Name: **MARK SCHEME....................................** Register no: .... Class: ....

NGEE ANN SECONDARY SCHOOL

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PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **28 August 2026**

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Total	**/90**

Checked by student: ………………………………. Date: …………………

This document consists of **21** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n\,(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The percentage Gross Domestic Product (GDP) growth of a country from 2014 to 2024 can be modelled by the quadratic equation $\mathrm{G}(t)=0.7t^2-7t+11.2$, where t is the number of years after 2014.

(a) By completing the square, find the year in which the percentage growth is the lowest, and state this minimum percentage GDP growth. [4]

$$0.7t^2-7t+11.2=0.7\left(t^2-10t\right)+11.2$$

$$=0.7\left(t-5\right)^2-6.3------\text{B1 for } 0.7\left(t-5\right)^2 \text{ and B1 for } -6.3$$

Hence, the year in which the percentage growth is the lowest is **2019** ----- A1
and the **minimum percentage GDP growth is** –6.3%----- A1

Marker's Report:
1. This part of the question was generally well done. Most candidates were able to complete the square correctly.
2. A number of candidates did not read the question carefully and left their answer as (t = 5) instead of stating the year, 2019. Some candidates also omitted the negative sign when stating the minimum percentage GDP growth.

(b) Explain why this model may not be suitable for predicting the country's percentage GDP growth in the long run. [1]

The model is **quadratic,** hence it will **eventually increase without bound as *t* becomes large.** This is unlikely to reflect real GDP growth in the long run as economic growth is affected by many changing factors. -----B1

Marker's Report:
1. **This part of the question was generally poorly done.**
2. **Candidates did not fully understand the question, "Explain why this model may not be suitable," and instead gave responses that were not based on mathematical reasoning. Candidates should also note that the increase is not exponential. For a quadratic model, the rate of increase is linear.**

2 **Do not use a calculator in this question.**

It is given that $\sqrt{2a} = \sqrt{b} + 12$, where a and b are positive real numbers.

(a) Show that $2a - 3b = 216 - 2\left(\sqrt{b} - 6\right)^2$. [4]

$2a = \left(\sqrt{b} + 12\right)^2 = b + 24\sqrt{b} + 144$ – – – – – M1 for squaring both sides

$2a - 3b = -2b + 24\sqrt{b} + 144$

$= 144 - 2(b - 12\sqrt{b})$ – – – – – M1 for factorising

$= 144 - 2(b - 12\sqrt{b} + (-6)^2 - (-6)^2)$ – – – – – M1 for completing the square

$= 144 - 2[(\sqrt{b} - 6)^2 - 36]$ – – – – – M1 for simplifying

$= 144 + 72 - 2(\sqrt{b} - 6)^2$

$= 216 - 2(\sqrt{b} - 6)^2$ – – – – – AG

Alternative Method:

$2a = \left(\sqrt{b} + 12\right)^2 = b + 24\sqrt{b} + 144$ – – – – – M1 for expansion

$2a - 3b = -2b + 24\sqrt{b} + 144$

$216 - 2\left(\sqrt{b} - 6\right)^2 = 216 - 2(b - 12\sqrt{b} + 36)$ – – – – – M1 for expansion

$= 216 - 2b + 24\sqrt{b} - 72$

$= 144 - 2b + 24\sqrt{b}$ – – – – – (2) – – – – – M1 for simplifying

Since (1) = (2), – – – – – M1 for comparing

$2a - 3b = 216 - 2\left(\sqrt{b} - 6\right)^2$ – – – – – AG

Marker's Report:

1. **This question was generally poorly done.**
2. **Among candidates who attempted the question, some omitted relevant working, especially since this was a "show" question. As a result, they were not awarded full credit for this part of the question.**

(b) Hence state the least possible positive value of $\dfrac{1}{2a - 3b}$. [1]

Maximum value of $2a - 3b = 216$

Least value of $\dfrac{1}{2a - 3b} = \dfrac{1}{216}$ – – – – – B1

Marker's Report:

1. **This part of the question was either not attempted or was generally poorly done.**
2. **Candidates should recognise that even without successfully completing part (a), they could still have answered part (b) correctly if they understood that the least possible value occurs when the denominator is at its maximum.**

3 **(a)** Express $\dfrac{1}{n(n+1)}$ in partial fractions. [3]

Let $\dfrac{1}{n(n+1)} = \dfrac{A}{n} + \dfrac{B}{n+1}$ ----- M1

$1 = A(n+1) + Bn$

When $n = 0, A = 1$ ----- A1

When $n = -1, B = -1$ ----- A1

$\dfrac{1}{n(n+1)} = \dfrac{1}{n} - \dfrac{1}{n+1}$ ----- minus 1 mark without final statement

Marker's Report:

1. **This part of the question was generally well done, although a number of candidates made careless mistakes when writing the final statement.**

(b) Hence show that $\dfrac{1}{3(4)} + \dfrac{1}{4(5)} + \dfrac{1}{5(6)} + ... + \dfrac{1}{(n-1)n} + \dfrac{1}{n(n+1)} = \dfrac{n-2}{3(n+1)}$. [3]

$\dfrac{1}{3(4)} + \dfrac{1}{4(5)} + \dfrac{1}{5(6)} + ... + \dfrac{1}{n(n-1)} + \dfrac{1}{n(n+1)}$

$= \dfrac{1}{3} - \dfrac{1}{4} + \dfrac{1}{4} - \dfrac{1}{5} + \dfrac{1}{5} - \dfrac{1}{6} + ... + \dfrac{1}{n-1} - \dfrac{1}{n} + \dfrac{1}{n} - \dfrac{1}{n+1}$ ----- M1 for writing specific terms

$= \dfrac{1}{3} - \dfrac{1}{n+1}$ ----- M1 for simplifying

$= \dfrac{n+1-3}{3(n+1)}$ ----- M1 for same denominator

$= \dfrac{n-2}{3(n+1)}$ (shown) ----- AG

Marker's Report:

1. **This part of the question was generally poorly done.**
2. **Candidates did not fully understand what was meant by "Hence show" and, as a result, did not make use of their answers from part (a) to proceed with this part of the question.**
3. **A handful of candidates incorrectly thought that this was a Binomial Theorem question.**

4 **(a)** Find the coefficient of x^3 in the binomial expansion of $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}$. [3]

$$T_{r+1} \text{ in } \left(\frac{1}{x}-\frac{x^2}{2}\right)^{12} = \binom{12}{r}\left(x^{-1}\right)^{12-r}\left(-\frac{x^2}{2}\right)^{r} \text{-----M1 for general term}$$

$$=\binom{12}{r}(x)^{-12+r}(-2)^{-r}(x)^{2r}$$

$$=\binom{12}{r}(-2)^{-r}x^{-12+3r}$$

$$-12+3r=3 \text{-----M1 for equating power of } x \text{ to } 3$$

$$r=5$$

$$\text{Coefficient of } x^3=\binom{12}{5}(-2)^{-5}=-24\frac{3}{4} \text{-----A1}$$

Marker's Report:

1. **This question was generally quite well done.**
2. **A number of candidates expanded the expression fully when they could have used the general term instead.**
3. **A number of candidates did not read the question carefully and therefore did not state the coefficient of the required term.**
4. **Candidates who did not arrive at the correct answer generally made algebraic slips or omitted the negative sign of the coefficient.**

(b) The coefficients of x and x^2 in the binomial expansion of $(3+2x)^n$ are equal. Find the value of *n*. [4]

$$(3+2x)^n=3^n+\binom{n}{1}3^{n-1}(2x)+\binom{n}{2}3^{n-2}(2x)^2+\ldots \text{-----M1 for general term}$$

$$\binom{n}{1}3^{n-1}(2)=\binom{n}{2}3^{n-2}(4) \text{-----M1 for equating}$$

$$n3^{n-1}(2)=\frac{n(n-1)}{2}3^{n-2}(4) \text{-----M1 for simplifying both sides}$$

$$\frac{3^n}{3}=\frac{3^n}{9}(n-1)$$

$$1=\frac{n-1}{3}$$

$$n=4 \text{-----A1}$$

Marker's Report:

1. **This part of the question was very poorly done.**
2. **Candidates who did not arrive at the correct answer were generally unable to simplify the expression when (n) was unknown.**

5 **(a)** The graph of $y = a + \log_b x$, where a and b are constants, passes through the points with coordinates $(9, 0)$ and $(243, 3)$.
Find the values of a and b, and hence express x in terms of y. [5]

$0 = a + \log_b 9$
$a = -\log_b 9$
$3 = a + \log_b 243 -----$ M1 for substituting coordinates for either pair
$\log_b 243 - \log_b 9 = 3 -----$ M1 for reducing to one unknown
$\log_b 27 = 3$
$b^3 = 27$
$b = 3$
$a = -2\log_b 3 = -2\log_3 3 = -2$
$b = 3, a = -2 -----$ A1 for both
$y = -2 + \log_3 x -----$ M1
$y + 2 = \log_3 x$
$x = 3^{y+2} -----$ A1

Alternative Method:
$0 = a + \log_b 9$
$a = -2\log_b 3 -----(1)$
$3 = a + \log_b 243$
$a = 3 - 5\log_b 3 -----(2) -----$ either (1) or (2) $-----$ M1
$-2\log_b 3 = 3 - 5\log_b 3 -----$ M1 for reducing to one unknown
$3\log_b 3 = 3$
$\log_b 3 = 1$
$b = 3, a = -2 -----$ A1
$y = -2 + \log_3 x -----$ M1
$y + 2 = \log_3 x$
$x = 3^{y+2} -----$ A1

Marker's Report:
1. Most candidates were able to substitute the 2 points into the equation to form 2 equations.
2. A number of candidates did not attempt the last part to express *x* in terms of *y*.

(b) One student was infected with HFMD in a school of 2900 students.
The spread of the disease through the school population was given by $N = \dfrac{2900}{1+2899e^{-0.65t}}$, where N was the total number of students infected after t days. The school would suspend lessons for a week when more than 40% of the students were affected.

Find, correct to the nearest whole number, the number of days after the first student was infected with HFMD that the school would suspend lessons. [4]

40% of $2900 = 1160$ ----- M1

$\dfrac{2900}{1+2899e^{-0.65t}} = 1160$ ----- M1

$1+2899e^{-0.65t} = \dfrac{5}{2}$

$e^{-0.65t} = \dfrac{3}{5798}$

$\ln e^{-0.65t} = \ln\left(\dfrac{3}{5798}\right)$

$-0.65t = \ln\left(\dfrac{3}{5798}\right)$ ----- M1

$t = \ln\left(\dfrac{3}{5798}\right) \div -0.65t$

$t = 11.641$

The number of days is estimated to be 12. ----- A1

Marker's Report:
1. This part of the question was generally well done.
2. Most students are aware that they have to round up the number of days to be 12.

6 **(a)** Prove that $\tan x+\dfrac{1}{\tan x}=\dfrac{1}{\sin x\cos x}$. [2]

$$\tan x+\frac{1}{\tan x}$$
$$=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}-----\text{M1}$$
$$=\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}-----\text{M1}$$
$$=\frac{1}{\sin x\cos x}-----\text{AG}$$

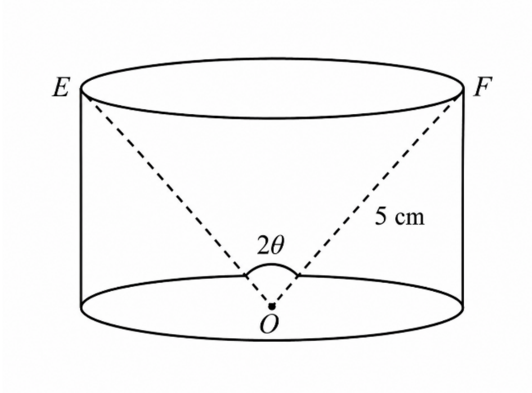
Marker's Report:
This part of the question was generally well done.

(b) Hence solve $\dfrac{2}{\sin x\cos x}=1+3\tan x$ for $0\le x\le\pi$. [4]

$$2\tan x+\frac{2}{\tan x}=1+3\tan x-----\text{M1}$$
$$-\tan x+\frac{2}{\tan x}=1$$
$$-\tan^2 x-\tan x+2=0$$
$$\tan^2 x+\tan x-2=0$$
$$(\tan x+2)(\tan x-1)=0-----\text{M1}$$
$$\tan x=-2 \text{ or } \tan x=1$$
$$\text{Basic angle}=1.1071 \quad \text{or } x=\frac{\pi}{4}-----\text{A1}$$
$$x=\pi-1.1071$$
$$=2.03-----\text{A1}$$

Marker's Report:
1. Common mistake: A handful of students stated that $x=\frac{\pi}{4}$ is the reference angle.
2. Remind students not to give the answer in degrees first then convert to radians.

7



The diagram shows an open cylinder tank with O as the centre of the base and $0° < \theta < 90°$. It is given that angle $EOF = 2\theta$ and OF = 5 cm.

(a) Prove that the total surface area of the cylinder is
$S = \dfrac{25\pi}{2}\left(2\sin 2\theta - \cos 2\theta + 1\right)$. [4]

Method 1

Let r be the radius of the cylinder and h be the height of the cylinder.

$\sin\theta = \dfrac{r}{5}$ ----- (1) ----- M1

$r = 5\sin\theta$

$\cos\theta = \dfrac{h}{5}$ ----- (2) or (1) ----- M1

$h = 5\cos\theta$

$S = \pi\left(5\sin\theta\right)^2 + 2\pi\left(5\sin\theta\right)\left(5\cos\theta\right)$ ----- M1

$= 25\pi\sin^2\theta + 50\pi\sin\theta\cos\theta$

$= 25\pi\sin^2\theta + 25\pi\sin 2\theta$

$= 25\pi\left(\dfrac{1-\cos 2\theta}{2}\right) + 25\pi\sin 2\theta$ ----- M1

$= \dfrac{25\pi}{2}\left(2\sin 2\theta - \cos 2\theta + 1\right)$ ----- AG

Method 2

$$EF^2 = 5^2 + 5^2 - 2(5)(5)cos2\theta$$
$$= 50 - 50\cos 2\theta$$
$$EF = \sqrt{50 - 50\cos 2\theta}$$
$$r = \frac{1}{2}\sqrt{50 - 50\cos 2\theta} \qquad \text{M1}$$
$$h^2 = 5^2 - \left(\frac{1}{2}\sqrt{50 - 50\cos 2\theta}\right)^2$$
$$= \frac{1}{4}\sqrt{50 + 50\cos 2\theta}$$
$$h = \frac{1}{2}\sqrt{50 + 50\cos 2\theta} \qquad \text{M1}$$
$$A = \frac{\pi}{4}\left(\sqrt{50 - 50\cos 2\theta}\right)^2 + 2\pi\frac{1}{2}\sqrt{50 - 50\cos 2\theta}\,\frac{1}{2}\sqrt{50 + 50\cos 2\theta} \qquad \text{M1}$$
$$= \frac{50}{4}(1 - \cos 2\theta) + \frac{\pi\sqrt{50}}{2}\sqrt{1 - \cos 2\theta}\sqrt{50}\sqrt{1 + \cos 2\theta}$$
$$= \frac{25\pi}{2}(1 - \cos 2\theta) + 25\pi\sqrt{\left(1 - \cos^2 2\theta\right)}$$
$$= \frac{25\pi}{2}(1 - \cos 2\theta) + 25\pi\sqrt{\sin^2 2\theta}$$
$$= \frac{25\pi}{2}(1 - \cos 2\theta) + 25\pi\sin 2\theta$$
$$= \frac{25\pi}{2}(2\sin 2\theta - \cos 2\theta + 1) \qquad \text{AG1}$$

Marker's Report:
1. This part of the question was quite badly done.
2. Some students did not realise that its an open cylinder.
3. A handful of students used cosine rule to find *EF* and the working becomes very tedious as shown in the alternative solution.

(b) Express S in the form $\frac{25\pi}{2}\left(R\sin\left(2\theta-\alpha\right)+1\right)$, where $R > 0$ and α is an acute angle. [3]

$$\text{Let } R\sin\left(2\theta-\alpha\right)=2\sin 2\theta-\cos 2\theta$$
$$R=\sqrt{2^2+1^2}=\sqrt{5}-----\text{M1}$$
$$\tan\alpha=\frac{1}{2}$$
$$\alpha=\tan^{-1}\frac{1}{2}=26.565°-----\text{M1}$$
$$S=\frac{25\pi}{2}\left(\sqrt{5}\sin\left(2\theta-26.6°\right)+1\right)-----\text{A1}$$

Marker's Report:
1. Some students did not read the question and gave the angle in radians which is wrong. A handful of students either did not give the angle in 1 decimal place or did not indicate the units.

(c) Hence find the maximum total surface area of the cylindrical tank and the corresponding value of θ. [2]

$$\text{Maximum } S=\frac{25\pi}{2}\left(\sqrt{5}+1\right)=127\ (3\text{ s.f.})-----\text{B1}$$
$$2\theta-26.565°=90°$$
$$\theta=58.3°\ (1\text{d.p.})-----\text{B1}$$

8 The chickens in a farm are infected by a certain bird flu. The number of chickens (in thousands) in the farm is modelled by $N=\dfrac{27}{2+\alpha t e^{\beta t}}$, where t is the number of days elapsed since the start of the spread of the bird flu and α and β are constants.

(a) Express $\ln\left(\dfrac{27-2N}{Nt}\right)$ as a linear function of t in terms of α and β. [4]

$$N=\frac{27}{2+\alpha t e^{\beta t}}$$
$$2N+N\alpha t e^{\beta t}=27\text{-----M1}$$
$$27-2N=N\alpha t e^{\beta t}$$
$$\frac{27-2N}{Nt}=\alpha e^{\beta t}\text{-----M1}$$
$$\ln\left(\frac{27-2N}{Nt}\right)=\ln\alpha e^{\beta t}\text{-----M1}$$
$$=\ln\alpha+\beta t\text{-----A1}$$

Marker's Report:
1. This question is quite badly done.
2. A number of students did not attempt the question.

(b) It is given that the slope and the intercept on the horizontal axis of the graph of the linear function obtained in **(a)** are -0.1 and $10\ln 0.03$ respectively.

Find α and β. [3]

$\beta = -0.1 ----- \text{B1}$

$0 = -0.1(10\ln 0.3) + \ln\alpha ----- \text{M1}$

$\ln\alpha = \ln 0.03$

$\alpha = 0.03 ----- \text{A1}$

9 The points A and B have coordinates $(5,7)$ and $(13,1)$ respectively. A point P moves such that $PA = PB$. The set of all possible positions of P satisfying $PA = PB$ forms a straight line, L.

(a) Show that the equation of L is given by $y = \frac{4}{3}x - 8$. [3]

Since $PA = PB, P$ lies on the perpendicular bisector of AB.

Midpoint of $AB = \left(\frac{5+13}{2}, \frac{7+1}{2}\right) = (9,4)$ ----- M1

Gradient of $AB = \frac{1-7}{13-5} = -\frac{3}{4}$ ----- M1

Equation of L:

$y - 4 = \frac{4}{3}(x-9)$ ----- M1

$y = \frac{4}{3}x - 8$ ----- AG

Alternative Method:

Let $P(x,y)$

$\sqrt{(x-5)^2+(y-7)^2} = \sqrt{(x-13)^2+(y-1)^2}$ ----- M1

$(x-5)^2+(y-7)^2 = (x-13)^2+(y-1)^2$

$x^2-10x+25+y^2-14y+49 = x^2-26x+169+y^2-2y+1$ ----- M1

$-10x-14y+74 = -26x-2y+170$

$-16x+12y+96 = 0$

$-4x+3y+24 = 0$

$3y = 4x-24$ ----- M1

$y = \frac{4}{3}x - 8$ ----- AG1

Marker's Report:
1. This question is generally well done. Most students are able to do this question.

(b) L intersects the x-axis at H and the y-axis at K.
A circle C passes through the three points O, H and K, where O is the origin.
Linabell claims that the circumference of C is greater than 30 units.
Determine if Linabell is correct. [4]

$H(6,0)$ ------ B1

$K(0,-8)$ ------ B1

Diameter of $C = HK$

$=\sqrt{(6-0)^2+(0-(-8))^2}$ ------ M1

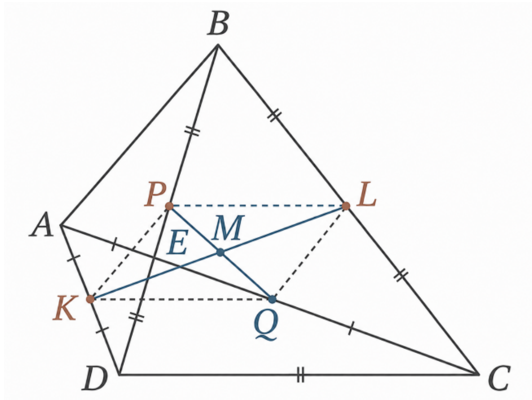
$=10$

Circumference of $C = \pi d = 10\pi = 31.416 > 30$

Hence, Linabell is correct. ------ A1 with working

Marker's Report:
1. This question is generally well done.
2. Students must indicate that the circumference is more than 30 with conclusion in order to get the final mark.

10



In the diagram, AB, BC, CD, DA, AC, and DB are walkways connecting four blocks of apartments. An urban planner plans to build some facilities at K, L, P and Q, which are midpoints of AD, BC, BD and AC respectively.

Show that

(a) $PK = LQ$, [3]

In ΔADB,

$AK = KD$ and $BP = PD$ ----- M1

By Midpoint Theorem, $KP = \frac{1}{2}AB$ and $KP // AB$. ----- M1

In ΔACB,

$AQ = QC$

$BL = LC$

By Midpoint Theorem, $QL = \frac{1}{2}AB$ and $QL // AB$. ----- M1

$KP = QL$ (proven) ----- AG

Marker's Report:
1. This question is quite badly done as a number of students did not attempt the question.
2. Students need to know the 2 results of Midpoint Theorem.

(b) KL bisects PQ. [4]

From (a), $KP = QL$

$KP // AB$ and $QL // AB$. Hence, $KP // QL$ $-----$ M1

Hence, $KPLQ$ is a parallelogram $-----$ M1

(a pair of opp. sides equal and parallel) $-----$ M1

Hence, KL bisects PQ. (diagonal bisects in parallelogram) $-----$ A1 with property

Marker's Report:

1. This question is quite badly done.

2. A number of students did not attempt the question.

3. A lot of students did not explain why *KPLQ* is a parallelogram.

11 A curve, C, has equation $y=(2x+8)^{\frac{3}{2}}+3x^2$, where $x>-4$.

(a) Find $\frac{dy}{dx}$. [1]

$$\frac{dy}{dx}=3(2x+8)^{\frac{1}{2}}+6x-----\text{B1}$$

(b) Duffy claims that two of the tangents to C are parallel to the straight line $6x+y+4=0$. Determine if Duffy is correct. Show your working clearly. [6]

$6x+y+4=0$

$y=-6x-4$

$\frac{dy}{dx}=-6-----\text{M1}$

$3\sqrt{2x+8}+6x=-6$

$\sqrt{2x+8}+2x=-2$

$\sqrt{2x+8}=-2-2x, x\leq-1-----\text{M1}$

$2x+8=4x^2+8x+4-----\text{M1}$

$2x^2+3x-2=0$

$(2x-1)(x+2)=0-----\text{M1}$

$x=\frac{1}{2}$ or $x=-2$

When $x=\frac{1}{2}$, $3\sqrt{2x+8}+6x=6\neq-6$.

When $x=-2$, $3\sqrt{2x+8}+6x=-6$.

Hence, reject $x=\frac{1}{2}$ $-----$ A1 with rejection

Hence, Duffy is incorrect as there is only one solution. $-----$ A1

Marker's Report:
1. (a) As this part is a proving question, students need to show how they differentiate to get the answer.
2. (b) A lot of students simply concluded that Duffy is correct as it is a quadratic equation with is incorrect.

12 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^3 2x\right) = 6\sin^2 2x\cos 2x$. [1]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^3 2x\right) = 3\sin^2 2x\cos 2x(2) \text{-----M1}$$
$$= 6\sin^2 2x\cos 2x \text{-----AG}$$

(b) Hence find $\int \cos^3 2x \, \mathrm{d}x$. [5]

$$\int 6\sin^2 2x\cos 2x \, \mathrm{d}x = \sin^3 2x + c_1 \text{-----M1}$$
$$\int 6\left(1-\cos^2 2x\right)\cos 2x \, \mathrm{d}x = \sin^3 2x + c_1 \text{-----M1}$$
$$\int 6\cos 2x - 6\cos^2 2x\cos 2x \, \mathrm{d}x = \sin^3 2x + c_1$$
$$\int 6\cos 2x - 6\cos^3 2x \, \mathrm{d}x = \sin^3 2x + c_1$$
$$\int 6\cos^3 2x \, \mathrm{d}x = \int 6\cos 2x \, \mathrm{d}x - \sin^3 2x + c_1$$
$$\int \cos^3 2x \, \mathrm{d}x = \frac{1}{6}\left[\int 6\cos 2x \, \mathrm{d}x - \sin^3 2x + c_1\right] \text{-----M1}$$
$$= \frac{1}{6}\left(\frac{6\sin 2x}{2}\right) - \frac{\sin^3 2x}{6} + c_2 \text{-----M1}$$
$$= \frac{1}{2}\sin 2x - \frac{1}{6}\sin^3 2x + c \text{------A1}$$

Marker's Report:
1. (a) As this part is a proving question, students need to show how they differentiate to get the answer.
2. (b) This part is badly done. Students are reminded to add d*x* or +*c* whenever they have an integral sign or have performed indefinite integration.

13 A particle is travelling in a straight line. It passes a fixed point A with a velocity of -7 m/s. Its acceleration, after passing A is given by $a = 8 - 2t$ for $0 \le t \le T$.
On reaching its maximum speed at time T seconds, the particle begins to decelerate at a constant rate of 1.5 m/s^2 until it comes to a rest at point B.

(a) Show that the particle first came to instantaneous rest when $t = 1$. [3]

$a = 8 - 2t$

$v = \int 8 - 2t \, \mathrm{d}t - - - - - \text{M1}$

$= 8t - t^2 + c$

When $v = -7, t = 0, c = -7$

$v = 8t - t^2 - 7$

$t^2 - 8t + 7 = 0$

$(t-1)(t-7) = 0 - - - - - \text{M1}$

$t = 1$ or $t = 7$.

Hence, the particle first came to instantaneous rest at $t = 1$. (shown) $- - - - -$ A1

(b) Find the maximum velocity attained by the particle and the value of T. [2]

Maximum velocity attained when $a = 0 - - - - - \text{M1}$

$T = 4$

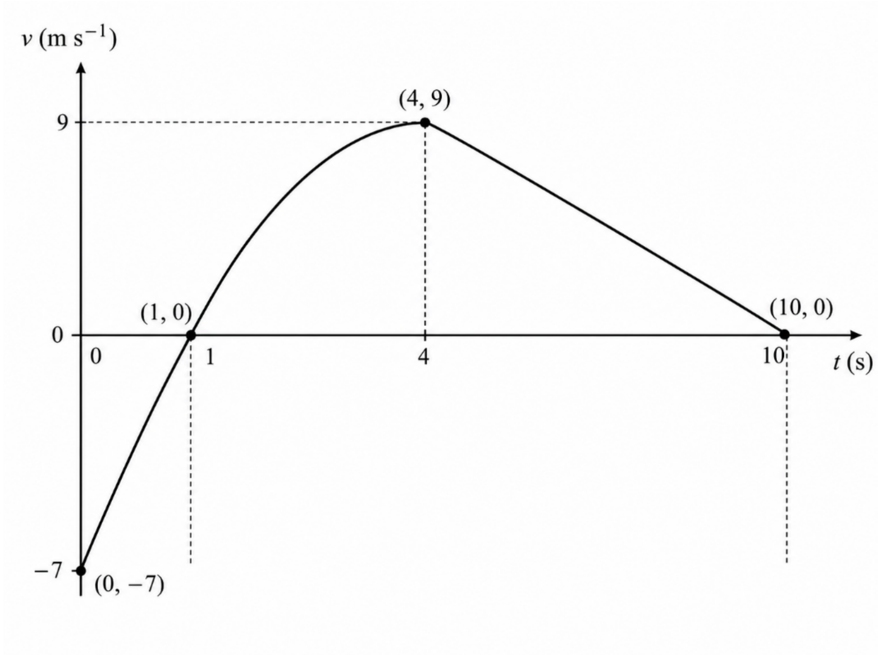
$v = 32 - 16 - 7$

$= 9$ m/s $- - - - -$ A1

Marker's Report:
1. (a) The common mistake is that students substitute *t* = 1 and showed that *v* = 0. This is incorrect as it may not be the first time the article comes to instantaneous rest.
2. (b) A number of students did not mention *T* = 4.

(c) Find the total distance travelled by the particle from A to B. [4]

Method 1



Total distance travelled

$$= -\int_0^1 8t - t^2 - 7 \text{ d}t + \int_1^4 8t - t^2 - 7 \text{ d}t + \left(\frac{1}{2} \times 6 \times 9\right) ------ \text{M1}$$

$$= -\left[\frac{8t^2}{2} - \frac{t^3}{3} - 7t\right]_0^1 + \left[\frac{8t^2}{2} - \frac{t^3}{3} - 7t\right]_1^4 + 27 ------ \text{M1,B1 for 27}$$

$$= \frac{10}{3} + 18 + 27$$

$$= 48\frac{1}{3}\text{m} ------ \text{A1}$$

Method 2

$$s=\int(8t-t^2-7)\ \mathrm{d}t$$

$$=4t^2-\frac{t^3}{3}-7t+c$$

When $t=0,\ s=0,\quad \therefore c=0$

$$s=4t^2-\frac{t^3}{3}-7t+c$$

When $t=1,\ s=-\dfrac{10}{3}$

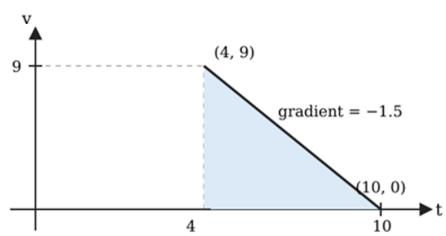
When $t=4,\ s=\dfrac{44}{3}$

Thus, from $t=0$ to $t=1$, the particle travels a distance of $\frac{10}{3}$ m. From $t=1$ to $t=4$, it travels from $s=-\frac{10}{3}$ to $s=\frac{44}{3}$.

At $t=4$,

$$v=8(4)-4^2-7=9$$

For $t\geq 4$, the velocity-time graph is a straight line with gradient -1.5.



Velocity-time graph

Let the velocity become zero at $t=T$.

$$\frac{9-0}{4-T}=-1.5$$

$$4-T=-6$$

$$T=10$$

Hence, the total distance travelled is

$$\frac{10}{3}+\left(\frac{10}{3}+\frac{44}{3}\right)+\frac{1}{2}(6)(9)$$

$$=\frac{145}{3}$$

Final answer:

$$\text{Total distance}=\mathbf{48\frac{1}{3}}\ \text{m}$$

Marker's Report:
1. This question is badly done.

2. Common mistakes made by students: First, students continue to use $s = 4t^2 - \frac{1}{3}t^3 - 7t$ after $t = 4$. Students also fail to recognise that from $t = 4$, the acceleration changes to the constant value -1.5 m/s^2. Hence, the velocity–time graph from $t = 4$ is a straight line.

END OF PAPER