)}{r} = 2mg(1-\cos\theta)$

$\Rightarrow N = mg(3\cos\theta - 2)$.

6(b)(i) The object losses contact with the surface when $N = 0$:

$$N = mg(3\cos\theta - 2) = 0 \Rightarrow \cos\theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\frac{2}{3} = 48.2^\circ$$

6(b)(ii) Using Eq. (*) in part **(a)**,

$$v_1 = \sqrt{2gr(1-\cos\theta_1)} = \sqrt{2gr(1-\frac{2}{3})} = \sqrt{\frac{2gr}{3}} = 0.816\sqrt{gr}\ .$$

6(b)(iii) $v_{\text{hori}} = v_1\cos\theta_1 = \frac{2}{3}v_1 = \frac{2}{3}\sqrt{\frac{2gr}{3}} = 0.544\sqrt{gr}$,

$$v_{\text{vert}} = v_1\sin\theta_1 = \sqrt{1-\left(\frac{2}{3}\right)^2}\,v_1 = \frac{\sqrt{5}}{3}\sqrt{\frac{2gr}{3}} = 0.609\sqrt{gr}\ .$$

6(b)(iv) Let $r = 1$,

$$v_{\text{hori}} = \frac{2}{3}\sqrt{\frac{2g}{3}} = 1.70\text{ m s}^{-1},\ v_{\text{vert}} = \frac{\sqrt{5}}{3}\sqrt{\frac{2g}{3}} = 1.91\text{ m s}^{-1}.$$

$$h = v_{\text{vert}}t + \frac{1}{2}gt^2 = r\cos\theta_1 = \frac{2}{3} \Rightarrow \frac{1}{2}\times 9.81t^2 + 1.91t - \frac{2}{3} = 0$$

$t = 0.222$ s (negative root is rejected)

$s = v_{\text{hori}}t = 1.70\times 0.222 = 0.377$ m

$$d = s - r(1-\sin\theta_1) = 0.377 - (1-\frac{\sqrt{5}}{3}) = 0.122 \approx 0.125 = \frac{1}{8}r\ .$$

6(c)(i) **diameter:** The moment of inertia $I = kmR^2$, k being a constant and R the radius of the object. For pure rolling, $\omega = \frac{v}{R}$. Hence, KE due to rotation about its centre of mass ($= \frac{1}{2}I\omega^2 = \frac{1}{2}kmR^2\omega^2 = \frac{1}{2}kmv^2$) is independent of R. For the same decrease in GPE, the increase in KE due to linear motion, hence the linear velocity, is also independent of R. Hence, d will remain the same.

Note: If rotational KE is taken into account,

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}kmr^2\omega^2 = \frac{1}{2}(1+k)mv^2 = \text{decrease in GPE}\ \cdots\cdots\cdots\ (**)$$

Hence, v (hence d) depends on k (which depends on the composition (solid or hollow) of the object), rather than the radius.

mass: d does not depend on the mass, since mass cancels out in the above calculations (N is proportional to mass). Hence, d will remain the same.

composition: Composition affects the moment of inertia (through k in $I = kmR^2$, see Eqn (**) above). Hollow sphere has larger moment of inertia (since mass is distributed farther away from centre of mass), hence rotational KE will be more significant, KE due to linear motion is smaller, d will be smaller as compared to solid spheres.

6(c)(ii) **property:** varying density at different parts of the object
explanation: varying density implies varying distribution of mass, which affects the moment of inertia (through k in $I = kmR^2$, similar to composition. See Eqn.(**)). Linear velocity v, hence d, is affected by varying density.
Note: For example, if the outer shell of a spherical object is extremely dense compared to the inner parts, the object is similar in mass distribution to a hollow sphere. If the density varies little in the object, it is similar to a solid sphere.

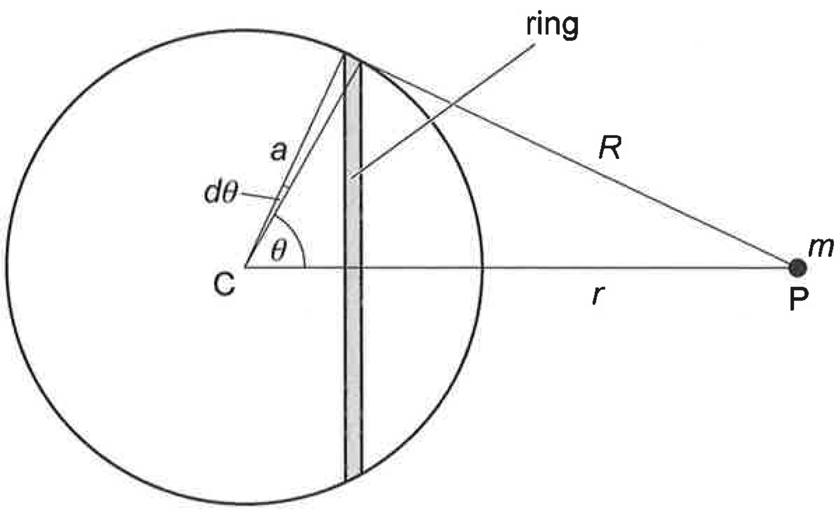
6(d)(i) By tracking the position of the point of crossing, the student can determine the centre of mass motion of the object.

6(d)(ii) **problem:** The ball may not move in the plane perpendicular to the line of sight of the camera lens.
solution: Blue-Tack can be used to create a very short 'track' at the starting point, to restrict the initial motion of the ball to be in the said plane.

7(a)(i) mass of ring = surface density × area = $\dfrac{M}{4\pi a^2} \times \left[a d\theta \times 2\pi a \sin\theta\right] = \dfrac{M}{2}\sin\theta d\theta$

7(a)(ii) $\phi = \int d\phi = -\int G\dfrac{dM}{R} = -\int_0^{\pi} G\dfrac{1}{R}\dfrac{M}{2}\sin\theta d\theta = -GM\int_0^{\pi}\dfrac{\sin\theta}{2R}d\theta$

7(a)(iii)



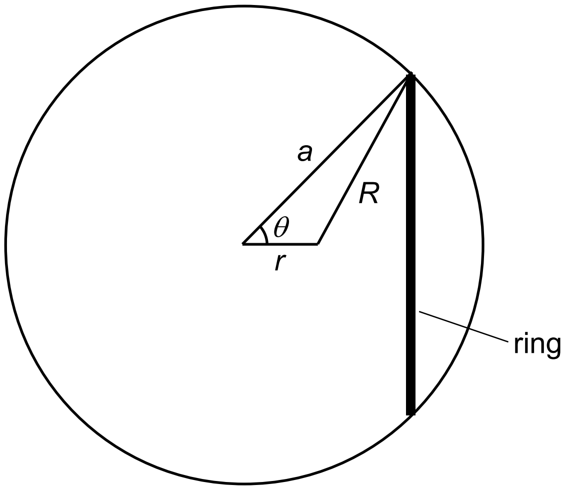
Consider the triangle formed by a, r and R. Using cosine rule,
$R^2 = a^2 + r^2 - 2ar\cos\theta$.
Differentiating both sides, noting that a and r are constants,
$2RdR = 2ar\sin\theta d\theta \Rightarrow \dfrac{\sin\theta}{R}d\theta = \dfrac{1}{ar}dR$

7(a)(iv) Note that, $R = r - a$ at $\theta = 0$ and $R = r + a$ at $\theta = \pi$.
$\phi = -GM\int_0^{\pi}\dfrac{\sin\theta}{2R}d\theta == -\dfrac{GM}{2}\int_0^{\pi}\dfrac{\sin\theta}{R}d\theta = -\dfrac{GM}{2ar}\int_{r-a}^{r+a}dR$
Hence, $k = \dfrac{GM}{2ar}$, $Y = r - a$, $Z = r + a$.

7(a)(v) $\phi = -\dfrac{GM}{2ar}\int_{r-a}^{r+a}dR = -\dfrac{GM}{2ar}[(r+a)-(r-a)] = -\dfrac{GM}{2ar}\times 2a = -\dfrac{GM}{r}$
$g = -\dfrac{d\phi}{dr} = -\dfrac{GM}{r^2}$
$F = mg = -\dfrac{GMm}{r^2}$

7(a)(vi) The gravitational force due to a uniform shell on a point mass outside of the shell is as if the shell is a point mass located at the centre of the shell.

7(a)(vii)



From the above graph, it is clear that, for a point within the shell (a distance r from the centre), $R = a - r$ at $\theta = 0$ and $R = a + r$ at $\theta = \pi$. Hence, the integration in **(a)(iv)** becomes

$\phi = -\dfrac{GM}{2ar}\displaystyle\int_{a-r}^{a+r} dR = -\dfrac{GM}{2ar}[(a+r)-(a-r)] = -\dfrac{GM}{2ar}\times 2r = -\dfrac{GM}{a}$, which is independent of r.

Hence,

$$F = mg = m\times\left(-\frac{d\phi}{dr}\right) = 0 .$$

7(b)(i) According to **7(a)**, for a mass located a distance r (< R_E) from the centre of the Earth, only the spherical part of the Earth with radius r from the centre of the Earth exerts a gravitational force on the mass:

$F = mg = -m\left(G\dfrac{m_{\text{inner}}}{r^2}\right) = -m\left(G\dfrac{\rho\frac{4}{3}\pi r^3}{r^2}\right) = -Gm\rho\dfrac{4\pi r}{3}$, where density $\rho = \dfrac{M}{\frac{4}{3}\pi {R_E}^3}$.

Hence, $F = -Gm\rho\dfrac{4\pi r}{3} = -Gm\dfrac{M}{\frac{4}{3}\pi {R_E}^3}\dfrac{4\pi r}{3} = -G\dfrac{mMr}{{R_E}^3}$ **.**

7(b)(ii) Dividing the above equation by m, and using Newton's Second Law,

$$\frac{F}{m} = a = -G\frac{M}{{R_E}^3}r\ .$$

Comparing with the defining equation of simple harmonic motion, $a = -\omega^2 x$,

$$\omega = \sqrt{G\frac{M}{{R_E}^3}} \ \cdots\cdots\cdots\cdots \ (***)\,.$$

The time to go from Pantoja to Singapore is half of a period of the oscillation, hence

$$t = \frac{T}{2} = \frac{1}{2}\frac{2\pi}{\omega} = \pi\sqrt{\frac{{R_E}^3}{GM}} = \pi\sqrt{\frac{(6.37\times10^6)^3}{6.67\times10^{-11}\times5.97\times10^{24}}} = 2531\text{ s} = 42.2\text{ min}\,.$$

7(b)(iii) For a satellite in low Earth orbit with orbital radius r,

$$m\omega^2 r = \frac{GMm}{r^2} \Rightarrow \omega = \sqrt{G\frac{M}{r^3}}$$

Comparing this angular speed with the angular frequency in Eqn (***), they should be similar in value if $r \approx R_E$ (for low Earth orbit). The period T, hence the time to travel from Pantoja to Singapore, should be approximately the same as well.

8(a) As the rubber belt rubs against the lower plastic roller, the belt becomes positively charged (losing electrons) and the roller negatively charged (gaining electrons). At the air gap between the top metal comb and the belt, the electric field is strong enough to ionise the air, which becomes conducting, and electrons move from the metal dome to the belt through the comb, by induction. The metal dome is left positively charged, with all excess positive charges residing on its outer surface. At the air gap between the lower metal comb and the belt, the electrons are discharged to the Earth (through the ionised air).

8(b)(i) Since the rod and the sphere are made from the same material (aluminium), and has the same mass, they must have the same volume of aluminium. Thus,

$$\frac{4}{3}\pi\left[\left(\frac{d}{2}\right)^3-\left(\frac{d}{2}-0.15\right)^3\right]=\pi\times\left[\left(\frac{1.0}{2}\right)^2-\left(\frac{1.0}{2}-0.15\right)^2\right]\times 50.0$$

$$3d^2-0.90d-127.41=0$$

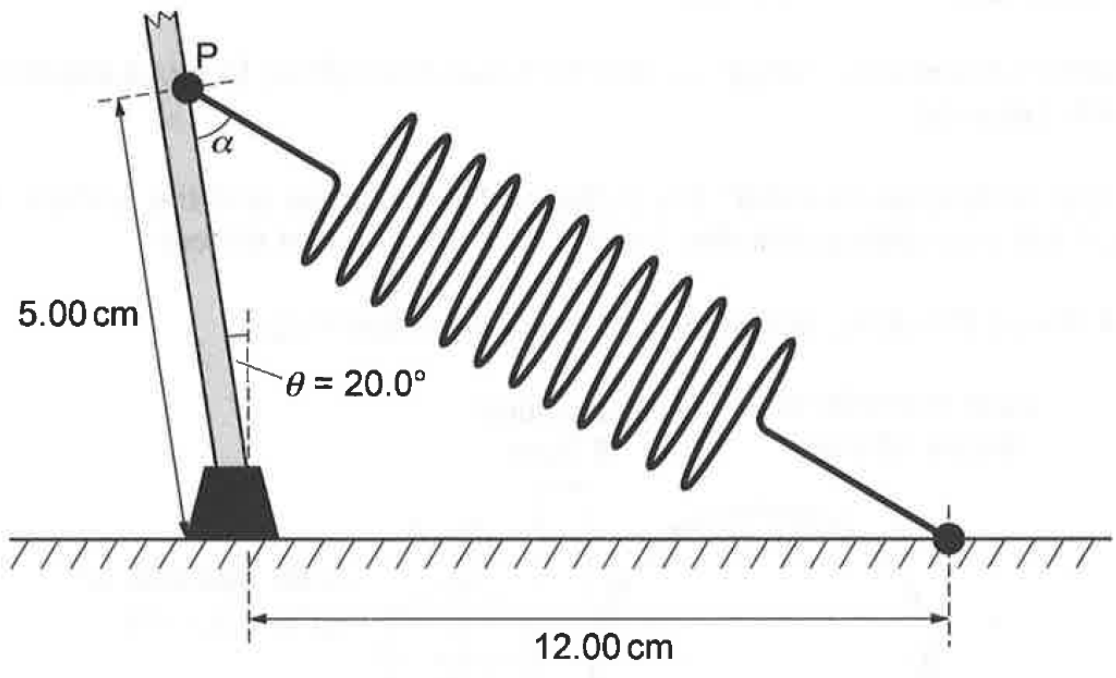
$$d=6.668=6.67\text{ cm}$$

Note 1: Equation is simplified using $x^3-y^3=(x-y)(x^2+xy+y^2)$.
Note 2: As a show question, you must show 6.668 before rounding to 6.67.

8(b)(ii) $\text{volume}=\pi\times\left[\left(\frac{1.0}{2}\right)^2-\left(\frac{1.0}{2}-0.15\right)^2\right]\times 50.0=20.0\text{ cm}^3$

mass = volume × density = 20.0 × 2.7 = 54 = 50 g.

8(b)(iii)



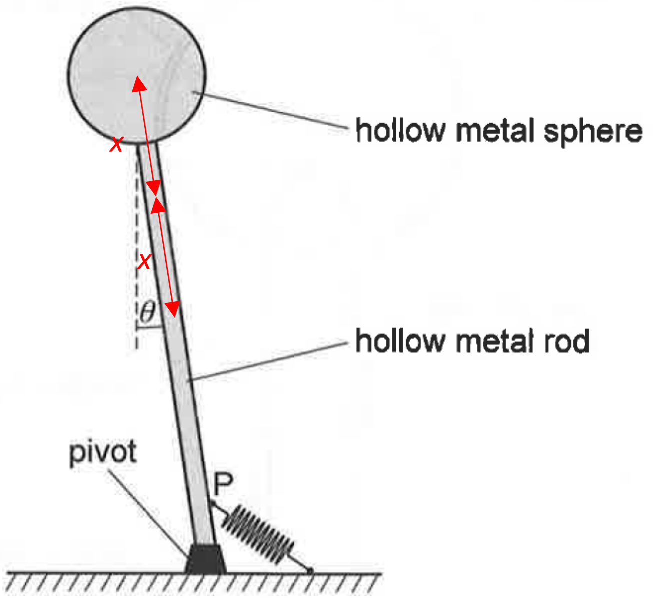
Using the cosine rule,

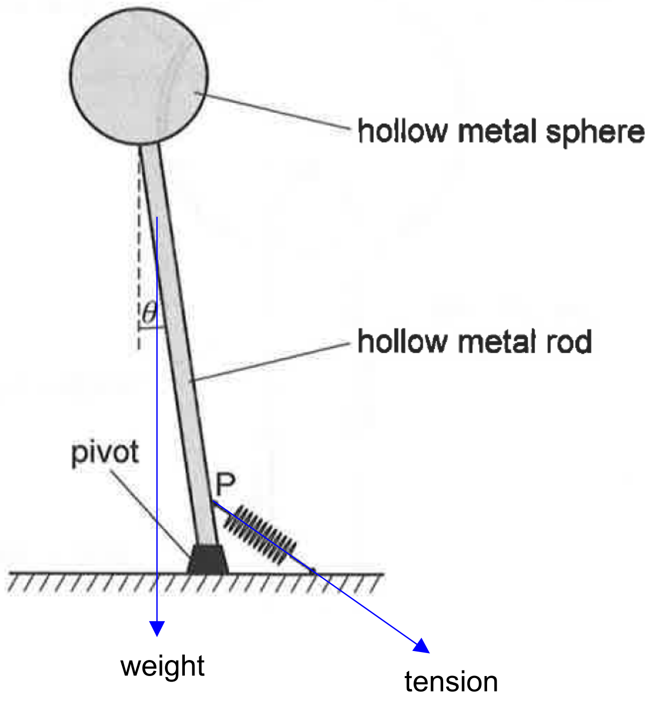
$$(14.34+x)^2=5.00^2+12.00^2-2\times 5.00\times 12.00\times\cos(110.0°)$$

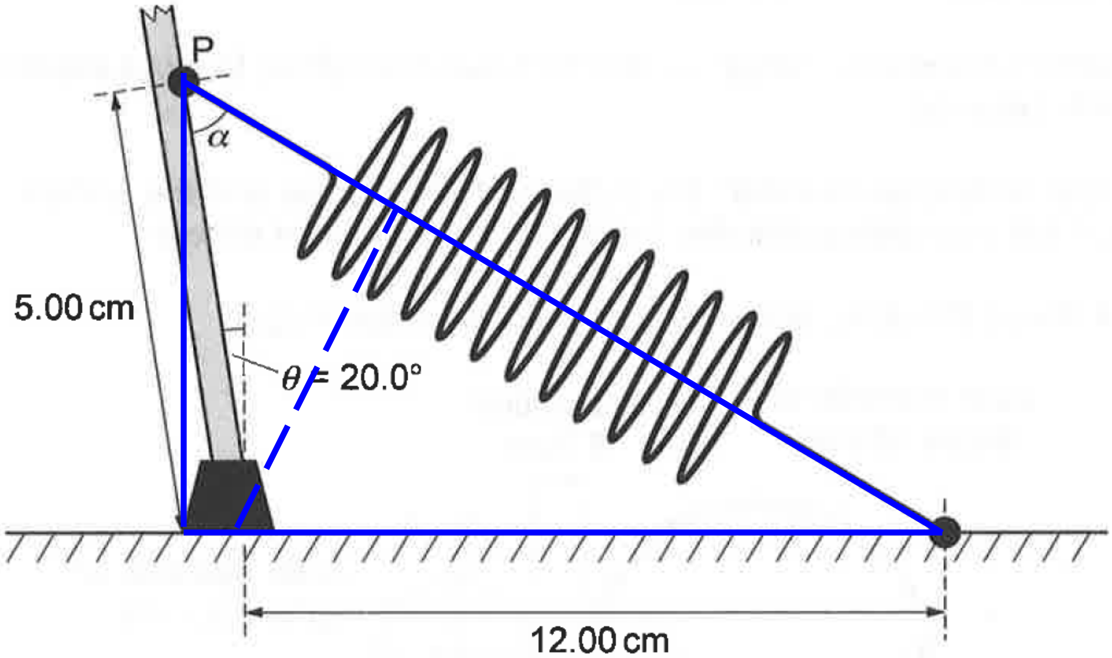
$$x=0.1528=0.15\text{ cm}$$

Note: Reminder to show more d.p. before rounding off.

8(b)(iv) To determine the position of the centre of mass (CoM), note that the CoM of the sphere and rod are at their respective geometric centres. Since they are of the same mass, the CoM of the rod+sphere system is halfway between their respective CoM. Hence,
$2x = 6.67 + 25 \Rightarrow x = 15.8$ cm.







Using the above diagrams, and applying the principle of moments,
clockwise moment = anticlockwise moment

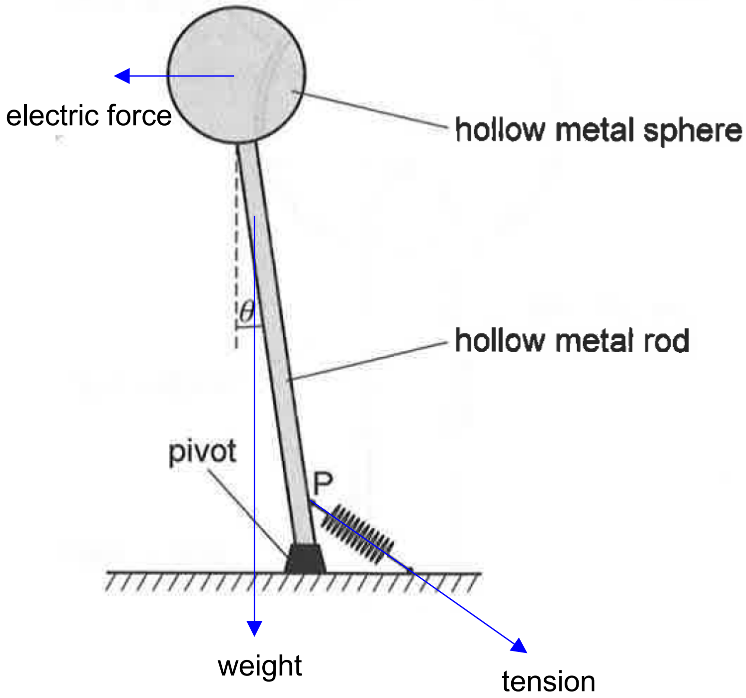
$$2 \times 0.050 \times 9.81 \times (0.158 + 0.250) \times \sin 20.0° = k \times 0.0015 \times 0.120 \times \frac{5.00 \cos 20.0°}{14.49}$$

$k = 2345$ N m^{-1}

8(c)(i) As the dome charges up, charges of opposite sign (to those on the dome) are induced on the side of the sphere close to the dome, and charges of the same sign on the far side of the sphere. There is net attraction between the dome and sphere, since the attraction between the charges of opposite signs is stronger, due to the smaller separation.

After sparking, the excess charge on the dome is neutralized, so both the dome and the sphere are neutral. There is no more attraction, and the dome and the sphere will be back to their original separation.

8(c)(ii)



The force diagram in this case is similar to that in **(b)(iv)**, except the additional electric force given by

$$F_E = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2},$$

r being the distance between the two charges:

$$r = 3.3 + \frac{30.0}{2} + \frac{6.7}{2} = 21.7 \text{ cm}.$$

The extension of the spring is 14.80 − 14.34 = 0.46 cm.
The corresponding moments equation is

$$F_E \times (0.500 + \frac{0.067}{2}) \times \cos 24.6° + 2 \times 0.050 \times 9.81 \times (0.158 + 0.250) \times \sin 24.6°$$

$$= 2345 \times 0.0046 \times 0.120 \times \frac{5.00 \cos 24.6°}{14.49}$$

$$\Rightarrow F_E = 0.495 \text{ N}$$

Hence,

$$Q_1 Q_2 = F_E \times 4\pi\varepsilon_0 r^2 = 0.495 \times 4\pi \times \frac{10^{-9}}{36\pi} \times 0.217^2 = 2.59 \times 10^{-12} \text{ C}^2$$

8(c)(iii) Because of induction, the centres of the charges will not be at the geometric centres, but will be closer (i.e. the actual value of *r* is smaller than 21.7 cm). Hence, Q_1Q_2 as calculated above is likely to be an over-estimate.