



RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Additional Practice Questions for

Chapter 7B: Arithmetic Progression and Geometric Progression

- 1 The sum of the first 100 terms of an arithmetic progression is 15050; the first, third and eleventh terms of this progression are three consecutive terms of a geometric progression. Find the first term, a and the non-zero common difference, d , of the arithmetic progression. [5]
- 2 Jack starts working in a company with an annual salary of \$16 000 in the first year. He will receive an annual salary increase of 4% each year. Assuming that he works in the company till his retirement, find
- (i) the amount he will earn in his 25th year, [2]
 - (ii) the total amount that he will earn over the 25-year period. [2]
 - (iii) the minimum number of years he has to work for his total earnings to exceed \$1 000 000. [4]
- 3 An infinite geometric series has first term a and common ratio r , where $r > 0$. The third term is 36 and the sum to infinity is 243.
- (i) Find the value of a and r . [3]
- An arithmetic series has first term 1 and common difference d . The sum of the first 6 terms of the arithmetic series is equal to the sum of the first 3 terms of the geometric series.
- (ii) Find the value of d . [3]
 - (iii) Find the least value of n for which the n th term of the arithmetic series is more than the sum of the first $2n$ terms of the geometric series. [3]
- 4 (a) An infinite geometric progression has first term a and common ratio r , where a and r are non-zero. The sum of all the terms after the n th term of the progression is equal to twice the n th term. Show that the sum to infinity of the progression is three times the first term. [3]
- (b) The positive integers, starting at 1, are grouped into sets, as follows.
- $$\{1\}, \{2, 3\}, \{4, 5, 6\}, \dots$$
- (i) Find, in terms of r , the first integer and the last integer in the r th set. [3]
 - (ii) Prove that the sum of the integers in the r th set is $\frac{1}{2}r(1+r^2)$. [2]

- 5 An arithmetic progression has first term 3. The sum of the first 13 terms of the progression is 156.
(i) Find the common difference. [2]
A geometric progression has first term 3 and common ratio r . The sum of the first 13 terms of the progression is 156.
(ii) Show that $r^{13} - 52r + 51 = 0$. Show that the common ratio cannot be 1 even though $r = 1$ is a root of this equation. Find the possible values of the common ratio. [4]
(iii) It is given that the common ratio of the geometric progression is positive, and that the n th term of this geometric progression is more than 100 times the n th term of the arithmetic progression. Write down an inequality, and hence find the smallest possible value of n . [3]
- 6 The sum of the first n terms of a series, G , is given by $S_n = c(2 - 2^{1-n})$, where c is a positive constant.
(i) Show that the n th term of the series G , $T_n = c2^{1-n}$. [2]
(ii) Hence, prove that the series G is a geometric series. [2]
(iii) It is given that the sixth term of G is $\frac{3}{8}$. Show that $T_1 = 12$. [2]
(iv) The first term and common difference of another arithmetic series A are the same as the first term and the common ratio of G respectively. Find the least value of n such that the sum of the first n terms of the arithmetic series A is more than one-quarter the sum of the first n terms of the series G by at least 250. [3]
- 7 Two athletes are training for a long distance race. During a training session, they run in 10-minute intervals and the distances they run in each 10-minute interval are recorded. Athlete A runs 2000 metres in the first 10-minute interval. On each subsequent 10-minute interval, she runs 45 metres less than in the previous interval.
(i) What distance is run on the 8th 10-minute interval? [1]
When Athlete A runs less than 1200 metres in a 10-minute interval, she will stop running after completing that interval.
(ii) Find the number of 10-minute intervals that Athlete A runs. [3]
(iii) Hence find the total distance run by Athlete A . [2]
Athlete B runs 2500 metres in the first 10-minute interval. On each subsequent 10-minute interval, she runs $\frac{11}{12}$ of the distance run on the previous interval.
(iv) Determine which athlete runs a longer distance on the 8th 10-minute interval. [2]
(iv) How many complete 10-minute intervals does it take for Athlete B to first exceed 95% of the theoretical maximum distance? [4]

- 8** A bank offers both an ordinary account and a savings account.
On 1 January 2020, Perlin puts \$100 into an ordinary account. On the first day of each subsequent month, she saves \$5 more than in the previous month, so that she saves \$105 on 1 February 2020, \$110 on 1 March 2020, and so on. This account pays no interest.

(i) On what date will the amount in Perlin's account first exceed \$5,000? [5]

On 1 January 2020, Pauline puts \$100 into a savings account, and on the first day of each subsequent month she puts another \$ x into the account. The interest rate is 0.5% per month, so that on the last day of each month the amount in the account on that day is increased by 0.5%.

(ii) Show that the expression for the amount in Pauline's account on the last day of the n th month (where January 2020 is the 1st month, February 2020 is the 2nd month, and so on) is given by

$$1.005^n (200x + 100) - 201x. \quad [3]$$

(a) If $x = 150$, find the number of complete months for the total amount in Pauline's account to first exceed \$5,000. [2]

(b) Pauline wishes for the amount in her account to be at least \$5,000 on 31 October 2021 in order to purchase a first-class flight to Japan for her year-end vacation. What is the smallest integer value of x would achieve this? [2]

- 9** (i) On 1 January 2020, Ms Eu took a study loan of \$20,000 from BOPS bank to fund her part-time studies. Based on the loan agreement, Ms Eu will make a yearly repayment of \$ x on 30 December of each year, starting from 30 December 2020. Furthermore, an interest rate of 4% per year is applied on the amount outstanding as at 31 December of each year, starting from 31 December 2020.

(a) Show that the amount outstanding at the end of the n^{th} year is

$$1.04^n (20000) - kx(1.04^n - 1),$$

where k is a constant to be determined. [3]

(b) Determine the value of x if Ms Eu intends to repay the loan fully at the end of 2024. [3]

(ii) While embarking on her part-time studies, Ms Eu started a new job and received her first monthly salary of \$3,300 on 1 July 2020. She saved \$1,600 from her first pay and deposited it into an account that paid no interest. Ms Eu intends to increase the amount saved from her salary by \$50 per month, until her account balance exceeds \$35,000. She will then transfer the full amount in this account into a fixed deposit in BOPS Bank immediately. She will keep the money for 12 months in the fixed deposit which earns a simple interest of 1.5% per year.

(a) Find the month and year in which Ms Eu will withdraw the fixed deposit. [3]

(b) Determine the value of the fixed deposit that Ms Eu will withdraw. [1]

(iii) After withdrawing the fixed deposit, Ms Eu has two options:

Option 1: Deposit \$30,000 in a fixed deposit that earns a simple interest of 1.5% per year, assuming that the amount in the fixed deposit account remains at \$30,000 each year.

Option 2: Invest \$30,000 in a fund that will yield a compound interest of 1.2% per year, assuming that no withdrawal was made.

Showing your working clearly, explain which option would be more suitable for Ms Eu if she wants to grow the value of this \$30,000 in the long term. [2]

10 A mask production factory produces 5000 reusable masks in the first week. In each subsequent week, the number of reusable masks produced is 200 more than the previous week.

(i) Find the number of reusable masks produced in the 10th week. [2]

(ii) Find the minimum number of weeks required by the factory to produce a total number of at least 100 000 reusable masks. [3]

Due to a certain flu pandemic, the demand for reusable masks has increased drastically. If the demand for reusable masks in the preceding week is M , it would become $100 + aM$ this week.

(iii) Given that the demand for reusable masks in the first week is 500, show that the demand for reusable masks in the N th week can be written as
$$\frac{100}{a-1}(5a^N - 4a^{N-1} - 1).$$
 [3]

It is now given that $a = 1.15$.

To meet the growing demand of reusable masks, the factory adjusts its production capacity such that the number of reusable masks produced from the 10th week onwards is k more than the previous week.

(iv) Find the least value of k required by the factory in order to meet the total demand for the first 25 weeks. [4]

11 A famous artist wants to create an artwork that uses squares only. On the first day, he draws the perimeter of a square with length x m as shown in Figure 1. On the second day, he divides the square into 9 equal squares and draws the perimeter of the middle square as shown in Figure 2. On the third day, he divides the remaining 8 empty squares from Day 2 into 9 equal squares, and draws the perimeters of the middle squares as shown in Figure 3. This procedure of “drawing the perimeters of the middle squares” is then repeated for subsequent days (see Figure 4 for his finished effort on Day 4).

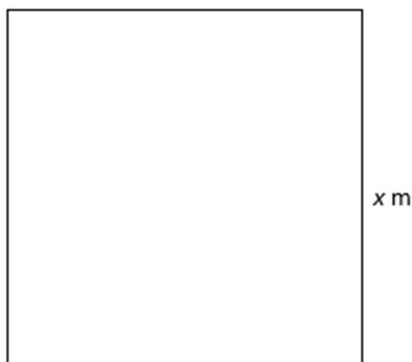


Figure 1 (Day 1)

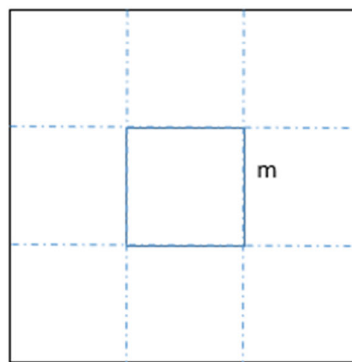


Figure 2 (Day 2)

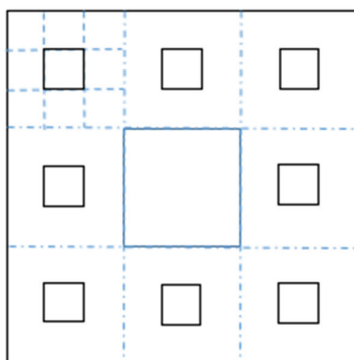


Figure 3 (Day 3)

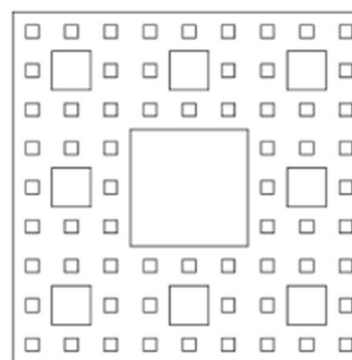


Figure 4 (Day 4)

- (i) Write down the number of new squares the artist draws on Day 3 and Day 4.
Hence write down the number of new squares the artist draws on Day N ($N \geq 2$). [2]
- (ii) Write down the length of one side of a new square the artist draws on Day 3 and Day 4.
Hence write down the length of one side of a new square the artist draws on Day N ($N \geq 2$). [2]
- (iii) Show that the total perimeter of new squares the artist draws on Day N ($N \geq 2$) is $\frac{x}{2} \left(\frac{8}{3} \right)^{N-1}$. [1]
- (iv) Hence find the total perimeter of all the squares the artist would have drawn by the end of the N -th day ($N \geq 2$). [3]
- (v) A company wants to support the artist. It decides to pledge \$10 for the drawing of the first square. For each subsequent square the artist draws, the company will pledge \$1 more than what was pledged for the previous square. (For the second square drawn, the artist will receive \$11. For the third square drawn, he will receive \$12, and so on.) The artist decides to stop drawing after he completes all the squares on Day 6.
A director of the company hears about the pledge, and he decides to personally pay for the 1st, 4th, 7th, 10th, ... square that the artist draws. Find the amount of money the director will personally pay. [4]

THE END