YISHUN JUNIOR COLLEGE
2015 JC2 PRELIMINARY EXAMINATION

MATHEMATICS **9740/01**
Higher 2

Paper 1 **17 AUGUST 2015**
MONDAY 0800h – 1100h

Additional materials :
Answer paper
Graph paper
List of Formulae (MF15)

TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and CTG in the spaces provided on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 A confectionery bakes 3 types of cupcakes; blueberry, strawberry and chocolate. Each type is prepared in trays of 12. The time taken to prepare a tray of cupcakes, the mass of flour used for each tray and the selling price of each cupcake are given in the following table.

Type of cupcake	Preparation Time (min) for each tray	Mass of flour (g) for each tray	Selling price of a cupcake
Blueberry	8	600	\$1.00
Strawberry	7	600	\$0.90
Chocolate	6	800	\$0.80

On a particular day, 17 hours of preparation time was spent and 96 kg of flour was used to make the cupcakes. All the cupcakes were sold and the total amount collected was \$1572. By forming a system of linear equations, determine the number of trays of each type of cupcakes made on that day. [4]

2 **(a)** Given that $\mathbf{a}\cdot\mathbf{b} = 0$, what can be deduced about the non-zero vectors $\mathbf{a}$ and $\mathbf{b}$? [1]

(b) If a non-zero vector $\mathbf{a}$ lies on the x-axis, find a unit vector $\mathbf{m}$ such that $\mathbf{m} \times \mathbf{a} = \mathbf{0}$. [1]

(c) Given a quadrilateral *ABCD* in which its diagonals bisect each other, prove that quadrilateral *ABCD* is a parallelogram. [2]

(d) Three points P, Q and R have position vectors $p\mathbf{x}$, $q\mathbf{y}$ and $r\mathbf{x} + s\mathbf{y}$ respectively, where $\mathbf{x}$ and $\mathbf{y}$ are non-parallel and non-zero vectors and p, q, r and s are scalars. If the points are collinear, prove that $ps + qr = pq$. [3]

3 A manufacturer constructs a cylindrical tub with a capacity of 1000 cm^3. The tub is open at the top and is made of material 1 cm in thickness. If the internal radius, the internal height and the amount of material used are r cm, h cm and V cm^3 respectively, show that

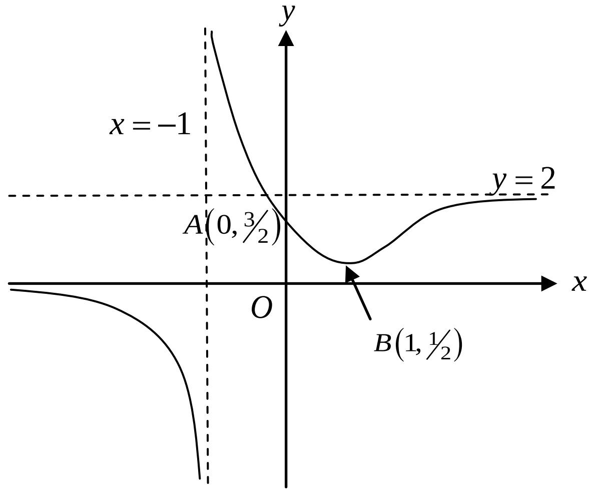
(i) $V = \pi(r+1)^2\left(\dfrac{1000}{\pi r^2} + 1\right) - 1000$, [2]

(ii) V is minimum when $r = h = \dfrac{10}{\sqrt[3]{\pi}}$ cm. [4]

4 Find the solution of the differential equation $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in the form $y = \mathrm{f}(x)$, given that $y = 0$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$ when $x = 0$. [4]

Sketch the solution curve, stating the equations of any asymptotes and the coordinates of any points of intersection with the axes. [2]

5



The diagram shows the graph of $y = \mathrm{f}(x)$. The curve crosses the y-axis at the point A and has a minimum point at B. The lines $y = 2$ and $x = -1$ are asymptotes to the curve. On separate diagrams, sketch the graphs of

(i) $y^2 = \mathrm{f}(x)$, [2]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$, [2]

(iii) $y = \mathrm{f}'(x)$, [2]

labelling (if applicable) the coordinates of the corresponding points of A and B, the coordinates of any points of intersection with the axes and the equations of any asymptotes.

6 **(a)** By using the substitution $x = \dfrac{1}{2}\left(1+\sin\theta\right)$, show that $\displaystyle\int_{\frac{1}{4}}^{\frac{3}{4}} \frac{x}{\sqrt{x-x^2}}\,\mathrm{d}x = \frac{\pi}{6}$. [5]

(b) **(i)** Obtain a formula for $\int_0^m x\mathrm{e}^{-3x}\,\mathrm{d}x$ in terms of m, where $m > 0$. [3]

(ii) Hence evaluate $\int_0^\infty x\mathrm{e}^{-3x}\,\mathrm{d}x$, justifying your answer. [2]

7 **(a)** Verify that $\dfrac{1}{1+(n-1)a} - \dfrac{1}{1+na} = \dfrac{a}{[1+(n-1)a](1+na)}$. [1]

Hence show that, for $a > 0$, $\displaystyle\sum_{n=1}^{N} \frac{a}{[1+(n-1)a](1+na)} = \frac{Na}{1+Na}$. [2]

Deduce that the infinite series $\dfrac{1}{(1)\left(\frac{3}{2}\right)} + \dfrac{1}{\left(\frac{3}{2}\right)(2)} + \dfrac{1}{(2)\left(\frac{5}{2}\right)} + \cdots$ is convergent and find its sum to infinity. [3]

(b) Prove by mathematical induction that for all positive integers n,

$$\sum_{r=1}^{n} \frac{r(2^r)}{(r+2)!} = 1 - \frac{2^{n+1}}{(n+2)!}.$$ [4]

8 **(a)** Mary had 1922 marbles and she decided to pack them into different bags. She placed 6 marbles in the first bag. Each subsequent bag that she packed contained 6 marbles more than the previous bag. She continued to fill the bags until there was not enough marbles to fill the next bag. Calculate the number of marbles that were left unpacked. [4]

(b) On the first day of February 2015, a bank loans a man \$10,000 at an interest rate of 1.5% per month. This interest is added on the last day of each month and is calculated based on the amount due on the first day of the month. The man agrees to make repayments on the first day of each subsequent month. Each repayment is \$1200 except for the final repayment which is less than \$1200.
The amount that he owes at the start of each month is taken to be the amount he still owes just after the monthly repayment has been made. Find the date and amount of the final repayment to the nearest cent. [4]

9 A curve C has parametric equations

$$x = 2\sin^3\theta,\ y = \cos^3\theta, \quad \text{for } 0 \le \theta \le \frac{\pi}{2}.$$

(i) Find the exact equations of the tangent and the normal to the curve at the point where $x = 0.25$. [6]

(ii) Find the rate at which y is decreasing at the instant when $x = 0.25$, given that θ increases at a constant rate of $\frac{1}{18}$ radians per second. [2]

10 Given that $y = \cos[\ln(1+x)]$, prove that

(i) $(1+x)\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\sin[\ln(1+x)]$, [1]

(ii) $(1+x)^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + (1+x)\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = 0$. [2]

Hence find the Maclaurin's series for y, up to and including the term in x^3. [4]
Verify that the same result is obtained if the standard series expansion for $\ln(1+x)$ and $\cos x$ are used. [2]

11 **(a)** Sketch on an Argand diagram, the set of points representing all complex numbers z satisfying

$$|z| \le 2,\ \ z + z^* \ge 2 \ \text{ and } \ 0 \le \arg(z) \le \frac{\pi}{4}.$$

Hence, find the greatest exact values of $|z - 4 - 4\mathrm{i}|$ and $\arg(z-4)$. [6]

(b) Solve $w^3 + 1 = 0$ and hence deduce the solutions of $\left(\dfrac{z+1}{z}\right)^3 = -1$, giving each answer in the form $a + \mathrm{i}b$, where a and b are real. [6]

12 Relative to the origin O, two points A and B have position vectors $\mathbf{a} = 2\mathbf{i} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ respectively.

(i) Find $\mathbf{a} \times \mathbf{b}$ and give the geometrical meaning of $|\mathbf{a} \times \mathbf{b}|$. [2]

Hence write down the area of triangle OAB. [1]

(ii) Find a vector equation of the line l passing through A and B. [2]

(iii) The perpendicular to l from the point C with position vector $-13\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ meets the line at the point M. Show that the position vector of M is $\mathbf{i} + \mathbf{j} - 2\mathbf{k}$. [3]

(iv) Find a cartesian equation of the plane containing O, A and B and the exact length of projection of $\overrightarrow{CM}$ onto this plane. [4]

(v) Find the acute angle between the line OC and the triangle OAB. [2]

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