2017 H2 Mathematics Paper 1 (9758/01) – Suggested Solutions

1

By using standard expansions of e^x and $\ln(1+x)$,

$$\mathrm{e}^{2x}\ln(1+ax)$$
$$=\left(1+(2x)+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\ldots\right)\left(ax-\frac{(ax)^2}{2}+\frac{(ax)^3}{3}-\ldots\right)$$
$$=ax-\frac{a^2x^2}{2}+\frac{a^3x^3}{3}+2ax^2-a^2x^3+2ax^3+\ldots$$
$$=ax+\left(2a-\frac{a^2}{2}\right)x^2+\left(\frac{a^3}{3}-a^2+2a\right)x^3+\ldots$$

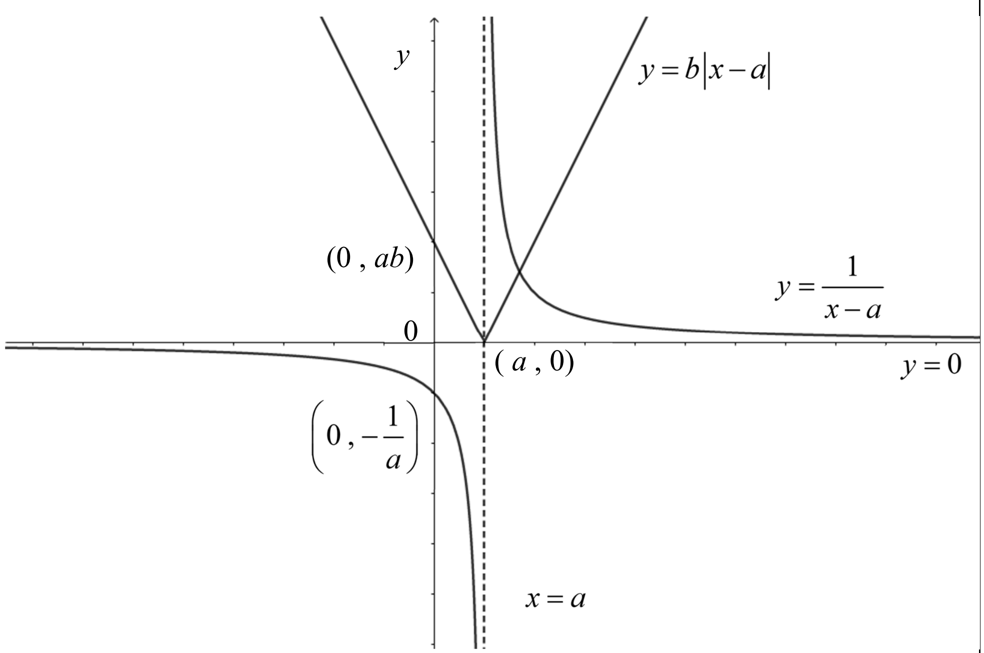
The expansion for $\mathrm{e}^{2x}\ln(1+ax)$ up to the term in x^3 is

$$ax+\left(\frac{a^2}{2}+2a\right)x^2+\left(\frac{a^3}{3}-a^2+2a\right)x^3$$

Given that there is no term in x^2 , $-\frac{a^2}{2}+2a=0$

$\Rightarrow 4a=a^2$
$\Rightarrow a=0$ or $a=4$
Since $a\neq 0$, $\therefore a=4$

2(i)



(ii)	Point of intersection occurs when $\frac{1}{x-a}=b\lvert x-a\rvert$. From graph, point of intersection occurs at $x>a$. $\therefore \lvert x-a\rvert = x-a$ $\Rightarrow \frac{1}{x-a}=b(x-a)$ $\Rightarrow 1=b(x-a)^2$ $\Rightarrow x-a=\pm\frac{1}{\sqrt{b}} \Rightarrow x=a\pm\frac{1}{\sqrt{b}}$ Since $x>a \Rightarrow$ Point of intersection occurs at $x=a+\frac{1}{\sqrt{b}}$ Hence, using the graph in (i), $\frac{1}{x-a}<b\lvert x-a\rvert$ when $x<a$ or $x>a+\frac{1}{\sqrt{b}}$.
3(i)	$y^2-2xy+5x^2-10=0$ Differentiating with respect to x, $2y\frac{dy}{dx}-2x\frac{dy}{dx}-2y+10x=0$ $\frac{dy}{dx}(2y-2x)=2y-10x---(1)$ At the stationary points of C, $\frac{dy}{dx}=0$ $\Rightarrow 2y-10x=0$ $\Rightarrow y=5x$ Substitute $y=5x$ into the equation of C: $(5x)^2-2x(5x)+5x^2-10=0$ $20x^2=10$ $x^2=\frac{1}{2}\Rightarrow x=\pm\frac{1}{\sqrt{2}}$ x-coordinates of stationary points of C are $x=\frac{1}{\sqrt{2}}$ and $x=-\frac{1}{\sqrt{2}}$
(ii)	For stationary point where $x>0$, $x=\frac{1}{\sqrt{2}}$ From (1),

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$$\frac{dy}{dx}(2y-2x)=2y-10x$$

$$\frac{dy}{dx}(y-x)=y-5x\text{ --(2)}$$

Differentiating (2) with respect to x,

$$\frac{d^2y}{dx^2}(y-x)+\frac{dy}{dx}\left(\frac{dy}{dx}-1\right)=\frac{dy}{dx}-5$$

At $x=\frac{1}{\sqrt{2}}, y=\frac{5}{\sqrt{2}}$ and $\frac{dy}{dx}=0$.

Substituting these values into the above equation:

$$\frac{d^2y}{dx^2}\left(\frac{5}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)+(0)(-1)=0-5$$

$$\frac{d^2y}{dx^2}\left(\frac{4}{\sqrt{2}}\right)=-5$$

$$\frac{d^2y}{dx^2}=\frac{-5\sqrt{2}}{4}<0$$

Hence the stationary point at $x=\frac{1}{\sqrt{2}}$ is a maximum.

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<tr><td>4(i)</td><td>

$$y=\frac{4x+9}{x+2}=4+\frac{1}{x+2}$$

Differentiating with respect to x,

$$\frac{dy}{dx}=-\frac{1}{(x+2)^2}$$

C is defined for all values of x, except for $x=-2$

$$(x+2)^2>0 \quad \forall x\in\mathbb{R}\backslash\{-2\}$$

$$\frac{dy}{dx}=-\frac{1}{(x+2)^2} \quad \forall x\in\mathbb{R}\backslash\{-2\}$$

$\therefore$ Gradient is negative for all points of C.

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$y=4+\frac{1}{x+2}$, where $a=4, b=1$.

Equations of asymptotes are $y=4$ and $x=-2$.

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The graph of C undergoes the following transformations:

1) Translation of 4 units in the negative direction of the y-axis,
2) Translation of 2 units in the positive direction of the x-axis.

Note that the order mentioned above can be interchangeable.

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5(i)	When $x=1$, $1+a+b+c=8$ $\Rightarrow a+b+c=7 \quad -(1)$ When $x=2$, $8+4a+2b+c=12$ $\Rightarrow 4a+2b+c=4 \quad -(2)$ When $x=3$, $27+9a+3b+c=25$ $\Rightarrow 9a+3b+c=-2 \quad -(3)$ Using the GC to solve equations $(1),(2),(3)$: $a=-\frac{3}{2}, b=\frac{3}{2}, c=7$
(ii)	$\mathrm{f}(x)=x^3-\frac{3}{2}x^2+\frac{3}{2}x+7, \quad x\in\mathbb{R}$ $\mathrm{f}'(x)=3x^2-3x+\frac{3}{2}$ $=3\left(x^2-x+\frac{1}{2}\right)$ $=3\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$ Since $3\left(x-\frac{1}{2}\right)^2\geq 0 \ \forall x\in\mathbb{R}$, $\mathrm{f}'(x)=\left(x-\frac{1}{2}\right)^2+\frac{3}{4}\geq\frac{3}{4}>0 \quad \forall x\in\mathbb{R}$. The gradient of the curve is always positive for all real values of x, Hence the function f is a strictly increasing function. This implies that f is one-to-one and therefore the line $y=0$ will cut the graph exactly once, which implies that $\mathrm{f}(x)=0$ has exactly one real root. Using the GC, the root is at $x=1.33$ (3 s.f.).
(iii)	When tangent is parallel to the line $y=2x-3$, the tangent has gradient 2.

	$\mathrm{f}'(x)=2$ $\Rightarrow 3\left(x-\frac{1}{2}\right)^2+\frac{3}{4}=2$ $\Rightarrow \left(x-\frac{1}{2}\right)^2=\frac{5}{12}$ $x=\frac{1}{2}\pm\sqrt{\frac{5}{12}}$ $=\frac{1}{2}\pm\frac{\sqrt{5}}{2\sqrt{3}}$ $=\frac{1}{2}+\frac{\sqrt{15}}{6}$ or $\frac{1}{2}-\frac{\sqrt{15}}{6}$ x-coordinates of the required points are $\frac{1}{2}+\frac{\sqrt{15}}{6}$ or $\frac{1}{2}-\frac{\sqrt{15}}{6}$.
6(i)	The equation describes the set of points with position vector **r**, where the points lie on a line parallel to the vector **b**, passing through a fixed point with position vector **a**.
(ii)	The equation describes the set of points with position vector **r,** where the points lie on a plane with a normal vector **n**, at a distance $\lvert d\rvert$ units away from the origin.
(iii)	Since $\underset{\sim}{b}\bullet\underset{\sim}{n}\neq 0$, the line is not parallel to the plane, hence there is a point of intersection between the line and the plane. $\mathbf{r}=\mathbf{a}+t\mathbf{b},\ \mathbf{r}\cdot\mathbf{n}=d$ Substitute $\mathbf{r}=\mathbf{a}+t\mathbf{b}$ into $\mathbf{r}\cdot\mathbf{n}=d$: $(\mathbf{a}+t\mathbf{b})\cdot\mathbf{n}=d$ $\Rightarrow \mathbf{a}\cdot\mathbf{n}\ +t\mathbf{b}\cdot\mathbf{n}=d$ Since $\mathbf{b}\cdot\mathbf{n}\neq 0$, $t\mathbf{b}\cdot\mathbf{n}=d-\mathbf{a}\cdot\mathbf{n}$ $t=\frac{d-\mathbf{a}\cdot\mathbf{n}}{\mathbf{b}\cdot\mathbf{n}}$ $\mathbf{r}=\mathbf{a}+t\mathbf{b}=\mathbf{a}+\left(\frac{d-\mathbf{a}\cdot\mathbf{n}}{\mathbf{b}\cdot\mathbf{n}}\right)\mathbf{b}$ **r** is the position vector of the point of intersection between the line with direction vector **b**, passing through the point with position vector **a**, and the plane with normal **n**, at a distance $\lvert d\rvert$ units from the origin.

7(i)	$\sin 2mx \sin 2nx$ $= -\frac{1}{2}(-2\sin 2mx \sin 2nx)$ [By factor formula] $= -\frac{1}{2}\left(\cos\left(2x(m+n)\right) - \cos\left(2x(m-n)\right)\right)$ $\int \sin 2mx \sin 2nx \ \mathrm{d}x$ $= \int -\frac{1}{2}\left(\cos\left(2x(m+n)\right) - \cos\left(2x(m-n)\right)\right) \ \mathrm{d}x$ $= -\frac{1}{2}\left[\frac{\sin\left(2x(m+n)\right)}{2(m+n)} - \frac{\sin\left(2x(m-n)\right)}{2(m-n)}\right] + C$ $= \frac{\sin\left(2x(m-n)\right)}{4(m-n)} - \frac{\sin\left(2x(m+n)\right)}{4(m+n)} + C,$ where C is an arbitrary constant
(ii)	$\int_0^{\pi} \left(\mathrm{f}(x)\right)^2 \ \mathrm{d}x$ $= \int_0^{\pi} \left(\sin 2mx + \sin 2nx\right)^2 \ \mathrm{d}x$ $= \int_0^{\pi} \left(\sin^2 2mx + \sin^2 2nx + 2\sin 2mx \sin 2nx\right) \ \mathrm{d}x$ $= \int_0^{\pi} \left(\sin^2 2mx + \sin^2 2nx\right) \ \mathrm{d}x + \int_0^{\pi} \left(2\sin 2mx \sin 2nx\right) \ \mathrm{d}x$ $= \int_0^{\pi} \left(\frac{1-\cos 4mx}{2} + \frac{1-\cos 4nx}{2}\right) \ \mathrm{d}x$ [Using $\cos 2A = 1 - 2\sin^2 A$] $+2\left[\frac{\sin\left(2x(m-n)\right)}{4(m-n)} - \frac{\sin\left(2x(m+n)\right)}{4(m+n)}\right]_0^{\pi}$ [Using part (i)] Note that when $x = 0$ or $x = \pi$, $\sin(2kx) = 0$. Hence $2\left[\frac{\sin\left(2x(m-n)\right)}{4(m-n)} - \frac{\sin\left(2x(m+n)\right)}{4(m+n)}\right]_0^{\pi} = 0$

	$\int_0^{\pi} \frac{1-\cos 4mx}{2}+\frac{1-\cos 4nx}{2}\ \mathrm{d}x+2\left[\frac{\sin\left(2x\left(m-n\right)\right)}{4\left(m-n\right)}-\frac{\sin\left(2x\left(m+n\right)\right)}{4\left(m+n\right)}\right]_0^{\pi}$ $=\int_0^{\pi} \frac{1-\cos 4mx}{2}+\frac{1-\cos 4nx}{2}\ \mathrm{d}x$ $=\int_0^{\pi} 1-\frac{1}{2}\left(\cos 4mx+\cos 4nx\right)\ \mathrm{d}x$ $=\left[x-\frac{1}{2}\left(\frac{\sin 4mx}{4m}+\frac{\sin 4nx}{4n}\right)\right]_0^{\pi}$ $=\left[\pi-\frac{1}{2}\left(0+0\right)\right]-\left[0-\frac{1}{2}\left(0+0\right)\right]=\pi$
8(a)	$z^2\left(1-i\right)-2z+\left(5+5i\right)=0$ Using the quadratic formula: $z=\frac{2\pm\sqrt{\left(-2\right)^2-4\left(1-i\right)\left(5+5i\right)}}{2\left(1-i\right)}$ $=\frac{2\pm\sqrt{4-20\left(1-i\right)\left(1+i\right)}}{2-2i}$ $=\frac{2\pm 2\sqrt{1-5\left(1-i^2\right)}}{2-2i}$ $=\frac{1\pm\sqrt{1-10}}{1-i}$ $=\frac{1\pm\sqrt{-9}}{1-i}$ $=\frac{1\pm 3i}{1-i}$ $z=\frac{1+3i}{1-i}=\frac{\left(1+3i\right)\left(1+i\right)}{1^2-i^2}=\frac{-2+4i}{2}=-1+2i$ or $z=\frac{1-3i}{1-i}=\frac{\left(1-3i\right)\left(1+i\right)}{1^2-i^2}=\frac{4-2i}{2}=2-i$ Hence the roots are $z=-1+2i$ and $z=2-i$

(b)(i)	$\omega^2 = (1-i)^2$ $= 1^2 + i^2 - 2i$ $= 1 - 1 - 2i$ $= -2i$ $\omega^3 = (-2i)(1-i)$ $= -2i + 2i^2$ $= -2 - 2i$ $\omega^4 = (-2-2i)(1-i)$ $= -2 + 2i - 2i + 2i^2$ $= -2 - 2 = -4$ $\omega^4 + p\omega^3 + 39\omega^2 + q\omega + 58 = 0$ Substituting the above information, $-4 + p(-2-2i) + 39(-2i) + q(1-i) + 58 = 0$ $-4 - 2p - 2pi - 78i + q - qi + 58 = 0$ $(54 - 2p + q) + i(-78 - 2p - q) = 0$ Comparing real and imaginary coefficients: $\begin{matrix} 54 - 2p + q = 0 \\ -78 - 2p - q = 0 \end{matrix} \Rightarrow \begin{matrix} 2p - q = 54 \\ 2p + q = -78 \end{matrix} \Rightarrow \begin{matrix} 2p - q = 54 \\ 2p + q + 2p - q = -78 + 54 \end{matrix}$ $4p = -24 \Rightarrow p = -6$ $q = -78 - 2(-6) = -66$ $p = -6, q = -66$
(ii)	$\omega^4 - 6\omega^3 + 39\omega^2 - 66\omega + 58 = 0$ $1-i$ is a root of the above equation. Since the equation has only real coefficients, hence, the complex conjugate, $1+i$ is also a root. $\therefore (\omega-(1-i))(\omega-(1+i))$ is a factor of $\omega^4 - 6\omega^3 + 39\omega^2 - 66\omega + 58$. $(\omega-(1-i))(\omega-(1+i))$ $= \omega^2 - (1 - i + 1 + i)\omega + (1^2 - i^2)$ $= \omega^2 - 2\omega + 2$ $\omega^2 - 2\omega + 2$ is a quadratic factor. $\therefore \omega^4 - 6\omega^3 + 39\omega^2 - 66\omega + 58 = (\omega^2 - 2\omega + 2)(a\omega^2 + b\omega + c), a, b, c$ real values $= a\omega^4 + (b - 2a)\omega^3 + (2a - 2b + c)\omega^2 + (2b - 2c)\omega + 2c$ Comparing coefficients:

	$a=1$ $2c=58 \Rightarrow c=29$ $b-2a=-6 \Rightarrow b-2=-6$ $b=-4$ Hence $\omega^4-6\omega^3+39\omega^2-66\omega+58=\left(\omega^2-2\omega+2\right)\left(\omega^2-4\omega+29\right)$
9(a)(i)	Given that $S_n=An^2+Bn$, $u_n=S_n-S_{n-1}$ $=An^2+Bn-A\left(n-1\right)^2-B\left(n-1\right)$ $=A\left(n^2-n^2+2n-1\right)+B$ $=A\left(2n-1\right)+B$
(ii)	$u_{10}=\left(20-1\right)A+B$ $=19A+B=48$ $u_{17}=\left(34-1\right)A+B$ $=33A+B=90$ Using the GC, $A=3, B=-9$
(b)	$\text{LHS}=r^2\left(r+1\right)^2-\left(r-1\right)^2 r^2$ $=r^2\left[\left(r+1\right)^2-\left(r-1\right)^2\right]$ $=r^2\left[\left(r+1+r-1\right)\left(r+1-r+1\right)\right]$ $=r^2\left[\left(2r\right)\left(2\right)\right]$ $=4r^3$ $=kr^3=\text{RHS}$, where $k=4$

	$\sum_{r=1}^{n} r^3 = \frac{1}{4}\sum_{r=1}^{n} 4r^3$ $= \frac{1}{4}\sum_{r=1}^{n}\left(r^2(r+1)^2-(r-1)^2 r^2\right)$ $= \frac{1}{4}\left(\begin{array}{l}(1^2\cdot 2^2-0)\\ +(2^2\cdot 3^2-1^2\cdot 2^2)\\ +(3^2\cdot 4^2-2^2\cdot 3^2)\\ \vdots\\ +\left[(n-2)^2\cdot(n-1)^2-(n-3)^2\cdot(n-2)^2\right]\\ +\left[(n-1)^2\cdot n^2-(n-2)^2\cdot(n-1)^2\right]\\ +\left[n^2\cdot(n+1)^2-(n-1)^2\cdot n^2\right]\end{array}\right)$ $= \frac{n^2(n+1)^2}{4}$
(c)	Let $a_r = \frac{x^r}{r!}, x \in \mathbb{R}$ To show that $\sum_{r=0}^{\infty}\frac{x^r}{r!} = \sum_{r=0}^{\infty} a_r$ converges for all real values of x: $\lim_{n\to\infty}\left\|\frac{a_{n+1}}{a_n}\right\| = \lim_{n\to\infty}\left\|\frac{x^{n+1}}{(n+1)!}\cdot\frac{n!}{x^n}\right\|$ $= \lim_{n\to\infty}\left\|\frac{x}{n+1}\right\|$ Since x is fixed, hence $\lim_{n\to\infty}\left\|\frac{x}{n+1}\right\| = 0$ Since $\lim_{n\to\infty}\left\|\frac{x}{n+1}\right\| = 0 < 1$, by D'Alembert's ratio test, the series converges for all real values of x. The sum to infinity is e^x, since by Maclaurin expansion, $\mathrm{e}^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^r}{r!} + ...$ $\quad$ (which holds for all real x) $= \sum_{r=0}^{\infty}\frac{x^r}{r!}$

10(i)	Let the cable C lie on line l_1, which has vector equation $l_1 : \mathbf{r} = \lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}, \lambda \in \mathbb{R}$ $\overrightarrow{OP} = \begin{pmatrix}1\\2\\-1\end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix}5\\7\\a\end{pmatrix}$ Let the new cable lie on line l_2. l_2 passes through $(1,2,-1)$ and is parallel to vector $\overrightarrow{PQ} = \begin{pmatrix}5-1\\7-2\\a+1\end{pmatrix} = \begin{pmatrix}4\\5\\a+1\end{pmatrix}$. Hence, $l_2 : \mathbf{r} = \begin{pmatrix}1\\2\\-1\end{pmatrix} + \mu\begin{pmatrix}4\\5\\a+1\end{pmatrix}, \mu \in \mathbb{R}$ At point of intersection, for some $\lambda, \mu \in \mathbb{R}$ $3\lambda = 1 + 4\mu$ --(1) $\lambda = 2 + 5\mu$ --(2) $-2\lambda = -1 + (a+1)\mu$ --(3) Using the GC to solve (1) and (2), $\lambda = -\frac{3}{11}, \mu = -\frac{5}{11}$. Substituting that into (3), $-2\left(-\frac{3}{11}\right) = -1 + (a+1)\left(-\frac{5}{11}\right)$ $\frac{6}{11} = -1 - \frac{5}{11}(a+1)$ $\frac{5}{11}(a+1) = -1 - \frac{6}{11}$ $= -\frac{17}{11}$ $(a+1) = -\frac{17}{11}\left(\frac{11}{5}\right)$ $a = -\frac{17}{5} - 1$ $= -\frac{22}{5}$

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Given that $a=-3$, $\overrightarrow{OQ}=\begin{pmatrix}5\\7\\-3\end{pmatrix}$.

Let R be a point on C. Then $\overrightarrow{OR}=\lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}$ for some $\lambda\in\mathbb{R}$.

If the situation is not possible, we want to show that angle PRQ cannot equal $90°$. (condition to reduce the length of cable required)

Equivalently, we can show that $\overrightarrow{PR}\cdot\overrightarrow{QR}\neq 0$ for all possible points R.

$\overrightarrow{PR}=\begin{pmatrix}3\lambda-1\\\lambda-2\\-2\lambda+1\end{pmatrix}$ $\qquad$ $\overrightarrow{QR}=\begin{pmatrix}3\lambda-5\\\lambda-7\\-2\lambda+3\end{pmatrix}$

$$\begin{aligned}\overrightarrow{PR}\cdot\overrightarrow{QR}&=\begin{pmatrix}3\lambda-1\\\lambda-2\\-2\lambda+1\end{pmatrix}\cdot\begin{pmatrix}3\lambda-5\\\lambda-7\\-2\lambda+3\end{pmatrix}\\&=(3\lambda-1)(3\lambda-5)+(\lambda-2)(\lambda-7)+(1-2\lambda)(3-2\lambda)\\&=9\lambda^2-18\lambda+5+\lambda^2-9\lambda+14+4\lambda^2-8\lambda+3\\&=14\lambda^2-35\lambda+22\\&=14\left(\lambda^2-\frac{5}{2}\lambda\right)+22\\&=14\left[\left(\lambda-\frac{5}{4}\right)^2-\frac{25}{16}\right]+22\\&=14\left[\left(\lambda-\frac{5}{4}\right)^2\right]+22-\frac{25(14)}{16}\\&=14\left[\left(\lambda-\frac{5}{4}\right)^2\right]+22-\frac{25(14)}{16}\\&=14\left[\left(\lambda-\frac{5}{4}\right)^2\right]+\frac{1}{8}\end{aligned}$$
Since

$14\left[\left(\lambda-\frac{5}{4}\right)^2\right]\geq 0,\ \forall\lambda\in\mathbb{R},$

$14\left[\left(\lambda-\frac{5}{4}\right)^2\right]+\frac{1}{8}\geq\frac{1}{8}>0$
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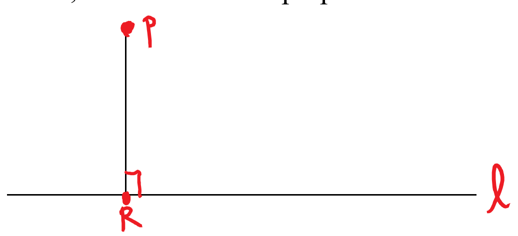
	$\overrightarrow{PR}\cdot\overrightarrow{QR}>0 \quad \forall\lambda\in\mathbb{R}$ Since $\overrightarrow{PR}\cdot\overrightarrow{QR}\neq 0$ for all points *R* on *C*, hence it is not possible for angle *PRQ* to equal $90°$.
(iii)	$\overrightarrow{PR}=\begin{pmatrix}3\lambda-1\\ \lambda-2\\ -2\lambda+1\end{pmatrix}$ for some $\lambda\in\mathbb{R}$ $\lvert\overrightarrow{PR}\rvert=\sqrt{(3\lambda-1)^2+(\lambda-2)^2+(1-2\lambda)^2}$ $=\sqrt{9\lambda^2-6\lambda+1+\lambda^2-4\lambda+4+4\lambda^2-4\lambda+1}$ $=\sqrt{14\lambda^2-14\lambda+6}$ To minimize $\lvert\overrightarrow{PR}\rvert$, it suffices to minimize $14\lambda^2-14\lambda+6$. $14(\lambda^2-\lambda)+6=14\left[(\lambda-\frac{1}{2})^2-\frac{1}{4}\right]+6$ $=14\left(\lambda-\frac{1}{2}\right)^2+\frac{5}{2}$ Minimum $\lvert\overrightarrow{PR}\rvert=\sqrt{\frac{5}{2}}$ occurs at $\lambda=\frac{1}{2}$. Coordinates of *R* in this case would be $\left(\frac{3}{2},\frac{1}{2},-1\right)$.
	Alternatively, $\overrightarrow{RP}=\begin{pmatrix}1-3\lambda\\ 2-\lambda\\ -1+2\lambda\end{pmatrix}$ for some $\lambda\in\mathbb{R}$ For the distance of *PR* to be the shortest, $\overrightarrow{RP}$ needs to be perpendicular to the line , l_1 , representing cable *C*.  $\begin{pmatrix}1-3\lambda\\ 2-\lambda\\ -1+2\lambda\end{pmatrix}\bullet\begin{pmatrix}3\\ 1\\ -2\end{pmatrix}=0$ $3-9\lambda+2-\lambda+2-4\lambda=0$ $7-14\lambda=0$ $\lambda=\frac{1}{2}$

	$\overrightarrow{OR} = \frac{1}{2}\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ Hence, the coordinates of R is $\left(\frac{3}{2}, \frac{1}{2}, -1\right)$. Minimum length is $\left\|\overrightarrow{RP}\right\|$ $= \left\|\begin{pmatrix} 1-\frac{3}{2} \\ 2-\frac{1}{2} \\ -1+1 \end{pmatrix}\right\|$ $= \left\|\begin{pmatrix} -\frac{1}{2} \\ \frac{3}{2} \\ 0 \end{pmatrix}\right\|$ $= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + 0}$ $= \sqrt{\frac{1}{4} + \frac{9}{4}}$ $= \frac{\sqrt{10}}{2} m$
11(i)(a)	Let v m/s be the velocity of the object at time t seconds. $\frac{\mathrm{d}v}{\mathrm{d}t} = c$ ---(1)
(b)	Integrating (1) with respect to t on both sides, $\int 1 \, \mathrm{d}v = \int c \, \mathrm{d}t$ $v = ct + A,$ where A is a constant At $t = 0$, $v = 4$. $\Rightarrow A = 4$ At $t = 2.5$, $v = 29$. $\Rightarrow 29 = \frac{5}{2}c + 4$ $c = 10$ $\therefore v = 10t + 4,$ where $c = 10$

(ii)	Since the rate of change of velocity is less than 10, $\frac{dv}{dt}=10-kv$, where $k>0$. $\Rightarrow \frac{1}{10-kv}\cdot\frac{dv}{dt}=1$ Integrating both sides with respect to t, $\int \frac{1}{10-kv}\ dv=\int 1\ dt$ $-\frac{1}{k}\ln(10-kv)=t+C$, where C is an arbitrary constant $\ln(10-kv)=-kt-kC$ $(10-kv)=e^{-kt-kC}$ $10-kv=Be^{-kt}$, where $B=\pm e^{-kC}$ $v=\frac{10-Be^{-kt}}{k}$ At $t=0$, $v=0$, $\Rightarrow 0=10-B\Rightarrow B=10$ $\therefore v=\frac{10-10e^{-kt}}{k}$
(iii)	As $t\to\infty, v\to 40$. $\lim_{t\to\infty}\frac{10-10e^{-kt}}{k}=\frac{10}{k}=40$, since $k>0$ and $e^{-kt}\to 0$ $\therefore k=\frac{1}{4}$ Want to find t at $v=0.9(40)=36$: $36=\frac{10-10e^{-\frac{t}{4}}}{\frac{1}{4}}$ $9=10-10e^{-\frac{t}{4}}$ $e^{-\frac{t}{4}}=\frac{10-9}{10}=\frac{1}{10}$ Taking ln on both sides: $-\frac{t}{4}=-\ln 10$ $t=4\ln 10=9.21\ (3\text{ s.f.})$ It takes 9.21 seconds to reach 90% of terminal velocity.