H3 Electromagnetism & Electric Field

1 **(a)** Two long straight parallel wires are a distance 2*a* apart in a vacuum. One carries a current *I* into the plane of the paper, and the other a current of the same magnitude but out of the plane of the paper as shown in Fig. 1.1.

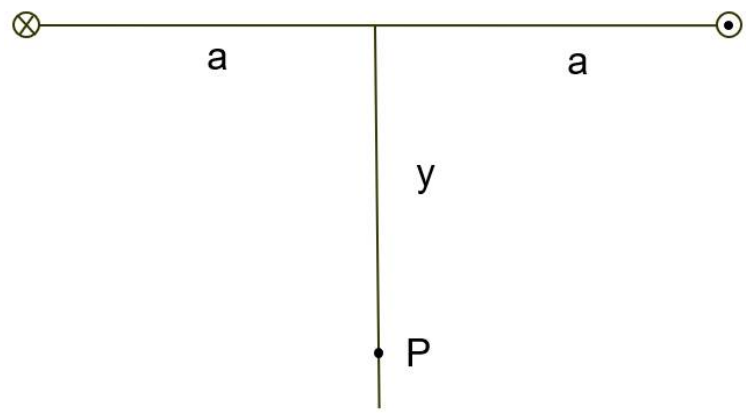

Fig. 1.1

P is a point equidistant from the two wires, a distance *y* form the line joining the wires.

(i) Derive an expression for the magnitude *B* of the flux density at P, in terms of μ_0, *I*, *a*, *y* and state the direction of this flux density.

[5]

(ii) Sketch a graph to show how *B* depends on *y*. State the value of *y* where *B* a maximum.

[2]

(b) The flux density due to a long, straight wire is given by the expression

$$B = \frac{\mu_0 I}{2\pi d}$$

Figure 1.2 shows a long straight vertical wire WX, carrying a current of 9.0 A downwards. A second long straight wire YZ is placed horizontally, and carries a current of 12.0 A in the direction shown. ABCD is a horizontal, rectangular table-top: the wire YZ is parallel to the side BC of this table, and the wire WX passes through a small hole in the table. The perpendicular distance between the wires is 100 mm. P is the point 50 mm from YZ along the perpendicular between the wires.

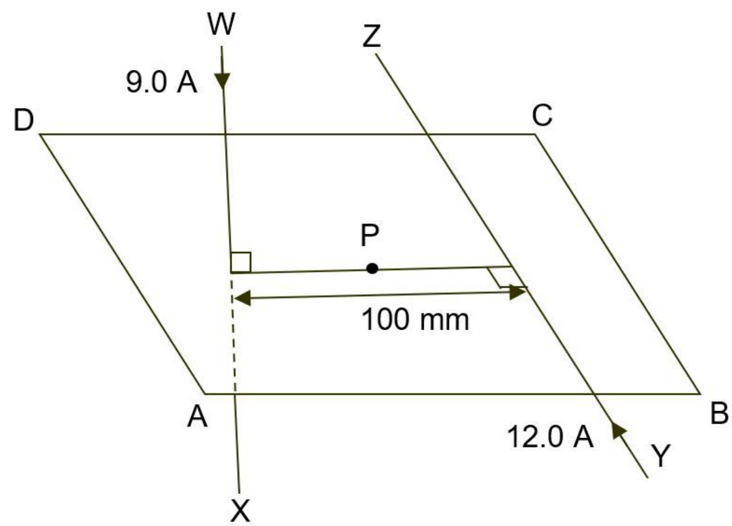

Fig. 1.2

(i) Calculate the magnitude and direction of the magnetic flux density at the point P. (Relate the direction of the flux density to the sides AB and BC of the table-top, as appropriate.)

magnetic flux density at P = …………………………………… [6]

(ii) The wire YZ is to remain fixed in position, but the orientation and position of the wire WX can be changed. The currents in the two wires remain at the same values as before, and the point P remains 50 mm away from YZ in the place ABCD. How should WX be arranged so as to produce *zero* magnetic flux density at P? (Again relate the required position of WX to the table-top ABCD.)

2 A *bubble chamber* is a device for tracing the paths followed by charged particles. The particle passes through a tank of liquid hydrogen, leaving a trail of tiny bubbles, which can be photographed. A magnetic field is applied across the chamber, and the speed of the particle can be determined from the radius of curvature of its track.

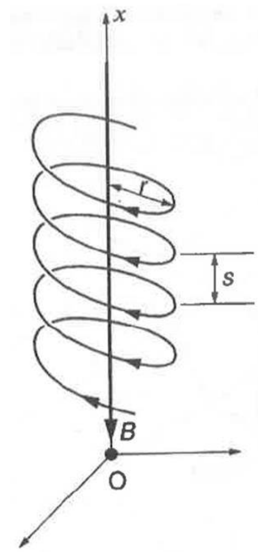

Fig. 2.1

(a) Fig 2.1 shows a track in a bubble chamber caused by a particle of mass m and charge of magnitude q. The shape of this track is called a *helix:* it is like the coils of a spring. The radius of the helix is r, and the distance between corresponding points on adjacent turns is s. The axis of the helix is parallel to the Ox axis, and the uniform magnetic field, of flux density B, is in the negative Ox direction. When viewed in the direction of the field, the motion of the particle is clockwise.

(i) State and explain the sign of the charge of the particle.

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(ii) Determine, in terms of r and s, the angle θ with the Ox axis at which the particle was moving when it entered the magnetic field.

(iii) Obtain an expression for the speed of v of the particle in terms of B, r, s and q/m.

Electric Field

3 **(a)** An isolated solid metal sphere of radius *R* is given a total charge +*Q* by repeatedly bringing up to it small quantities δq of charge. The final charge is found to exist entirely on the surface of the sphere.

(i) Explain why the charge on the sphere is entirely on its surface.

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……………………………………………………………………………………………… [2]

(ii) For points on the surface of, or outside, a sphere carrying a charge +*q*, the electric field and potential are exactly the same as if the sphere were a point charge +*q* situated at the centre of the sphere. Show that the electric potential energy E_p of the sphere when it carries the total charge +*Q* is given by

$$E_P = \frac{Q^2}{8\pi\varepsilon_0 R}.$$

(hint: consider the electric PE between the existing charge q, and the additional small charge *dq*, then sum up over all the charges added)

[4]

(b) Another sphere of radius R is made by bringing together a very large number of tiny pieces of insulating material. Each piece is positively charged. The charge density (the charge per unit volume) of the material is such that the overall charge on the completed sphere is $+Q$. Because the material is a perfect insulator, the sphere remains uniformely charged throughout its volume. Show that the electric potential energy E_S of the fully-assembled sphere is given by

$$E_S = \frac{3Q^2}{20\pi\varepsilon_0 R}.$$

(hint: similar to (b)(ii), but express q in terms of r because the electric PE varies with r, not just with q this time; use integration by substitution)

(c) Suppose that the material used to assemble the sphere in **(b)** was not perfectly insulating, so that in time all the charge migrated to the surface of the sphere.

(i) Find an expression for the loss of electric potential energy of the sphee as a result of the migration of charge.

[1]

(ii) Suggest what might have happened to this amount of energy.

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…………………………………………………………………………………………………… [1]

<u>Answer Key</u>

1 (b)(i) $B = 6.0 \times 10^{-5}$ T ; θ = 53.1°

H3 Electromagnetism & Electric Field

1 **(a)** Two long straight parallel wires are a distance 2*a* apart in a vacuum. One carries a current *I* into the plane of the paper, and the other a current of the same magnitude but out of the plane of the paper as shown in Fig. 1.1.

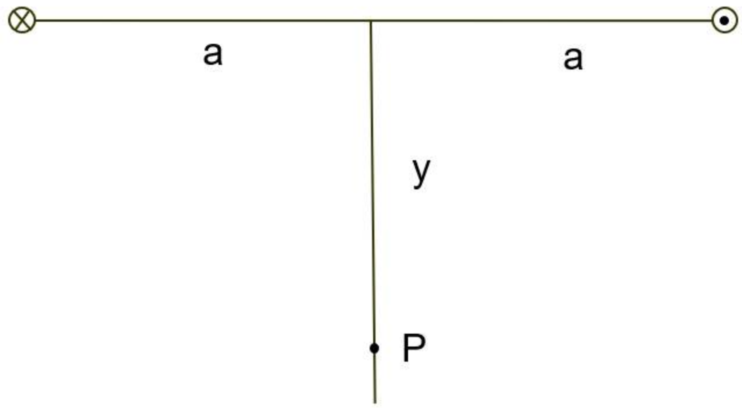


Fig. 1.1

P is a point equidistant from the two wires, a distance *y* form the line joining the wires.

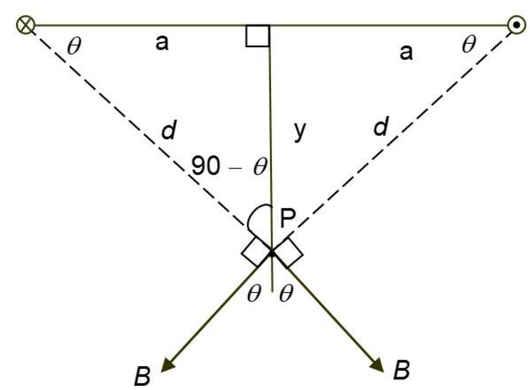
(i) Derive an expression for the magnitude *B* of the flux density at P, in terms of μ_0, *I*, *a*, *y* and state the direction of this flux density.

Applying Pythagoras' Theorem,
distance from each wire to P is

$d = \sqrt{a^2 + y^2}$ [B1]

magnetic flux density *B* due to each wire at P:

$B = \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I}{2\pi\sqrt{a^2 + y^2}}$ [B1]



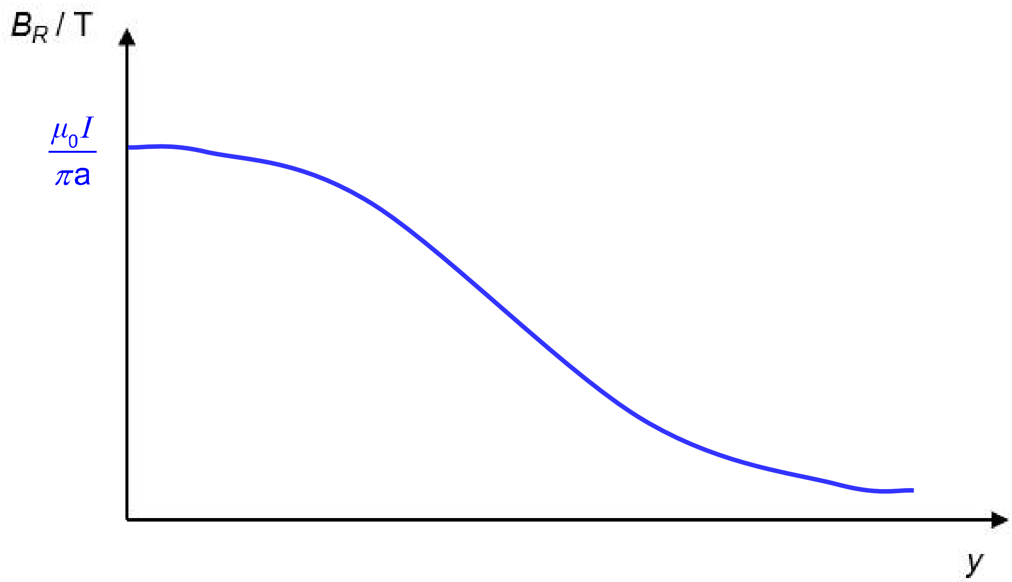
Since the horizontal components of the 2 magnetic fields are equal and opposite in direction, they cancel each other.
The vertical components are equal in magnitude and in the same direction, thus reinforcing each other.
The resultant magnetic field is thus

$B_R = 2 \times B\cos\theta = 2\left(\frac{\mu_0 I}{2\pi\sqrt{a^2 + y^2}}\right)\left(\frac{a}{\sqrt{a^2 + y^2}}\right)$ [B1]

$B_R = \frac{\mu_0 I a}{\pi\left(a^2 + y^2\right)}$ [A1]

B_R is acting vertically downwards. [B1]

(ii) Sketch a graph to show how *B* depends on *y*. State the value of *y* where *B* a maximum.



[2]

To find maximum B, differentiate with respect to y

$$B_R = \frac{\mu_0 I a}{\pi\left(a^2+y^2\right)}$$

$$\frac{dB_R}{dy} = \frac{-2y\mu_0 I a}{\pi\left(a^2+y^2\right)^2}$$

$$\frac{dB_R}{dy} = 0$$

$$y = 0$$

Hence, maximum B occurs when $y = 0$,where $B = \frac{\mu_0 I}{\pi a}$.

(b) The flux density due to a long, straight wire is given by the expression

$$B = \frac{\mu_0 I}{2\pi d}$$

Figure 1.2 shows a long straight vertical wire WX, carrying a current of 9.0 A downwards. A second long straight wire YZ is placed horizontally, and carries a current of 12.0 A in the direction shown. ABCD is a horizontal, rectangular table-top: the wire YZ is parallel to the side BC of this table, and the wire WX passes through a small hole in the table. The perpendicular distance between the wires is 100 mm. P is the point 50 mm from YZ along the perpendicular between the wires.

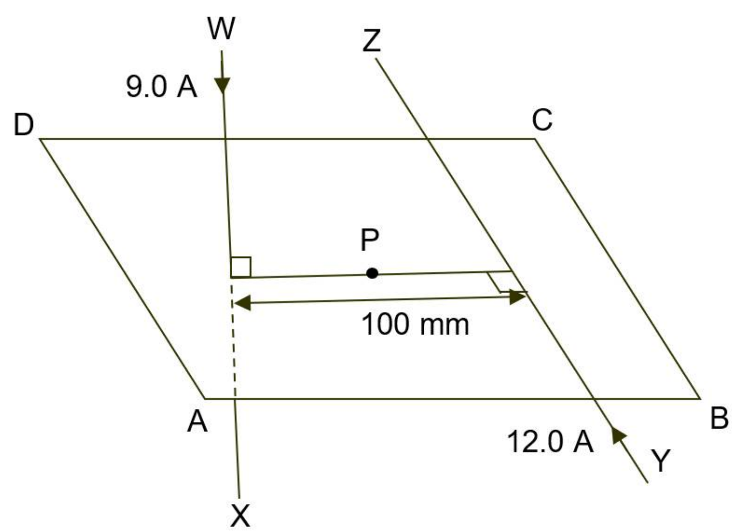


Fig. 1.2

(i) Calculate the magnitude and direction of the magnetic flux density at the point P. (Relate the direction of the flux density to the sides AB and BC of the table-top,

Magnetic flux density at P due to current in wire WX is:

$$B_{WX} = \frac{\mu_0 I}{2\pi d} = \frac{4\pi \times 10^{-7} \times 9.0}{2\pi \times 50 \times 10^{-3}} = 3.6 \times 10^{-5} \text{ T}$$ [B1]

Using right hand grip rule, the direction is from C to B. [B1]

Magnetic flux density at P due to current in wire YZ is:

$$B_{YZ} = \frac{\mu_0 I}{2\pi d} = \frac{4\pi \times 10^{-7} \times 12.0}{2\pi \times 50 \times 10^{-3}} = 4.8 \times 10^{-5} \text{ T}$$ [B1]

Using right hand grip rule, the direction vertically upwards. [B1]

Resultant magnetic flux density, $B_R = \sqrt{\left(3.6 \times 10^{-5}\right)^2 + \left(4.8 \times 10^{-5}\right)^2}$

$= 6.0 \times 10^{-5}$ T [B1]

Let θ be the angle that B_R makes with the plane ABCD.

$$\tan\theta = \frac{4.8 \times 10^{-5}}{3.6 \times 10^{-5}} = 1.333$$

$\theta = \tan^{-1}(1.333) = 53.1°$ [B1]

The direction of the magnetic flux density is 53.1° above the plane of the table, and in the vertical plane parallel to the side BC, pointing towards the line AB

(ii) The wire YZ is to remain fixed in position, but the orientation and position of the wire WX can be changed. The currents in the two wires remain at the same values as before, and the point P remains 50 mm away from YZ in the place ABCD. How should WX be arranged so as to produce *zero* magnetic flux density at P? (Again relate the required position of WX to the table-top ABCD.)

In order to cancel the upward magnetic flux density of 4.8×10^{-5} T due to the current in wire YZ at point P, the current in wire WX must flow in the same direction as YZ [B1] (and on the same plane as the table top ABCD), to the left of P. [B1]

The perpendicular distance from wire WX to point P can be found from:

$$B=\frac{\mu_0 I}{2\pi d}$$

$$d=\frac{\mu_0 I}{2\pi B}=\frac{4\pi\times10^{-7}\times9.0}{2\pi\times4.8\times10^{-5}} \quad \text{[M1]}$$

$$=0.0375\text{ m} \quad \text{[A1]}$$

2 A *bubble chamber* is a device for tracing the paths followed by charged particles. The particle passes through a tank of liquid hydrogen, leaving a trail of tiny bubbles, which can be photographed. A magnetic field is applied across the chamber, and the speed of the particle can be determined from the radius of curvature of its track.

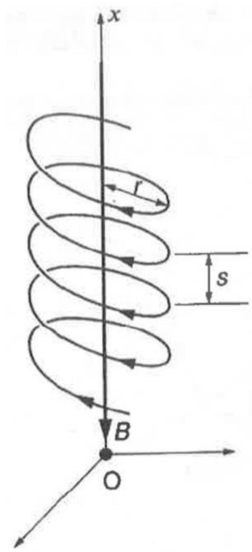


Fig. 2.1

(a) Fig 2.1 shows a track in a bubble chamber caused by a particle of mass m and charge of magnitude q. The shape of this track is called a *helix:* it is like the coils of a spring. The radius of the helix is r, and the distance between corresponding points on adjacent turns is s. The axis of the helix is parallel to the Ox axis, and the uniform magnetic field, of flux density B, is in the negative Ox direction. When viewed in the direction of the field, the motion of the particle is clockwise.

(i) State and explain the sign of the charge of the particle.

By applying Fleming's Left Hand Rule, where the centripetal force provided by the magnetic force points towards the axis Ox, the direction of the conventional current is opposite to the velocity of the charged particle. [B1]

Hence, the particle has a negative charge. [B1]

.... [2]

(ii) Determine, in terms of *r* and *s*, the angle θ with the O*x* axis at which the particle was moving when it entered the magnetic field.

The velocity component that is perpendicular to the magnetic field produces the circular motion. [B1]

$$v_{\perp} = r\omega$$

$$v\sin\theta = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v\sin\theta} \text{ -------- eqn 1 [B1]}$$

The velocity component that is parallel to the magnetic field pulls the particle forward by *s* every cycle. [B1]

$$s = v_{\parallel} T \frac{x-\mu}{\sigma}$$

$$s = (v\cos\theta)T \text{ -------- eqn 2 [B1]}$$

Sub eqn 1 into eqn 2:

$$s = v\cos\theta \frac{2\pi r}{v\sin\theta} = \frac{2\pi r\cos\theta}{\sin\theta} \quad \text{[B1]}$$

Rearranging, we get:

$$\tan\theta = \frac{2\pi r}{s}$$

$$\theta = \tan^{-1}\left(\frac{2\pi r}{s}\right) \text{ [A1]}$$

(iii) Obtain an expression for the speed of *v* of the particle in terms of *B*, *r*, *s* and *q/m*.

Centripetal force is provided by magnetic force. By Newton's Second Law,

$$F_B = F_C$$

$$Bqv_{\perp} = \frac{mv_{\perp}^{2}}{r} \quad \text{[B1]}$$

$$Bq(v\sin\theta) = \frac{m(v\sin\theta)^2}{r}$$

$$v = \frac{Bqr}{m\sin\theta} \text{ -------- eqn 3 [B1]}$$

Since $\tan\theta = \frac{2\pi r}{s}$,

$$\sin\theta = \frac{2\pi r}{\sqrt{(2\pi r)^2 + s^2}}$$

$$\sin\theta = \frac{2\pi r}{\sqrt{4\pi^2 r^2 + s^2}} \text{ -------- eqn 4 [B1]}$$

Sub eqn 4 into eqn 3:

$$v = \frac{Bqr}{m} \cdot \frac{\sqrt{4\pi^2 r^2 + s^2}}{2\pi r}$$

$$= \frac{Bq\sqrt{4\pi^2 r^2 + s^2}}{2m\pi} \quad \text{[B1]}$$

Electric Field

3 (a) An isolated solid metal sphere of radius R is given a total charge $+Q$ by repeatedly bringing up to it small quantities δq of charge. The final charge is found to exist entirely on the surface of the sphere.

(i) Explain why the charge on the sphere is entirely on its surface.

The like charges will repel one another until they are furthest from one another [B1] in order to achieve the lowest possible energy state [B1].

Hence, the charges only reside on the surface of the sphere.

.... [2]

(ii) For points on the surface of, or outside, a sphere carrying a charge $+q$, the electric field and potential are exactly the same as if the sphere were a point charge $+q$ situated at the centre of the sphere. Show that the electric potential energy E_p of the sphere when it carries the total charge $+Q$ is given by

$$E_P = \frac{Q^2}{8\pi\varepsilon_0 R}.$$

(hint: consider the electric PE between the existing charge q, and the additional small charge dq, then sum up over all the charges added)

When a small charge δq is deposited on the surface of the sphere of radius R with an initial charge q, the EPE of the sphere is = $dE_P = \frac{q}{4\pi\varepsilon_0 R}dq$ [B1]

When the total charge on the surface of the sphere builds up to Q, the total EPE stored by the sphere is

$$E_P = \int_0^{E_P} dE_P = \int_0^Q \frac{q}{4\pi\varepsilon_0 R}dq = \frac{1}{4\pi\varepsilon_0 R}\int_0^Q q\,dq = \frac{1}{4\pi\varepsilon_0 R}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{8\pi\varepsilon_0 R}$$. [B3]

[4]

(b) Another sphere of radius R is made by bringing together a very large number of tiny pieces of insulating material. Each piece is positively charged. The charge density (the charge per unit volume) of the material is such that the overall charge on the completed sphere is $+Q$. Because the material is a perfect insulator, the sphere remains uniformely charged throughout its volume. Show that the electric potential energy E_S of the fully-assembled sphere is given by

$$E_S = \frac{3Q^2}{20\pi\varepsilon_0 R}.$$

(hint: similar to (b)(ii), but express q in terms of r because the electric PE varies with r, not just with q this time; use integration by substitution)

When a small charge δq is deposited on the surface of the sphere of radius r with an initial charge q, the change in EPE of the sphere is = $\delta E_P = \frac{q}{4\pi\varepsilon_0 r}\delta q$ [B1]

Assuming that the sphere has a uniform charge density ρ,

$$q = \rho\frac{4\pi}{3}r^3 \qquad \text{[B1]}$$

$$\text{Therefore, } \delta q = \rho\frac{4\pi}{3}\left(3r^2\,\delta r\right) = 4\pi\rho r^2\,\delta r \qquad \text{[B1]}$$

$$\text{Hence, } \delta E_P = \frac{1}{4\pi\varepsilon_0 r}q\delta q = \frac{1}{4\pi\varepsilon_0 r}\left(\rho\frac{4\pi}{3}r^3\right)\left(4\pi\rho r^2\,\delta r\right) = \frac{4\pi}{3\varepsilon_0}\rho^2 r^4\,\delta r\text{ . [B1]}$$

When finally the total charge on the solid insulating sphere is Q, its EPE is =

$$E_S = \int_0^{E_P} dE_P = \int_0^R \frac{4\pi}{3\varepsilon_0}\rho^2 r^4\,dr = \frac{4\pi}{3\varepsilon_0}\rho^2\frac{R^5}{5} = \frac{4\pi}{15\varepsilon_0}\left(\frac{Q}{\frac{4\pi}{3}R^3}\right)^2 R^5 = \frac{3Q^2}{20\pi\varepsilon_0 R} \text{ (shown)}$$

(c) Suppose that the material used to assemble the sphere in **(b)** was not perfectly insulating, so that in time all the charge migrated to the surface of the sphere.

(i) Find an expression for the loss of electric potential energy of the sphee as a result of the migration of charge.

$$\begin{aligned}\text{Loss in electric potential energy } &= E_i - E_f \\ &= \frac{3Q^2}{20\pi\varepsilon_0 R} - \frac{Q^2}{8\pi\varepsilon_0 R} \\ &= \left(\frac{3}{20} - \frac{1}{8}\right)\frac{Q^2}{\pi\varepsilon_0 R} \\ &= \frac{1}{40}\frac{Q^2}{\pi\varepsilon_0 R} \qquad \text{[B1]}\end{aligned}$$

[1]

(ii) Suggest what might have happened to this amount of energy.

When the charges move towards the surface of the sphere, this flow of charge is equivalent to an electric current. This current produces joule heating ($P = I^2R$) or ohmic losses, and the heat energy produced is eventually lost to the surroundings. [B1]

.... [1]

Answer Key

1 (b)(i) $B = 6.0 \times 10^{-5}$ T ; $\theta = 53.1°$