Name _____ ( )	Class 4 _____

ANGLICAN HIGH SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY FOUR

S4

MATHEMATICS **4048/01**
Paper 1 **Monday 10 September 2018**
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10
Marks										
Question	11	12	13	14	15	16	17	18	19	20
Marks										

Table of Penalties		**Qn. No.**		
Presentation	–1			
Units	–1			
Significant Figures	–1		**Parent's/ Guardian's Name/ Signature/ Date**	**80**

This question paper consists of 17 printed pages.

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 One solution of $2x^2 + kx - 12 = 0$ is $x = -4$. Find

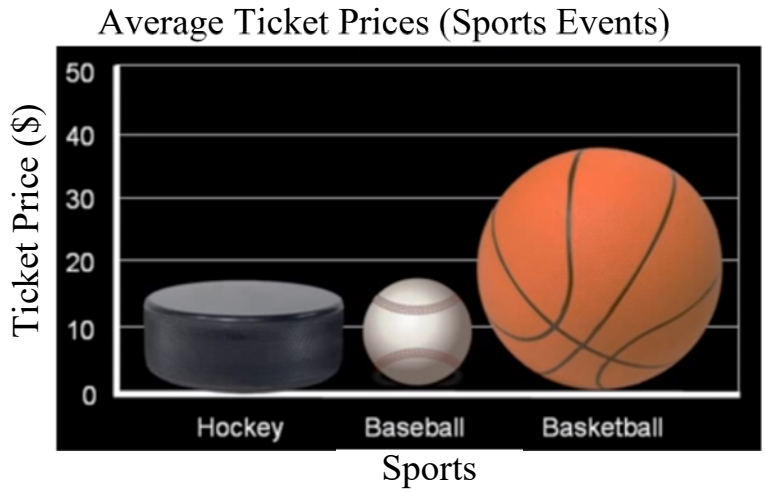
(a) the value of k,

Answer $k =$ ……..…….. [1]

(b) the other solution of the equation.

Answer ……..…… [1]

2



Adapted from https://www.youtube.com/watch?v=ETbc8GIhfHo.

State one aspect of the above graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer ……

……

…….. [2]

---

3 **(a)** Express $\sqrt[5]{121}$ in index form with base 11.

Answer ……[1]

(b) Evaluate $3^{\frac{2}{3}} \times 24^{\frac{1}{3}}$ without using a calculator.

Answer ……[2]

(c) Simplify $ab \div \left(\frac{2}{a}\right)^{-2}$

Answer ……[1]

---

4 **(a)** Solve the inequality $-\frac{17-8x}{4} < 2 - \frac{4-3x}{2} < 5\frac{1}{3}$.

Answer ……[2]

(b) **Hence**, state the smallest prime number that satisfies the inequality.

Answer ……[1]

---

5 It is given that $a = \sqrt[3]{\dfrac{2b+c}{c-b}}$.

(a) Express b in terms of a and c.

Answer ……. [2]

(b) Find the value of b when $a = 2$ and $c = 5$.

Answer b = ……...…… [1]

---

6 **(a)** Simplify $5(3x-5)^2 - 3(5-3x)$.

Answer ……. [2]

(b) Factorise completely $-25y^2 - 5xy + x + 1$.

Answer ……. [2]

---

7 **(a)** It is given that

ξ = {x: x is an integer between 0 and 9 inclusive}
$A \subset \xi$ and $B \subset \xi$
$\{0, 2\} \subset (A' \cap B)$, $7 \in A \cap B$, $\{1, 4, 5, 8\} \subset ((A \cup B) \cap B')$ and $3, 6, 9 \notin (A \cup B)$

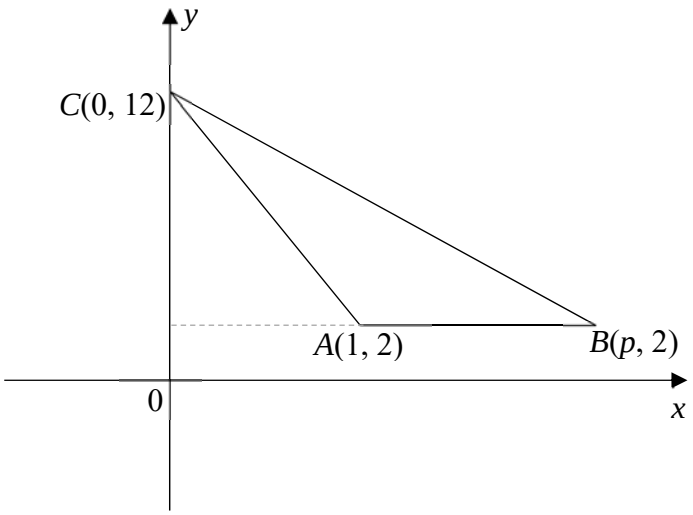
Draw a Venn diagram to represent the information given. [3]

Answer

(b) List down all the proper subsets of the set $\{a, b, c\}$.

Answer……..… [1]

8 In the diagram below, not drawn to scale, A is the point (1, 2), B is the point (p, 2) and C is the point (0, 12).



(a) Find the length of the line AC.

Answer …….……units [2]

(b) Write down the value of $\cos \angle BAC$.

Answer ……..…… [1]

(c) Given that the length of the line BC is $5\sqrt{5}$ units, find the value of p.

Answer p = ……..…..…… [2]

9 **(a)** Mr Tan wants to change $4 000 Singapore dollars to US dollars for a holiday trip to the USA.
The exchange rate in Singapore is 1 SGD = 0.736 USD.
The exchange rate in USA is 1 USD = 1.352 SGD.
In which country should he change his money and how much more USD can he get?

Answer…… [2]

(b) In 2017, Matthew earned an annual income of $80 000. He is required to pay tax based on net income. His net income is obtained after deducting CPF contribution of $16 000 and personal expenses relief of $3000 from the annual income. The tax rate is $200 for the first $30 000 of net income and 5% for the remaining net income. Calculate Matthew's

(i) net income,

Answer $ ……..…… [1]

(ii) income tax.

Answer $ ……..…… [2]

10 Amanda wrote down four numbers.

The mean of these numbers is 15, the median is 12 and the mode is 8.

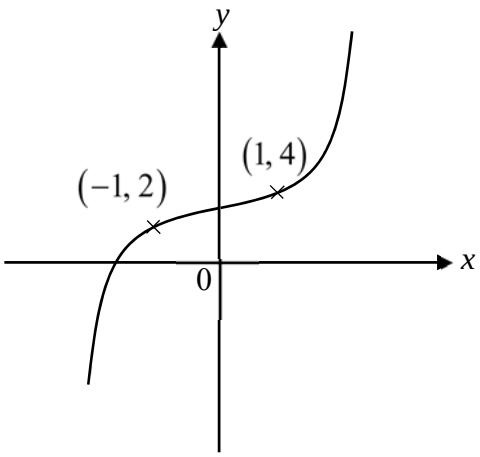
Find the four numbers.

Answer ……., ……., ……., ……. [2]

---

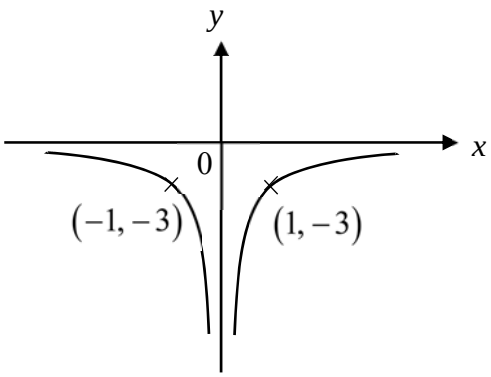
11 Write down a possible equation for each of the graphs shown below.

(a)



Answer …...……..……. [1]

(b)



Answer …...……..……. [1]

---

12 **(a)** Find the greatest integer that will divide both 126 and 2100.

Answer ……..…… [2]

(b) A rectangular field is measured as 49.9 and 24.5 correct to the nearest 0.1 metre. Find,

(i) the least possible perimeter in metres.

Answer ……..…..…..…… m [1]

(ii) the greatest possible area in square metres. Express your answer in standard form correct to 4 significant figures.

Answer ……..…..…… m^2 [2]

---

13 The surface area of a sphere is directly proportional to the square of its radius. If the radius increases by 15%, find the percentage increase in the surface area.

Answer …...…...……..……. % [2]

---

14 In the following sequence,

$$(1\times 2)-2=0$$
$$(2\times 3)-4=2$$
$$(3\times 4)-6=6$$
$$\cdots$$
$$\cdots$$
$$(a\times 13)-b=c$$
$$\cdots$$
$$\cdots$$
$$(d\times e)-f=g$$

(a) Find the values of a, b and c.

Answer a = …..…..…..……

b = …..…..…..……

c = …..…..…..…… [2]

(b) Express g in terms of d only.

Answer ……..…… [2]

(c) Explain why 343 cannot be the result of an equation in this sequence.

Answer ……...……

……

…..…….

…….

…… [1]

15 At an online supermarket, a 0.5 kg bag of carrots costs \$0.90, a 0.5 kg of onions costs \$1.50 and a 0.5 kg bag of local tomatoes costs \$1.30. On Monday, there were 730 orders for carrots, 421 orders for onions and 279 orders for tomatoes. On Tuesday, there were x orders for carrots, 355 orders for onions and 249 orders for tomatoes.

(a) Write down a 2×3 matrix, $\boldsymbol{M}$, representing the orders over Monday and Tuesday.

Answer $\boldsymbol{M} = \begin{pmatrix} & & \\ & & \end{pmatrix}$ [1]

(b) Find, in terms of x, the matrix $\boldsymbol{P} = \boldsymbol{M}\begin{pmatrix} 0.90 \\ 1.50 \\ 1.30 \end{pmatrix}$.

Answer $\boldsymbol{P} =$ [2]

(c) Explain clearly what each element in matrix P represents.

Answer ……..……...

……... [1]

(d) If the total cost of orders on Tuesday is about 10% less than the total cost of orders on Monday,

(i) calculate the value of x.

Answer $x =$ …...……. [1]

(ii) use a matrix method to compute the total cost of orders on Monday and Tuesday.

Answer \$ …...……. [1]

16 The stem-and-leaf diagram below shows the times of two groups of students, Group A and Group B, doing shuttle run.

Group A		Group B
	9	4 9 9
7	10	0 1 3 4 4 4 5 6 9
1 1 1 2 2 3 5 7	11	0 1
0 2 3 6 7	12	

Key (Group A)
7 | 10 means 10.7 seconds

Key (Group B)
9 | 4 means 9.4 seconds

(a) Write down the modal timing of Group B.

Answer ……..…… seconds [1]

(b) Write down the median of Group A.

Answer …… seconds [1]

(c) Explain briefly which group of students ran faster.

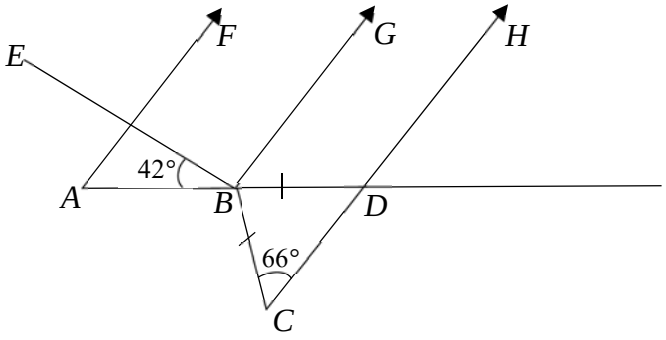
Answer ……..……...

……..……...

……..……...

……..... [1]

17 In the diagram, $BC=BD$, $\angle ABE = 42°$ and $\angle BCD = 66°$. AF, BG and CH are parallel.



Show your working and give reasons, calculate

(a) $\angle CBD$,

Answer …...……..…..…… ° [1]

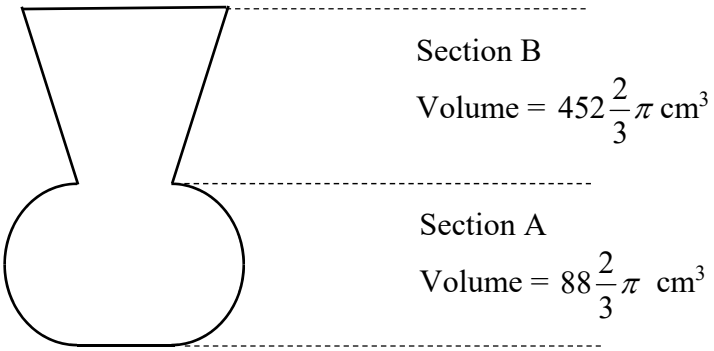
(b) $\angle GBE$,

Answer …...……..…..…… ° [2]

(c) $\angle BAF$.

Answer …...……..…..…… ° [1]

18 The diagram shows the cross section of a vase.



A factory produces 3 geometrically similar vases, Small, Medium and Large. The sketch above shows the dimensions of the Large vase. The volume of the Small vase is $67\frac{2}{3}\pi$ cm^3 with height 14 cm. The Medium vase has a height 50% more than the Small vase.

(a) Calculate

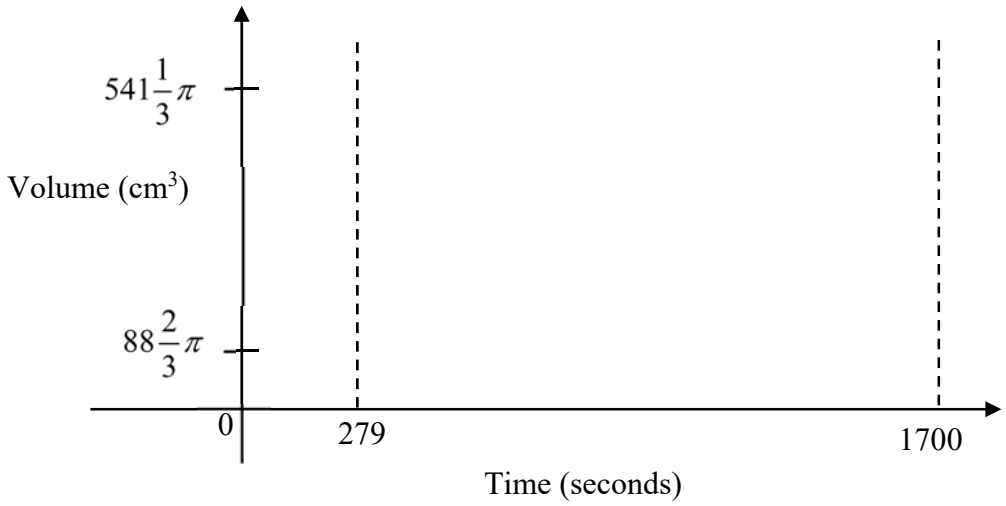
(i) the height of the Large vase, and

Answer …...……...…… cm [2]

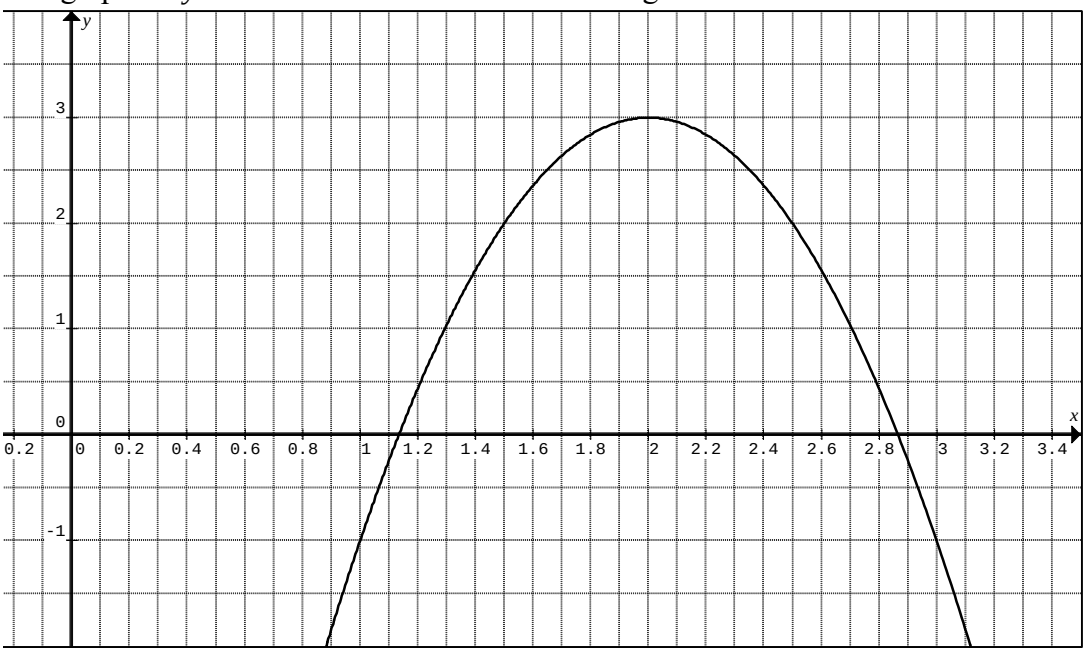
(ii) the volume of the Medium vase, in terms of π.

Answer ……...…… cm^3 [2]

(b) Water is poured into the Large vase at 1 cm^3/s. Sketch the volume-time graph of the Large vase.
[3]



19 The graph of $y = -4x^2 + 16x - 13$ is drawn on the grid.



(a) Explain why $y = 3.7$ has no solution.

Answer ...……

……...…… [1]

(b) The points $(1.5, 2)$ and $(2.5, 2)$ are the intersection points for this curve and another curve, $y = ax^2 + bx + c$. Given that $a > 0$, find a possible equation for this second curve.

Answer ……..…...……... [1]

(c) The equation $-2x^2 + 7x - 5 = 0$ can be solved by adding a straight line to the grid above. Find the equation of this line.

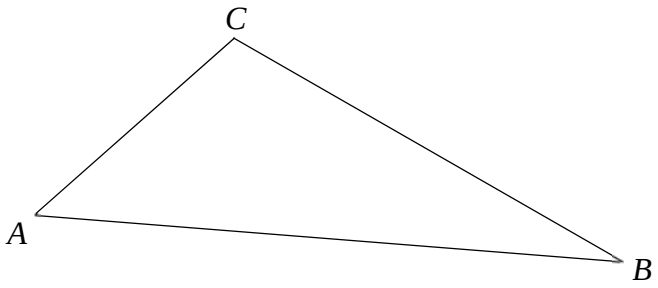
Answer ……..…...……... [1]

(d) By drawing this straight line, solve the equation $-2x^2 + 7x - 5 = 0$.

Answer x = ……. or x = ……... [3]

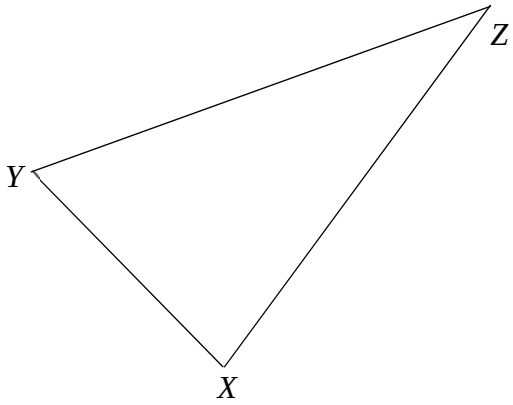
20 **(a)** Find, by construction, the point *P*, that is equidistant from the points *A*, *B* and *C*.

Hence, draw a circle passing through *A*, *B* and *C*. Measure the radius of the circle.



Answer …..……..…….. [3]

(b) Find, by construction, the point *Q*, that is equidistant from the lines *XY, YZ* and *XZ.*
Hence, or otherwise, draw a circle that is tangent to the lines *XY, YZ* and *XZ.*
Measure the radius of the circle.



Answer …..……..…….. [3]

End of Paper

Answer key

1 (a) $k = 5$ (b) $x = 1.5$ or $\frac{3}{2}$.

2 The three objects are all not the same shape. One is a cylinder and the other two are spheres.

It is not clear how ticket prices can be determined. For Hockey, one can look at the centre of the top of the cylinder or the top of the diagram. For Baseball and Basketball, one may consider the centre of the objects or the top of the circles. The curved tops make finding the highest point inaccurate.

3 (a) $11^{\frac{2}{5}}$ (b) 6 (c) $\frac{4b}{a}$

4 (a) $x < 3\frac{5}{9}$

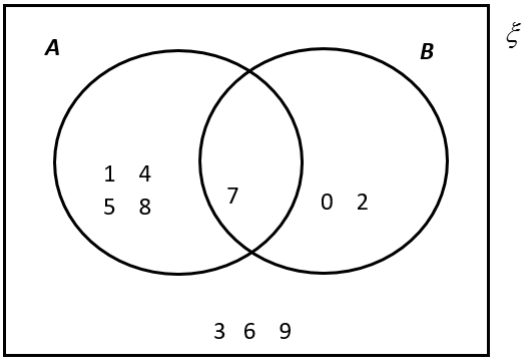
(b) Smallest prime number = 2

5 (a) $b = \frac{c\left(a^3 - 1\right)}{2 + a^3}$ (b) $b = 3.5$

6 (a) $45x^2 - 141x + 110$

(b) $(1 - 5y)(1 + 5y + x)$

7 (a)



(b) $\{\ \}$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$, $\{a, c\}$

8 **(a)** 10.0 units **(b)** ≈ -0.0995

(c) $p = 5$ only

9 (a) He should change in USA. He will get 14.58 US dollars more.

(b)(i) \$61 000 (ii) \$1750

10 8, 8, 16 and 28.

11 (a) $y = x^n + 3$, where n must be odd

(b) $y = -\frac{3}{x^n}$ where n must be even

12 (a) 42

(b)(i) 148.6m (ii) $1.226 \times 10^3 \text{ m}^2$

13 32.25%

14 (a) $a = 12,\ b = 24,\ c = 132$ (b) $g = d(d - 1)$

(c) Since d cannot be negative and non-integer, therefore 343 cannot be the result of an equation in this sequence.

Alternatively,

g is the product of an odd and an even number. Therefore g must be even. Since 343 is odd, 343 cannot be the result of an equation.

Alternatively,

From the pattern shown above, g is always even. Therefore 343 cannot be the answer as it is odd

15 (a) $\begin{pmatrix} 730 & 421 & 279 \\ x & 355 & 249 \end{pmatrix}$

(b) $\begin{pmatrix} 1651.20 \\ 0.9x + 856.20 \end{pmatrix}$

(c) The elements show that on Monday, the total cost of orders was \$1651.20 and on Tuesday, the total cost of orders was $\$0.9x + 856.20$.

(d) (i) 700 (ii) \$3137.29

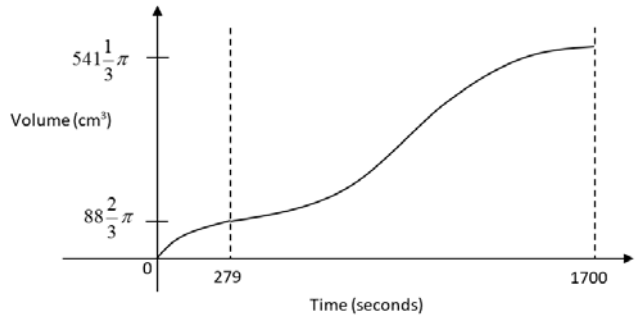
16 (a) 10.4 seconds (b) 11.4 seconds

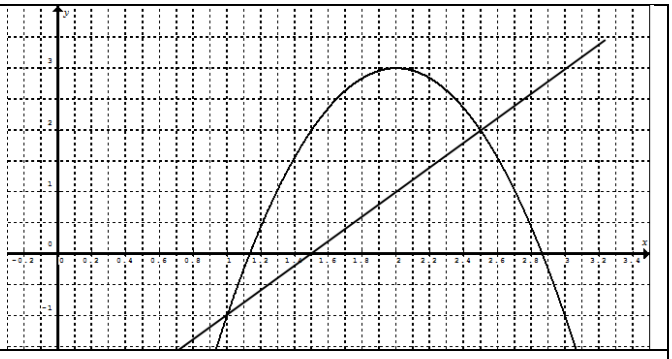
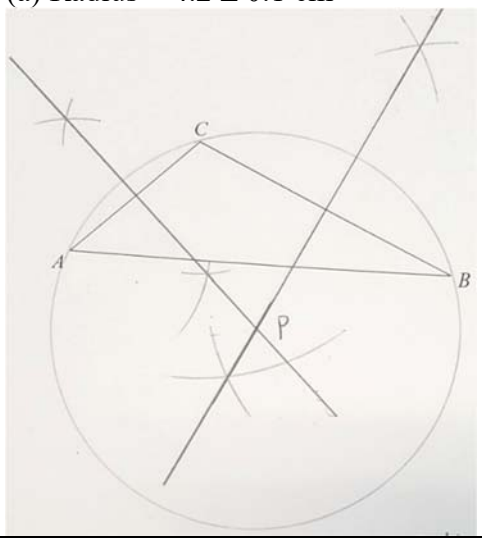
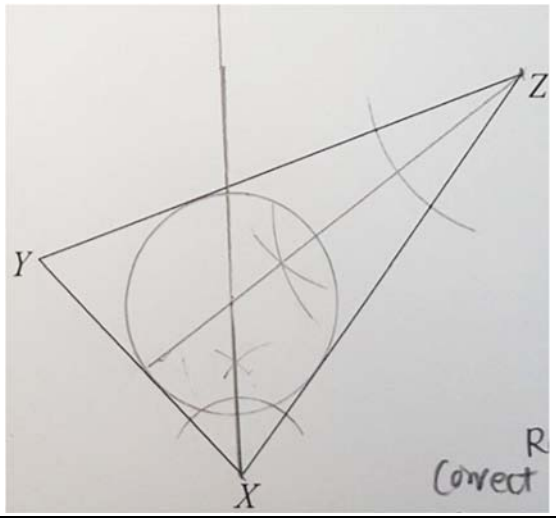
(c) Since Group B has a smaller median than Group A, so Group B ran faster.

17 (a) 48° (b) 72° (c) 66°

18 (a)(i) 28cm (ii) $228\frac{3}{8}\pi \text{ cm}^3$

(b)



<table>
<tr>
<td>19</td>
<td>(a) The maximum value for y is 3.
Hence there is no solution for $y = 3.7$.
(b) $y = 4x^2 - 16x + 17$
(c) $y = 2x - 3$
(d) $x = -1$ or $x = 2.5$</td>
<td>(d)</td>
<td>  </td>
</tr>
<tr>
<td>20</td>
<td>(a) Radius = 4.2 ± 0.1 cm
 
</td>
<td>(b)</td>
<td>Radius = 1.3 ± 0.1 cm
 
</td>
</tr>
</table>

Answer Scheme for Sec 4 Math Prelim Paper 1 2018

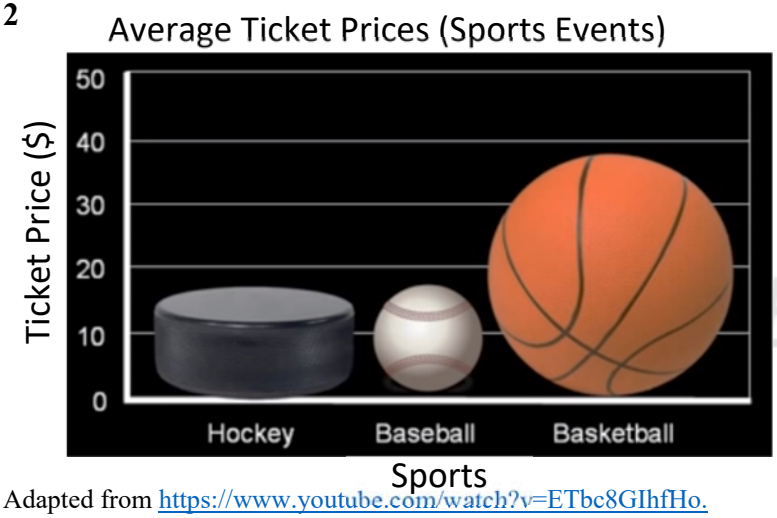
1 One solution of $2x^2 + kx - 12 = 0$ is $x = -4$. Find

(a) the value of k, [1]

(b) the other solution of the equation. [1]

1a	Sub $x = -4$ into $2x^2 + kx - 12 = 0$, $2(-4)^2 + k(-4) - 12 = 0$ $20 - 4k = 0$ $k = 5$	[B1]
1b	$2x^2 + 5x - 12 = 0$ $(x + 4)(2x - 3) = 0$ The other solution is $x = 1.5$ or $\frac{3}{2}$.	[B1]

2



Adapted from https://www.youtube.com/watch?v=ETbc8GIhfHo.

State one aspect of the above graph that may be misleading and explain how this may lead to a misinterpretation of the graph. [2]

2	The three objects are all not the same shape. One is a cylinder and the other two are spheres. It is not clear how ticket prices can be determined. For Hockey, one can look at the centre of the top of the cylinder or the top of the diagram. For Baseball and Basketball, one may consider the centre of the objects or the top of the circles. The curved tops make finding the highest point inaccurate.	B1 B1

3 **(a)** Express $\sqrt[5]{121}$ in index form with base 11. [1]

(b) Evaluate $3^{\frac{2}{3}} \times 24^{\frac{1}{3}}$ without using calculator. [2]

(c) Simplify $ab \div \left(\frac{2}{a}\right)^{-2}$. [1]

3a	$\sqrt[5]{121} = 121^{\frac{1}{5}}$ $= \left(11^2\right)^{\frac{1}{5}}$ $= 11^{\frac{2}{5}}$	B1
3b	$3^{\frac{2}{3}} \times 24^{\frac{1}{3}}$ $= 3^{\frac{2}{3}} \times (3\times 8)^{\frac{1}{3}}$ $= 3^{\frac{2}{3}} \times 3^{\frac{1}{3}} \times 8^{\frac{1}{3}}$ $= 3\times(2^3)^{\frac{1}{3}}$ $= 3\times 2$ $= 6$	M1 for any of the first 3 steps. A1
3c	$ab \div \left(\frac{2}{a}\right)^{-2}$ $= ab \times \left(\frac{2}{a}\right)^{2}$ $= ab \times \frac{4}{a^2}$ $= \frac{4b}{a}$	B1

4 **(a)** Solve the inequality $-\frac{17-8x}{4} < 2 - \frac{4-3x}{2} < 5\frac{1}{3}$. [2]

(b) **Hence**, state the smallest prime number that satisfies the inequality. [1]

4a	$-\frac{17-8x}{4} < 2-\frac{4-3x}{2}$ $-17+8x < 2\times 4 - 2(4-3x)$ $-17 < 8-8+6x-8x$ $-17 < -2x$ $x < 8.5$ Therefore, $x < 3\frac{5}{9}$.	or	$2-\frac{4-3x}{2} < \frac{16}{3}$ $2\times 6 - 3(4-3x) < 32$ $12-12+9x < 32$ $9x < 32$ $x < 3\frac{5}{9}$	M1 splitting A1
4b	Smallest prime number = 2			B1

5 It is given that $a = \sqrt[3]{\dfrac{2b+c}{c-b}}$.

(a) Express b in terms of a and c. [2]

(b) Find the value of b when $a = 2$ and $c = 5$. [1]

5a	$a = \sqrt[3]{\dfrac{2b+c}{c-b}}$ $a^3 = \dfrac{2b+c}{c-b}$ $a^3c - a^3b = 2b + c$ $a^3c - c = 2b + a^3b$ $b\left(2+a^3\right) = c\left(a^3-1\right)$ $b = \dfrac{c\left(a^3-1\right)}{2+a^3}$	M1 for any first 3 steps A1
5b	$b = \dfrac{c\left(a^3-1\right)}{2+a^3}$ $b = \dfrac{5(8-1)}{2+8}$ $b = \dfrac{35}{10}$ $b = 3.5$	 B1

6 **(a)** Simplify $5(3x-5)^2 - 3(5-3x)$. [2]

(b) Factorise completely $-25y^2 - 5xy + x + 1$. [2]

6a	$5(3x-5)^2 - 3(5-3x) = 5\left(9x^2 - 30x + 25\right) - 15 + 9x$ $= 45x^2 - 150x + 125 - 15 + 9x$ $= 45x^2 - 141x + 110$	M1 A1
6b	$1 - 25y^2 + x - 5xy = (1+5y)(1-5y) + x(1-5y)$ $= (1-5y)(1+5y+x)$	M1 A1

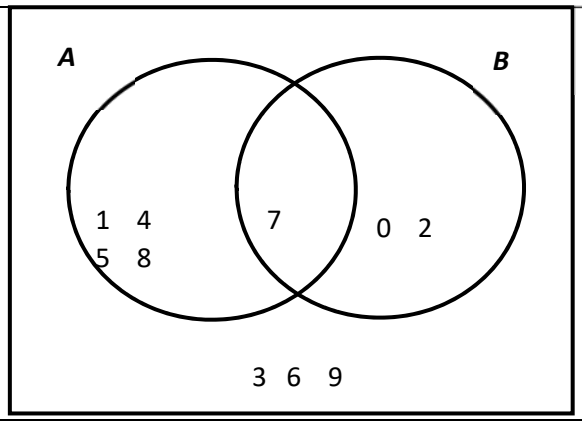
7 **(a)** It is given that
ξ = { x: x is an integer between 0 and 9 inclusive }

$A \subset \xi$ and $B \subset \xi$

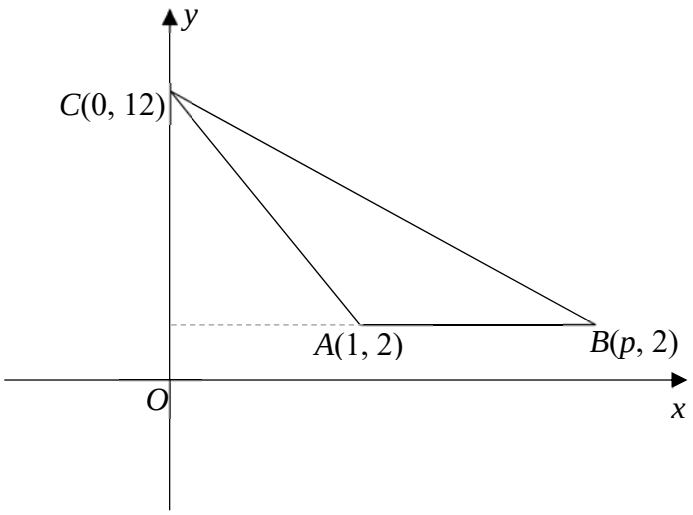
$\{0, 2\} \subset (A' \cap B)$, $7 \in A \cap B$, $\{1, 4, 5, 8\} \subset ((A \cup B) \cap B')$ and
$3, 6, 9 \notin (A \cup B)$

Draw a Venn diagram to represent the information given. [3]

(b) List down all the proper subsets of the set $\{a, b, c\}$. [1]

7a		B3 Deduct one mark for each error.
7b	{ }, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$, $\{a, c\}$	B1

8 In the diagram below, not drawn to scale, A is the point (1, 2), B is the point (p, 2) and C is the point (0, 12).



(a) Find the length of the line AC. [2]

(b) Write down the value of $\cos \angle BAC$. [1]

(c) Given that the length of the line BC is $5\sqrt{5}$ units, find the value of p. [2]

8a	$AC^2 = 1^2 + 10^2$ $= 101$ $AC = \sqrt{101}$ $= 10.0499$ ≈ 10.0 units	M1 A1
8b	$\cos \angle BAC = -\cos \alpha$ $= -\frac{1}{\sqrt{101}} \approx -0.0995$	 B1
8c	$(p-0)^2 + (2-12)^2 = \left(5\sqrt{5}\right)^2$ $p^2 + 100 = 125$ $p^2 = 25$ $p = 5$ only	M1 A1

9 **(a)** Mr Tan wants to change \$4 000 Singapore dollars to US dollars for a holiday trip to the USA.
The exchange rate in Singapore is 1 SGD = 0.736 USD.
The exchange rate in USA is 1 USD = 1.352 SGD.
In which country should he change his money and how much more USD can he get? [2]

(b) In 2017, Matthew earned an annual income of \$80 000. He is required to pay tax based on net income. His net income is obtained after deducting CPF contribution of \$16 000 and personal expenses relief of \$3000 from the annual income. The tax rate is \$200 for the first \$30 000 of net income and 5% for the remaining net income. Calculate Matthew's

(i) net income, [1]

(ii) income tax. [2]

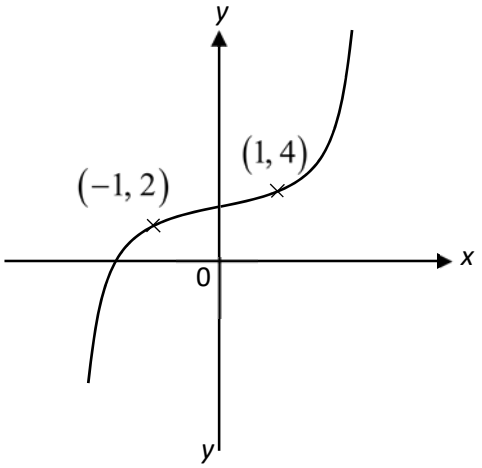
9a	In Singapore, Mr Tan will get $4000 \times 0.736 = 2944$ US dollars In USA, Mr Tan will get $4000 \div 1.352 = 2958.58$ US dollars He should change in USA. He will get 2958.57 - 2944 =14.58 US dollars more.	M1 for changing both into US dollars A1
9bi	Net Income = 80 000 – 16 000 – 3000 = \$61 000	 B1
9bii	Income Tax = 200 + (61 000 – 30 000) x 5% = \$1750	M1 A1

10 Amanda wrote down four numbers.
The mean of these numbers is 15, the median is 12 and the mode is 8.
Find the four numbers. [2]

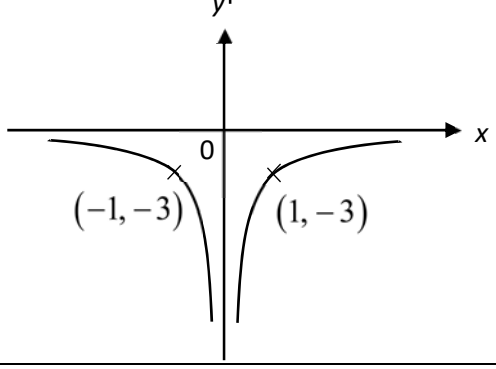
10	Let the four numbers be a, b, c and d in ascending order. Since the mode is smaller than median, so a and b will be 8. Since median is 12, so $\frac{8+c}{2}=12$ $c=24-8$ $c=16$ Since mean is 15, $8+8+16+d=15\times 4$ $d=28$ The four numbers are 8, 8, 16 and 28. Note: Award B2 for 4 correct answers if students do not show any working at all.	M1 for either mode, median or mean A1

11 Write down a possible equation for each of the graphs shown below.

(a)



(b)



11a	$y=x^n+3$, where n must be odd	B1
11b	$y=-\frac{3}{x^n}$ where n must be even	B1

12 **(a)** Find the greatest integer that will divide both 126 and 2100. [2]

(b) A rectangular field is measured as 49.9 and 24.5 correct to the nearest 0.1 metre. Find,

(i) the least possible perimeter in metres. [1]

(ii) the greatest possible area in square metres. Express your answer in standard form correct to 4 significant figures. [2]

12a	$126 = 2\times 3^2 \times 7$ $2100 = 2^2 \times 3 \times 5^2 \times 7$ HCF value is greatest integer to divide both numbers. HCF $= 2\times 3\times 7$ $= 42$	M1 B1
12bi	Least Perimeter $= 2(49.85) + 2(24.45) = 148.6\text{m}$	B1
12bii	Greatest area $= 49.95 \times 24.55$ $= 1226.273$ $= 1.226\times 10^3 \text{m}^2$	M1 A1

13 The surface area of a sphere is directly proportional to the square of its radius. If the radius increases by 15%, find the percentage increase in the surface area. [2]

13	$S = kr^2$ Let $S_1 = k(r_1)^2$ $r_2 = 1.15r_1$ $S_2 = k(r_2)^2$ $= k(1.15r_1)^2$ Percentage Increase $= \dfrac{S_2 - S_1}{S_1}\times 100$ $= \dfrac{k(1.15r_1)^2 - (kr_1)^2}{k(r_1)^2}\times 100$ $= \left((1.15)^2 - 1\right)\times 100$ $= 32.25\%$	 M1 A1

14 In the following sequence,

$$(1\times 2)-2=0$$
$$(2\times 3)-4=2$$
$$(3\times 4)-6=6$$
$$\cdots$$
$$\cdots$$
$$(a\times 13)-b=c$$
$$\cdots$$
$$\cdots$$
$$(d\times e)-f=g$$

(a) Find the values of a, b and c. [2]
(b) Express g in terms of d. [2]
(c) Explain why 343 cannot be the result of an equation in this sequence. [1]

14a	$a = 12,\ b = 24, c = 132$	B2
14b	$e = d+1$ $f = 2d$ $g = d(d+1)-2d$ $= d^2+d-2d$ $= d^2-d$ $= d(d-1)$	M1 A1
14c	$d^2-d=343$ $d^2-d-343=0$ $d=\dfrac{1\pm\sqrt{1-4(1)(-343)}}{2}$ $=\dfrac{1\pm 37.054}{2}$ $=19.03$ or -18.03 Since d cannot be negative and non-integer, therefore 343 cannot be the result of an equation in this sequence. **Alternatively,** g is the product of an odd and an even number. Therefore g must be even. Since 343 is odd, 343 cannot be the result of an equation. **Alternatively,** From the pattern shown above, g is always even. Therefore 343 cannot be the answer as it is odd.	 B1 or B1 or B1

15 At an on-line supermarket, a 0.5 kg bag of carrots costs \$0.90, a 0.5 kg of onions costs \$1.50 and a 0.5 kg bag of local tomatoes costs \$1.30. On Monday, there were 730 orders for carrots, 421 orders for onions and 279 orders for tomatoes. On Tuesday, there were x orders for carrots, 355 orders for onions and 249 orders for tomatoes.

(a) Write down a 2×3 matrix, M, representing the orders over Monday and Tuesday. [1]

(b) Find, in terms of x, the matrix $P = M\begin{pmatrix}0.90\\1.50\\1.30\end{pmatrix}$. [2]

(c) Explain clearly what each element in matrix P represents. [1]

(d) If the total cost of orders on Tuesday is about 10% less than the total cost of orders on Monday,

(i) calculate the value of x. [1]

(ii) use a matrix method to compute the total cost of orders on Monday and Tuesday. [1]

15a	$M = \begin{pmatrix}730 & 421 & 279\\ x & 355 & 249\end{pmatrix}$	B1
15b	$P = \begin{pmatrix}730 & 421 & 279\\ x & 355 & 249\end{pmatrix}\begin{pmatrix}0.90\\1.50\\1.30\end{pmatrix}$ $= \begin{pmatrix}1651.20\\0.9x+856.20\end{pmatrix}$	M1 A1
15c	The elements show that on Monday, the total cost of orders was \$1651.20 and on Tuesday, the total cost of orders was \$ $0.9x+856.20$.	B1
15di	$0.9x+856.20 = 0.9\times1651.20$ $x = 699.88$ $x = 700$	B1
15dii	$\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1651.20\\0.9\times699.88+856.20\end{pmatrix} = \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1651.20\\1486.092\end{pmatrix}$ $= (1\times1651.20+1\times1486.092)$ $= (3137.292)$ The total cost of orders for Monday and Tuesday was \$3137.29.	B1

16 The stem-and-leaf diagram below shows the times of two groups of students, Group A and Group B, doing shuttle run.

Group A		Group B
	9	4 9 9
7	10	0 1 3 4 4 4 5 6 9
1 1 1 2 2 3 5 7	11	0 1
0 2 3 6 7	12	

Key (Group A)
7 | 10 means 10.7 seconds

Key (Group B)
9 | 4 means 9.4 seconds

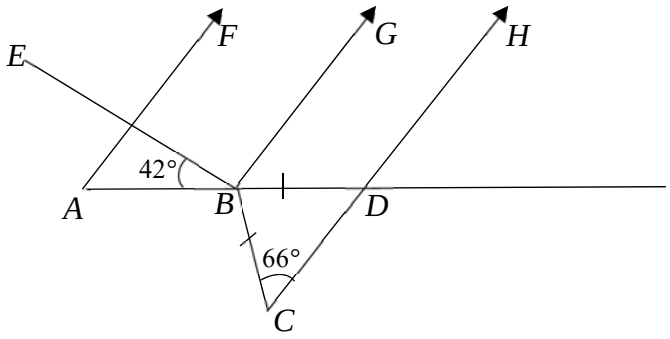
(a) Write down the modal timing of Group B. [1]

(b) Write down the median of Group A. [1]

(c) Explain briefly which group of students ran faster. [1]

16a	Mode = 10.4 seconds	B1
16b	Median = $\frac{11.3+11.5}{2} = 11.4$ seconds	B1
16c	Median for Group A = 11.4 seconds Median for Group B = $\frac{10.4+10.4}{2} = 10.4$ seconds. Since Group B has a smaller median than Group A, so Group B ran faster. Note: Mode is not acceptable in this answer as not many students recorded the modal values.	B1

17 In the diagram, $BC=BD$, $\angle ABE = 42°$ and $\angle BCD = 66°$. AF, BG and CH are parallel.



Showing your working and giving reasons, calculate

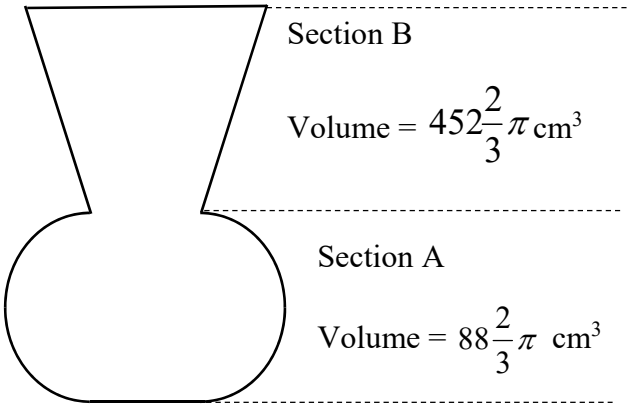
(a) $\angle CBD$, [1]

(b) $\angle GBE$, [2]

(c) $\angle BAF$. [1]

17a	$\angle CBD = 180° - 66° - 66°$(angle sum of isosceles triangle) $= 48°$	B1
17b	$\angle HDB = 180° - 66°$(adjacent angle on a straight line) $= 114°$ $\angle GBE + 42° = \angle HDB$(corr angles, BG // CH) $\angle GBE = 114° - 42°$ $= 72°$	M1 A1
17c	$\angle BAF = 180° - \angle HDB$ (interior angles, AF // DH) $= 180° - 114°$ $= 66°$ Deduct 1 mark from the whole of question for not stating reason or incorrect reason.	 B1

18 The diagram shows the cross section of a vase.



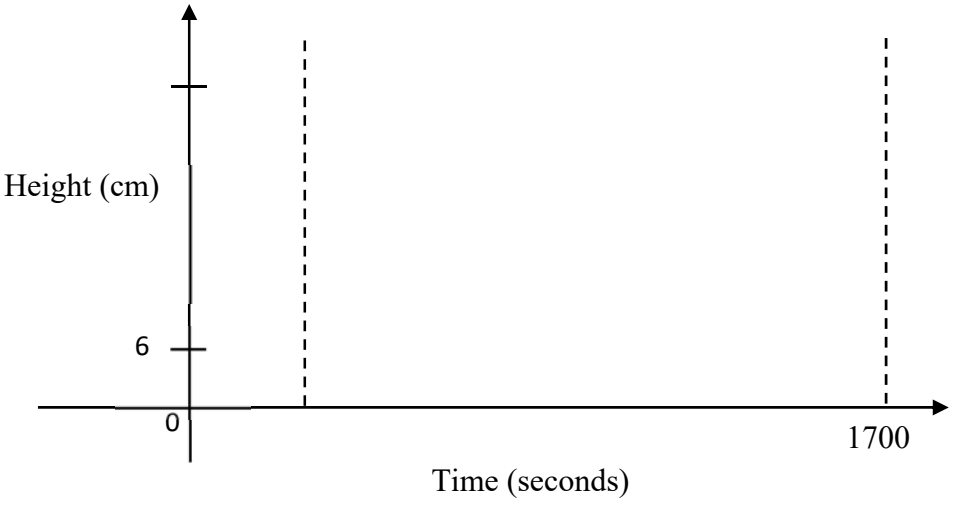
A factory produces 3 geometrically similar vases, Small, Medium and Large. The sketch above shows the dimensions of the Large vase. The volume of the Small vase is $67\frac{2}{3}\pi$ cm^3 with height 14 cm. The Medium vase has a height 50% more than the Small vase.

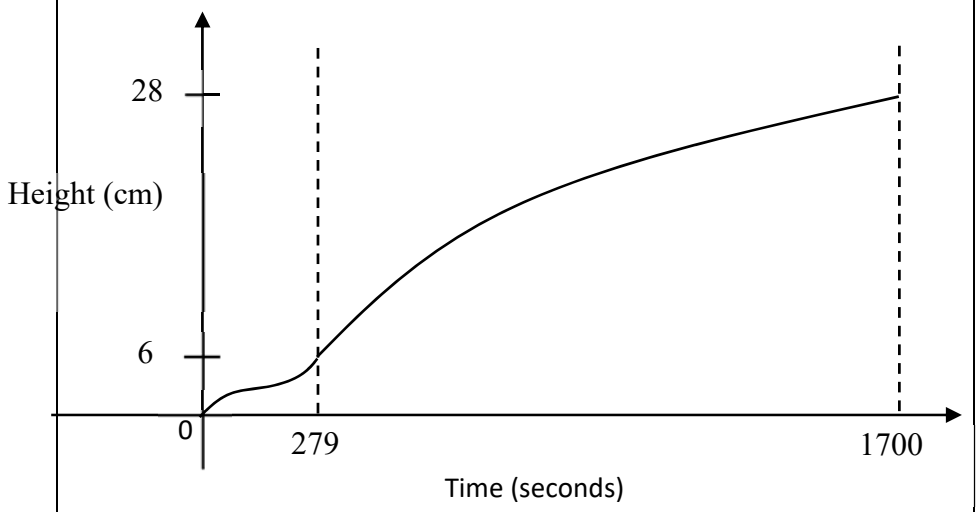
(a) Calculate

(i) the height of the Large vase, and [2]

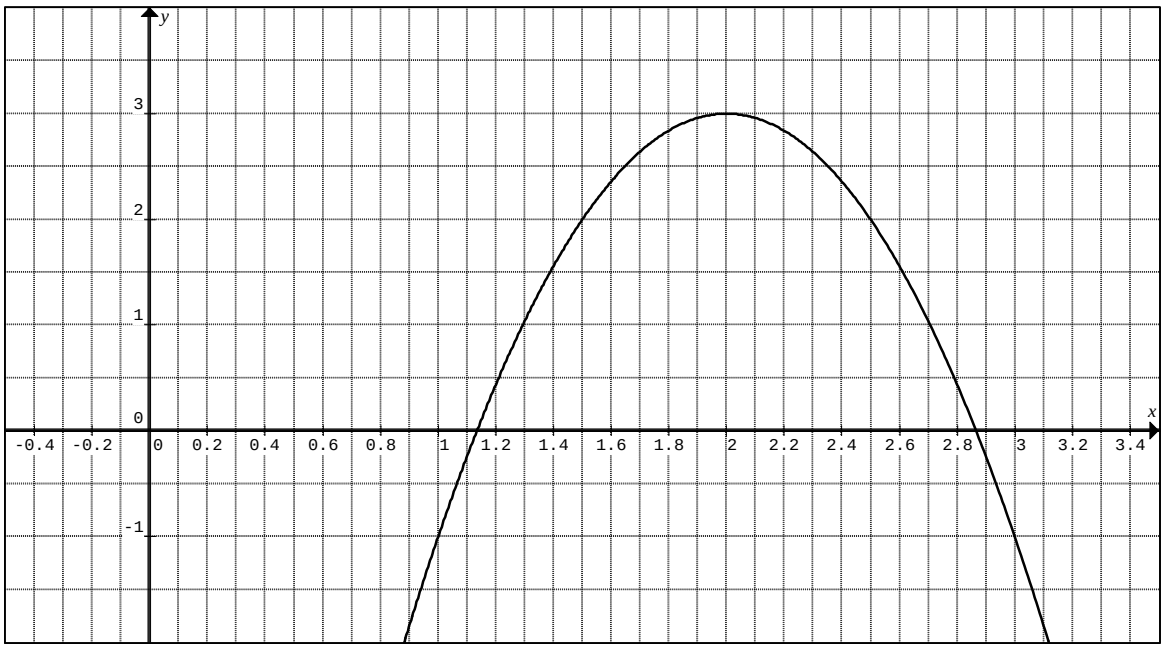
(ii) the volume of the Medium vase, in terms of π. [2]

(b) Water is poured into the Large vase at 1 cm^3/s. The height of Section A is approximately 6 cm. Sketch the height-time graph of the water in the Large vase. [3]



18a	Volume of the Small vase $= 67\frac{2}{3}\pi \text{ cm}^3$ Volume of Large vase $= 452\frac{2}{3}\pi + 88\frac{2}{3}\pi = 541\frac{1}{3}\pi \text{ cm}^3$	
18ai	Height of the Small vase $= 14$ cm $\left(\frac{\text{Height of Large vase}}{14}\right)^3 = \frac{541\frac{1}{3}\pi}{67\frac{2}{3}\pi}$ $= 8$ $\frac{\text{Height of Large vase}}{14} = 2$ Height of Large vase $= 28$ cm	M1 A1
18aii	Height of the Medium vase $= 14 + 7 = 21$ cm $\frac{\text{Volume of Medium vase}}{\text{Volume of Small vase}} = \left(\frac{21}{14}\right)^3$ $\frac{\text{Volume of Medium vase}}{67\frac{2}{3}\pi} = \left(\frac{3}{2}\right)^3$ Volume of Medium vase $= \left(\frac{3}{2}\right)^3 \times 67\frac{2}{3}\pi$ $= 228\frac{3}{8}\pi \text{ cm}^3$	 M1 A1
18b	Time taken to fill Section A $= 88\frac{2}{3}\pi \times 1 = 279$ sec (3 significant figures) Time taken to fill Section B $= 452\frac{2}{3}\pi \times 1 = 1420$ sec (3 significant figures)  	B1 B1 for A B1 for B

19 The graph of $y = -4x^2 + 16x - 13$ is drawn on the grid.



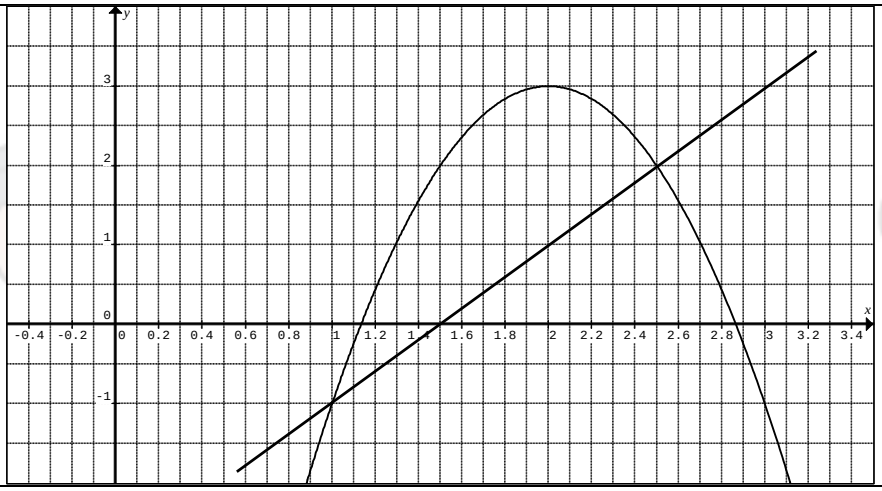
(a) Explain why $y = 3.7$ has no solution. [1]

(b) The points $(1.5, 2)$ and $(2.5, 2)$ are the intersection points for this curve and another curve, $y = ax^2 + bx + c$. Given that $a > 0$, find a possible equation for this second curve. [1]

(c) The equation $-2x^2 + 7x - 5 = 0$ can be solved by adding a straight line to the grid above. Find the equation of this line. [1]

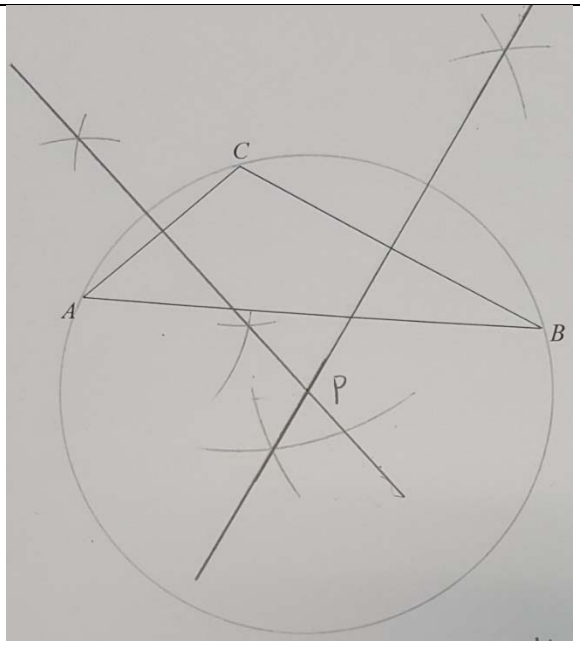
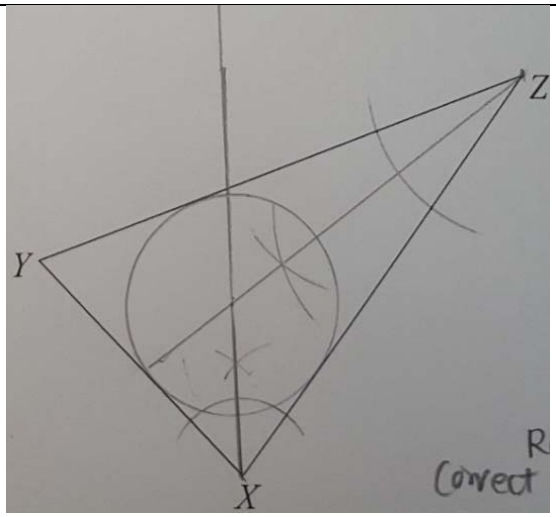
(d) By drawing this straight line, solve the equation $-2x^2 + 7x - 5 = 0$. [3]

19a	The maximum value for y is 3. Hence there is no solution for $y = 3.7$.	B1
19b	The solutions $x = 1.5$ and $x = 2.5$ come from the equation $(x-1.5)(x-2.5) = 0$. $(x-1.5)(x-2.5) = 0$ $\left(x-\frac{3}{2}\right)\left(x-\frac{5}{2}\right) = 0$ $(2x-3)(2x-5) = 0$ $4x^2 - 16x + 15 = 0$ When $x = 1.5$, $y = 2$, when $x = 2.5$, $y = 2$ One possible equation is $y = 4x^2 - 16x + 15 + 2$ $y = 4x^2 - 16x + 17$ Or	B1 or

	Reflect the curve $y = -4x^2 + 16x - 13$ in the $x-$axis. The new equation is $y = -\left(-4x^2 + 16x - 13\right)$ The points become $(1.5, -2)$ and $(2.5, -2)$. To get back $(1.5, 2)$ and $(2.5, 2)$, translate the graph by 4 units. The equation becomes $y = -\left(-4x^2 + 16x - 13\right) + 4$ $y = 4x^2 - 16x + 17$	B1
19c	$-2x^2 + 7x - 5 = 0$ $-4x^2 + 14x - 10 = 0$ $-4x^2 + 16x - 10 = 2x$ $-4x^2 + 16x - 13 = 2x - 3$ The equation of the straight line is $y = 2x - 3$	B1
19d		
19(d)	Drawing of the line $y = 2x - 3$. $x = -1$ or $x = 2.5$	M1A1 A1

20 **(a)** Find, by construction, the point *P*, that is equidistant from the points *A*, *B* and *C*. Hence, draw a circle passing through *A*, *B* and *C*. Measure the radius of the circle.

(b) Find, by construction, the point *Q*, that is equidistant from the lines *XY, YZ* and *XZ*. Hence, or otherwise, draw a circle that is tangent to the lines *XY, YZ* and *XZ*. Measure the radius of the circle.

20a	 Radius = 4.2 ± 0.1 cm Correct construction of two perpendicular bisectors. Circle passing through *A*, *B* and *C*.	B1 B1 B1
20b	 Radius = 1.3 ± 0.1 cm Correct construction of two angle bisectors Circle tangent to *XY*, *XZ* and *YZ*.	B1 B1 B1

Name _____ ( )	Class 4 _____

ANGLICAN HIGH SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY FOUR

S4

MATHEMATICS **4048/02**
Paper 2 **Thursday 13 September 2018**
2 hours 30 minutes

Additional Materials
Writing Paper × 7
Graph Paper × 1

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10	11
Marks											

Table of Penalties		Qn. No.		
Presentation	–1			
Units	–1			
Significant Figures	–1		Parent's/ Guardian's Name/ Signature/ Date	100

This question paper consists of 12 printed pages.

Mathematical Formulae

Compound Interest

Total amount = $P\left(1+\frac{r}{100}\right)^n$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

Statistics

Mean = $\frac{\sum fx}{\sum f}$

Standard deviation = $\sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$

1 **(a)** Express as a single fraction in its simplest form $\frac{3x}{2-3y}+\frac{6x}{9y^2-4}$. [2]

(b) Simplify $\frac{21p^2q^3r^0}{2r^5}\div\frac{7pq}{4p^2}$, leaving your answer in positive indices. [2]

(c) **(i)** Simplify $\frac{6x^2-x-12}{3x^2-11x-20}$. [2]

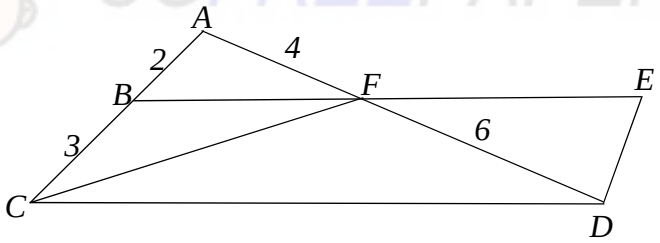
(ii) Hence, or otherwise, solve $\frac{6x^2-x-12}{3x^2-11x-20}=3$. [2]

---

2 **(a)** Each exterior angle of a regular polygon is $24°$. Find the number of sides of the polygon. [1]

(b) Interior angles of a hexagon are $(x+20)°, 120°, 53°, (2x-24)°, 3x°$ and $17°$. Find the value of x. [2]

(c) In the diagram, $AB = 2$ cm, $BC = 3$ cm, $AF = 4$ cm and $FD = 6$ cm.



(i) Show that ΔACD is similar to ΔABF. [2]

(ii) Explain why BE is parallel to CD. [2]

(iii) Given that the area of ΔABF is 20 cm^2, calculate the area of $BCDF$. [2]

---

3 A wardrobe has 3 white, 1 black and 2 pink shirts. Two shirts are drawn at random, one after another, without replacement.

(a) Draw the possibility diagram to show the outcome of the draw. [2]

(b) Find, as a fraction in its simplest form, the probability that

(i) both shirts are white, [1]

(ii) both shirts are of different colours, [1]

(iii) at least one of the shirts is pink. [1]

---

4 **Answer the whole of this question on a sheet of graph paper.**

A population of flies increases according to the formula

$$N = 30 \times 2^t$$

where N is the population of flies after t days.
The table shows some corresponding values of variables N and t.

t	1	1.5	2	2.5	3	3.5	4
N	60	84.9	120	k	240	339	480

(a) Find the value of k. [1]

(b) Determine the initial number of flies. [1]

(c) Using a scale of 2 cm to represent 1 unit, draw a horizontal scale for $0 \le t \le 4$.
Using a scale of 2 cm to represent 100 units, draw a vertical scale for $0 \le N \le 500$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

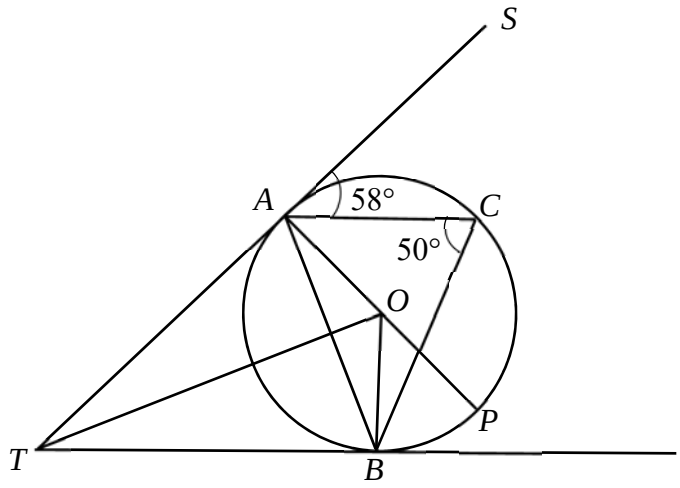
(d) Use your graph to determine the time when the population reaches 250. [1]

(e) By drawing a tangent, find the gradient of the curve at $t = 2$. Explain what this gradient represents. [2]

(f) Use your graph to determine the time when the population is increasing at 200 flies per day. [2]

---

5 In the diagram, O is the centre of the circle. SAT and BT are tangents to the circle. AP is the diameter. $\angle SAC = 58°$ and $\angle ACB = 50°$.



(a) Show that triangle AOT is congruent to triangle BOT. [2]

(b) Find

(i) $\angle CAO$, [1]

(ii) $\angle AOB$, [1]

(iii) $\angle BAO$, [1]

(iv) $\angle ATB$, [1]

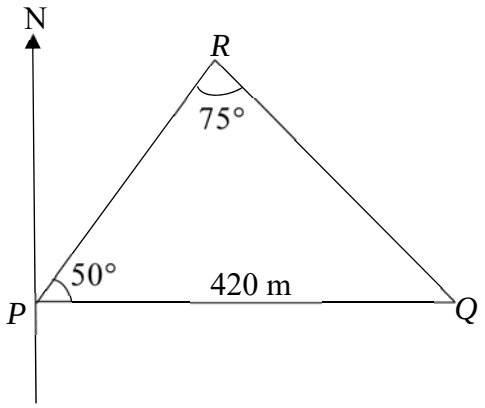
(v) $\angle OBC$, [2]

(vi) $\angle OPB$. [1]

Show your working and give reasons.

(c) A point D is such that $ACBD$ is a quadrilateral where $\angle ADB = 130°$.
Determine whether D lies on the circumference of the circle. [1]

6 In the diagram below, not drawn to scale, P, Q and R are on level ground and Q is due east of P. $PQ = 420$ m, $\angle RPQ = 50°$ and $\angle PRQ = 75°$.



(a) Find the distance PR. [2]

(b) A flag pole is erected at point P such that the angle of depression from the top of the flag pole to point R is $1.93°$. Calculate the height of the flag pole. [2]

(c) **(i)** Find the area of ΔPQR. [1]

(ii) Hence, find the shortest distance from R to PQ. [2]

(d) The bearing of point S from point P is $120°$. Given $PS = 200$ m, find

(i) angle RPS, [1]

(ii) the distance RS. [2]

7 Peter bought m kiwi fruits for \$64.

(a) Find an expression, in terms of m, for the cost of one kiwi fruit in cents. [1]

(b) Given that 5 of the kiwi fruits were rotten and could not be sold, Peter sold each remaining kiwi fruit at 40 cents more than he paid for it.

Without simplifying, write down an expression in terms of m, for the total amount he received from the sale of the kiwi fruits. [1]

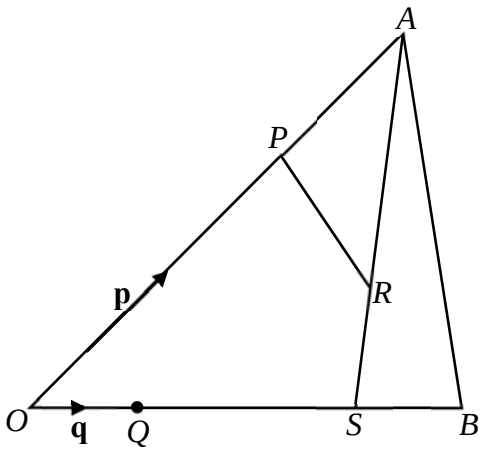
(c) He made a profit of \$26 from the sale of the kiwi fruits.
Write down an equation in m to represent this information, and show that it reduces to $m^2 - 70m - 800 = 0$. [3]

(d) Solve the equation $m^2 - 70m - 800 = 0$. [3]

(e) Find the selling price of a kiwi fruit. [2]

---

8 In the diagram below, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$. It is given that $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OA}$, $\overrightarrow{OQ} = \frac{1}{3}\overrightarrow{OS}$, $OQ = SB$ and $SA = 3SR$.



(a) Express, as simply as possible, in terms of **p** and **q**,

(i) $\overrightarrow{SA}$, [1]

(ii) $\overrightarrow{PB}$, [1]

(iii) $\overrightarrow{PR}$. [2]

(b) Prove that P, R and B are collinear. [2]

(c) Find the numerical value of

(i) $\dfrac{\text{Area of } \Delta APR}{\text{Area of } \Delta ARB}$, [1]

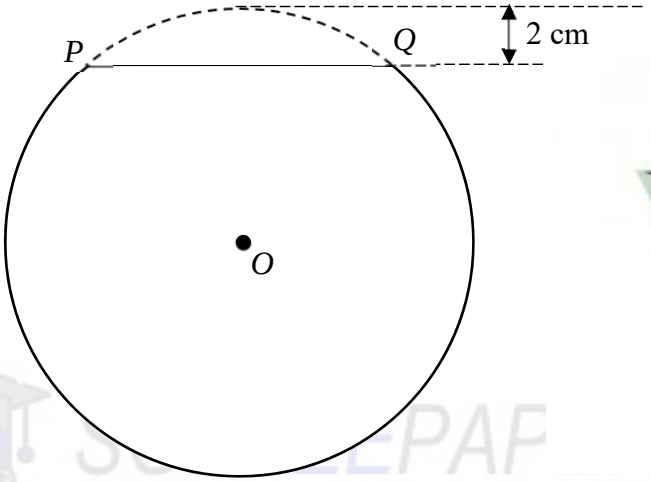
(ii) $\dfrac{\text{Area of } \Delta RSB}{\text{Area of } \Delta APR}$. [2]

9 **(a)** The diagram below shows a circle with centre O. The major arc PQ is 30.4 cm. Given that the straight line PQ is 9.4 cm and the minor segment has a vertical height of 2 cm from the centre of line PQ.

(i) Show that the radius of the circle is 6.5225 cm. [3]

(ii) Calculate the reflex angle POQ. [2]

(iii) Find the area of the minor segment POQ. [2]



(b) The diagram in **(a)** is a 2-dimensional view of the body of a teapot with the minor segment being the lid of the teapot.

(i) The volume of the teapot can be calculated using the formula,

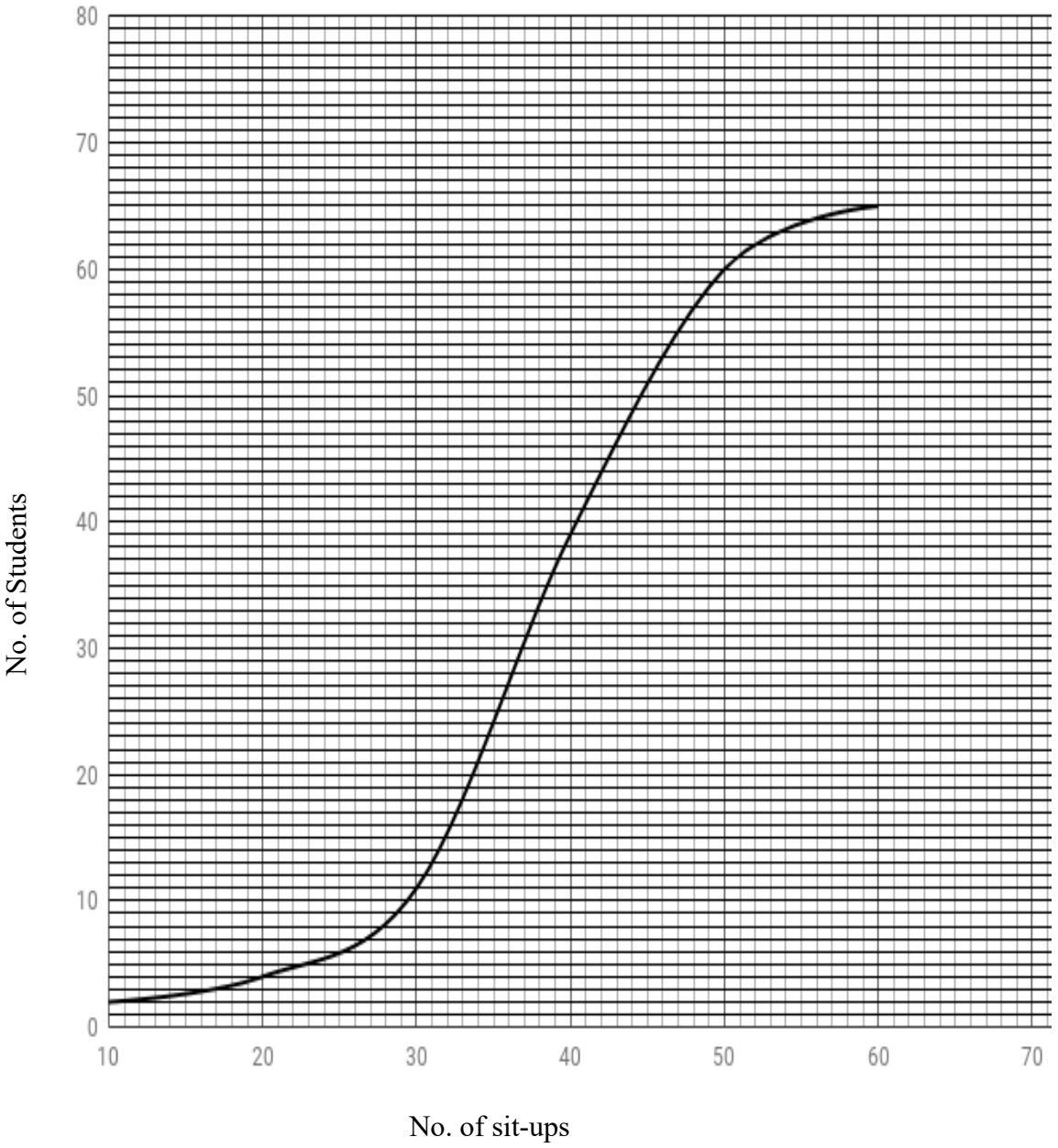
$$V = \frac{\pi}{6} h\left(3c^2 + h^2\right) .$$

h is the vertical height of the teapot measured from the bottom to the opening and passing through the centre, O.
c is radius of the top opening of the teapot.

Calculate the volume of the teapot. [3]

(ii) How many 250ml teacups are needed to contain all the tea in the teapot in part **(b)(i)** given that each cup should be only 60% full? [2]

10 One of the NAPFA test stations is to record the number of sit-ups that each student can do in one minute. The cumulative frequency curve below shows the number of sit-ups by a group of 65 students.



(a) By using the cumulative frequency curve, find the value of a and of b. [2]

Number of sit-ups	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Number of students	2	a	7	28	21	b

10 **(b)** Find the percentage of students who did between 35 and 42 sit-ups inclusive. [1]

(c) Calculate an estimate of

(i) the mean number of sit-ups, [1]

(ii) the standard deviation. [1]

(d) The number of sit-ups by a second group of 65 students is recorded in the table shown below.

Number of sit-ups	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Number of students	12	10	7	14	12	5

(i) Given that the standard deviation for the second group of students is about 16.3, explain briefly which group is more consistent in their performance. [1]

(ii) If the two groups were compared, explain whether the mean or median would be a better measure of central tendency. [1]

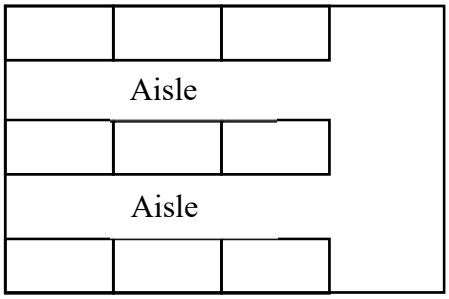
11 For a Parent Teacher Conference, a school has to convert the parade square into a parking lot. The parade square is a rectangular plot of land 40 m by 30 m.

ENTRANCE / EXIT

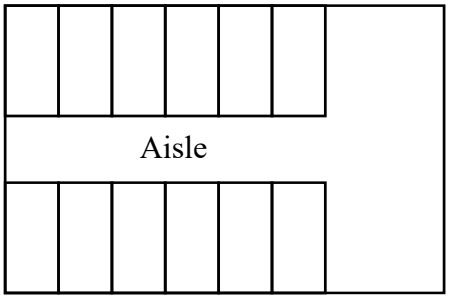
There is an 6 m wide entrance / exit at one corner of the parade square as shown in the sketch above.
You are required to do the parking arrangements.

The Land Transport Authority recommends the following guidelines.
A **Parking Stall** refers to the space for parking of one motorcar, that is, a car parking lot. The space of the stall should be rectangular. The longer side is known as the length and the shorter side is the width.
A **Parking Aisle** refers to an access lane or driveway with adjacent parking stalls.

In parallel parking, the longer side is parallel to the parking aisle or driveway. The aisle for cars to move must be at least 3.6 m.
For two-way traffic flow, the width of the aisle must be at least 6 m.
Each parking stall is 5.4 m by 2.4 m.



In 90° parking, the longer side is perpendicular to the parking aisle or driveway.
The aisle or lane for cars to move must be at least 6 m for one-way traffic flow and at least 6.6 m for two-way traffic flow.
Each parking stall is 4.8 m by 2.4 m.



Propose a possible parking arrangement that would maximise the use of space, showing your calculations clearly. Your proposal must include a sketch, not drawn to scale, indicating the location of the parking stalls, the aisles and the type(s) of parking. You should allow for cars to enter and leave the parade square at any time. You can assume that the cars will not leave in large numbers at any one time, and the parking will be supervised by security guards. [10]

End of Paper

Answer key

1

(a) $\dfrac{9xy}{(2+3y)(2-3y)}$ (b) $\dfrac{6p^3q^2}{r^5}$

(c)(i) $\dfrac{2x-3}{x-5}$ (ii) $x=12$

2

(a) $n=15$
(b) $x=89$
(c)(ii)
They are corresponding angles to lines BE and CD, hence BE and CD are parallel
(iii) 105cm^2

3(a)

Colours	W_1	W_2	W_3	B	P_1	P_2
W_1		W_2, W_1	W_3, W_1	B, W_1	P_1, W_1	P_2, W_1
W_2	W_1, W_2		W_3, W_2	B, W_2	P_1, W_2	P_2, W_2
W_3	W_1, W_3	W_2, W_3		B, W_3	P_1, W_3	P_2, W_3
B	W_1, B	W_2, B	W_3, B		P_1, B	P_2, B
P_1	W_1, P_1	W_2, P_1	W_3, P_1	B, P_1		P_2, P_1
P_2	W_1, P_2	W_2, P_2	W_3, P_2	B, P_2	P_1, P_2	

3(b)(i) $\dfrac{1}{5}$
(ii) $\dfrac{11}{15}$
(iii) $\dfrac{3}{5}$

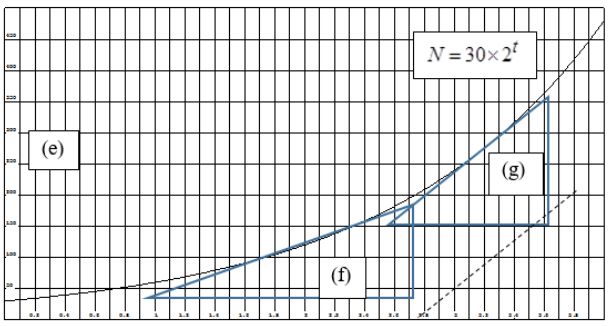
4

(a) 170 (b) 30 (d) $2.9 \le t \le 3.15$

(e) $78 \le$ gradient ≤ 85. At $t=2$, the flies are increasing at 83 flies per day.

(f) $2.8 \le t \le 3.8$

4(c)



5(b) (i) 32° (ii) 100° (iii) 40° (iv) 80° (v) 18° (vi) 50°

5(c) By angles in opposite segment property, D is a point on the circle.

6

(a) 356 m (b)12.0 m
(c)(i) 57300m^2 **(ii)** 273m

(d)(i) $\angle RPS = 80°$ **(ii)** 377m

7

(a) $\dfrac{6400}{m}$ cents (b) $(m-5)\left(\dfrac{6400}{m}+40\right)$ cents

(d) $m=80$ *or* $m=-10$

(e) 120 cents or \$1.20

8

(a)(i) $-3\mathbf{q}+\dfrac{3}{2}\mathbf{p}$ **(ii)** $-\mathbf{p}+4\mathbf{q}$

(iii) $2\mathbf{q}-\dfrac{1}{2}\mathbf{p}$

(b)
Since $\overrightarrow{PR}=\dfrac{1}{2}\overrightarrow{PB}$,
$\therefore P, R$ and B are collinear.

(c) (i) 1 (ii) $\dfrac{1}{2}$

9

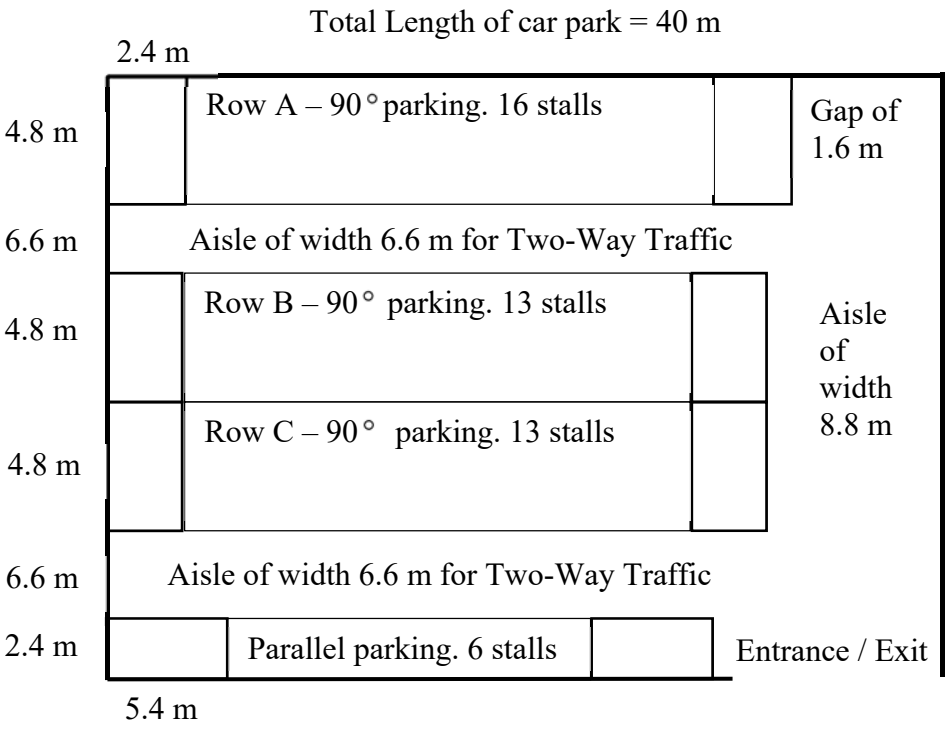
(a) (ii) 4.66 rad or 267.0° (iii) 13.3 cm^2
(b)(i) $\approx 1090\,\text{cm}^3$ (ii) 8

10

(a) $a=2, b=5$
(b) 30.8%
(c) (i) 37.2 (ii) 10.5
(d) (i) Since first group has a smaller standard deviation, so the students in first group are more consistent in their performance.
(ii) The median is a better measure due to the outliers.

Q11

Possible Solution



B1 – sketch is clearly labelled
B1 – layout of parking stalls using parallel parking
B1 – layout of parking stalls using 90° parking
B1 – layout of aisles clearly showing that cars can move in or out at any time.
B1 – aisles are at appropriate width, either 6 m for one-way or 6.6 m for two-way.

	Number of lots in Row A $= \frac{40}{2.4} = 16\frac{2}{3} \approx 16$ Number of lots in Row B or C $= \frac{40-6.6}{2.4} = 13\frac{11}{12} \approx 13$ Length of Row A $= 16 \times 2.4 = 38.4 \text{ m} < 40 \text{ m}$ Length of Row B $= 13 \times 2.4 = 31.2 \text{ m} < 40 \text{ m}$ Length of Row C $= 13 \times 2.4 = 31.2 \text{ m} < 40 \text{ m}$ Number of lots for parallel parking $= \frac{40-6}{5.4} = 6\frac{8}{27} \approx 6$ Length of parallel parking $= 6 \times 5.4 = 32.4 \text{ m} < 36 \text{ m}$	Considering the length M1 A1
	Width of the rows of parking stalls and aisles $= 4.8 \times 3 + 2.4 + 2 \times 6.6$ $= 30 \text{ m}$	Considering the width M1 A1
	Total number of parking stalls $= 16 + 2 \times 13 + 6 = 48$	A1 – Accept minimum as 42
	Remarks – Students who consider only parallel parking or 90° parking but not both get a maximum of 8 marks.	

1 **(a)** Express as a single fraction in its simplest form $\frac{3x}{2-3y}+\frac{6x}{9y^2-4}$. [2]

(b) Simplify $\frac{21p^2q^3r^0}{2r^5}\div\frac{7pq}{4p^2}$, leaving your answer in positive indices. [2]

(c) **(i)** Simplify $\frac{6x^2-x-12}{3x^2-11x-20}$. [2]

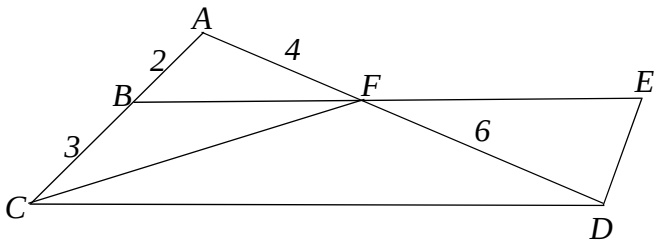
(ii) Hence, or otherwise, solve $\frac{6x^2-x-12}{3x^2-11x-20}=3$. [2]

1(a)	$\frac{3x}{2-3y}+\frac{6x}{9y^2-4}=\frac{3x}{2-3y}-\frac{6x}{4-9y^2}$ $=\frac{3x}{2-3y}-\frac{6x}{(2+3y)(2-3y)}$ $=\frac{3x(2+3y)-6x}{(2+3y)(2-3y)}$ $=\frac{9xy}{(2+3y)(2-3y)}$	M1 A1
1(b)	$\frac{21p^2q^3r^0}{2r^5}\div\frac{7pq}{4p^2}=\frac{21p^2q^3}{2r^5}\times\frac{4p^2}{7pq}$ $=\frac{3p^2q^3}{r^5}\times\frac{2p}{q}$ $=\frac{6p^3q^2}{r^5}$	M1 A1
1(c)(i)	$\frac{6x^2-x-12}{3x^2-11x-20}=\frac{(3x+4)(2x-3)}{(3x+4)(x-5)}$ $=\frac{2x-3}{x-5}$	M1 A1
1(c)(ii)	$\frac{6x^2-x-12}{3x^2-11x-20}=3$ $\frac{2x-3}{x-5}=\frac{3}{1}$ $2x-3=3x-15$ $x=12$	 M1 A1

2 **(a)** Each exterior angle of a regular polygon is $24°$. Find the number of sides of the polygon. [1]

(b) Interior angles of a hexagon are $(x+20)°, 120°, 53°, (2x-24)°, 3x°$ and $17°$. Find the value of x. [2]

(c) In the diagram, $AB = 2$ cm, $BC = 3$ cm, $AF = 4$ cm and $FD = 6$ cm.



(i) Show that ΔACD is similar to ΔABF. [2]

(ii) Explain why BE is parallel to CD. [2]

(iii) Given that the area of $\Delta\ ABF$ is 20 cm^2, calculate the area of $BCDF$. [2]

2(a)	Let n be the number of sides $24n = 360$ $n = 15$	 B1
2(b)	Total interior of hexagon $= (6-2)\times 180° = 720°$ $x + 20° + 120° + 53° + 2x - 24° + 3x + 17° = 720°$ $6x + 186 = 720$ $6x = 534$ $x = 89$	M1 A1
2(c)(i)	$\angle CAD = \angle BAF$ (*common angle*) $\frac{AC}{AB} = \frac{AD}{AF} = \frac{5}{2}$ By SAS Similarity Test, ΔACD is similar to ΔABF.	 M1 A1
2(c)(ii)	Since $\Delta\ ACD$ is similar to $\Delta\ ABF$, $\angle ABF = \angle ACD$ They are corresponding angles to lines BE and CD, hence BE and CD are parallel	 M1 A1
2(c)(iii)	Area of triangle $ACD = \left(\frac{5}{2}\right)^2 \times 20$ $= 125\ cm^2$ Therefore area of quadrilateral $BCDF = 125 - 20$ $= 105\ cm^2$	 M1 A1

3 A wardrobe has 3 white, 1 black and 2 pink shirts. Two shirts are drawn at random, one after another, without replacement.

(a) Draw the possibility diagram to show the outcome of the draw. [2]

(b) Find, as a fraction in its simplest form, the probability that

(i) both shirts are white, [1]

(ii) both shirts are of different colours, [1]

(iii) at least one of the shirts is pink. [1]

3(a)		B2 Award 1 mark if 5 or more cells are correct .

Colours	**W_1**	**W_2**	**W_3**	**B**	**P_1**	**P_2**
W_1		W_2, W_1	W_3, W_1	B, W_1	P_1, W_1	P_2, W_1
W_2	W_1, W_2		W_3, W_2	B, W_2	P_1, W_2	P_2, W_2
W_3	W_1, W_3	W_2, W_3		B, W_3	P_1, W_3	P_2, W_3
B	W_1, B	W_2, B	W_3, B		P_1, B	P_2, B
P_1	W_1, P_1	W_2, P_1	W_3, P_1	B, P_1		P_2, P_1
P_2	W_1, P_2	W_2, P_2	W_3, P_2	B, P_2	P_1, P_2	

o.e.

3(bi)	P (both shirts are white) $= \frac{3}{6} \times \frac{2}{5} = \frac{1}{5}$ Or read directly from the table: P (both shirts are white) $= \frac{6}{30} = \frac{1}{5}$	B1
3(bii)	P (both shirts are of different colours) = 1 – P (both shirts are white or pink) $= 1 - \left(\frac{1}{5} + \frac{2}{6} \times \frac{1}{5}\right) = \frac{11}{15}$ Or read directly from the table: P (both shirts are of different colours) $= \frac{22}{30} = \frac{11}{15}$	B1
3(biii)	P (at least one of the shirts is pink) = 1 – P (no shirts are pink) $= 1 - \left(\frac{4}{6} \times \frac{3}{5}\right) = \frac{3}{5}$ Or read directly from the table: P (at least one of the shirts is pink) = $= \frac{18}{30} = \frac{3}{5}$	B1

4 **Answer the whole of this question on a sheet of graph paper.**

A population of flies increases according to the formula

$$N = 30 \times 2^t$$

where N is the population of flies after t days.

The table shows some corresponding values of variables N and t .

t	1	1.5	2	2.5	3	3.5	4
N	60	84.9	120	k	240	339	480

(a) Find the value of k. [1]

(b) Determine the initial number of flies. [1]

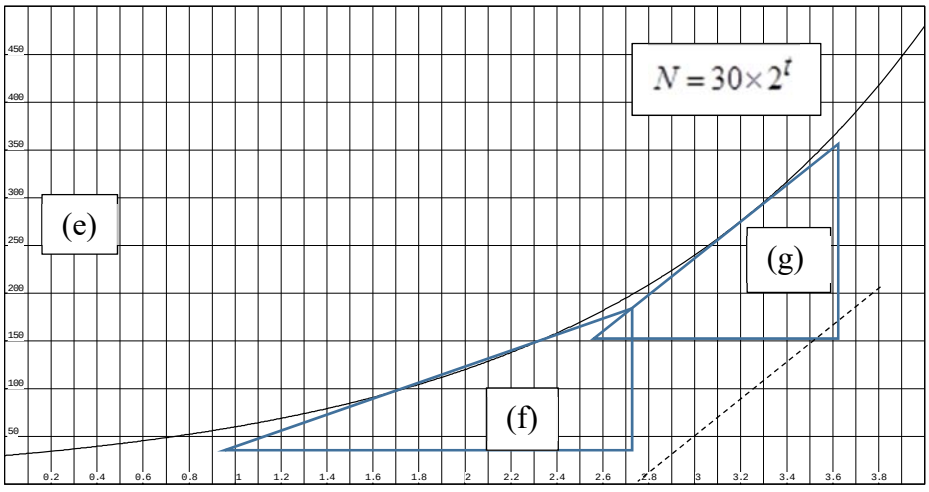
(c) Using a scale of 2 cm to represent 1 unit, draw a horizontal scale for $0 \le t \le 4$.
Using a scale of 2 cm to represent 100 units, draw a vertical scale for $0 \le N \le 500$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

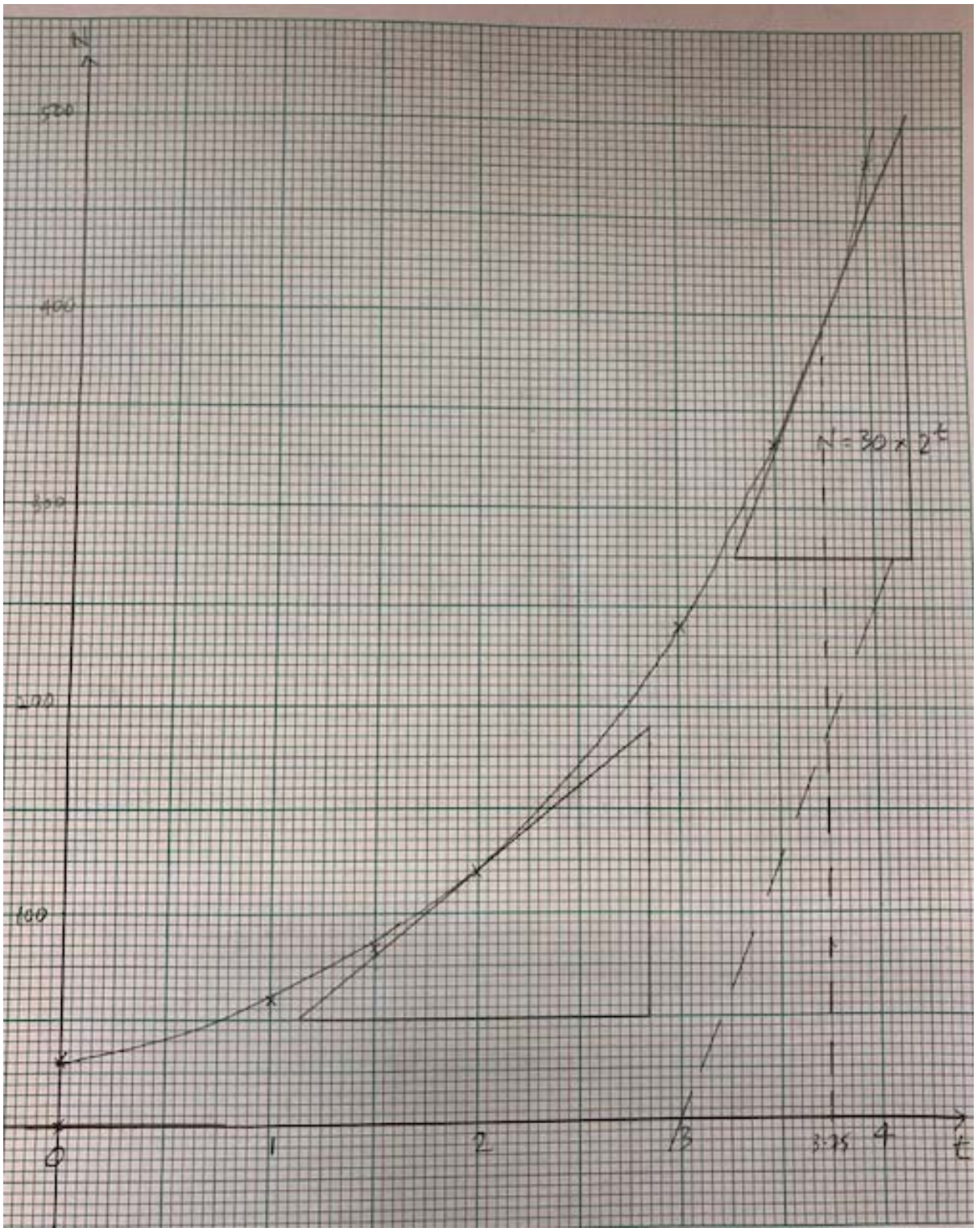
(d) Use your graph to determine the time when the population reaches 250. [1]

(e) By drawing a tangent, find the gradient of the curve at $t = 2$. Explain what this gradient represents. [2]

(f) Use your graph to determine the time when the population is increasing at 200 flies per day. [2]

4(a)	$k = 30 \times 2^{2.5} = 169.706 = 170$	B1
4(b)	When $t = 0$, $N = 30 \times 2^0 = 30$ Initial number of flies is 30.	B1
4(c)	B1 – correct plotting B1 – smooth curve B1 – labelling and correct scale	
4(d)	$2.9 \le t \le 3.15$	B1
4(e)	$78 \le$ gradient ≤ 85. At $t = 2$, the flies are increasing at 83 flies per day.	B1 – tangent B1 - statement
4(f)	$2.8 \le t \le 3.8$	B1- tangent with gradient 200 B1 - value





5 In the diagram, O is the centre of the circle. SAT and BT are tangents to the circle. AP is the diameter. $\angle SAC = 58°$ and $\angle ACB = 50°$.

(a) Show that triangle AOT is congruent to triangle BOT. [2]

(b) Find

(i) $\angle CAO$, [1]

(ii) $\angle AOB$, [1]

(iii) $\angle BAO$, [1]

(iv) $\angle ATB$, [1]

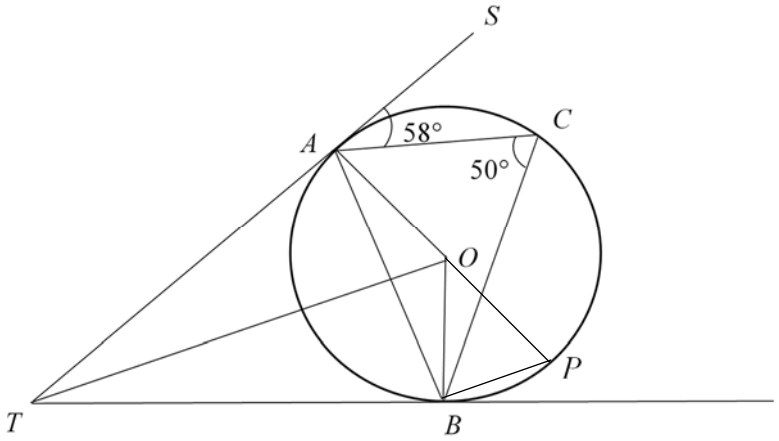
(v) $\angle OBC$, [2]

(vi) $\angle OPB$. [1]

Show your working and give reasons.

(c) A point D is such that $ACBD$ is a quadrilateral where $\angle ADB = 130°$.

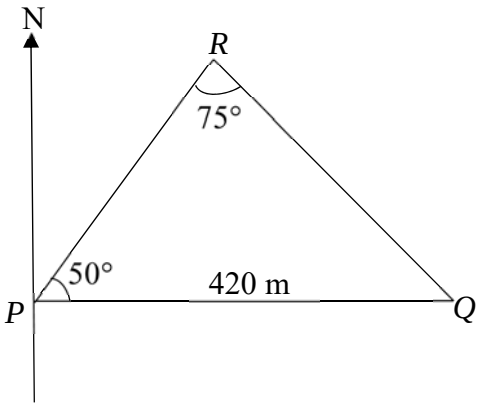
Determine whether D lies on the circumference of the circle. [1]



5(a)	$AO = OB$(radii of circle) $\angle TAO = \angle TBO = 90°$(radius perpendicular to tangent) o.e. $OT = OT$(common side, hypotenuse) By RHS, $\Delta\ AOT$ is congruent to $\Delta\ BOT$.	M1 A1
5(b)(i)	$\angle CAO = 90° - 58°$(radius perpendicular to tangent) $= 32°$	B1
5(b)(ii)	obtuse $\angle AOB = 50° \times 2$($\angle$ at centre $= 2 \times \angle$ at circumference) $= 100°$	B1
5(b)(iii)	$\angle BAO = \frac{180° - 100°}{2}$(isosceles ΔAOB) $= 40°$	B1

5(b)(iv)	$\angle ATB = 360° - \angle TAO - \angle TBO - \text{obtuse}\angle AOB$ $= 360° - 90° - 90° - 100°$(property ofquadrilateral ATBO) $= 80°$ *OR* $\angle AOT = 100° \div 2$ $= 50°$ $\angle ATO = 180° - 90° - 50°$(sum of angles of triangle) $\angle ATB = 40° \times 2$ $= 80°$	B1
5(b)(v)	$\angle ACO = \angle CAO = 32°$(isosceles ΔAOC) $\angle OCB = \angle ACB - \angle ACO$ $= 50° - 32°$ $= 18°$ $\angle OBC = \angle OCB$(isosceles ΔCOB) $= 18°$	M1 A1
5(b)(vi)	$\angle OPB = 50°$($\angle$s in the same segment)	B1
5(c)	By angles in opposite segment property, *D* is a point on the circle. Deduct 1 mark from the whole of question for not stating reason or incorrect reason.	B1

6 In the diagram below, not drawn to scale, *P*, *Q* and *R* are on level ground and *Q* is due east of *P*. $PQ = 420$ m, $\angle RPQ = 50°$ and $\angle PRQ = 75°$.



(a) Find the distance *PR*. [2]

(b) A flag pole is erected at point *P* such that the angle of depression from the top of the flag pole to point *R* is $1.93°$. Calculate the height of the flag pole. [2]

(c) **(i)** Find the area of ΔPQR. [1]

(ii) Hence, find the shortest distance from R to PQ. [2]

The bearing of point S from point P is $120°$. Given $PS = 200$ m, find

(d) **(i)** angle RPS [1]

(ii) the distance RS. [2]

6(a)	$\angle RQP = 180° - 75° - 50° = 55°$ $\frac{\sin 55°}{PR} = \frac{\sin 75°}{420}$ $PR = 356.18$ ≈ 356 m	M1 A1
6(b)	$\tan 1.93° = \frac{\text{height of flagpole}}{356.18}$ height of flagpole $= 12.002$ ≈ 12.0m	M1 A1
6(c)(i)	Area of $\Delta PQR = \frac{1}{2}(356.18)(420)\sin 50°$ $= 57298.4$ $\approx 57300\text{m}^2$	 B1
6(c)(ii)	$\frac{1}{2}(420)h = 57298.4$ $h = 272.85$ $h \approx 273$m	M1 A1
6(d)(i)	$\angle RPS = 80°$	B1
6(d)(ii)	$RS^2 = 356.18^2 + 200^2 - 2(356.18)(200)\cos 80°$ $RS^2 = 142124.19$ $RS = 376.99$ $RS \approx 377$m	M1 A1

7 Peter bought m kiwi fruits for \$64.

(a) Find an expression, in terms of m, for the cost of one kiwi fruit in cents. [1]

(b) Given that 5 of the kiwi fruits were rotten and could not be sold, Peter sold each remaining kiwi fruit at 40 cents more than he paid for it.
Without simplifying, write down an expression in terms of m, for the total amount he received from the sale of the kiwi fruits. [1]

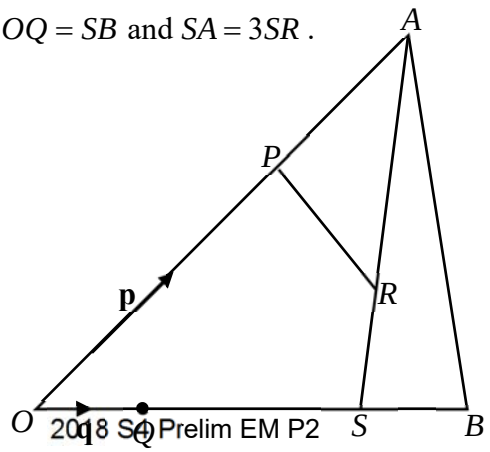
(c) He made a profit of \$26 from the sale of the kiwi fruits.
Write down an equation in m to represent this information, and show that it reduces to $m^2 - 70m - 800 = 0$. [3]

(d) Solve the equation $m^2 - 70m - 800 = 0$. [3]

(e) Find the selling price of a kiwi fruit. [2]

7(a)	Cost of each kiwi fruit = $\frac{6400}{m}$ cents	B1
7(b)	Total sum received = $(m-5)\left(\frac{6400}{m}+40\right)$ cents	B1
7(c)	$(m-5)\left(\frac{6400}{m}+40\right)-6400=2600$ $6400+40m-\frac{32000}{m}-200-6400-2600=0$ $40m-2800-\frac{32000}{m}=0$ $40m^2-2800m-32000=0$ $m^2-70m-800=0$	M1form equations M1expansion A1
7(d)	$m^2-70m-800=0$ $(m-80)(m+10)=0$ $m=80 \text{ or } m=-10$	M1 factorisation A2
7(e)	Selling price of each kiwi = $\frac{6400}{80}+40$ $=120$ cents or \$1.20	M1 A1

8 In the diagram below, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$. It is given that $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OA}$, $\overrightarrow{OQ} = \frac{1}{3}\overrightarrow{OS}$, $OQ = SB$ and $SA = 3SR$.



(a) Express, as simply as possible, in terms of **p** and **q**,

(i) $\overrightarrow{SA}$, [1]

(ii) $\overrightarrow{PB}$, [1]

(iii) $\overrightarrow{PR}$. [2]

(b) Prove that P, R and B are collinear. [2]

(c) Find the numerical value of

(i) $\dfrac{\text{Area of } \Delta APR}{\text{Area of } \Delta ARB}$, [1]

(ii) $\dfrac{\text{Area of } \Delta RSB}{\text{Area of } \Delta APR}$. [2]

8(a)(i)	$\overrightarrow{SA} = \overrightarrow{SO} + \overrightarrow{OA}$ $= -3\mathbf{q} + \frac{3}{2}\mathbf{p}$	 B1
8(a)(ii)	$\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB}$ $= -\mathbf{p} + 4\mathbf{q}$	B1
8(a)(iii)	$\overrightarrow{PA} = \frac{1}{2}\mathbf{p}$ $\overrightarrow{AR} = \frac{2}{3}\overrightarrow{AS}$ $= -\frac{2}{3}\left(-3\mathbf{q} + \frac{3}{2}\mathbf{p}\right)$ $= 2\mathbf{q} - \mathbf{p}$ $\overrightarrow{PR} = \frac{1}{2}\mathbf{p} + 2\mathbf{q} - \mathbf{p} = 2\mathbf{q} - \frac{1}{2}\mathbf{p}$	 M1 A1
8(b)	$\overrightarrow{PB} = -\mathbf{p} + 4\mathbf{q}$ $\overrightarrow{PR} = 2\mathbf{q} - \frac{1}{2}\mathbf{p}$ $= \frac{1}{2}(4\mathbf{q} - \mathbf{p})$ $= \frac{1}{2}\overrightarrow{PB}$ Since $\overrightarrow{PR} = \frac{1}{2}\overrightarrow{PB}$, $\therefore P, R$ and B are collinear.	 M1 A1
8(c)(i)	$\dfrac{\text{Area of } \Delta APR}{\text{Area of } \Delta ARB} = 1$	B1

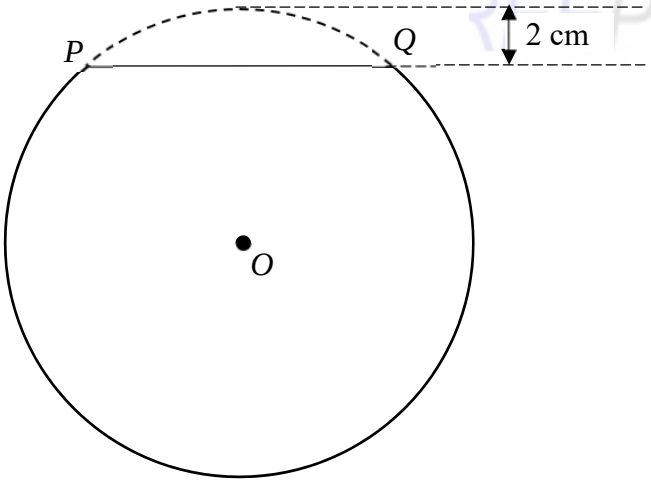
8(c)(ii)	$\frac{\text{Area of } \Delta RSB}{\text{Area of } \Delta ARB} = \frac{1}{2}$ $\frac{\text{Area of } \Delta RSB}{\text{Area of } \Delta APR} = \frac{\text{Area of } \Delta RSB}{\text{Area of } \Delta ARB} \times \frac{\text{Area of } \Delta ARB}{\text{Area of } \Delta APR}$ $= \frac{1}{2} \times \frac{1}{1}$ $= \frac{1}{2}$	M1 A1

9 **(a)** The diagram below shows a circle with centre O. The major arc PQ is 30.4 cm. Given that the straight line PQ is 9.4 cm and the minor segment has a vertical height of 2 cm from the centre of line PQ.

(i) Show that the radius of the circle is 6.5225 cm. [3]

(ii) Calculate the reflex angle POQ. [2]

(iii) Find the area of the minor segment POQ. [2]



(b) The diagram in **(a)** is a 2-dimensional view of the body of a teapot with the minor segment being the lid of the teapot.

(i) The volume of the teapot can be calculated using the formula,

$$V = \frac{\pi}{6} h\left(3c^2 + h^2\right) .$$

h is the vertical height of the teapot measured from the bottom to the opening and passing through the centre, O.

c is radius of the top opening of the teapot.

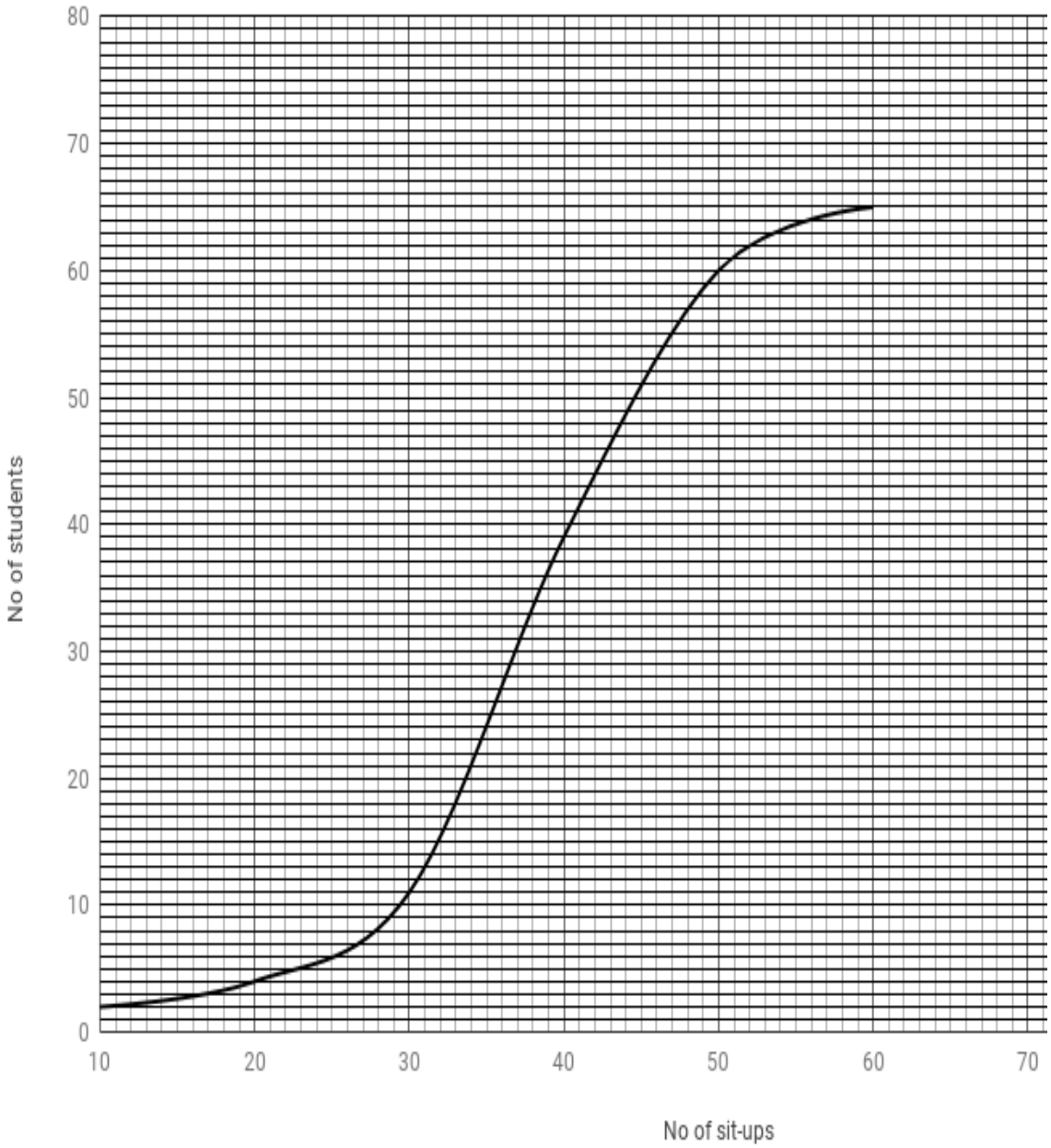
Calculate the volume of the teapot. [3]

(ii) How many 250ml teacups are needed to contain all the tea in the teapot in part **(b)(i)** given that each cup should be only 60% full? [2]

9(ai)	Let the radius of the circle be r cm. $r^2-(r-2)^2=\left(\frac{9.4}{2}\right)^2$ $(r+r-2)(r-r+2)=22.09$ $2(2r-2)=22.09$ $4(r-1)=22.09$ $r=\frac{22.09}{4}+1$ $r=6.5225$ The radius is 6.5225 cm.	M1 forming equation M1 simplify equation A1
9(aii)	$6.5225\times\angle POQ=30.4$ $\angle POQ=\frac{30.4}{6.5225}$ $\angle POQ\approx 4.66078$ rad $\angle POQ\approx 4.66$ rad Or $\frac{\angle POQ}{360°}\times 2\times 6.5225\times\pi=30.4$ $\angle POQ=\frac{30.4\times 360°}{2\times 6.5225\times\pi}$ $\angle POQ\approx 267.043°$ $\angle POQ\approx 267.0°$	M1 A1
9(aiii)	Area of the minor segment POQ $=\frac{1}{2}(6.5225)^2\left[(2\pi-4.66078)-\sin(2\pi-4.66078)\right]$ ≈ 13.268 cm^2 ≈ 13.3 cm^2 Or Area of the minor segment POQ $=\frac{360°-267.043°}{360°}\times\pi\times(6.5225)^2-\frac{1}{2}(6.5225)^2\times\sin(360°-267.043°)$ ≈ 13.268 cm^2 ≈ 13.3 cm^2	M1 A1

9(bi)	$h = 2 \times .65225 - 2$ $h = 11.045\,\text{cm}$ Volume of the teapot $= \frac{\pi}{6}(11.045)\left[3\left(\frac{9.4}{2}\right)^2 + 11.045^2\right]$ ≈ 1088.747 $\approx 1090\,\text{cm}^3$	M1 M1 A1
9(bii)	Number of teacups needed $= \frac{1088.747}{0.6 \times 250}$ ≈ 7.2583 ≈ 8	M1 A1

10 One of the NAPFA test station is to record the number of sit-ups that each student can do in one minute. Below shows the cumulative frequency curve of the number of sit-ups by a group of 65 students.



(a) By using the cumulative frequency curve, find the value of a and of b. [2]

Number of sit-ups	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Number of students	2	a	7	28	21	b

(b) Find the percentage of students who did between 35 and 42 sit-ups inclusive. [1]

(c) Calculate an estimate of

(i) the mean number of sit-ups, [1]

(ii) the standard deviation. [1]

(d) The number of sit-ups by a second group of 65 students is recorded in the table shown below.

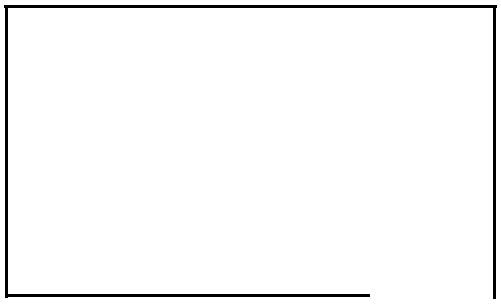
Number of sit-ups	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Number of students	12	10	7	14	12	5

(i) Given that the standard deviation for the second group of students is about 16.3, explain briefly which group is more consistent in their performance. [1]

(ii) If the two groups were compared, explain whether the mean or median would be a better measure of central tendency. [1]

10(a)	$a = 2, b = 5$	B2
10(b)	Percentage of students $= \frac{44-24}{65} \times 100\% = 30.8\%$	B1
10(c)(i)	Mean number of sit-ups $= 37\frac{2}{13}$ or $\approx 37.154 \approx 37.2$	B1
10(c)(ii)	Standard deviation $= = \sqrt{\frac{96825}{65} - \left(37\frac{2}{13}\right)^2}$ ≈ 10.4502 ≈ 10.5	 B1
10(d)(i)	Since first group has a smaller standard deviation, so the students in first group are more consistent in their performance.	B1
10(d)(ii)	The median is a better measure due to the outliers. Note: Award mark as long as the description is similar to outlier. No mark at all if students compare mean.	B1

11 For a Parent Teacher Conference, a school has to convert the parade square into a parking lot. The parade square is a rectangular plot of land 40 m by 30 m.



ENTRANCE / EXIT

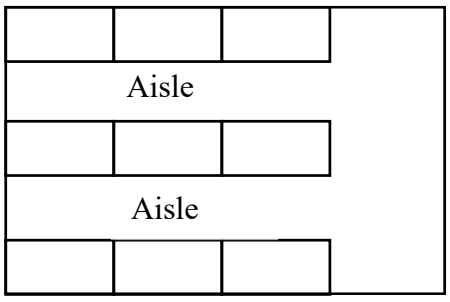
There is an 6 m wide entrance / exit at one corner of the parade square as shown in the sketch above.
You are required to do the parking arrangements.

The Land Transport Authority recommends the following guidelines.

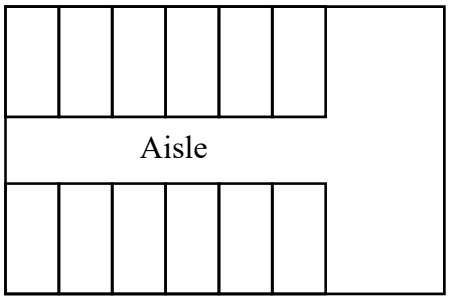
A Parking Stall refers to the space for parking of one motorcar, that is, a car parking lot. The space of the stall should be rectangular. The longer side is known as the length and the shorter side is the width.

A Parking Aisle refers to an access lane or driveway with adjacent parking stalls.

In parallel parking, the longer side is parallel to the parking aisle or driveway.
The aisle for cars to move must be at least 3.6 m. For two-way traffic flow, the width of the aisle must be at least 6 m. Each parking stall is 5.4 m by 2.4 m.

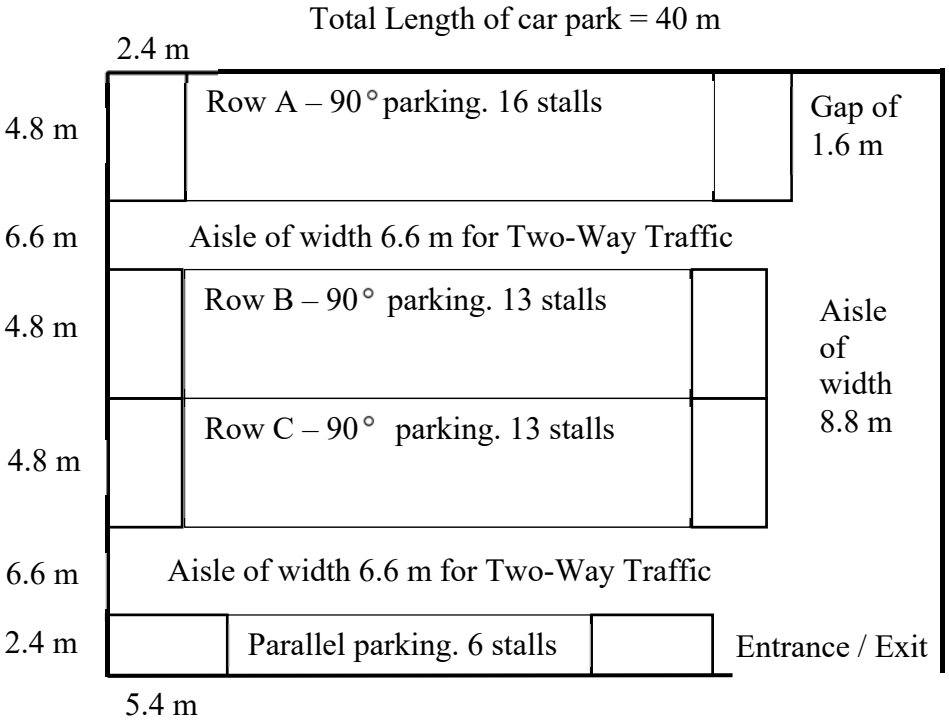


In 90° parking, the longer side is perpendicular to the parking aisle or driveway.
The aisle or lane for cars to move must be at least 6 m for one-way traffic flow and at least 6.6 m for two-way traffic flow.
Each parking stall is 4.8 m by 2.4 m.



Propose a possible parking arrangement that would maximise the use of space, showing your calculations clearly. Your proposal must include a sketch, not drawn to scale, indicating the location of the parking stalls, the aisles and the type(s) of parking. You should allow for cars to enter and leave the parade square at any time. You can assume that the cars will not leave in large numbers at any one time, and the parking will be supervised by security guards. [10]

Possible Solution



B1 – sketch is clearly labelled
B1 – layout of parking stalls using parallel parking
B1 – layout of parking stalls using 90° parking
B1 – layout of aisles clearly showing that cars can move in or out at any time.
B1 – aisles are at appropriate width, either 6 m for one-way or 6.6 m for two-way.

	Number of lots in Row A $= \frac{40}{2.4} = 16\frac{2}{3} \approx 16$ Number of lots in Row B or C $= \frac{40-6.6}{2.4} = 13\frac{11}{12} \approx 13$ Length of Row A $= 16 \times 2.4 = 38.4$ m < 40 m Length of Row B $= 13 \times 2.4 = 31.2$ m < 40 m Length of Row C $= 13 \times 2.4 = 31.2$ m < 40 m Number of lots for parallel parking $= \frac{40-6}{5.4} = 6\frac{8}{27} \approx 6$ Length of parallel parking $= 6 \times 5.4 = 32.4$ m < 36 m	Considering the length M1 A1
	Width of the rows of parking stalls and aisles $= 4.8 \times 3 + 2.4 + 2 \times 6.6$ $= 30$ m	Considering the width M1 A1
	Total number of parking stalls $= 16 + 2 \times 13 + 6 = 48$	A1 – Accept minimum as 42
	Remarks – Students who consider only parallel parking or 90° parking but not both get a maximum of 8 marks.	

Candidate's Name _____**Class** _____**Index No**____

BUKIT PANJANG GOVERNMENT HIGH SCHOOL
PRELIMINARY EXAMINATION 2018
SEC FOUR EXPRESS / FIVE NORMAL

MATHEMATICS **4048 / 01**

Paper 1 **Date: 15 Aug 2018**

Duration: 2 hours

Time: 0745 - 0945

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

FOR EXAMINER'S USE
/ 80

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1. (a) Factorise $9x^2 - 12xy + 4y^2$.
   (b) Hence, for $9x^2 - 12xy + 4y^2 = 0$, find the ratio $x : y$.

Answer (1a) _____[2]

(1b) _____[1]

2. Red paint was mixed with blue paint in different combinations in an attempt to obtain purple paint. The table shows the combinations of red paint and blue paint.

Red paint (litres)	1	2	3	4
Blue paint (litres)	3	5	7	9

State and explain your answer, whether the amount of red paint used is proportional to the amount of blue paint used for the combinations.

Answer (2) _____[1]

3. A group of students line up. If they lined up in 2s or 6s or 9s, there will be one student without a partner. Calculate the least number of students in the contingent.

Answer (3) _____[2]

4. Arthur, Clement and John are to share a bag of sweets amongst themselves in the ratio $2:3:4$ John obtained 6 sweets more than Arthur, Find

(a) the total number of sweets received by Arthur,
(b) the number of sweets received by Clement.

Answer (4a) _____[2]

(4b) _____[1]

5. The rectangular floor of a room measuring 456 m by 696 m is to be laid with square tiles.

(a) Calculate the highest common factor of 456 and 696.
(b) Hence, or otherwise, find the least number of identical square tiles that is required to cover the floor.

Answer (5a) _____[1]

(5b) _____[2]

6. The table below shows the number of boys and girls in a class with their dietary preferences.

NUMBER OF			
BOYS WHO PREFER		GIRLS WHO PREFER	
CHILI	TOMATO SAUCE	CHILI	TOMATO SAUCE
12	8	10	10

(a) A pupil is selected at random from the class. Calculate the probability that the pupil
(i) is a boy who prefers chili,
(ii) is a girl.

(b) Two pupils are selected at random from the class. Calculate the probability that
(i) both are boys,
(ii) neither is a girl who prefers tomato sauce.

Answer (6ai) _____[1]

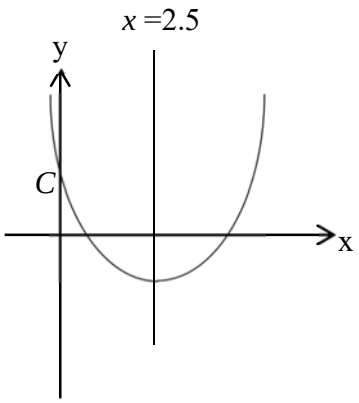
(6aii) _____[1]

(6bi) _____[2]

(6bii) _____ [2]

7. The diagram shows the graph of $y = x^2 + bx + c$. The line of symmetry is $x = 2.5$. The graph cuts the y- axis at $C(0,4)$. Calculate the value of

(a) b,
(b) c,
(c) the minimum y – value of the graph.



Answer (7a) _____[2]

(7b) _____[1]

(7c) _____[1]

8. Simplify $(4h^{\frac{1}{2}} + 2k^{\frac{3}{2}})(4h^{\frac{1}{2}} - 2k^{\frac{3}{2}})$

Answer (8) _____ _[2]

9. Consider the sequence 2, 5, 8, 11, …… .

(a) State the
(i) 6^{th} term of the sequence,
(ii) n^{th} term of the sequence.

(b) If the p^{th} term of the sequence is 56, find the value of p.

Answer (9ai) _____[1]

(9aii) _____[1]

(9b) _____[1]

10. Given that x and y are integers such that $-2 \leq x \leq 5$ and $3 \leq y \leq 8$, find the

(a) greatest value of $x^2 - y^2$,
(b) smallest value of $\frac{y+x}{x}$.

Answer (10a) _____[2]

(10b) _____[2]

11. p is directly proportional to q^2.
If q is decreased by 75 %, find the percentage decrease in p.

Answer (11) _____[2]

12. Adrian, Belle and Cindy were having a conversation, when Denzyl comes along. Commenting on a statistical finding that 1 in 4 Singaporeans in their 50s suffer from disease *X*, Adrian said," Since all of us are in our 50s and the 3 of us do not have disease *X*, Denzyl must be suffering from disease *X*." State with reason as to whether Adrian was right in his conclusion.

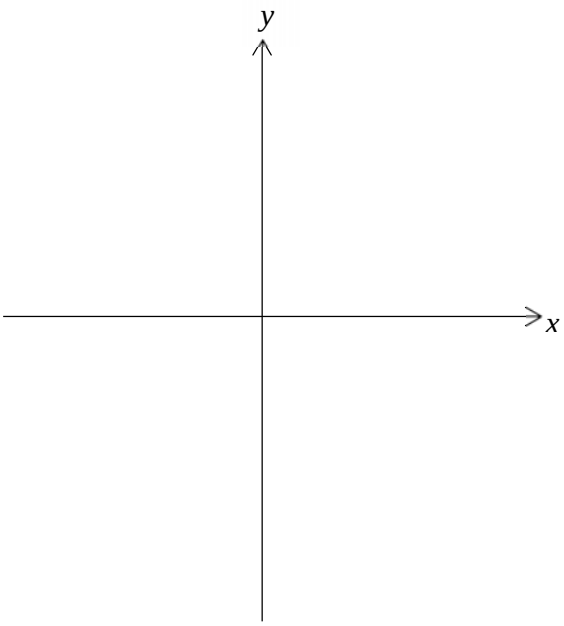
Answer (12) _____

_____

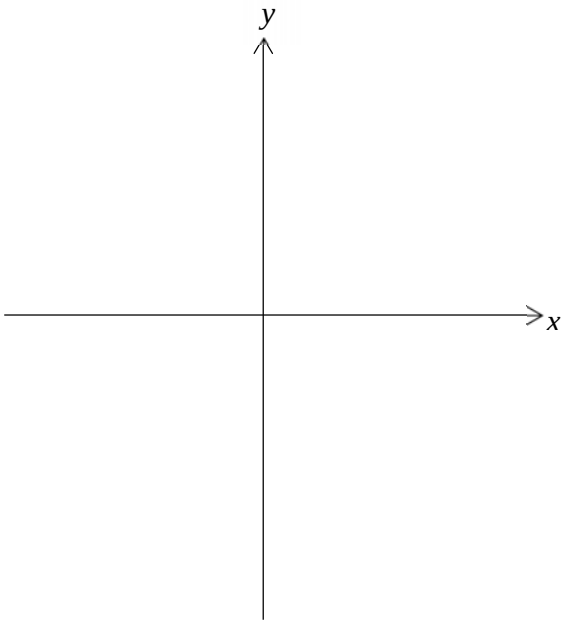
_____[2]

13. Sketch the graphs of

(a) $y = -\frac{1}{x} + 2$ [1]

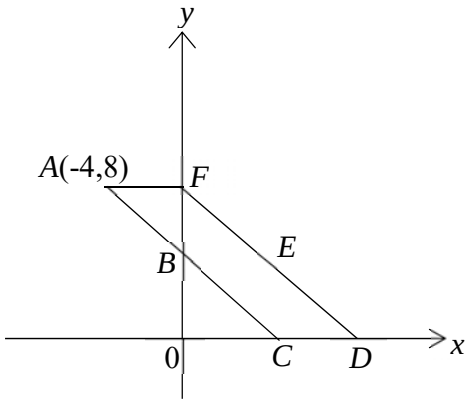


(b) $y = x^3 + 3$ [1]



14. *ABC* is a straight line. Point *B* (on the y – axis) bisects line *AC*. Point *C* lies on the *x*-axis. *FED* is a straight line having the same length as *AC* and is parallel to *AC*. *E* is the mid-point of *FD*.

(Not drawn to scale)



(a) Show that point *C* has coordinates (4,0).

(b) State the coordinates of point *B*.

(c) Show that $\overrightarrow{FD} = \begin{pmatrix} 8 \\ -8 \end{pmatrix}$.

(d) Hence, or otherwise, calculate the column vector $\overrightarrow{OE}$.

(e) Calculate area of parallelogram *ACDF*.

Answer (14a) _____[2]

(14b) _____[1]

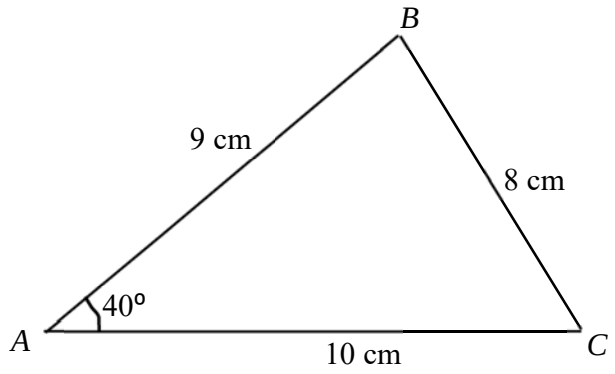
(14c) _____[1]

(14d) _____ [1]

(14e) _____ [1]

15. May set a mathematics question to test her classmates.

Calculate the area of triangle *ABC*.



(a) Describe and explain what is wrong with the question.

(b) Based on your identified error, calculate the
(i) correct area of triangle *ABC*,
(ii) shortest distance from point A to the line *BC*.

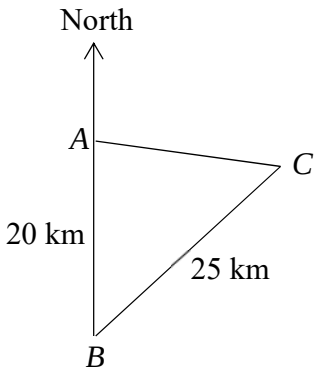
Answer (15a) _____[2]

(15bi) _____[1]

(15bii) _____[1]

16. The points *A*, *B* and *C* rest on level ground. Point *A* lies 20 km to the north of point *B*. Point *C* is at a bearing of 100^0 from point *A*. *BC* is 25 km.

(a) Calculate $\angle ACB$.
(b) Calculate the bearing of *C* from *B*.

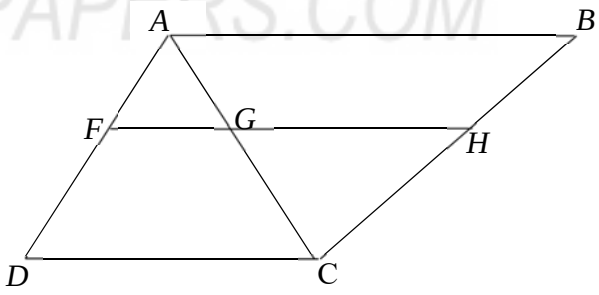


Answer (16a) _____[2]

(16b)_____[1]

17. In the diagram, *ABCD* is a trapezium, where *AB* is parallel to *CD*. *FGH* is a straight line where *FH* is parallel to *AB*. *Given 3AF = 2FD*, calculate the ratio of

(a) *FG : DC* ,
(b) *GH : AB* .

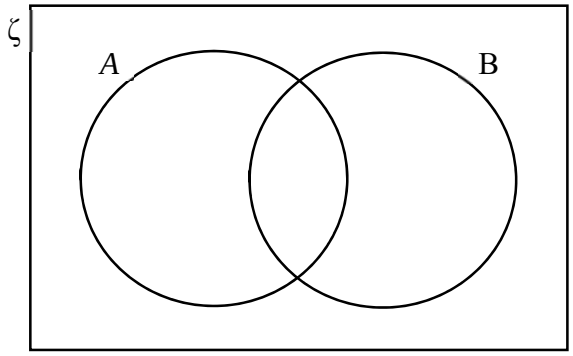


Answer (17a) _____[1]

(17b) _____[2]

18. $\zeta = \{x : x \text{ is an integer and } 3 < x \leq 15\}$
$A = \{x : x \text{ is a multiple of } 5\}$
$B = \{x : x \text{ is a multiple of } 3\}$

(a) List the elements of A.
(b) Fill in the members of ζ, A and B in the spaces in the Venn diagram below.
(c) List all possible subsets of A.



[2]

Answer (18a) _____[1]

(18c) _____[2]

19. The table below shows the number of fishes kept by students.

Number of fishes	0	1	2	3	4
Number of students	10	12	x	2	3

(a) If the mean is 1.25, find the value of x.
(b) If the median is 1, find the possible range of values of x.
(c) If the mode is 1, find the highest possible value of x.

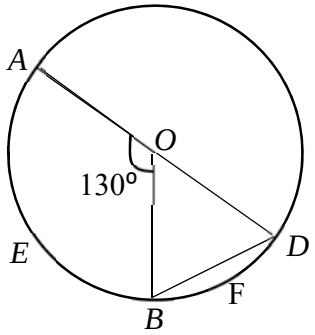
Answer (19a) _____[2]

(19b) _____[2]

(19c) _____ [1]

20. The diagram shows a circle with centre O. AD is the diameter of circle.
If radius OA is 5 cm, and $\angle AOB = 130^{\circ}$, calculate the

(a) area of ***major*** sector AOB,
(b) arc length AEB,
(c) angle OBD,
(d) area of minor segment BDF.



Answer (20a) _____ [1]

(20b) _____ [1]

(20c) _____ [1]

(20d) _____[2]

21. Three points P, Q and R lie on the circumference of a circle.

(a) Draw the perpendicular bisectors of PR and QR. [1]

(b) Label the intersection of these two perpendicular bisectors as X. Using X as the centre and XP as the radius, draw a circle to pass through P, Q and R. [1]

(c) Complete the sentence. X is eqiuidistant from _____ , _____ and _____. [2]

(d) Measure the radius of the circle.

Answer (21d) _____[1]

22. The position vector of P, relative to O, is $\overrightarrow{OP} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and the coordinates of Q are $(5, -10)$.

a. Find the coordinates of R such that $\overrightarrow{OR} = 3\overrightarrow{OP} + \overrightarrow{OQ}$.

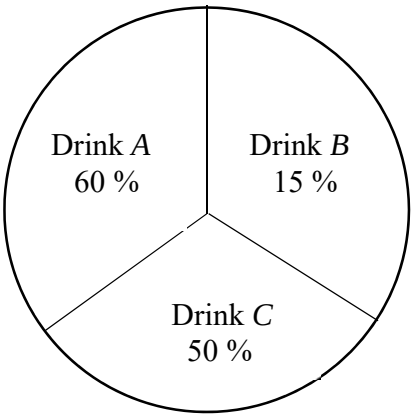
b. Given that M is the midpoint of PQ, express $\overrightarrow{OM}$ as a column vector.

Answer (22a) _____ [1]

(22b) _____ [2]

23. During a vote for the favorite drink sold in the canteen, a pie – chart was displayed to show the percentage of votes for each of the 3 drinks. State two reasons why the pie-chart is misleading.

State two reasons why the pie-chart is misleading.



Answer (23a)_____

_____

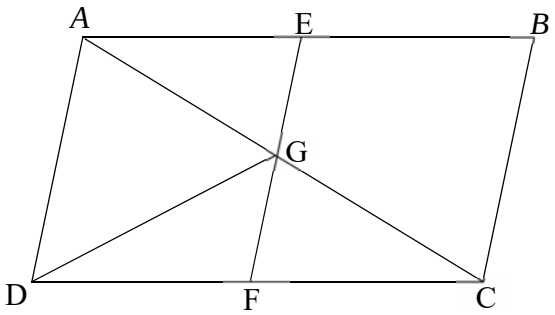
_____[1]

(23b) _____

_____

_____ [1]

24. In the diagram, ABCD is a parallelogram. *AG* : *GC* is 1:1. *EF*, *BC* and *AD* are parallel to each other. Is triangle *DGF* congruent to triangle *CGF* ? Explain.



Answer (24) _____

_____

_____[2]

END OF PAPER 1

2018 ELEMENTARY MATHEMATICS PRELIMINARY EXAMINATIONS PAPER 1 ANSWERS

1. (a) $(3x-2y)^2$ (b) $x:y=2:3$

2. No, as the quotients of red paint to blue paint are not constant or equal for each and every quotient.

3. Least number of students in the contingent is 18 + 1 = 19.

4. (a) 6 sweets. (b) 9 sweets.

5. (a) $HCF = 24$ (b) 51

6. (ai) $\frac{3}{10}$ (aii) $\frac{1}{2}$

   (bi) $\frac{19}{78}$ (bii) $\frac{29}{52}$

7. (a) $b=-5$ (b) $c=4$ (c) -2.25

8. $16h-4k^3$

9. (ai) 17
   (aii) $3n-1$
   (b) $p=19$

10. (a) 16
    (b) -7

11. Percentage decrease in P $= 93.75\%$

12. No, The probability figure of ¼ is for a much larger sample size.

14. (a) $\frac{GB}{XC}=\frac{1}{2}$

    $XC = 8\ units$
    x-coordinates of C = -4 + 8
    = 4 units
    C (4.0)

    (b) B (0,4)
    (c) $\overrightarrow{FD}=\begin{pmatrix}8\\-8\end{pmatrix}$
    (d) $\overrightarrow{OE}=\begin{pmatrix}4\\4\end{pmatrix}$
    (e) Area = 32 units^2.

15.(a) Assuming angle $BAC = 40^{\circ}$, $BC = \sqrt{9^2 + 10^2 - 2(9)(10)cos40^0}$
$= 6.57$ cm

But question stated that $BC = 8$ cm.

OR: Assuming BC = 8 cm,
$8^2 = 9^2 + 10^2 - 2(9)(10)cosA$
Angle $BAC = 49.5^{\circ}$, but Angle $BAC = 40^{\circ}$ in question.

(bi) Assuming angle BAC = 40°, area = $\frac{1}{2}(9)(10)sin40^0 = 28.9\ cm^2$

OR: Assuming Angle BAC = 49.4584°, Area = $\frac{1}{2}(9)(10)sin49.4584^0$ = 34.2 cm^2

bii) $\frac{1}{2}(8)h = 34.19704$
h = 8.55 cm

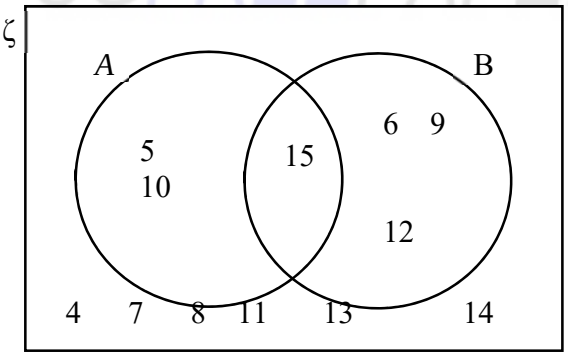
OR: $\frac{1}{2}(6.56597)h=28.92544$
H = 8.81 cm

16. (a) Angle $ACB = 52.0^{\circ}$
(b) Bearing is 048.0°

17. (a) $FG : DC = 2: 5$

(b) $GH : AB = 3: 5$

18. (b)



(a) 5, 10, 15

(c) {5}, {10}, {15}, {5,10}, {5,15}, {10,15}, {5,10,15}, { }

19. (a) $x = 5$
(b) $0 \leq x \leq 17$
(c) 11.

20.
(a) Area = 50.2 cm^2
(b) Arc length = 11.3 cm
(c) Angle $OBD = 65^{\circ}$
(d) Area = 1.33 cm^2.

21. (c) P, Q and R
(d) 3.0 cm

22. (a) *R* (14,4)

(b) $\overrightarrow{OM} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$

23a The sum of percentages is 125 % which is not equal to 100 %.

23b. The percentages for each of the 3 drinks are not in proportion to the percentage of area of circle

24. *GF* is common,
DF = CF,
but there is no information to suggest *DG = CG* thus *SSS property for congruency*
angle *GFC* = angle *GFD thus* SAS property for congruency.

Candidate's Name: _____Class_____ Index No____

BUKIT PANJANG GOVERNMENT HIGH SCHOOL

Preliminary Examination 2018

SECONDARY 5 (NORMAL(ACADEMIC))

SECONDARY 4 (EXPRESS STREAM)

MATHEMATICS **4048/02**

Paper 2

Date: 14 August, 2018
Duration: 2h 30 min
Time: 0745 – 1015 h

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
At the end of the examination, fasten all your work securely together.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This paper consists of 13 printed pages.

Setter: Ms Nurdiana **[Turn over]**

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1. (a) Given that $m = 2k\sqrt{\frac{n-5}{3+n}}$, express n in terms of k and m. [3]

   (b) Express $1 + \frac{8}{x^2-9} - \frac{1}{3-x}$ as a single fraction in its lowest term. [3]

   (c) Given that $4^{k-1} = 2\sqrt{8^k}$, determine the value of k. [3]

2. A pond is filled up with 60 m^3 of water. There are two pumps, *A* and *B* which can be used to drain the pond. Pump *A* can drain the water at a rate of x m^3 per minute, while Pump *B* can drain the water at a rate of $(x + 0.3)$ m^3 per minute.

   (a) (i) Write down the time taken, in minutes, for Pump *A* to drain out all the water from the pond completely. [1]

   (ii) Write down the time taken, in minutes, for Pump *B* to drain out all the water from the pond completely. [1]

   (b) Given that Pump *A* takes 3 minutes and 20 seconds more than Pump *B*, form an equation in x and show that it reduces to $10x^2 + 3x - 54 = 0$. [3]

   (c) Solve the equation, $10x^2 + 3x - 54 = 0$ and hence write down the time taken in minutes for Pump *A* to drain out all the water from the pond completely. [3]

   (d) If both pumps are turned on together, will the pumps be able to drain out all the water completely within 12 minutes? Explain your answer. [2]

3. The first three terms in a sequence of numbers, T_1, T_2, T_3, …, are given below.

$$T_1 = 3^0 + 1 + 2^2 = 6$$
$$T_2 = 3^1 + 4 + 3^2 = 16$$
$$T_3 = 3^2 + 7 + 4^2 = 32$$

   (a) Find T_4. [1]

   (b) Find an expression, in terms of n, for the nth term, T_n of the sequence. [3]

   (c) Consider the following sequence.

$$S_1 = -T_1$$
$$S_2 = T_2$$
$$S_3 = -T_3$$

   Using your answer from (b), find an expression for the n^{th} term, S_n of the sequence. [1]

4. **Answer the whole of this question on a sheet of graph paper.**

A manufacturer makes a profit of $\$y$ for x toys sold, where

$$y = 250 - \frac{4800}{x} - \frac{x}{2}.$$

Some corresponding values of x and y are given in the table below.

$\boldsymbol{x}$	10	30	40	50	100	150	200	250	300
$\boldsymbol{y}$	–235	75	110	129	152	$\boldsymbol{p}$	126	105	84

(a) (i) Find the value of $\boldsymbol{p}$. [1]

(ii) Using a scale of 2 cm to represent 50 toys, draw a horizontal x-axis for $0 \leq x \leq 300$.

Using a scale of 2 cm to represent \$50, draw a vertical y-axis for $-250 \leq y \leq 200$.

On your axes, plot the points given in the table and join them in a smooth curve. [3]

(b) Use your graph to find the

(i) number of toys the manufacturer needs to sell so as to break even, [1]

(ii) maximum profit earned by the toy manufacturer and the corresponding number of toys sold. [2]

(c) (i) By drawing a tangent, find the gradient of the curve at the point where $x = 160$. [2]

(ii) Describe briefly what your answer in (c)(i) represents. [1]

(d) By drawing a suitable straight line on the same axes, solve the equation [3]

$$\frac{x}{2} + \frac{4800}{x} - 250 + 100 = 0$$

5. A confectionary shop sells 2 different gift hampers, *Deluxe* and *Premiere*, each comprising of chocolate bars, bags of candy and packets of biscuits. The contents of each box are as shown below.

Gift Box	Number of chocolate bars	Number of bags of candy	Number of packets of biscuits
Deluxe	3	5	2
Premiere	5	4	3

The above information can be represented by the matrix $\boldsymbol{P} = \begin{pmatrix} 3 & 5 & 2 \\ 5 & 4 & 3 \end{pmatrix}$.

The cost price and the selling price of each item are as shown.

	Cost price	Selling price
Chocolate bar	\$3	\$4
Bag of candy	\$2.80	\$4.20
Packet of biscuits	\$2	\$3.80

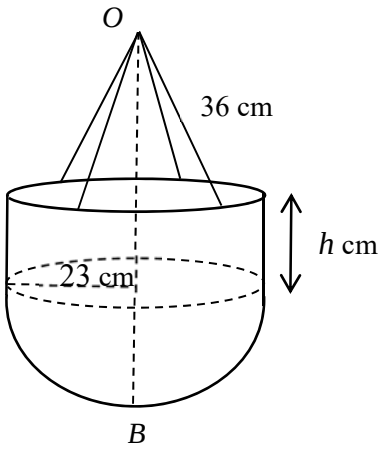
(a) (i) A customer, Mr Lee bought 12 *Deluxe* hampers and 20 *Premiere* hampers.
Given that $\boldsymbol{R} = (12 \quad 20)$, find the matrix $\boldsymbol{S}$ if $\boldsymbol{S} = \boldsymbol{RP}$. [2]

(ii) Describe what is represented by the elements of $\boldsymbol{S}$. [1]

(b) Using matrix multiplication, evaluate matrix $\boldsymbol{Q}$ such that the elements of $\boldsymbol{Q}$ informs the the confectionary shop owner of the total cost price and the total selling price respectively. [3]

(c) Given that $\boldsymbol{X} = \begin{pmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{pmatrix}$ and $\boldsymbol{QX}$ gives the profit made, state the value of $\boldsymbol{a}$ and $\boldsymbol{b}$, and hence find the total profit. [2]

6. (a) The diagram below shows a container which is made by attaching an open hemisphere of internal radius 23 cm to the rim of a hollow cylinder with the same internal radius and a height of h cm.



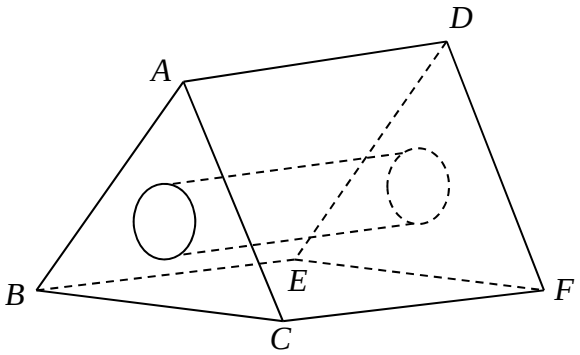
The container is suspended from O by four wires, each of length 36 cm, fastened symmetrically to the rim of the cylinder.

It is given that 43.7 litres of water is needed to completely fill the container.

(i) Show that $h = 10.96$ cm, corrected to 4 significant figures. [2]

(ii) Hence, find the vertical distance OB. [2]

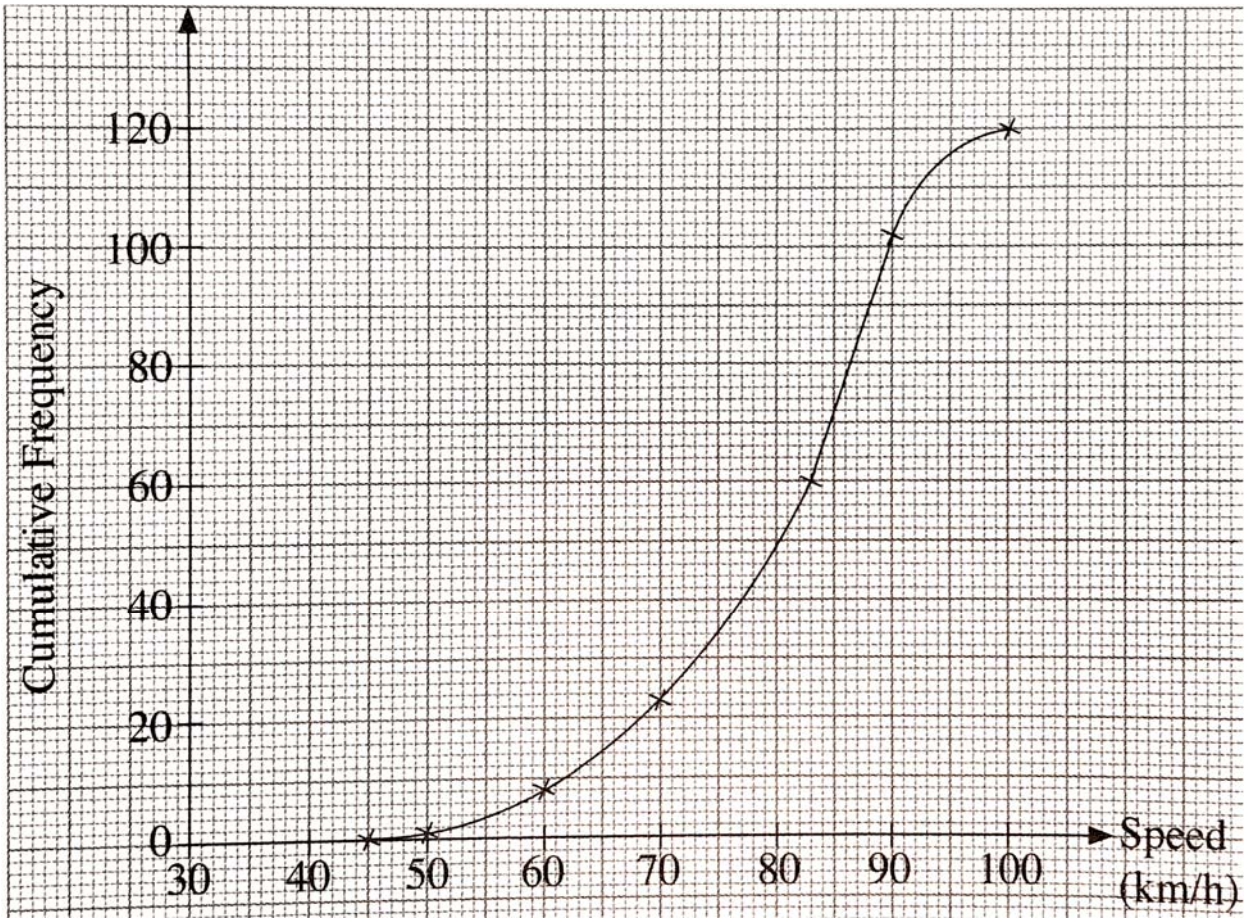
(b) The figure below shows a solid triangular prism where the cross section ABC and DEF are equilateral triangles. It has a cylindrical hole in the centre and a square base $BCFE$ of area 36 m^2. The volume of the cylindrical hole is 75.36 m^3.



1 can of paint covers 9 m^2 of the area. Find the number of cans of paint to purchase in order to paint the entire solid.

[5]

7. The diagram below shows the cumulative frequency curve of the speed of 120 cars passing through a certain point along an expressway at 11 am.



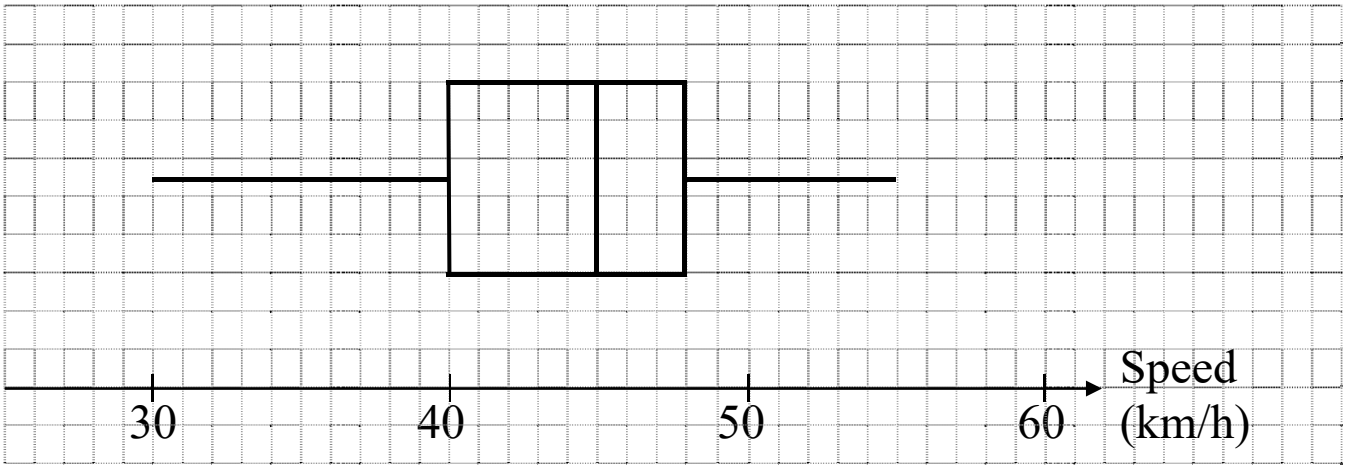
(a) Use the graph to find,

(i) the median speed, [1]

(ii) the interquartile range. [2]

(b) A speed camera is located at the point. Calculate the percentage of cars that will be fined for speeding if the speed limit is 90 km/h. [1]

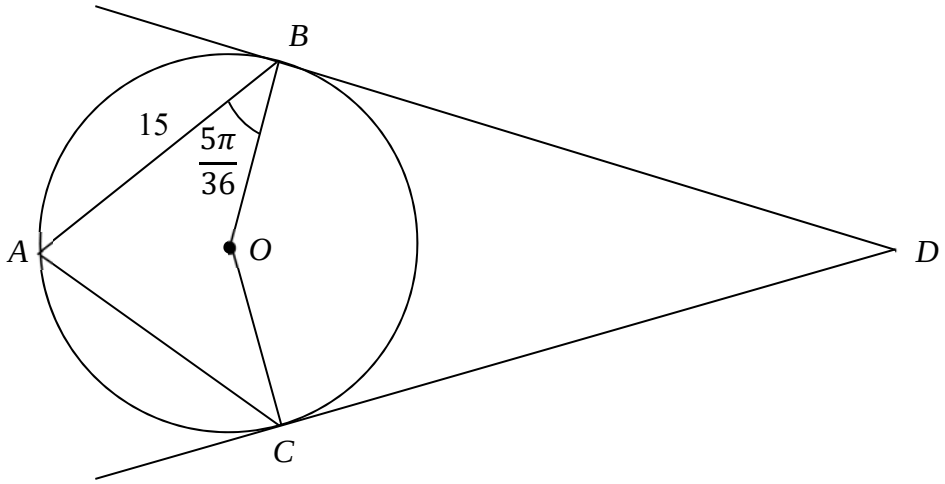
The box-and-whisker plot below shows the speed of another 100 cars along the same point of the expressway at 6:30 pm.



(c) Make 2 comparisons between the speeds of the cars at 11 am and 6:30 pm. [2]

(d) Suggest a reason for the difference in the speed of the cars measured along the same point of the expressway at 11 am and 6:30 pm. [1]

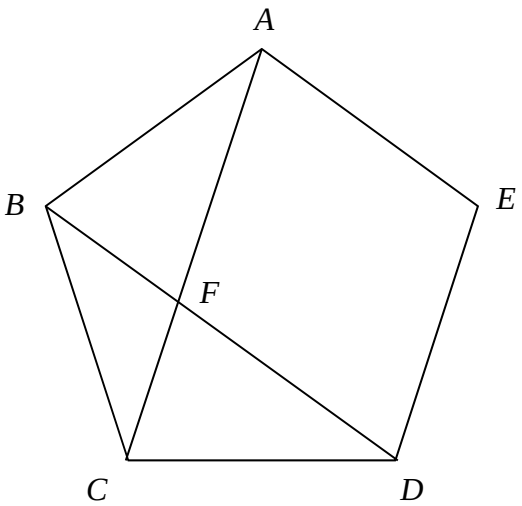
8. (a) In the figure, A, B and C are points on the circle with centre at O. BD and CD are tangents to the circle at points B and C respectively. It is given that $AB = 15$ cm and $\angle ABO = \frac{5\pi}{36}$ rad.



(i) Find the radius of the circle. [3]

(ii) Suppose that $\angle OAC = \frac{5\pi}{18}$ rad, find the area enclosed by the tangents BD and CD and minor arc BC. [4]

(b) The diagram shows a regular pentagon $ABCDE$. AC and BD intersect at F.

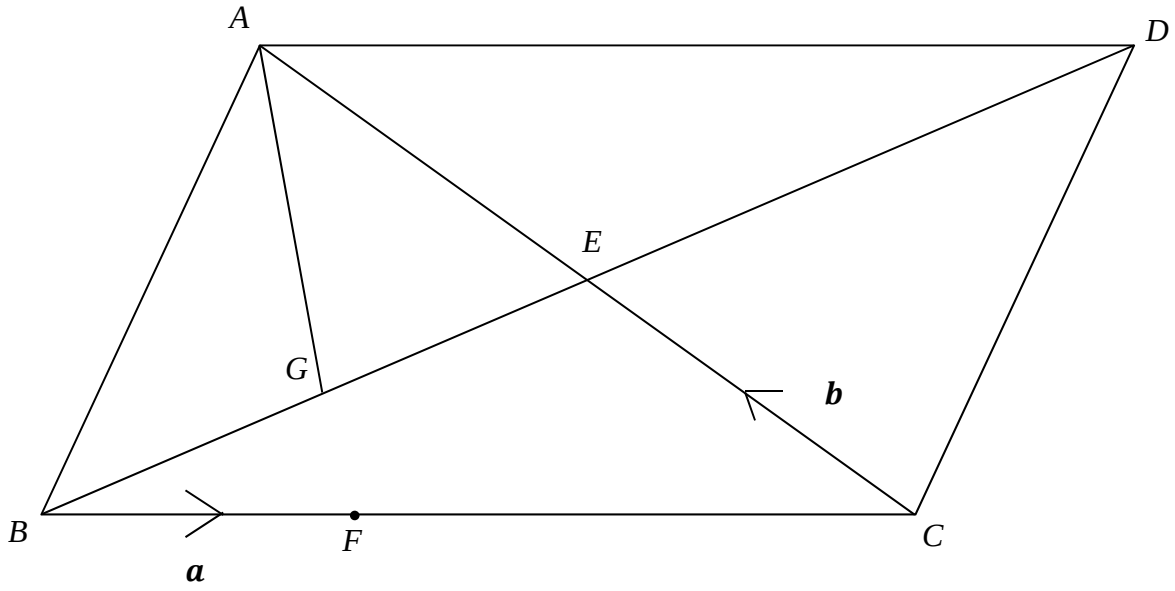


(i) Find the value of $\angle CDF$. [2]

(ii) Show that $\angle DFA = 108°$. [2]

9. (a) In the diagram, *ABCD* is a parallelogram. The diagonals *AC* and *BD* intersect at *E*. *F* is a point on *BC* such that $\overrightarrow{BC} = 3\overrightarrow{BF}$. *G* is the midpoint of *BE*.

It is given that $\overrightarrow{BF} = \boldsymbol{a}$ and $\overrightarrow{CE} = \boldsymbol{b}$.



(i) Express $\overrightarrow{BA}$ and $\overrightarrow{BD}$ as simply as possible, in terms of $\boldsymbol{a}$ and/or $\boldsymbol{b}$. [2]

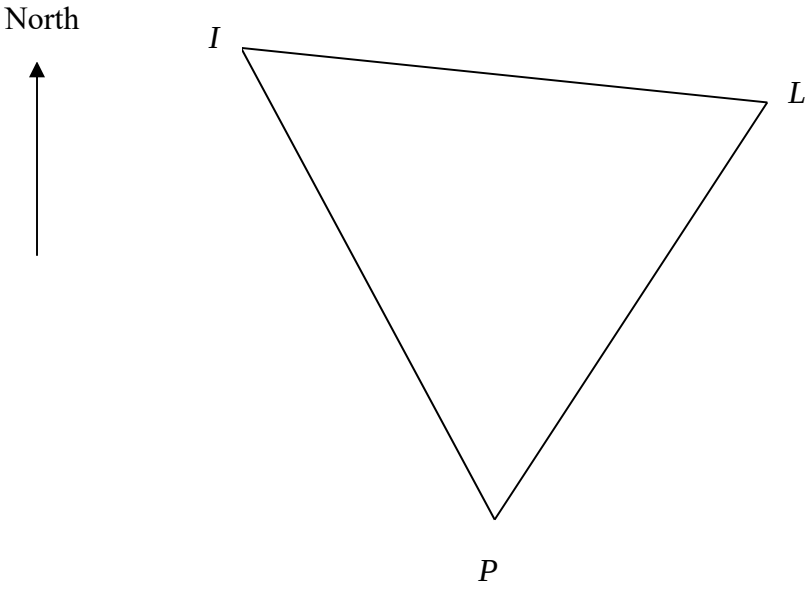
(ii) Show that $\overrightarrow{AG} = -\frac{3}{2}(\boldsymbol{a} + \boldsymbol{b})$. [2]

(iii) Express $\overrightarrow{AF}$ as simply as possible, in terms of $\boldsymbol{a}$ and/or $\boldsymbol{b}$.
Hence, show that *A*, *G* and *F* are on a straight line. [2]

(iv) Find the numerical value of $\frac{area\ of\ \Delta BFG}{area\ of\ quadrilateral\ ABCD}$. [1]

(b) It is given that the coordinates of *Q* are (5, −10) and the point *N* lies on *QO* produced such that $\left|\overrightarrow{ON}\right| = 4\sqrt{5}$ units. Express $\overrightarrow{ON}$ as a column vector. [3]

10. A ship leaves a port at P and sails 21 km towards a lighthouse, L. It then sails 28 km towards an island, I. It is given that the bearing of L from I is 116° and the bearing of P from I is 163°.



(a) Find the bearing of I from L. [1]

(b) Calculate the distance IP. [3]

(c) The ship then returns to the port P, travelling along the route IP. Calculate the distance from P when the ship is closest to the lighthouse, L. [2]

(d) Given that the height of the lighthouse is 500 m, calculate the angle of depression of P from the top of the lighthouse. [2]

11. The table below shows the time taken by the delivery men of a company, IXEA, to assemble each type of furniture at the delivery location.

Furniture	Time taken to assemble per piece (minutes)
Study table	45
Reading chair	3
Bedside drawer	12
Bunk bed	100

(a) Find the total time taken to assemble one study table and two reading chairs. [1]

(b) On a particular day, the planned delivery route is as shown below.

No.	Location	Order	Estimated time of delivery
1	Sunset Ville	• 1 study table • 2 reading chairs	0900 to 1030
2	Casa Ville	• 1 bedside drawer	1030 to 1200
3	Cloud Cove	• 1 study table • 1 bedside drawer • 2 bunk beds	1300 to 1500

The delivery men left the office at 0915 for the first location at Sunset Ville. After assembling the order, they proceeded to the second location at Casa Ville and arrived at 1030. Additional information that may be needed for the delivery is shown on the Annex.

(i) Calculate the average speed, in km/h, of the delivery van, leaving your answer to the nearest whole number. Do you think the answer is a reasonable estimate of the actual travelling speed of the van? Justify your answer. [3]

(ii) The daily working hours for the delivery men is 0830 to 1600 and they are entitled to have a 45 minutes lunch break. Using the answer found in (i), determine if the delivery men can leave the office punctually at 1630 for that day. Support your answer with appropriate calculations and state one reasonable assumption you made. [6]

Annex

Table A: Distance Chart between Various Locations

Distance (in km)	IXEA Office	Sunset Ville	Casa Ville	Cloud Cove
IXEA Office		13.8	18.1	9.7
Sunset Ville	13.8		4.7	3.8
Casa Ville	18.1	4.7		6.1
Cloud Cove	9.7	3.8	6.1	

Table B: Speed Limits for Vehicles

Source: https://www.lta.gov.sg/content/ltaweb/en/roads-and-motoring/road-safety-and-regulations/road-regulations.html

SPEED LIMITS FOR VEHICLES

Driving above the speed limit, or too fast for the current conditions, can have severe consequences for the people in the vehicle and other road users. The following speed limits are enforced by LTA to ensure everyone's safety:

Type of Vehicle	Roads	Expressways	Tunnels
Cars & motorcycles	50km/h	70-90km/h	50-80km/h
Buses & coaches	50km/h	60km/h	50-60km/h
Light commercial vehicles (includes Light Goods Vehicles and small buses not exceeding 3.5 tonnes and seating capacity of up to 15 passengers)	50km/h	60-70km/h	50-70km/h
Exceptions: Fire engines, Ambulances, and Government vehicles used by Singapore Police Force or the Singapore Civil Defence Force			

----------END OF PAPER 2----------

Answers:

1. (a) $n = \frac{3m^2+20k^2}{4k^2-m^2}$

   (b) $\frac{x^2+x+2}{(x+3)(x-3)}$

   (c) k = 6

2. (a)(i) $\frac{60}{x}$ minutes

   (ii) $\frac{60}{x+0.3}$ minutes

   (b) $\frac{60}{x} - \frac{60}{x+0.3} = 3\frac{1}{3}$

   (c) x = 2.1786 or –2.4786

   Time taken for Pump A = 27.5 mins

   (d) Time taken for Pump A and B

   $= \frac{60}{2.1786+(2.1786+0.3)} = 12.883$ minutes

   No, both pumps are not able to drain out all the water completely within 12 minutes.

3. (a) $3^3 + 10 + 5^2 = 62$

   (b) $T_n = 3^{n-1} + n^2 + 5n - 1$

   (c) $S_n = (-1)^n T_n$

4. (a)(i) p = 143

   (ii) Graph

   (b)(i) From graph, no. of toys = 20 toys (± 5)

   (ii) From graph, max. profit = \$152 (± 5)

   corr no. of toys = 98 toys (± 5)

   (c)(i) From graph, gradient = –0.35 (± 0.1)

   (ii) The gradient represents the change of profit over the change in number of toys at x = 160.

   (d)(i) Draw the line y = 100.

   From graph, x = 35.1 or x = 260. (± 5)

5. (a)(i) (136 140 84)

   (ii) The total number of chocolate bars, bags of candy and packets of biscuits bought respectively.

   (b) (968 1451.20)

   (c) a = –1, b = 1

   $Q\begin{pmatrix}1\\-1\end{pmatrix} = (483.20)$

   Total profit = \$483.20

6.(a)(i) (show question)

   (ii) 61.7 cm

6. (b) Total exposed surface area of prism = Area of 2 triangles (without circular holes) + 3 faces of prism + curved SA of cylinder = 189.4360239

   No. of cans of paint needed = 22 cans

7.(a)(i) 83 km/h

   (ii) 15 km/h

   (b) 15%

   (c) The median speed of the cars at 6:30 pm is 45 km/h which is lower than that at 11 am. Therefore the cars are travelling slower at 6:30 pm.

   The IQR of the speed of the cars at 6:30 pm is 8 km/h which is smaller than that at 11 am.Hence, the speed of the cars is more consistent at 6:30 pm / The spread of the speed of cars is wider at 11 am as compared to 6:30 pm.

   (d) Heavy traffic during peak hour.

8. (a)(i) 8.28 cm

   (ii) 166 cm^2

   (b)(i) 36°

   (ii) show question

9.(a)(i) $\overrightarrow{BA} = 3\mathbf{a} + 2\mathbf{b}$

   $\overrightarrow{BD} = 6\mathbf{a} + 2\mathbf{b}$

   (ii) show question

   (iii) $\overrightarrow{AF} = -2(\mathbf{a} + \mathbf{b})$

   $3\overrightarrow{AF} = 4\overrightarrow{AG}$, hence A, F and G are collinear with A as the common point.

   (iv) $\frac{1}{24}$

   (b) $\overrightarrow{ON} = \begin{pmatrix}-4\\8\end{pmatrix}$

10. (a) 296°

   (b) 23.7 km

   (c) 4.65 km

   (d) 1.4°

11. (a) 51 minutes

(b) (i) Distance from Office → Sunset Ville → Casa Ville = 13.8 + 4.7 = 18.5 km

Total time taken travelling = 75 mins – 51 mins (assembly) = 24 mins

Average travelling speed = 46.25 km/h = 46 km/h (nearest whole no.)

A reasonable estimate as it's within the speed limit. OR
Not a reasonable estimate as the time taken to move the furniture up/wait for the lift is not considered. As such, less time spent on the road and actual speed may be faster.

(ii) Total distance from Office → Sunset Ville → Casa Ville → Cloud Cove → Office
= 13.8 + 4.7 + 6.1 + 9.7
= 34.3 km

Total time taken (for travelling) = 34.3 km ÷ 46 km/h = 45 mins (to the nearest min)
Total time taken (for lunch) = 45 mins
Total time taken (for assembling) = 45 + 2(3) + 12 + 45 + 12 + 200 = 320 mins

Total time taken = 45 + 45 + 320 = 410 mins = 6 h 50 mins

0915 (time they left office) $\xrightarrow[6\text{h } 50 \text{ mins}]{}$ 1605

The delivery men will be able to leave punctually.

<u>Assumption:</u> (Any one)
1. Owners are at home when delivery men reach.
2. No traffic jams.
3. Traffic condition is more or less the same from one location to another such that average speed is 46 km/h.

Name:	Class:	Class Register Number:

中正中学
CHUNG CHENG HIGH SCHOOL (MAIN)

_____
Parent's Signature

PRELIMINARY EXAMINATION 2018

SECONDARY 4

MATHEMATICS 4048/01

Paper 1 **11 September 2018**

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

80

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 If $a = \frac{4}{3}b$ and $b = \frac{5}{6}c$, write down the ratio of $a : b : c$.

Answer ……... [1]

---

2 If x and y are integer values, such that $-8 \le x \le 7$ and $-1 \le y \le 5$, find

(a) the smallest possible value of $2y - x^2$,

Answer ……... [1]

(b) the largest possible value of $-(y^3 x)$.

Answer ……... [1]

---

3 Solve $\frac{2x-3}{2}-\frac{x+6}{6}=\frac{2}{9}$.

Answer x = ……... [3]

---

4 **(a)** Simplify $7x-4y-4(x-5y)$.

Answer ……... [1]

(b) Express $\frac{2x}{x^2-1}+\frac{3}{1-x}$ as a single fraction.

Answer ……... [3]

---

5 **(a)** Solve $49^{2x+1} \div \sqrt[3]{7} = \frac{1}{343}$.

Answer x = ……... [3]

(b) Without using a calculator, show that $3^{1161} + 3^{1158}$ is exactly divisible by 7.

Answer ……

……

……

……

……

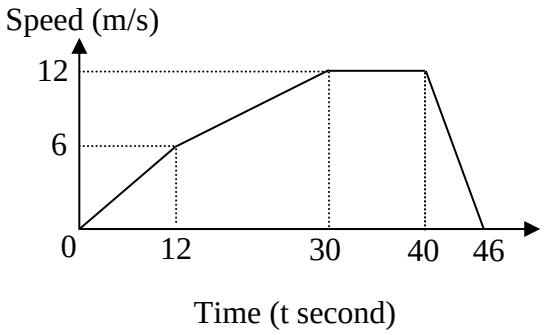
……

……

……

…… [2]

6 The diagram shows the speed–time graph for a car's journey.



Calculate

(a) the acceleration of the car during the first 12 seconds,

Answer ……... m/s^2 [1]

(b) the speed of the car when $t = 18$,

Answer …… m/s [2]

(c) the total distance travelled in the first 40 seconds.

Answer …… m [2]

7 $W = 6\sqrt{\dfrac{y^3 - x^2}{7}}$

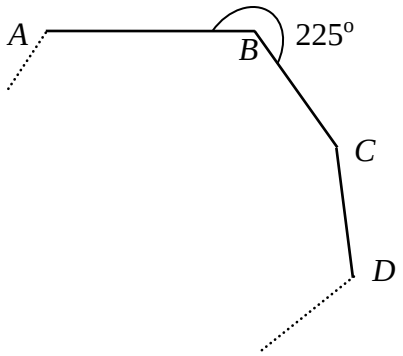
(a) Calculate the value of W when $y = 5$ and $x = -4$. Write your answer correct to two decimal places.

Answer ……... [1]

(b) Rearrange the formula to make x the subject.

Answer ……... [3]

8 *A, B, C , D*, …. is part of a regular polygon. Given that reflex angle *ABC* = 225°, how many sides does the polygon have?



Answer ……... [2]

---

9 In a particular school, the enrolment in 2016 is 900 students. In 2015, there were 20% more students than in 2016. There were 20% less students in 2016 as compared to 2017.

Calculate the enrolment in

(a) 2015,

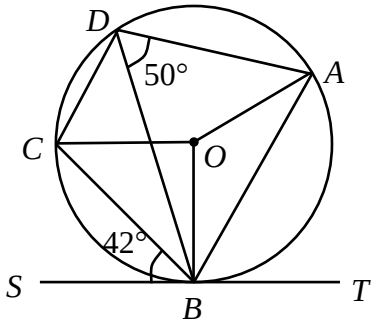
Answer ……... [1]

(b) 2017.

Answer ……... [1]

---

10 *A, B, C* and *D* are four points on the circumference of a circle with centre *O*. *ST* is a tangent to the circle at *B*. It is given that angle *ADB* = 50° and angle *CBS* = 42°. Calculate, showing your working clearly,



(a) angle *COB*,

Answer Angle *COB* =…..... [2]

(b) angle *CDB*,

Answer Angle *CDB* =……..... [1]

(c) angle *AOC*.

Answer Angle *AOC* =…….. [2]

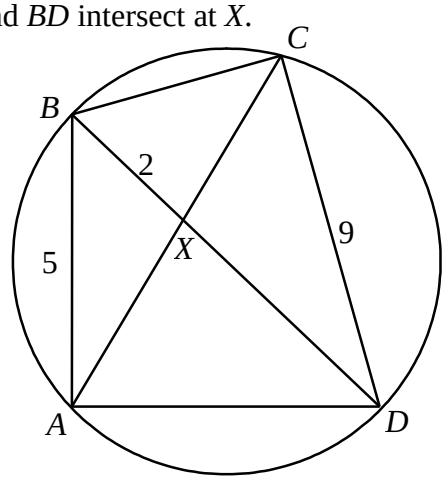
11 **(a)** Given that $4x^2 - 12xy + 9y^2 = 0$, find the value of $\frac{4x}{15y}$.

Answer …….. [3]

(b) Factorise $a^2 + 2ab + b^2 - 4b^2c^2$ completely.

Answer ……..……..... [2]

12 The diagram shows a circle *ABCD*. The diagonals *AC* and *BD* intersect at *X*.
It is given that *BX* = 2 cm, *AB* = 5 cm and *DC* = 9 cm.



(a) Prove, stating your reasons clearly, that triangle *ABX* is similar to triangle *DCX*.

Answer ……

……

……

……

…… [2]

(b) Find the length of *CX*.

Answer ……cm [1]

(c) Find $\frac{\text{area of triangle } ABX}{\text{area of triangle } DCX}$.

Answer ……... [1]

13 Twenty four boys took part in the high jump event in a school sports meet. Their records in centimetres are shown in the stem-and-leaf diagram.

Stem	Leaf							
11	0	2						
12	1	2	6					
13								
14	2	3	3	4	4	5	5	
15	6	6	7	7				
16	0	0	1	1	5	5	5	6

Key : 11 | 0 represents 110 cm

(a) Find the interquartile range.

Answer ……cm [1]

(b) Find

(i) the modal distance,

Answer ……cm [1]

(ii) the median distance,

Answer ……cm [1]

(iii) the mean distance.

Answer ……cm [1]

(c) Would you use the mode or the median as the most appropriate measure of average in this case? Justify your answer.

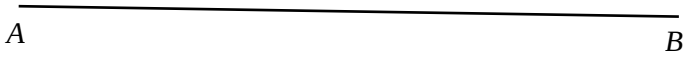
Answer ……

……

……

…… [1]

14 **(a)** Construct the quadrilateral *ABCD* such that angle $ABC = 70^{o}$, $BC = 11\,\text{cm}$, $AD = 6\,\text{cm}$ and angle $BAD = 110^{o}$. Line *AB* has already been drawn. [2]



(b) State the special name of this quadrilateral.

Answer …….... [1]

(c) Construct the perpendicular bisector of *BC*. [1]

(d) Construct the angle bisector of $\angle BCD$. [1]

(e) Label the intersection between the bisectors in **(c)** and **(d)** as *Y*. Name an isosceles triangle from the diagram.

Answer Triangle…….... [1]

15 $\xi = \{\text{integers } x : 1 \le x \le 15\}$

$A = \{\text{integers divisible by 2}\}$

$B = \{\text{integers divisible by 3}\}$

(a) Draw a Venn Diagram to illustrate this information. [2]

(b) List the elements contained in the set $(A \cup B)'$.

Answer ……..……... [1]

(c) Describe, as simply as possible, in words, the elements contained in the set $A \cap B$.

Answer ……

……

…… [1]

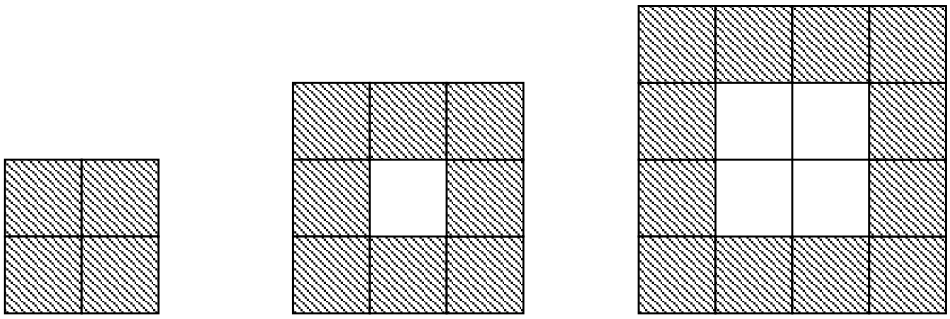
---

16 It is given that y is inversely proportional to the cube of x. If x is doubled, find the percentage decrease in y.

Answer …...…….……% [3]

---

17 Small squares are used to form a series of big squares as shown below.



(a) Complete the following table: [2]

Length of side of big square (n)	Number of shaded small squares (S)	Number of unshaded small squares (U)
2	4	0
3	8	1
4	12	4
5	16	9
6		

(b) Find an equation connecting U, the number of unshaded small squares, and n, the length of side of big square.

Answer ……..... [1]

(c) How many unshaded small squares are there in a big square if the length of the big square is 13?

Answer ……..... [1]

(d) Explain why the number of unshaded small squares (U) cannot be 288.

Answer ……

……

……

…… [1]

18 Ethan invested some money in a bank for 3 years. The rate of compound interest was fixed at 2% per annum. At the end of 3 years, Ethan has a total of $3077.50 in his account. What was the amount of money that Ethan invested at the beginning?

Answer $……... [3]

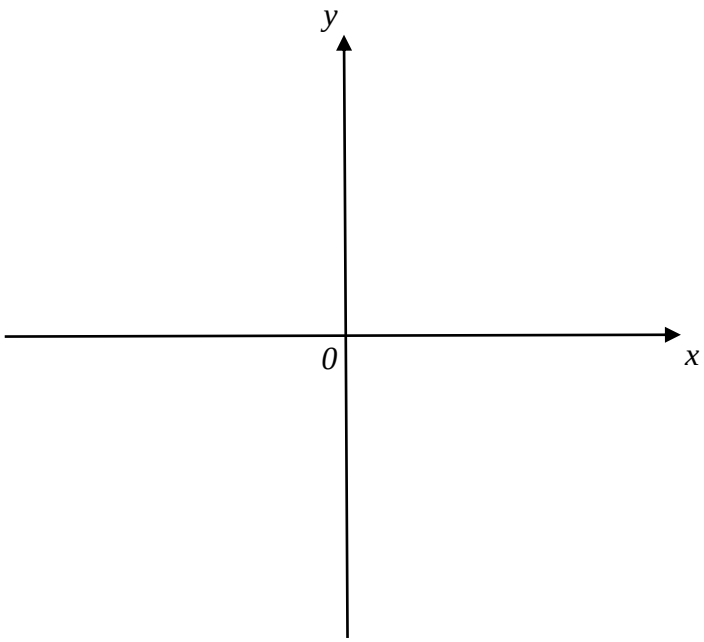
---

19 **(a)** Express $6x + x^2 + 15$ in the form of $a + (x + b)^2$.

Answer …….. [2]

(b) Hence sketch the graph of $y = 6x + x^2 + 15$.

[2]



20 The daily expenses of 1040 students in School *X* are recorded. The cumulative frequency curve in **Diagram I** below shows the distribution of their expenses.

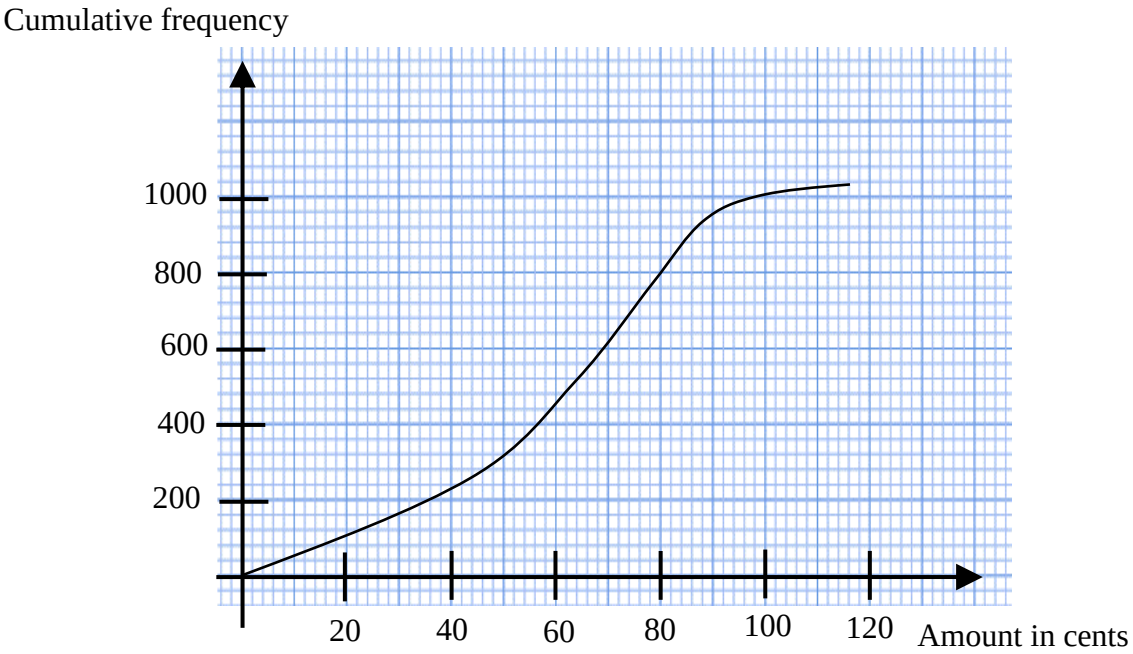


Diagram I

(a) Use the curve to estimate

(i) the median,

Answer ……cents [1]

(ii) the interquartile range.

Answer ……cents [1]

(b) The expenses of 1040 students in another School *Y* had the same median but a smaller interquartile range. Draw a possible cumulative curve to show the distribution of the students' expenses for School *Y* on **Diagram I**.

[1]

21 The table below shows the number of cups of four types of flavoured tea sold by a café on Monday, Tuesday and Wednesday, and the respective selling price and cost price of each type of flavoured tea.

	Green	Plum	Lemon	Assam
Monday	33	47	34	18
Tuesday	40	25	56	34
Wednesday	56	73	21	51
Selling price per cup	\$3.20	\$3.80	\$3.00	\$3.50
Cost price per cup	\$1.60	\$2.00	\$1.70	\$1.80

(a) Express the profit made per cup for each type of flavoured tea as a single row matrix **P**.

Answer **P =** ..... [1]

(b) Find using matrix multiplication, a 1×3 matrix **R** which represents the profit made from selling the flavoured tea on Monday, Tuesday and Wednesday respectively.

Answer **R =** ..... [2]

(c) Evaluate the matrix $\mathbf{Q}=\mathbf{R}\begin{pmatrix}1\\1\\1\end{pmatrix}$ and explain what the element in **Q** represents.

Answer **Q =** ..... [1]

Answer The element in **Q** represents ……

……

…… [1]

Answers

1 20 :15 :18

2 (a) -66 (b) 1000

3 $x = 3\frac{4}{15}$

4 (a) $3x+16y$ (b) $\frac{-x-3}{(x+1)(x-1)}$ OR $\frac{x+3}{(x+1)(1-x)}$ OR $-\frac{x+3}{(x+1)(x-1)}$

5 (a) $x = -1\frac{1}{6}$

(b) $3^{1161}+3^{1158} = 2^2 \times 7 \times 3^{1158}$

Since $3^{1161}+3^{1158}$ has a factor 7, therefore it is divisible by 7.

OR Since $3^{1161}+3^{1158}$ has a factor 28, and 28 is divisible by 7 therefore it is divisible by 7.

OR 28 is a multiple of 7 therefore $3^{1161}+3^{1158}$ is divisible by 7.

OR 7 is a factor of 28 therefore $3^{1161}+3^{1158}$ is divisible by 7.

Without using a calculator, show that is exactly divisible by 7.

6 (a) $\frac{1}{2}$ m/s^2 (b) 8 m/s (c) 318 m

7 (a) 23.68 (b) $x = \pm\sqrt{\frac{36y^3-7W^2}{36}}$

8 8

9 (a) 1080 (b) 1125

10 (a) 84^o (b) 42^o (c) 176^o

11 (a) $\frac{2}{5}$ (b) $(a+b+2bc)(a+b-2bc)$

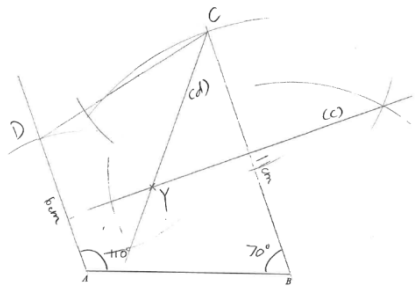
12 (a) triangle *ABX* is similar to triangle *DCX* (AA similarity)

(b) $3\frac{3}{5}$ cm (c) $\frac{25}{81}$

13 (a) 18 cm (b)(i) 165 cm (b)(ii) 150.5 cm (b)(iii) 147 or $146\frac{11}{12}$ cm

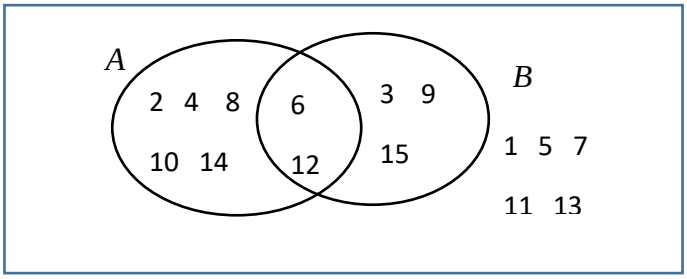
(c) The modal distance is not suitable as it is far away from the rest of the data.
Since the above distribution is skewed (not normal), thus I would use the median.
(Any reasonable answer)

14 (a)



14 (b) Trapezium (d) Triangle *CYB*

15 (a) ξ



(b) {1, 5, 7, 11, 13}

(c) Elements in set $A \cap B$ are positive integers that are less than or equal to 15 and is divisible by 6. .

16 87.5 %

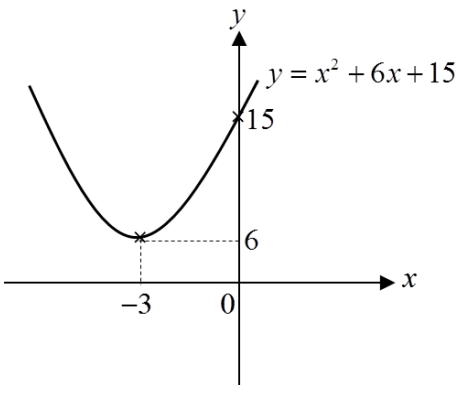
17 (a) 20 , 16 (b) $U = (n-2)^2$ or $U = n^2 - 4n + 4$ (c) 121

(d) The number of unshaded squares (*U*) cannot be 288 as 288 is not a perfect square.

18 $2900.00

19 (a) $6 + (x+3)^2$

(b)



20 (a)(i) 64 cents (a)(ii) 35 cents. (b) The interquartile range must be smaller with the same median.

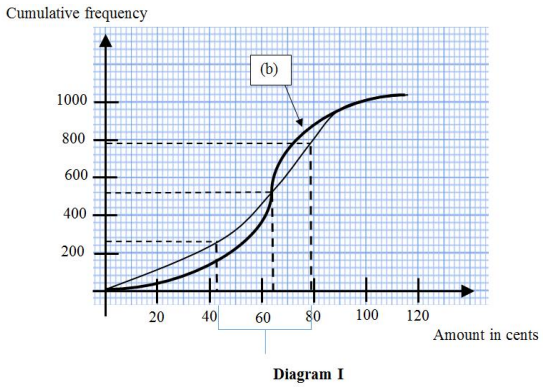


Diagram I

21 (a) $P = (1.6 \quad 1.8 \quad 1.3 \quad 1.7)$ (b) $R = \begin{pmatrix} 212.2 \\ 239.6 \\ 335 \end{pmatrix}$

(c) $Q = (786.8)$.

The element in Q represents the total profit made by this café from the sale of flavoured tea from these 3 days.

Name:	Class:	Class Register Number:

中正中学
CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2018

SECONDARY 4

MATHEMATICS 4048/01

Paper 1 **11 September 2018**

2 hours

Answer Scheme

This document consists of **19** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 If $a = \frac{4}{3}b$ and $b = \frac{5}{6}c$, write down the ratio of $a : b : c$.

$$\frac{a}{b} = \frac{4}{3}$$
$$= \frac{20}{15}$$
$$\frac{b}{c} = \frac{5}{6}$$
$$= \frac{15}{18}$$
$$\therefore a : b : c = 20 : 15 : 18$$

Answer 20 :15 :18 [1]

---

2 If x and y are integer values, such that $-8 \le x \le 7$ and $-1 \le y \le 5$, find

(a) the smallest possible value of $2y - x^2$,

smallest possible value of
$$2y - x^2 = 2(-1) - (-8)^2$$
$$= -66$$

Answer –66 [1]

(b) the largest possible value of $-(y^3x)$.

largest possible value of
$$-(y^3x) = -[(5)^3(-8)]$$
$$= 1000$$

Answer 1000 [1]

---

3 Solve $\frac{2x-3}{2}-\frac{x+6}{6}=\frac{2}{9}$.

$$\frac{2x-3}{2}-\frac{x+6}{6}=\frac{2}{9}$$

$$\frac{9(2x-3)-3(x+6)}{18}=\frac{4}{18} \quad \text{OR} \quad 9(2x-3)-3(x+6)=4$$

$$18x-27-3x-18=4$$

$$15x=49$$

$$x=\frac{49}{15}$$

$$=3\frac{4}{15}$$

Answer $x=3\frac{4}{15}$ [3]

4 **(a)** Simplify $7x-4y-4(x-5y)$.

$$7x-4y-4(x-5y)=7x-4y-4x+20y$$

$$=3x+16y$$

Answer $3x+16y$ [1]

(b) Express $\frac{2x}{x^2-1}+\frac{3}{1-x}$ as a single fraction.

$$\frac{2x}{x^2-1}+\frac{3}{1-x}=\frac{2x}{(x+1)(x-1)}-\frac{3}{x-1} \qquad x^2-1=(x+1)(x-1)$$

$$-\frac{3}{x-1}$$

$$=\frac{2x-3(x+1)}{(x+1)(x-1)}$$

$$=\frac{-x-3}{(x+1)(x-1)}$$

OR $\frac{x+3}{(x+1)(1-x)}$ OR $-\frac{x+3}{(x+1)(x-1)}$

OR

$$\frac{2x}{x^2-1}+\frac{3}{1-x}=\frac{2x(1-x)+3(x^2-1)}{(x^2-1)(1-x)}$$

$$=\frac{2x-2x^2+3x^2-3}{(x^2-1)(1-x)}$$

$$=\frac{x^2+2x-3}{(x^2-1)(1-x)}$$

$$=\frac{(x+3)(x-1)}{(x^2-1)(1-x)}$$

$$=\frac{(x+3)(x-1)}{(x+1)(x-1)(1-x)}$$

$$=\frac{x+3}{(x+1)(1-x)}$$

Answer $\frac{-x-3}{(x+1)(x-1)}$ [3]

5 **(a)** Solve $49^{2x+1} \div \sqrt[3]{7} = \frac{1}{343}$.

$$49^{2x+1} \div \sqrt[3]{7} = \frac{1}{343}$$
$$7^{2(2x+1)} \div 7^{\frac{1}{3}} = \frac{1}{7^3}$$
$$7^{4x+2-\frac{1}{3}} = 7^{-3}$$

By comparing,

$$4x + 2 - \frac{1}{3} = -3$$
$$x = (-3 - 2 + \frac{1}{3}) \div 4$$
$$= -1\frac{1}{6}$$

Answer $x = -1\frac{1}{6}$ [3]

(b) Without using a calculator, show that $3^{1161} + 3^{1158}$ is exactly divisible by 7.

Answer

$$3^{1161} + 3^{1158} = 3^{1158}(1 + 3^3)$$
$$= 3^{1158}(28)$$
$$= 2^2 \times 7 \times 3^{1158}$$

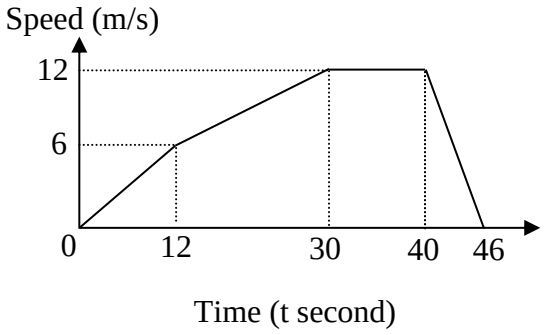
Since $3^{1161} + 3^{1158}$ has a factor 7, therefore it is divisible by 7.

OR Since $3^{1161} + 3^{1158}$ has a factor 28, and 28 is divisible by 7 therefore it is divisible by 7.

OR 28 is a multiple of 7 therefore $3^{1161} + 3^{1158}$ is divisible by 7.

OR 7 is a factor of 28 therefore $3^{1161} + 3^{1158}$ is divisible by 7. [2]

6 The diagram shows the speed–time graph for a car's journey.



Calculate

(a) the acceleration of the car during the first 12 seconds,

$$\text{Acceleration} = \frac{6-0}{12-0}$$
$$= \frac{1}{2}\text{m/s}^2$$

Answer $\frac{1}{2}$ m/s^2 [1]

(b) the speed of the car when t = 18,

Let the speed of the car be v at $t = 18$

$$\frac{v-6}{18-12} = \frac{12-6}{30-12}$$
$$v = \frac{6}{18} \times 6 + 6$$
$$= 8\text{m/s}$$

Answer 8 m/s [2]

(c) the total distance travelled in the first 40 seconds.

Total distance travelled in first 40 seconds

$$= \frac{1}{2} \times 6 \times 12 + \frac{1}{2}(6+12) \times 18 + 12 \times 10$$
$$= 318\text{m}$$

Answer 318 m [2]

7 $W = 6\sqrt{\dfrac{y^3 - x^2}{7}}$

(a) Calculate the value of W when $y = 5$ and $x = -4$. Write your answer correct to two decimal places.

$$W = 6\sqrt{\frac{y^3 - x^2}{7}}$$

$$W = 6\sqrt{\frac{5^3 - (-4)^2}{7}}$$

$$= 23.6763....$$

$$= 23.68 \text{ (correct to 2 dp.)}$$

Answer 23.68 [1]

(b) Rearrange the formula to make x the subject.

$$W = 6\sqrt{\frac{y^3 - x^2}{7}}$$

$$\frac{W}{6} = \sqrt{\frac{y^3 - x^2}{7}}$$

$$\left(\frac{W}{6}\right)^2 = \frac{y^3 - x^2}{7}$$

$$7\left(\frac{W}{6}\right)^2 = y^3 - x^2$$

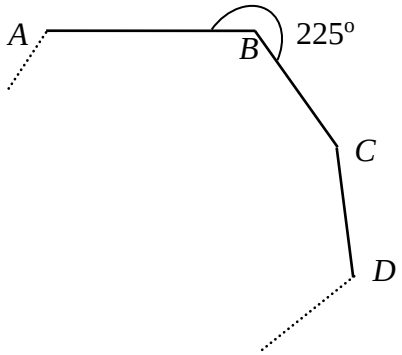
$$x^2 = y^3 - \frac{7W^2}{36}$$

$$= \frac{36y^3 - 7W^2}{36}$$

$$x = \pm\sqrt{\frac{36y^3 - 7W^2}{36}}$$

Answer $x = \pm\sqrt{\dfrac{36y^3 - 7W^2}{36}}$ [3]

8 *A, B, C , D*, …. is part of a regular polygon. Given that reflex angle *ABC* = 225º, how many sides does the polygon have?



$$\text{Interior angle} = 360^o - 225^o \text{ (angles at a point)}$$
$$= 135^o$$
$$(n-2)180 = 135n$$
$$180n - 135n = 360$$
$$n = \frac{360}{45}$$
$$= 8$$

OR

$$\text{Exterior angle} = 180^o - 135^o$$
$$= 45^o$$
$$n = \frac{360}{45}$$
$$= 8$$

$$\text{Number of sides} = 8$$

Answer 8 [2]

---

9 In a particular school, the enrolment in 2016 is 900 students. In 2015, there were 20% more students than in 2016. There were 20% less students in 2016 as compared to 2017.

Calculate the enrolment in

(a) 2015,

Number of students in 2015

$$= \frac{120}{100} \times 900$$
$$= 1080$$

Answer 1080 [1]

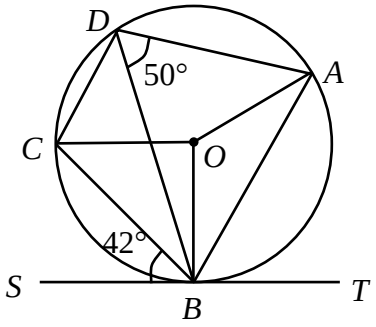
(b) 2017.

Number of students in 2017

$$= \frac{100}{80} \times 900$$
$$= 1125$$

Answer 1125 [1]

10 *A, B, C* and *D* are four points on the circumference of a circle with centre *O*. *ST* is a tangent to the circle at *B*. It is given that angle *ADB* = 50° and angle *CBS* = 42°. Calculate, showing your working clearly,



(a) angle *COB*,

angle $SBO = 90^o$ (tangent $\perp$ radius)

angle $CBO = 90^o - 42^o$

$= 48^o$

angle $BCO = 48^o$ (base angles of isosceles triangle)

angle $COB = 180^o - 48^o \times 2$ (angles sum of triangle)

$= 84^o$

Answer Angle $COB = 84^o$ [2]

(b) angle *CDB*,

angle $CDB = \frac{1}{2} \times 84^o$ (angle at centre is twice angle at circumference)

$= 42^o$

Answer Angle $CDB = 42^o$ [1]

(c) angle *AOC*.

angle $ABC = 180^o - (50^o + 42^o)$ (angles in opposite segment)

$= 88^o$

angle $AOC = 2 \times 88^o$ (angle at centre is twice angle at the circumference)

$= 176^o$

OR

angle $BOA = 2 \times 50^o$ (angle at centre is twice angle at the circumference)

$= 100^o$

angle $AOC = 360^o - 84^o - 100^o$ (angles at a point)

$= 176^o$

Answer Angle $AOC = 176^o$ [2]

11 **(a)** Given that $4x^2 - 12xy + 9y^2 = 0$, find the value of $\frac{4x}{15y}$.

$$\begin{aligned}
4x^2 - 12xy + 9y^2 &= 0 \\
(2x - 3y)^2 &= 0 \\
2x - 3y &= 0 \\
2x &= 3y \\
\frac{x}{y} &= \frac{3}{2} \\
\frac{4x}{15y} &= \left(\frac{3}{2}\right)\left(\frac{4}{15}\right) \\
&= \frac{2}{5}
\end{aligned}$$

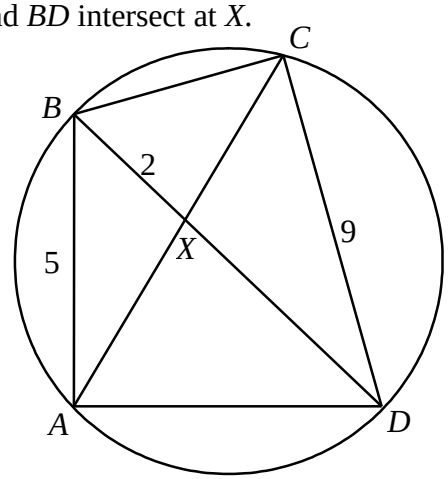
Answer $\frac{2}{5}$ [3]

(b) Factorise $a^2 + 2ab + b^2 - 4b^2c^2$ completely.

$$\begin{aligned}
a^2 + 2ab + b^2 - 4b^2c^2 &= (a+b)^2 - (2bc)^2 \\
&= (a+b+2bc)(a+b-2bc)
\end{aligned}$$

Answer $(a+b+2bc)(a+b-2bc)$ [2]

12 The diagram shows a circle *ABCD*. The diagonals *AC* and *BD* intersect at *X*.
It is given that *BX* = 2 cm, *AB* = 5 cm and *DC* = 9 cm.



(a) Prove, stating your reasons clearly, that triangle *ABX* is similar to triangle *DCX*.

Answer

angle BXA = angle CXD (vertically opposite angles)
angle ABX = angle DCX (angle in the same segment)
Therefore triangle *ABX* is similar to triangle *DCX* (AA similarity) [2]

(b) Find the length of *CX*.

$$\frac{CX}{BX} = \frac{CD}{BA} \text{ (ratios of corresponding side are equal)}$$

$$\frac{CX}{2} = \frac{9}{5}$$

$$CX = \frac{9}{5} \times 2$$

$$= 3\frac{3}{5} \text{cm}$$

Answer $3\frac{3}{5}$ cm [1]

(c) Find $\frac{\text{area of triangle } ABX}{\text{area of triangle } DCX}$.

$$\frac{\text{area of triangle } ABX}{\text{area of triangle } DCX} = \left(\frac{5}{9}\right)^2$$

$$= \frac{25}{81}$$

Answer $\frac{25}{81}$ [1]

13 Twenty four boys took part in the high jump event in a school sports meet. Their records in centimetres are shown in the stem-and-leaf diagram.

Stem	Leaf							
11	0	2						
12	1	2	6					
13								
14	2	3	3	4	4	5	5	
15	6	6	7	7				
16	0	0	1	1	5	5	5	6

Key : 11 | 0 represents 110 cm

(a) Find the interquartile range.

$$\text{Lower quartile} = \frac{142+143}{2}$$
$$= 142.5\,\text{cm}$$
$$\text{Upper quartile} = \frac{160+161}{2}$$
$$= 160.5\,\text{cm}$$
$$\text{Interquartile range} = 160.5 - 142.5$$
$$= 18\,\text{cm}$$

Answer 18 cm [1]

(b) Find

(i) the modal distance,

modal distance = 165 cm

Answer 165 cm [1]

(ii) the median distance,

$$\text{median distance} = \frac{145+156}{2}$$
$$= 150.5\,\text{cm}$$

Answer 150.5 cm [1]

(iii) the mean distance.

$$\text{mean distance} = \frac{3526}{24}$$
$$= 147\,\text{cm (3sf)}$$

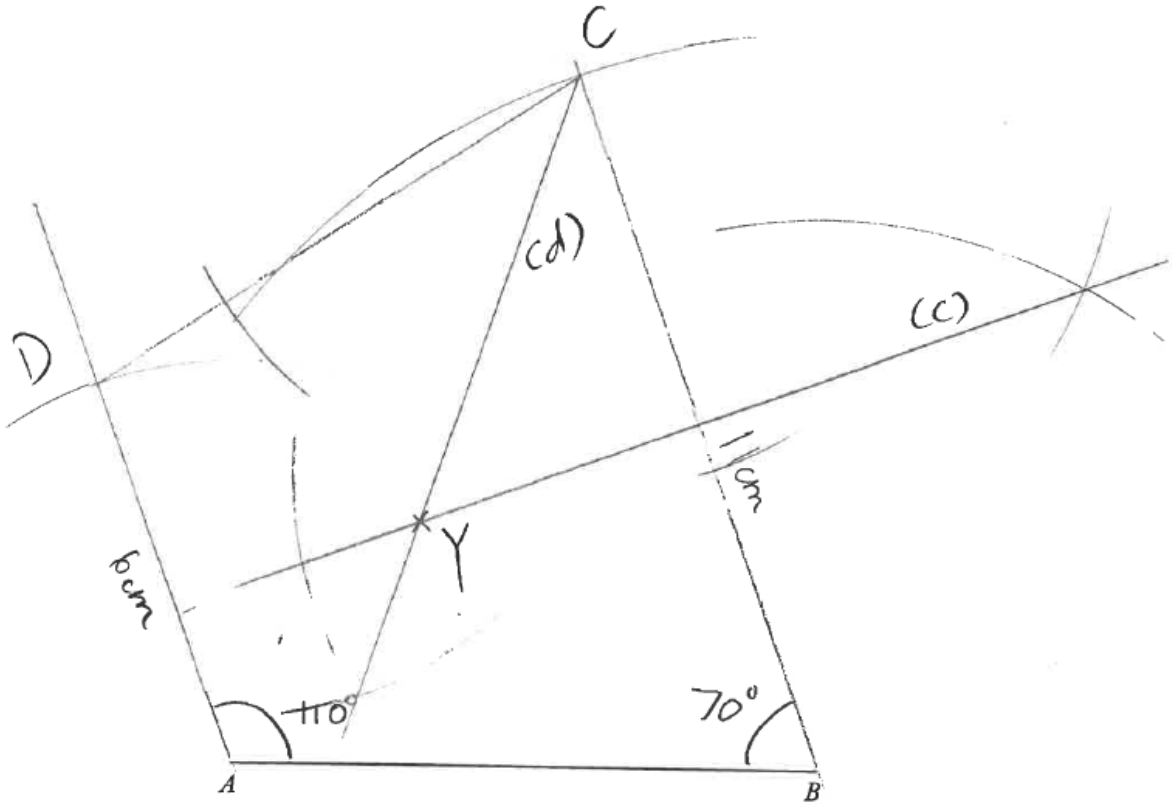
Answer 147 or $146\frac{11}{12}$ cm [1]

(c) Would you use the mode or the median as the most appropriate measure of average in this case? Justify your answer.

Answer

The modal distance is not suitable as it is far away from the rest of the data. Since the above distribution is skewed (not normal), thus I would use the median. (Any reasonable answer) [1]

14 **(a)** Construct the quadrilateral *ABCD* such that angle $ABC = 70^{o}$, $BC = 11\,\text{cm}$, $AD = 6\,\text{cm}$ and angle $BAD = 110^{o}$. Line *AB* has already been drawn. [2]



(b) State the special name of this quadrilateral.

Answer Trapezium [1]

(c) Construct the perpendicular bisector of *BC*. [1]

(d) Construct the angle bisector of $\angle BCD$. [1]

(e) Label the intersection between the bisectors in **(c)** and **(d)** as *Y*. Name an isosceles triangle from the diagram.

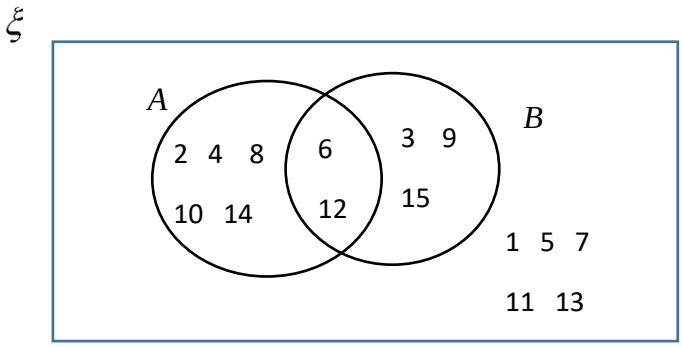
Answer Triangle *CYB* [1]

15 $\xi = \{\text{integers } x : 1 \le x \le 15\}$

$A = \{\text{integers divisible by 2}\}$

$B = \{\text{integers divisible by 3}\}$

(a) Draw a Venn Diagram to illustrate this information. [2]



(b) List the elements contained in the set $(A \cup B)'$.

$(A \cup B)' = \{1, 5, 7, 11, 13\}$ *Answer* $\{1, 5, 7, 11, 13\}$ [1]

(c) Describe, as simply as possible, in words, the elements contained in the set $A \cap B$.

Answer

Elements in set $A \cap B$ are positive integers that are less than or equal to 15 and is divisible by 6. [1]

---

16 It is given that y is inversely proportional to the cube of x. If x is doubled, find the percentage decrease in y.

$$\text{Let } y_1 = \frac{k}{x^3} \quad ------(1)$$

$$y_2 = \frac{k}{(2x)^3} \quad ------(1)$$

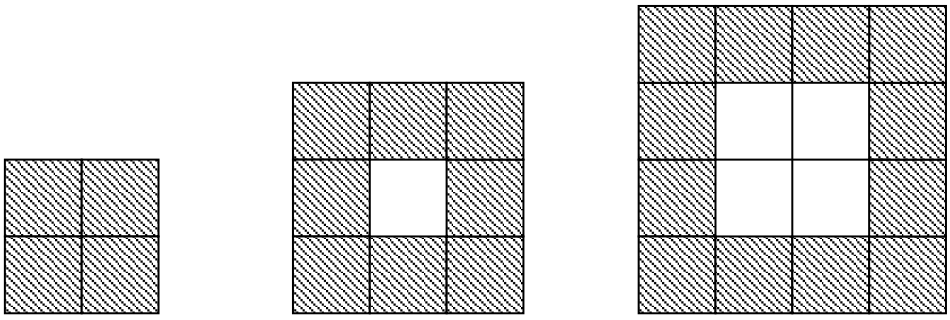
$$\text{Percentage decrease in } y = \frac{\frac{k}{x^3} - \frac{k}{(2x)^3}}{\frac{k}{x^3}} \times 100\%$$

$$= \frac{\frac{k}{x^3}\left(1 - \frac{1}{8}\right)}{\frac{k}{x^3}} \times 100\%$$

$$= 87.5\%$$

Answer 87.5 % [3]

17 Small squares are used to form a series of big squares as shown below.



(a) Complete the following table: [2]

Length of side of big square (n)	Number of shaded small squares (S)	Number of unshaded small squares (U)
2	4	0
3	8	1
4	12	4
5	16	9
6	**20**	**16**

(b) Find an equation connecting U, the number of unshaded small squares, and n, the length of side of big square.

Answer $U = (n-2)^2$ or
$U = n^2 - 4n + 4$ [1]

(c) How many unshaded squares are there in a big square if the length of the big square is 13?

$(13-2)^2 = 11^2$
$= 121$

Answer 121 [1]

(d) Explain why the number of unshaded squares (U) cannot be 288.

Answer The number of unshaded squares (U) cannot be 288 as 288 is not a perfect square.

[1]

18 Ethan invested some money in a bank for 3 years. The rate of compound interest was fixed at 2% per annum. At the end of 3 years, Ethan has a total of \$3077.50 in his account. What was the amount of money that Ethan invested at the beginning?

$$P\left(1+\frac{2}{100}\right)^3 = 3077.50$$

$$P = 3077.50 \div (1.02)^3$$
$$= 2899.996...$$
$$= 2900.00 \text{(nearest cent)}$$

The amount of money that Ethan invested at the beginning = \$2900.00

Answer \$2900.00 [3]

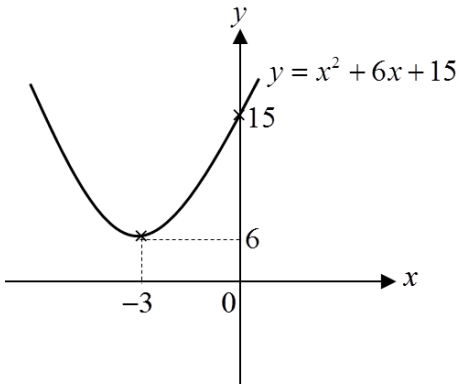
---

19 **(a)** Express $6x + x^2 + 15$ in the form of $a + (x+b)^2$.

$$6x + x^2 + 15 = x^2 + 6x + 3^2 - 3^2 + 15$$
$$= 6 + (x+3)^2$$

Answer $6 + (x+3)^2$ [2]

(b) Hence sketch the graph of $y = 6x + x^2 + 15$. [2]



---

20 The daily expenses of 1040 students in School *X* are recorded. The cumulative frequency curve in **Diagram I** below shows the distribution of their expense.

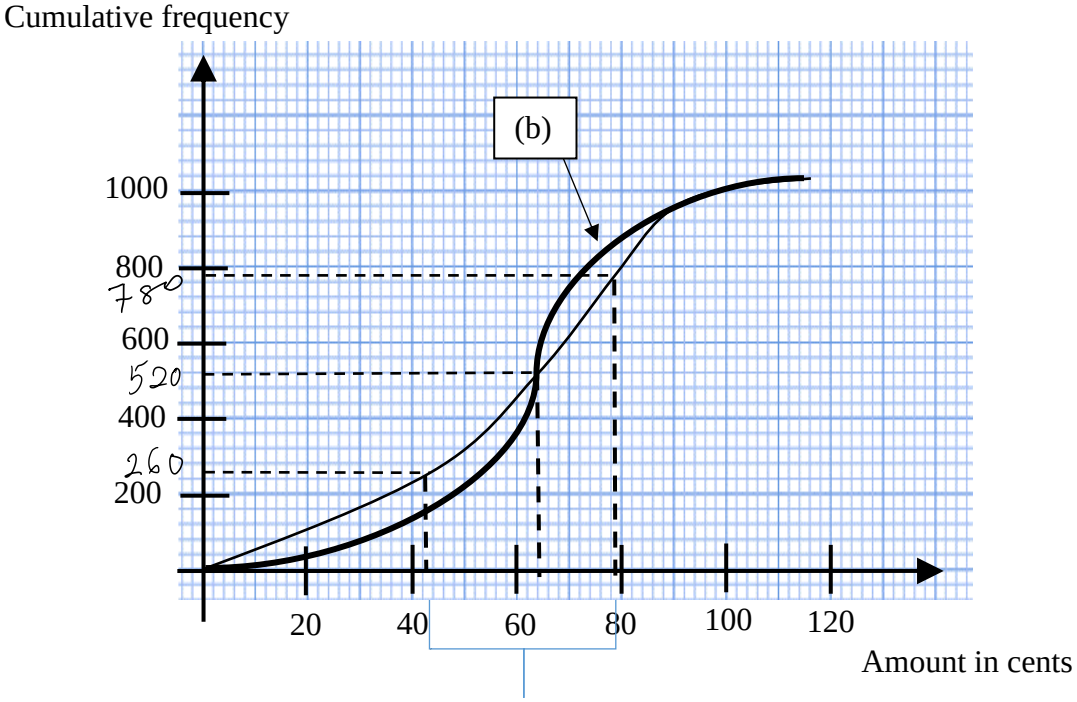


Diagram I

(a) Use the curve to estimate

(i) the median,

Median = 64 cents

Answer 64 cents [1]

(ii) the interquartile range.

Interquartile range = 79 – 44
= 35 cents
OR Interquartile range = 78 – 44
= 34 cents

Answer 35 cents [1]
(can accept 34, 36)

(b) The expenses of 1040 students in another School *Y* had the same median but a smaller interquartile range. Draw a possible cumulative curve to show the distribution of the students' expenses for School *Y* on **Diagram I**.
(The interquartile range must be smaller with the same median)

[1]

21 The table below shows the number of cups of different flavoured tea by a café on Monday, Tuesday and Wednesday, and the respective selling price and cost price of each kind of flavoured tea.

	Green	Plum	Lemon	Assam
Monday	33	47	34	18
Tuesday	40	25	56	34
Wednesday	56	73	21	51
Selling price per cup	\$3.20	\$3.80	\$3.00	\$3.50
Cost price per cup	\$1.60	\$2.00	\$1.70	\$1.80

(a) Express the profit made per cup for each type of flavoured tea as a single row matrix **P**.
(1.6 1.8 1.3 1.7)

Answer $\mathbf{P} = (1.6 \quad 1.8 \quad 1.3 \quad 1.7)$ [1]

(b) Find using matrix multiplication, a 1×3 matrix **R** which represents the profit made from selling the flavoured tea on Monday, Tuesday and Wednesday respectively.

$$R = \begin{pmatrix}1.6 & 1.8 & 1.3 & 1.7\end{pmatrix}\begin{pmatrix}33 & 40 & 56\\ 47 & 25 & 73\\ 34 & 56 & 21\\ 18 & 34 & 51\end{pmatrix}$$

$$= \begin{pmatrix}212.2 & 239.6 & 335\end{pmatrix}$$

Answer $\mathbf{R} = \begin{pmatrix}212.2 & 239.6 & 335\end{pmatrix}$ [2]

(c) Evaluate the matrix $\mathbf{Q} = \mathbf{R}\begin{pmatrix}1\\1\\1\end{pmatrix}$ and explain what the element in **Q** represents.

$$\begin{pmatrix}212.2 & 239.6 & 335\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}212.2 + 239.6 + 335\end{pmatrix}$$

$$= (786.8)$$

Answer $\mathbf{Q} = (786.8)$ [1]

Answer The element in **Q** represents the total profit made by this café from the sale of flavoured tea from these 3 days. [1]

End of Paper

Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Parent's Signature

_____

PRELIMINARY EXAMINATION 2018

SECONDARY 4

MATHEMATICS 4048/02

Paper 2

13 September 2018

2 hours 30 minutes

Additional Materials : Answer Paper
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

100

This document consists of **11** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions

1 **(a)** **(i)** Solve the inequality $\frac{7}{3} < \frac{x}{2} - \frac{x}{3} + 2 \leq 3$. [2]

(ii) Represent your solution to **(i)** on a number line. [1]

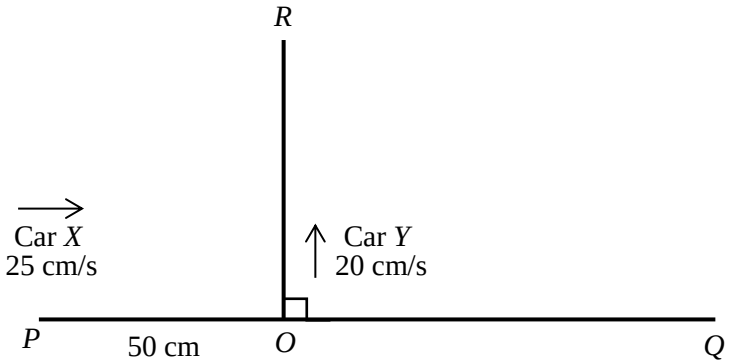
(iii) Write down the smallest prime number which satisfies $\frac{7}{3} < \frac{x}{2} - \frac{x}{3} + 2 \leq 3$. [1]

(b) Express as a single fraction in its simplest form $\frac{4x-5}{2x^2-5x-3} - \frac{3}{2x+1}$. [3]

(c) Simplify $\left(\frac{2p}{3q}\right)^2 \div \frac{\sqrt{q^5}}{\sqrt{p}}$. [2]

(d) Solve the equation $\frac{x}{3-x} = \frac{3}{x-3} + 5$. [3]

2



The diagram shows a toy car track *POQR* with *PQ* perpendicular to *OR* and $OP = 50$ cm. Toy car *X* starts from *P* and travels towards *Q* at a constant speed of 25 cm/s. At the same time, another toy car *Y* starts from *O* and travels towards *R* at a constant speed of 20 cm/s.

(a) Given that *t* is the time in seconds after the start of motion for both cars, write down expressions, in terms of *t*, for the distances of both cars from *O* when $t < 2$. [2]

(b) Show that when the two toy cars are 35 cm apart,

$$41t^2 - 100t + 51 = 0.$$ [3]

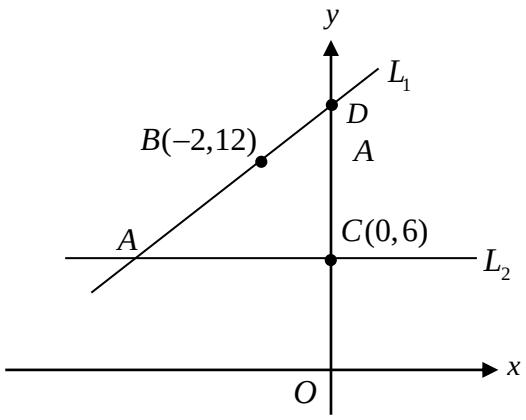
(c) Solve the equation $41t^2 - 100t + 51 = 0$, leaving your answers correct to 2 decimal places. [3]

(d) Hence, find the distance(s) of car *Y* from *O*, when the two cars are 35 cm apart. [2]

A stone was detected lying at a point along the track between *OQ*.

(e) Given that $OQ = 75$ cm, find the range of values of *t* for which car *X* will take to reach the stone. [2]

3 The diagram shows a line, L_1, drawn through point B (–2 , 12) and another line, L_2, drawn through point C (0, 6). L_1 cuts the y-axis at D and L_2 is parallel to the x-axis . L_1 and L_2 intersect at A.



(a) Write down the equation of the line AC. [1]

(b) The gradient of L_1 is 2. Find the equation of L_1. [2]

(c) Find the coordinates of A. [2]

(d) Find the area of triangle AOD, where O is the origin. [3]

(e) A student draws another line $y = -3x + 1$ on the diagram and claims that it passes through the point B.

Is he correct? Justify your answer with calculations. [2]

---

4 The table below shows the distances of four planets from the Sun.

Planet	Distance from the Sun (km)
Pluto	5.91 billion
Earth	1.46×10^{8}
Uranus	2.88×10^{9}
Mercury	5.79×10^{7}

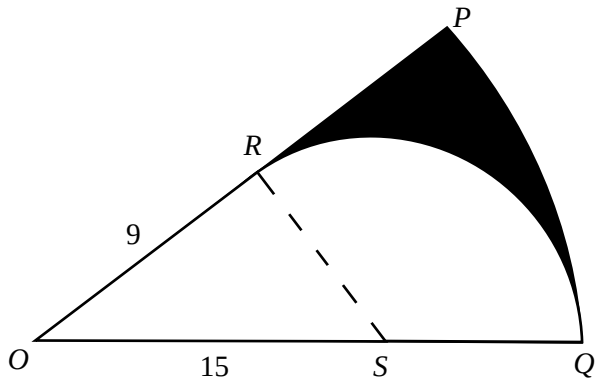
(a) Write 5.91 billion in standard form. [1]

(b) Calculate the distance between Earth and Mercury.
Give your answer in standard form. [1]

(c) Show that Uranus is approximately 50 times further away from the Sun as compared to Mercury. [2]

(d) Light travels 1 kilometre in 3.34×10^{-6} seconds.
How many seconds does light take to travel from the Sun to Earth? [2]

5



In the diagram, *OPQ* is the cross section of a wooden door stopper.
PQ is an arc of a circle, centre *O* and *RQ* is an arc of another circle, centre *S*.
OR = 9 cm, *OS* = 15 cm and *OP* is a tangent to arc *RQ* at *R*.

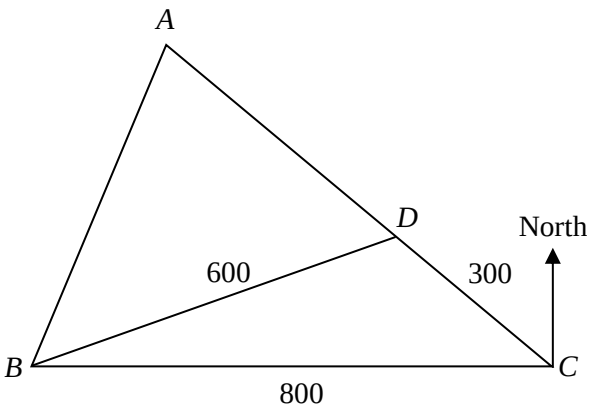
(a) Show that angle *ROS* = 0.927 radians, correct to 3 significant figures. [2]

The wooden door stopper is 30 mm thick. The shaded region represents the portion that will be cut off to remove its sharp edge.

(b) Calculate the perimeter of the shaded region. [5]

(c) Calculate the volume of wood, in cm^3, that needs to be cut off. [5]

6 *A*, *B*, *C* and *D* are four coastal guard posts on the Indian Ocean. *C* is 800 m due east of *B* and *AC* = *BC*. *D* is on line *AC* such that $CD = 300$ m and $BD = 600$ m.



(a) Calculate

(i) angle *BCD*, [2]

(ii) the bearing of *A* from *B*. [2]

(b) Find *AB*. [2]

(c) A ship sailing along *AC* stops at a point *X*, which is nearest to *B*.

(i) Find *BX*. [2]

(ii) The ship at point *X* sends a distress signal by shooting a red flame vertically up into the sky. It was spotted from point *B* when the red flame reached a height of 250 m.

Find the angle of elevation of the red flame from *B*. [2]

7 Answer the whole of this question on a sheet of graph paper.

A man sends a drone down a cliff next to the sea.
The height, *h* metres, of the drone above sea level *t* seconds after it is released can be modelled by the equation

$$h = 18 - 3t + 0.2t^2 .$$

The table shows some corresponding values of *t* and *h*.

t	0	2	4	7	8	10	12	13
h	18	12.8	9.2	*p*	6.8	8	10.8	12.8

(a) Find the value of *p*. [1]

(b) Using a scale of 2 cm to represent 2 seconds, draw a horizontal *t*-axis for $0 \le t \le 13$.
Using a scale of 2 cm to represent 2 metres, draw a vertical *h*-axis for $0 \le h \le 18$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to estimate

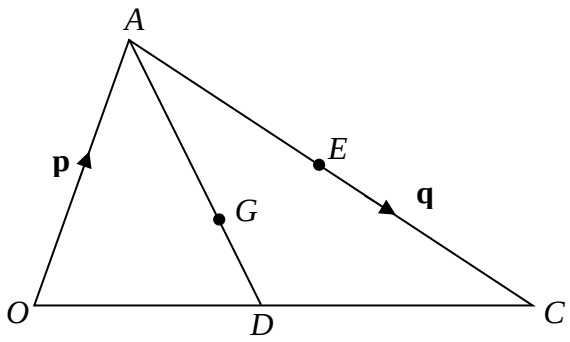
(i) the minimum height of the drone above the sea level, [1]

(ii) the length of time at which the drone is less than 9 m above the sea level. [2]

(d) (i) By drawing a tangent, find the gradient of the curve at $t = 10$. [2]

(ii) Explain the significance of your answer in **(d)(i)**. [1]

8



OAC is a triangle.
D and E are the midpoints of OC and AC respectively.
$\overrightarrow{OA} = \mathbf{p}$ and $\overrightarrow{AC} = \mathbf{q}$. G is a point along AD such that $AG : AD = 2 : 3$.

(a) Express, as simply as possible, in terms of **p** and **q**

(i) $\overrightarrow{OE}$, [1]

(ii) $\overrightarrow{AD}$, [2]

(iii) $\overrightarrow{AG}$, [2]

(iv) $\overrightarrow{GE}$. [2]

(b) Explain whether O, G and E are collinear. [2]

(c) If the area of triangle AOD is 14 cm^2, find the area of triangle AOC. [2]

9 The table below summarises the heights of 200 trees in Rainforest A.

Height (h metres)	$0 \le h < 10$	$10 \le h < 20$	$20 \le h < 30$	$30 \le h < 40$	$40 \le h < 50$
Frequency	10	25	80	65	20

(a) Calculate an estimate of

(i) the mean height, [1]

(ii) the standard deviation. [1]

(b) The mean height for another 200 trees in Tropical Forest B was 20.1 metres and the standard deviation was 4.53 metres.

Use this information to comment on two differences between the heights of the trees in the two forests. [2]

(c) A tree was selected at random from Rainforest A.
Find, as a fraction in its lowest terms, the probability that its height is

(i) 40 metres and above, [1]

(ii) below 30 metres. [1]

(d) Two trees are selected at random.
Find the probability that **both** trees will be less than 20 metres.
Give your answer as a decimal correct to 3 significant figures. [2]

---

10 The Central Provident Fund (CPF) is a comprehensive social security savings scheme funded by contributions from **both** employers and employees. It is a key pillar of Singapore's social security system, and serves to meet the citizen's retirement, housing and healthcare needs.

The table shows the current monthly CPF contribution rates for Singapore Citizens and Permanent Residents.

Employee's age (years)	Monthly Contribution Rates from 1 Jan 2016 (for gross monthly income ≥ $750)		
	By Employer (% of income)	By Employee (% of income)	Total (% of income)
55 and below	17	20	37
Above 55 to 60	13	13	26
Above 60 to 65	9	7.5	16.5
Above 65	7.5	5	12.5

(a) Brandon, a Singapore Citizen, aged 25 years, earns a gross monthly income of $3650.

Calculate the **total** amount that contributes to Brandon's CPF account every month. [1]

(b) The monthly CPF contribution goes into three accounts.

The table below provides information on the accounts and the ratio of contribution.

Account Type	**Ratio of Contribution**
Ordinary Account (OA) *Primarily for housing needs.*	0.6217
Special Account (SA) *Primarily for retirement needs.*	0.1621
Medisave Account (MA) *Primarily for healthcare needs.*	0.2162

Calculate the monthly amount that goes into Brandon's OA, giving your answer to the nearest cent. [1]

(c) Brandon is planning to purchase a new 3-room flat.

He found the following information online.

Average Price of New Flats in 3rd Quarter of 2018		
3-room	4-room	5-room/Executive
$270000	$350000	$410000

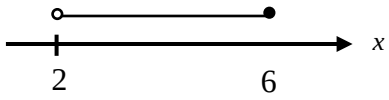
***Administration Fees Payable for Purchase of a New Flat**

Type of fee	**Amount**
Option Fee *Option Fee is paid when you book your flat. It is based on the flat type that is booked.*	4/5-room and Executive Flat : $2000 3-room : $1000 2-room : $1000
Downpayment	10% of purchase price of flat
Stamp Duty on Agreement for Lease *A duty levied on the legal recognition of certain documents. It is based on the **purchase price** of the flat.*	First $180,000 : 1% Next $180,000 : 2% Remaining Amount : 3%
Conveyancing fee for sale of flat *Legal costs involved when buying/selling a flat. . It is based on the **purchase price** of the flat.*	First $30,000 : $0.90 per $1000 Next $30,000 : $0.72 per $1000 Remaining Amount : $0.60 per $1000 Please note that conveyancing fees are subjected to 7% GST.
Caveat registration fee *A caveat is lodged with the Singapore Land Authority to protect your interest in the flat. This is done by giving notice of your interest pending the Lease registration.*	$64.45 (Inclusive of GST)
* These administration fees can be paid using funds in the CPF OA.	

Brandon wishes to pay the administration fees completely from his OA and has $6000 in his OA.

Suggest the number of years that Brandon will need to accumulate sufficient money in his OA to pay for the administration fees. [7]

Answer Key

1	**(a)(i)**	$2 < x \leq 6$
	(a)(ii)	
	(a)(iii)	3
	(b)	$\frac{x+4}{(2x+1)(x-3)}$
	(c)	$\frac{4p^{\frac{5}{2}}}{9q^{\frac{9}{2}}}$
	(d)	$x = 2$
2	**(a)**	Car X : distance from O = $(50 - 25t)$ cm Car Y : distance from O = $20t$ cm
	(b)	Show question
	(c)	$t = 1.71$ or $t = 0.73$
	(d)	When $t = 0.73$, distance of Y from O = 14.5 cm (3 sig.fig) When $t = 1.71$, distance of Y from O = 34.3 cm (3 sig.fig)
	(e)	$2 < t < 5$
3	**(a)**	$y = 6$
	(b)	$y = 2x + 16$
	(c)	$A(-5, 6)$
	(d)	40 units^2
	(e)	Since $B(-2,12)$ does not satisfy the equation $y = -3x + 1$, the point B does not lie on the line $y = -3x + 1$. The student is not correct.
4	**(a)**	5.91×10^9
	(b)	8.81×10^7
	(c)	Show question
	(d)	487.64 s
5	**(a)**	Show question
	(b)	73.0 cm (3 s.f.)
	(c)	312 cm^3
6	**(a)(i)**	39.6°
	(a)(ii)	019.8°
	(b)	542 m (3 s.f.)
	(c)(i)	510 m (3 s.f.)
	(c)(ii)	26.1° (1 d.p.)
7	**(a)**	$p = 6.8$
	(c)(i)	6.7 m
	(c)(ii)	6.6 s
	(d)(i)	0.938
	(d)(ii)	It represents the speed at which the drone is rising at $t = 10$, which is 0.938 m/s.

8	**(a)(i)**	$\underset{\sim}{p}+\frac{1}{2}\underset{\sim}{q}$
	(a)(ii)	$-\frac{1}{2}\underset{\sim}{p}+\frac{1}{2}\underset{\sim}{q}$
	(a)(iii)	$-\frac{1}{3}\underset{\sim}{p}+\frac{1}{3}\underset{\sim}{q}$
	(a)(iv)	$\frac{1}{3}\underset{\sim}{p}+\frac{1}{6}\underset{\sim}{q}$
	(b)	O, G and E are collinear since $\overrightarrow{GE}=\frac{1}{3}\overrightarrow{OE}$ with a common point E.
	(c)	28 cm^2
9	**(a)(i)**	28 m
	(a)(ii)	9.80 m
	(b)	The trees in Rainforest A are generally taller as their mean height of 28 m is higher than 20.1 m of Tropical Forest B. The heights of trees in Rainforest A are generally **more widespread** as they have a higher standard deviation of 9.80 as compared to Tropical Forest B of 4.53 m.
	(c)(i)	$\frac{1}{10}$
	(c)(ii)	$\frac{23}{40}$
	(d)	0.0299 (3 s.f.)
10	**(a)**	\$1350.50
	(b)	\$839.61 (nearest cents)
	(c)	Brandon will need about 3 years to accumulate sufficient money.

Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2018

SECONDARY 4

MATHEMATICS 4048/02

Paper 2 **13 September 2018**

2 hours 30 minutes

SOLUTION FOR STUDENTS

This document consists of **21** printed pages and **0** blank page.

1 (a)(i)

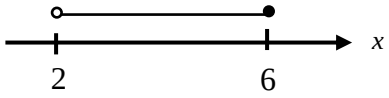
$$\frac{7}{3} < \frac{x}{2} - \frac{x}{3} + 2 \leq 3$$

$$\frac{7}{3} - 2 < \frac{x}{2} - \frac{x}{3} \leq 3 - 2$$

$$\frac{1}{3} < \frac{x}{6} \leq 1$$

$$2 < x \leq 6$$

(a)(ii)



(a)(iii) 3

(b)

$$\frac{4x-5}{2x^2-5x-3} - \frac{3}{2x+1}$$

$$= \frac{4x-5}{(2x+1)(x-3)} - \frac{3}{2x+1}$$

$$= \frac{4x-5}{(2x+1)(x-3)} - \frac{3(x-3)}{(2x+1)(x-3)}$$

$$= \frac{4x-5-3x+9}{(2x+1)(x-3)}$$

$$= \frac{x+4}{(2x+1)(x-3)}$$

(c)

$$\left(\frac{2p}{3q}\right)^2 \div \frac{\sqrt{q^5}}{\sqrt{p}}$$

$$= \frac{4p^2}{9q^2} \times \frac{\sqrt{p}}{\sqrt{q^5}}$$

$$= \frac{4p^2}{9q^2} \times \frac{p^{\frac{1}{2}}}{q^{\frac{5}{2}}}$$

$$= \frac{4p^{\frac{5}{2}}}{9q^{\frac{9}{2}}}$$

(d)

$$\frac{x}{3-x} = \frac{3}{x-3} + 5$$
$$-\frac{x}{x-3} = \frac{3}{x-3} + 5$$
$$-5 = \frac{3}{x-3} + \frac{x}{x-3}$$
$$-5 = \frac{3+x}{x-3}$$
$$-5(x-3) = 3+x$$
$$-5x+15 = 3+x$$
$$6x = 12$$
$$x = 2$$

2 (a) Car X : distance from O = (50 – 25t) cm

Car Y : distance from O = 20t cm

(b)

$$(50-25t)^2 + (20t)^2 = 35^2$$
$$2500 - 2500t + 625t^2 + 400t^2 = 1225$$
$$1025t^2 - 2500t + 1275 = 0$$
$$(\div 25) \quad 41t^2 - 100t + 51 = 0 \text{ (shown)}$$

(c)

$$41t^2 - 100t + 51 = 0$$
$$t = \frac{-(-100) \pm \sqrt{(-100)^2 - 4(41)(51)}}{2(41)}$$
$$= \frac{100 \pm \sqrt{1636}}{82}$$

$t = 1.7127...$ or $t = 0.72625....$

$= 1.71$ (2 dec.pl) $= 0.73$(2 dec.pl)

(d)

When $t = 0.76262\ldots$

Distance of Y from $O = 20(0.7262..)$

$= 14.525\ldots$

$= 14.5$ cm (3 sig.fig)

When $t = 1.7127\ldots$

Distance of Y from $O = 20(1.7127..)$

$= 34.255\ldots$

$= 34.3$ cm (3 sig.fig)

(e)

$$\text{Time to reach } O = \frac{50}{25}$$
$$= 2$$
$$\text{Time to reach } Q = \frac{75}{25}$$
$$= 3$$
$$2 < t < 5$$

---

3 (a) $y = 6$

(b)

$y = 2x + c$

subst $(-2, 12)$

$12 = 2(-2) + c$

$12 = -4 + c$

$c = 16$

$\therefore L_1 : y = 2x + 16$

(c)

$y = 6 - - - (1)$

$y = 2x + 16 - - - (2)$

subst (1) into (2),

$2x + 16 = 6$

$2x = -10$

$x = -5$

$y = 6$

$\therefore A(-5, 6)$

(d)

$L_1 : y = 2x + 16$

when $x = 0, y = 16$

Area of triangle AOD

$= \frac{1}{2} \times 16 \times 5$

$= 40 \text{ units}^2$

(e)

subst $x = -2$ into $y = -3x + 1$

$y = -3(-2) + 1$

$= 7 \neq 12$

Since $B(-2,12)$ does not satisfy the equation $y = -3x + 1$,

the point B does not lie on the line $y = -3x + 1$.

The student is not correct.

---

4 (a) 5.91 billion = 5.91×10^9

(b)

$1.46 \times 10^8 - 5.79 \times 10^7$

$= 8.81 \times 10^7$

(c)

$\frac{2.88 \times 10^9}{5.79 \times 10^7} = 49.7409...$

$= 50$ (2 sig.fig) (Shown)

(d)

Time taken

$= 1.46 \times 10^8 \times 3.34 \times 10^{-6}$

$= 487.64 \ s$

---

5 (a)

$\angle ORS = \frac{\pi}{2}$ (tangent perpendicular to radius)

$\cos\angle ROS = \frac{9}{15}$

$\angle ROS = \cos^{-1}\left(\frac{9}{15}\right)$

$= 0.927295...$

$= 0.927$ (3 sig.fig)

(b)

$RS = \sqrt{15^2 - 9^2}$

$= \sqrt{144}$

$= 12$ cm

$OP = OQ$

$= 15 + 12$

$= 27$ cm

$RP = 27 - 9$

$= 18$ cm

$\angle RSQ = \frac{\pi}{2} + 0.927295...$

$= 2.49809...$

Arc length $RQ = 12(2.49809...)$

$= 29.9770...$cm

Arc length $PQ = 27(0.927295...)$

$= 25.036965...$cm

Perimeter $= 18 + 29.97709 + 25.036965$

$= 73.0140...$

$= 73.0$ cm (3 sig.fig)

(c)

Area of triangle $ROS = \frac{1}{2} \times 9 \times 12$

$= 54 \text{ cm}^2$

Area of sector $RSQ = \frac{1}{2} \times (12)^2 \times 2.49809...$

$= 179.86259... \text{ cm}^2$

Area of sector $OPQ = \frac{1}{2} \times (15+12)^2 \times \left[\cos^{-1}\left(\frac{9}{15}\right) \right]$

$= 337.999... \text{ cm}^2$

Area of shaded region $= 337.999... - 54 - 79.86259$

$= 104.136517... \text{ cm}^2$

Volume of wood to be cut off

$= 104.136517... \times 3$

$= 312.4095....$

$= 312 \text{ cm}^3$ (3 sig.fig)

6 (a) (i)

$$600^2 = 800^2 + 300^2 - 2(800)(300)\cos \angle BCD$$

$$360000 = 730000 - 480000 \cos \angle BCD$$

$$\cos \angle BCD = \frac{360000 - 730000}{-480000}$$

$$\angle BCD = 39.571219...$$

$$= 39.6° \text{ (1 dec.pl)}$$

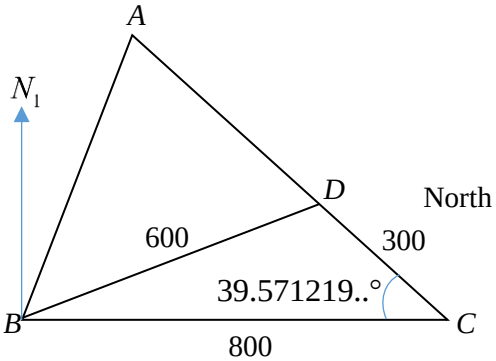
(ii)

$AC = BC$ (given)

$\angle ABC = \angle CAB$

$= \frac{180° - 39.571219°}{2}$

$= 70.21439...°$



$\angle N_1BA = 90° - 70.21439...°$

$= 19.7856..°$

$= 19.8°$ (1 dec.pl)

Bearing of A from $B = 019.8°$(1 dec.pl)

(b)

$$\frac{800}{\sin\angle BAC} = \frac{AB}{\sin\angle DCB}$$

$$\frac{800}{\sin 70.21439°} = \frac{AB}{\sin 39.57129°}$$

$$AB = \frac{800}{\sin 70.21439°} \times \sin 39.57129°$$

$$= 541.60336...$$

$$= 542 \text{ m (3 sig.fig)}$$

Using cosine rule

$$AB^2 = 800^2 + 800^2 - 2(800)(800)\cos 39.571219°$$

$$AB^2 = 293333.32...$$

$$AB = 541.60255...$$

$$= 542 \text{ m (3 sig.fig)}$$

(c)(i)

$$\frac{BX}{AB} = \sin 70.2143...°$$

$$BX = 541.60336... \times \sin 70.2143...°$$

$$= 509.629...$$

$$= 510 \text{ m (3 sig.fig)}$$

(ii)

Let the top of the flame be F

$$\tan\angle FBX = \frac{250}{509.629...}$$

$$\angle FBX = 26.1303...°$$

$$= 26.1° \text{ (1 dec.pl)}$$

7 (a)

$p = 6.8$

(b) All 8 points plotted

Smooth curve through plotted points

(c)(i) Minimum height = 6.7 m

(ii) length of time = 10.8 – 4.2

= 6.6 s

(d)(i)

$$\text{gradient} = \frac{11-5}{13.2-6.8}$$

$$= 0.938$$

(ii) It represents the speed at which the drone is rising at t = 10, which is 0.938 m/s.

8 (a)(i)

$$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$$
$$= \underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}$$

(ii)

$$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$$
$$= -\underset{\sim}{p} + \frac{1}{2}\overrightarrow{OC}$$
$$= -\underset{\sim}{p} + \frac{1}{2}\left(\underset{\sim}{p} + \underset{\sim}{q}\right)$$
$$= -\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}$$
$$= -\frac{1}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}$$

(iii)

$$\overrightarrow{AG} = \frac{2}{3}\overrightarrow{AD}$$
$$= \frac{2}{3}\left(-\frac{1}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}\right)$$
$$= -\frac{1}{3}\underset{\sim}{p} + \frac{1}{3}\underset{\sim}{q}$$

(iv)

$$\overrightarrow{GE} = \overrightarrow{GA} + \overrightarrow{AE}$$
$$= \frac{1}{3}\underset{\sim}{p} - \frac{1}{3}\underset{\sim}{q} + \frac{1}{2}\underset{\sim}{q}$$
$$= \frac{1}{3}\underset{\sim}{p} + \frac{1}{6}\underset{\sim}{q}$$

(b)

$$\overrightarrow{GE} = \frac{1}{3}\underset{\sim}{p} + \frac{1}{6}\underset{\sim}{q}$$
$$= \frac{1}{3}\left(\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}\right)$$
$$= \frac{1}{3}\overrightarrow{OE}$$

O, G and E are collinear since $\overrightarrow{GE} = \frac{1}{3}\overrightarrow{OE}$ with a common point E.

(c)

Both triangles have common height,

$$\frac{\text{Area of triangle } AOD}{\text{Area of triangle } AOC} = \frac{1}{2}$$

$$\frac{14}{\text{Area of triangle } AOC} = \frac{1}{2}$$

$$\text{Area of triangle } AOC = 14 \times 2$$

$$= 28 \text{ cm}^2$$

9 (a) (i) 28 m
(ii) 9.80 m

(b) The trees in Rainforest A are generally taller as their mean height of 28 m is higher than 20.1 m of Tropical Forest B.

The heights of trees in Rainforest A are generally **more widespread** as they have a higher standard deviation of 9.80 as compared to Tropical Forest B of 4.53 m.

(c) (i)

$$\text{P(height} \geq 40 \text{ m)} = \frac{20}{200}$$

$$= \frac{1}{10}$$

(ii)

$$\text{P(height} < 30 \text{ m)} = \frac{80 + 25 + 10}{200}$$

$$= \frac{23}{40}$$

(d)

$$\text{P(both trees} < 20 \text{ m)} = \frac{35}{200} \times \frac{34}{199}$$

$$= 0.0299 \text{ (3 sig.fig)}$$

10 (a)
monthly contribution

$= \$\frac{37}{100} \times 3650$

$= \$1350.50$

(b)
Amount in OA

$= \$\,0.6217 \times 1350.50$

$= \$839.605...$

$= \$839.61$ (nearest cents)

(c)
Administrative Fees for 3 room flat

(1) option fee : $\$1000$

(2) Downpayment $= \$\frac{10}{100} \times 270\,000$

$= \$27\,000$

(3) Stamp duty = $\$1800 + \left(\frac{2}{100} \times \$90\,000\right)$

$= \$3600$

(4) Conveyancing fee : First \$30000 : $\$0.90 \times 30 = \$27$

Next \$30000 : $\$0.72 \times 30 = \$21.60$

Remaining amount : $\$0.60 \times 210 = \$126$

Total including GST $= (\$27 + \$21.60 + \$126) \times 1.07$

$= \$186.82$

Total amount payable = $\$1000 + \$27\,000 + \$3600 + \$186.82 + \$64.45$

$= \$31851.27$

Amount that Brandon needs = $\$31851.27 - \$6000$

$= \$25851.27$

Number of years needed

$= \left(\frac{\$25851.27}{\$839.61}\right) \div 12$

=2.565.. years

≈ 3 years

Brandon will need about 3 years to accumulate sufficient money.

CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

CANDIDATE NAME	Sec 4 ( ) Reg. No: ( )

CENTRE NUMBER					INDEX NUMBER			

MATHEMATICS
Paper 1

4048/01
15 August 2018

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

This document consists of **16** printed pages.

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's Use

Answer **all** the questions.

1 **(a)** Expand and simplify $(2x-1)(2-3x)-3x(2x-5)$.

Answer ..... [2]

(b) Factorise completely $24ab-4ac+pc-6pb$.

Answer ..... [2]

---

2 Solve the equation $2^{3x} \times 125^{x} = 100$.

Answer $x =$ ..... [3]

For Examiner's Use

For Examiner's Use

3 $A = \{\text{points lying on the line } 2x + y = 8\}$
$B = \{\text{points lying on the line } 3x - 4y = 12\}$
$C = \{\text{points lying on the line } mx - 4y = c\}$

(a) Is $(-1,6) \in A$? Explain clearly.

Answer

_____

_____ [1]

(b) Find the element p such that $p \in (A \cap B)$.

Answer $p =$ _____ [2]

(c) Write down a possible value of m and of c such that $B \cap C = \varnothing$.

Answer $m =$ _____ [1]

$c =$ _____ [1]

For Examiner's Use

For Examiner's Use

4 **(a)** Solve the inequalities $8+x<10+\frac{3}{2}x\leq 15.5-2x$.

Answer ..... [3]

(b) Write down the largest rational number that satisfies $8+x<10+\frac{3}{2}x\leq 15.5-2x$.

Answer ..... [1]

---

5 Integers P and Q, written as products of their prime factors, are $P=2^2\times 3\times k^2$ and $Q=2^3\times 7\times k$, where k is a prime number.

(a) Express, in terms of k and as a product of its prime factors, the smallest integer which is divisible by both P and Q.

Answer ..... [1]

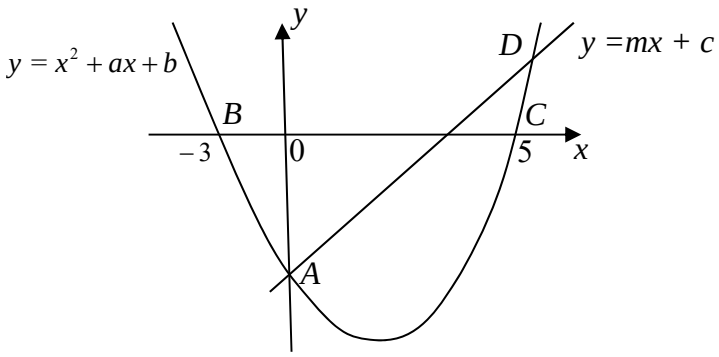
(b) Find the smallest integer, n, such that $(27k)n$ is a multiple of P. Give your answer in terms of k **if necessary**.

Answer $n =$ ..... [1]

For Examiner's Use

For Examiner's Use

6 The diagram shows the curve $y = x^2 + ax + b$ and the line $y = mx + c$ which intersect at A on the y-axis and D.
The curve intersects the x-axis at B and C. The coordinates of D are $(6, p)$.



(a) Find the value of a and of b.

Answer $a =$ _____ [1]

$b =$ _____ [1]

(b) Find the value of p.

Answer $p =$ _____ [1]

(c) Find the equation of the line parallel to BD and passing through C.

Answer _____ [2]

(d) Find the coordinates of E such that $ABCE$ is a parallelogram.

Answer _____ [2]

For Examiner's Use

For Examiner's Use

7 y is inversely proportional to the square root of x and the difference in the values of y when $x = 9$ and when $x = 16$ is 3.

Find

(a) an equation connecting x and y,

Answer _____ [3]

(b) the exact value of x when $y = 5$.

Answer $x =$ _____ [1]

For Examiner's Use

For Examiner's Use

8 Jon travels at 20 km/h for 50% of the journey and 25 km/h for the rest of the journey.
Calculate Jon's average speed, in km/h for the whole journey.

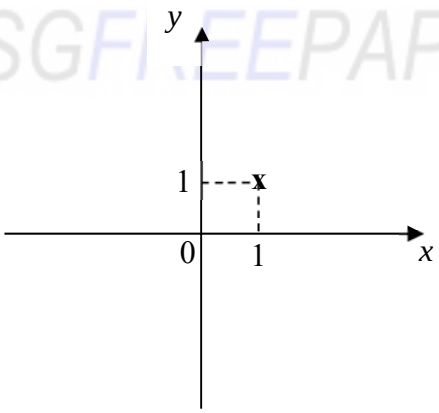
Answer _____ km/h [3]

---

9 The point (1,1) is marked on each diagram in the answer space below.
Sketch, for $x > 0$, the graph of

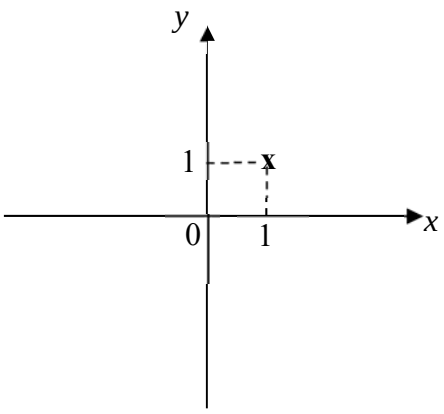
(a) $y = 3^x$,

Answer [1]



(b) $y = \frac{2}{x}$.

Answer [1]



For Examiner's Use

For Examiner's Use

10 Vehicles approaching a crossroad may go in one of these directions: left, right or straight ahead.

If the probability of a vehicle turning left is $\frac{7}{20}$, the probability of it turning right is $\frac{9}{20}$ and the probability of it going straight ahead is $\frac{1}{5}$, calculate the probability that for any three vehicles approaching the crossroads,

(a) all will go straight,

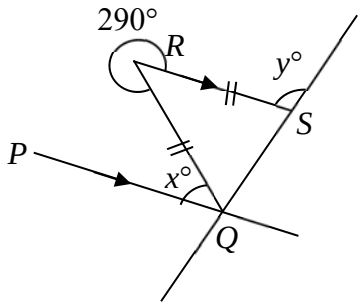
Answer _____ [1]

(b) at least one will turn right.

Answer _____ [2]

---

11 In the diagram, PQ is parallel to RS, $RQ = RS$ and reflex $\angle QRS = 290°$.
Calculate the value of



(a) x,

Answer $x =$ _____ [1]

(b) y.

Answer $y =$ _____ [2]

For Examiner's Use

For Examiner's Use

12 The cash price of a computer is $2 750. Mr Ong bought the computer by paying a 30% downpayment and monthly instalments of $90 over 2 years.

(a) How much is the downpayment?

Answer $ _____ [1]

(b) Calculate the total amount Mr Ong paid for the computer.

Answer $ _____ [2]

(c) Find the flat rate of interest per annum for the instalments.

Answer _____ % [3]

---

13 On a certain day, the exchange rate between the pound (£) and the Singapore dollar (S$) was S$1.95 = £1.

(a) Calculate the amount of pounds which May can buy with S$1170.

Answer £ _____ [1]

(b) After four weeks, she realised she has too much pounds and she now wants to change £200 back to Singapore dollars.
If the loss by this transaction is $6, what is the current exchange rate?

Answer £1= S$ _____ [2]

For Examiner's Use

For Examiner's Use

14 **(a)** The perimeter of a rectangular lawn is 72 m and its area is 323 m^2. Calculate the length of a diagonal of the lawn, without solving for its length and breadth.

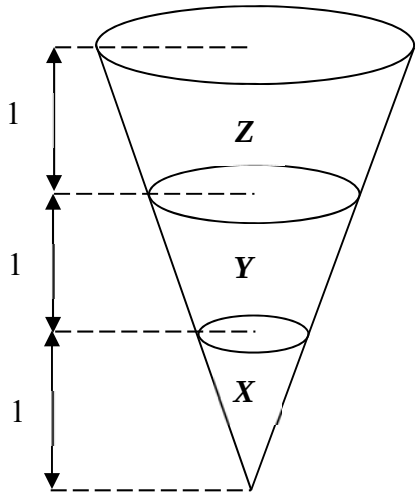
Answer _____ m [4]

(b) Another rectangular lawn is 18 m long and 7 m wide, correct to the nearest metre. Find the greatest possible area of this lawn.

Answer _____ m^2 [1]

---

15 A right circular cone is divided into 3 portions, *X*, *Y* and *Z* , as shown in the diagram below. The height of each portion is 1 unit.
Calculate volume of *Y* : volume of *Z*.



Answer _____ : _____ [3]

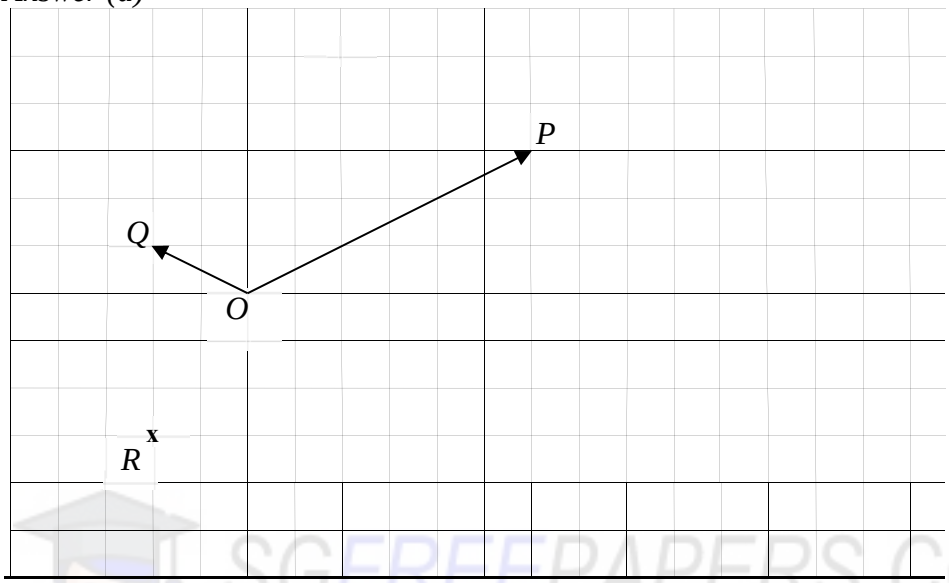
For Examiner's Use

For Examiner's Use

16 On the grid in the answer space, $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OQ} = \mathbf{q}$.
The point R is also marked.

(a) Draw and label clearly on the grids below, the vector, $\overrightarrow{OS}$, such that $\overrightarrow{OS} = \mathbf{p} + 2\,\mathbf{q}$.

Answer (a) [1]



(b) Express $\overrightarrow{OR}$ in terms of **p** and / or **q**.

Answer _____ [1]

(c) If the length of each small square is 1 unit, calculate $\left|\overrightarrow{OS}\right|$.

Answer _____ unit [2]

For Examiner's Use

For 2xaminer's Use

17 **(a)** Calculate the size of an exterior angle of a regular polygon with 12 sides.

Answer _____ [1]

(b) Tile *P* is in the shape of a regular 12-sided polygon.
Explain, showing your working clearly, whether tiles *P* will fit together on the floor without gaps.

Answer

_____

_____ [2]

---

18 On a map, a straight road measuring 600 m is represented by a line segment of length 7.5 cm.

(a) Express the scale of the map in the ratio $1:r$.

Answer 1 : _____ [1]

(b) The length of a canal on the map is 25 cm. Find the actual length in km.

Answer _____ km [1]

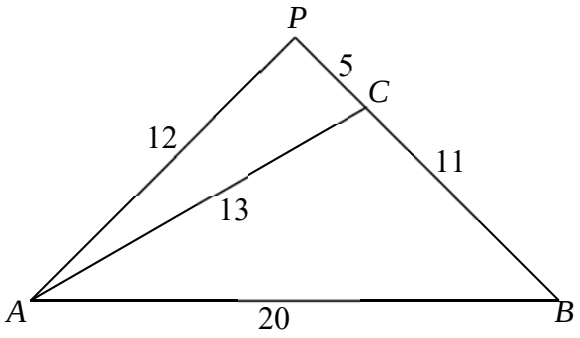
(c) The actual area of a school is 22 400 m^2. Find the area of the school on the map.

Answer _____ cm^2 [2]

For Examiner's Use

For Examiner's Use

19 ABC is a triangle in which $AC = 13$ cm, $BC = 11$ cm and $AB = 20$ cm. P is a point on BC produced, where $CP = 5$ cm and $AP = 12$ cm.



(a) Explain why $\angle APB = 90°$.

Answer

.....

..... [2]

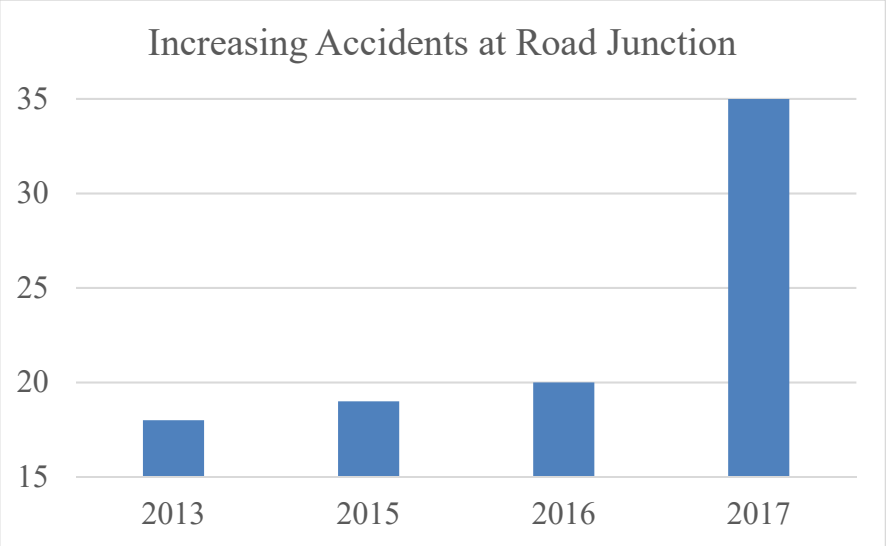
(b) Expressing your answer as a fraction in its simplest form, find $\dfrac{\tan \angle ABC}{\cos \angle ACB}$.

Answer ..... [3]

For Examiner's Use

For Examiner's Use

20 The graph shows the number of accidents occurring at a road junction over a number of years.
State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.



Answer

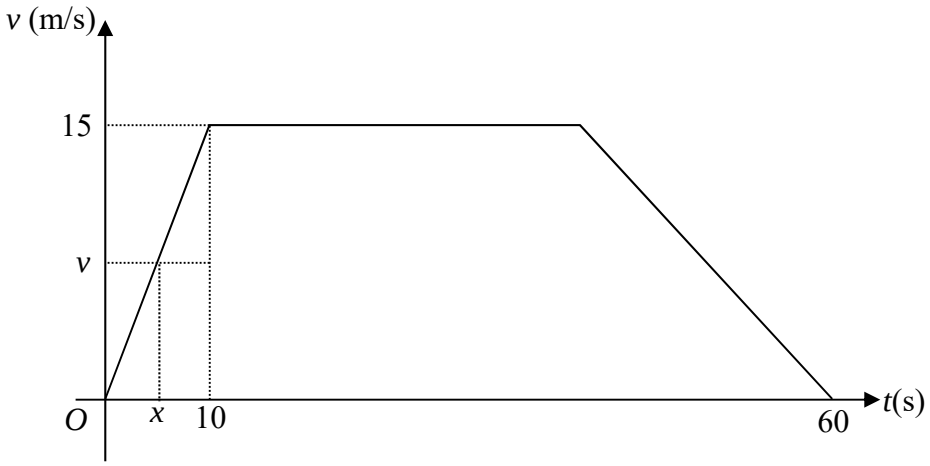
.....

..... [2]

For Examiner's Use

For Examiner's Use

21 The diagram shows the speed-time graph of a lorry. The lorry starts from rest and accelerates uniformly to a speed of 15m/s in 10 seconds. The lorry then travels at a constant speed for some time before it decelerates uniformly at 0.75 m/s^2 until it comes to rest.
The whole journey takes one minute.



(a) Given that the speed after x seconds is v m/s, express v in terms of x.

Answer $v =$ _____ [1]

(b) For how long does the lorry travel at the maximum speed ?

Answer _____ s [2]

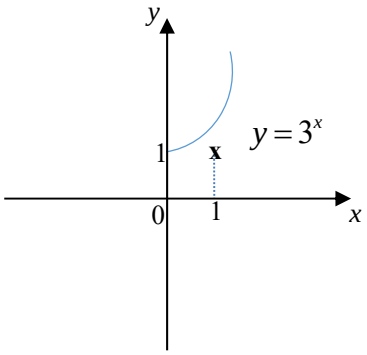
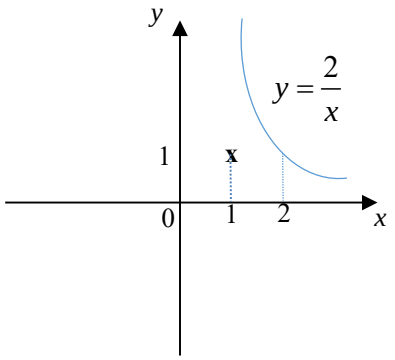
(c) Calculate the total distance travelled by the lorry during this 1 minute journey.

Answer _____ m [2]

End of Paper

CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 4 MATHEMATICS
Answer Key for 2018 Preliminary Examination 2

PAPER 4048/1

1a	$-12x^2+22x-2$	**15**	7 : 19
1b	$(6b-c)(4a-p)$	**16b**	$-\frac{2}{3}\mathbf{p}-\mathbf{q}$
2	$x=\frac{2}{3}$	**16c**	5.39 unit
3a	LHS $\neq$ RHS. $(-1,6)$ does not satisfy the equation so $(-1,6)\notin A$.	**17a**	30°
		17b	$\frac{360}{150}=2.4$ which is not a positive integer so the tiles will not fit together on the floor without gaps.
3b	(4, 0)		
3c	$m=3,\ c=1\ (c\neq 12)$		
4a	$-4<x\leq 1\frac{4}{7}$	**18a**	1:8000
		18b	2 km
4b	$1\frac{4}{7}$	**18c**	3.5 cm^2
5a	$2^3\times 3\times 7\times k^2$	**19a**	By converse of Pythagoras' Theorem, ΔAPC is a right-angled Δ. So $\angle APB=90^{\circ}$.
5b	$n=4k$	**19b**	$-39/20$
6a	$a=-2,\ b=-15$	**20**	The vertical axis did not start from 0, giving an exaggerated difference between the bars. (Any other sensible reasons.)
6b	$p=9$		
6c	$y=x-5$		
6d	$E=(8,\ -15)$	**21a**	$v=1.5x$
7a	$y=\frac{36}{\sqrt{x}}$	**21b**	30 s
		21c	675 m
7b	$x=51\frac{21}{25}$	**9a**	
8	22.2 km/h		
10a	$\frac{1}{125}$		
10b	0.834		
11a	$\angle x=70°$		
11b	$\angle y=125°$		
12a	\$825	**9b**	
12b	\$2 985		
12c	6.10%		
13a	£600		
13b	£1 = S\$ 1.92		
14a	25.5 m		
14b	138.75 m^2		

CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

CANDIDATE NAME	Sec 4 ( ) Reg. No: ( )

CENTRE NUMBER					INDEX NUMBER			

MATHEMATICS(MARK SCHEME) 4048/01

Paper 1 **15 August 2018**

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

This document consists of **16** printed pages.

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's Use

Answer **all** the questions.

1 **(a)** Expand and simplify $(2x-1)(2-3x)-3x(2x-5)$.

$$\begin{aligned}&(2x-1)(2-3x)-3x(2x-5)\\&=4x-6x^2-2+3x-6x^2+15x\\&=-12x^2+22x-2\end{aligned}$$

Answer $-12x^2+22x-2$ [2]

(b) Factorise completely $24ab-4ac+pc-6pb$.

$$\begin{aligned}&24ab-4ac+pc-6pb\\&=4a(6b-c)+p(c-6b)\\&=4a(6b-c)-p(6b-c)\\&=(6b-c)(4a-p)\\&\text{or } (c-6b)(p-4a)\end{aligned}$$

Answer $(6b-c)(4a-p)$ [2]

---

2 Solve the equation $2^{3x}\times 125^{x}=100$.

$$\begin{aligned}&2^{3x}\times 125^{x}=100\\&2^{3x}\times 5^{3x}=10^2\\&10^{3x}=10^2\\&3x=2\\&x=\frac{2}{3}\end{aligned}$$

Answer $x=\frac{2}{3}$ [3]

For Examiner's Use

For Examiner's Use

3 $A = \{\text{points lying on the line } 2x + y = 8\}$
$B = \{\text{points lying on the line } 3x - 4y = 12\}$
$C = \{\text{points lying on the line } mx - 4y = c\}$

(a) Is $(-1,6) \in A$? Explain clearly.

Answer
Substitute $(-1,6)$, LHS = $2(-1)+6=4$, RHS = 8 .

LHS $\neq$ RHS. $(-1,6)$ does not satisfy the equation so $(-1,6) \notin A$. [1]

(b) Find the element p such that $p \in (A \cap B)$.

$2x + y = 8$ .....(1)
$3x - 4y = 12$ .....(2)
From (1): $y = -2x + 8$ into (2)
$3x - 4(8 - 2x) = 12$
$3x - 32 + 8x = 12$
$11x = 44$
$x = 4$
$y = 0$

Answer $p =$ (4, 0) [2]

(c) Write down a possible value of m and of c such that $B \cap C = \varnothing$.

$y = \frac{3}{4}x - 3$ and $y = \frac{mx}{4} - \frac{c}{4}$
Since $B \cap C = \varnothing$
$\frac{m}{4} = \frac{3}{4}$
$m = 3$
$-\frac{c}{4} = -3$
$c = 12$

Answer $m =$ 3 [1]

$c =$ Any real value $\neq 12$ [1]

For Examiner's Use

For Examiner's Use

4 **(a)** Solve the inequalities $8+x<10+\frac{3}{2}x\le 15.5-2x$.

$8+x<10+\frac{3}{2}x$ and $10+\frac{3}{2}x\le 15.5-2x$

$8+x<10+\frac{3}{2}x$ and $\frac{7}{2}x\le 5.5$

$-\frac{1}{2}x<2$ and $x\le\frac{11}{7}$

$x>-4$ and $x\le\frac{11}{7}$

$-4<x\le\frac{11}{7}$ or $1\frac{4}{7}$

Answer $-4<x\le 1\frac{4}{7}$ [3]

(b) Write down the largest rational number that satisfies $8+x<10+\frac{3}{2}x\le 15.5-2x$.

Greatest rational value of x is $1\frac{4}{7}$.

Answer $1\frac{4}{7}$ [1]

---

5 Integers P and Q, written as products of their prime factor, are $P=2^2\times 3\times k^2$ and $Q=2^3\times 7\times k$, where k is a prime number.

(a) Express, in terms of k and as a product of its prime factors, the smallest integer which is divisible by both P and Q.

LCM $=2^3\times 3\times 7\times k^2$

Answer $2^3\times 3\times 7\times k^2$ [1]

(b) Find the smallest integer, n, such that $(27k)n$ is a multiple of P. Give your answer in terms of k **if necessary**.

$27kn=m(2^2\times 3\times k^2)$

$n=2^2\times k$

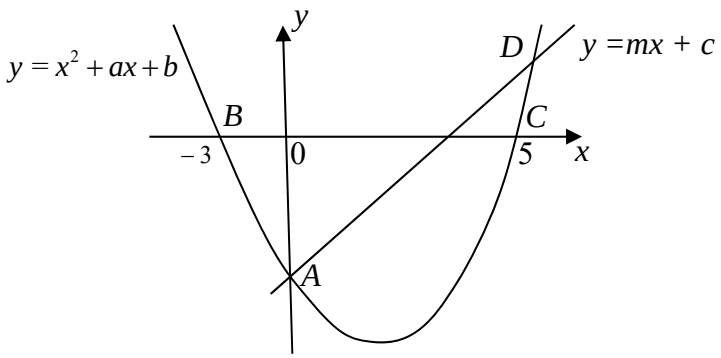
$n=4k$

Answer $n=$ $4k$ [1]

For Examiner's Use

For Examiner's Use

6 The diagram shows the curve $y = x^2 + ax + b$ and the line $y = mx + c$ which intersect at A on the y-axis and D.
The curve intersects the x-axis at B and C. The coordinates of D are $(6, p)$.



(a) Find the value of a and of b.

$y = (x+3)(x-5)$
$y = x^2 - 2x - 15$

Answer $a =$ -2 [1]

$b =$ -15 [1]

(b) Find the value of p.

$y = (x+3)(x-5)$
At $D(6, p)$,
$p = (6+3)(6-5) = 9$

Answer $p =$ 9 [1]

(c) Find the equation of the line parallel to BD and passing through C.

Gradient of $BD = \dfrac{9-0}{6-(-3)} = 1$
Equation of line is $y = x + c$
At $C(5,0)$
Equation is $y = x - 5$

Answer $y = x - 5$ [2]

(d) Find the coordinates of E such that $ABCE$ is a parallelogram.

Since BC is on the x-axis, y-coordinate of E is -15
Midpoint of AC=Midpoint of BE
$0+5 = x - 3$
$x = 8$
$E = (8, -15)$

Answer $(8, -15)$ [2]

For Examiner's Use

For Examiner's Use

7 y is inversely proportional to the square root of x and the difference in the values of y when $x = 9$ and when $x = 16$ is 3.

Find

(a) an equation connecting x and y,

$$y = \frac{k}{\sqrt{x}}$$

When $x = 9$, $y_1 = \frac{k}{3}$

When $x = 16$, $y_2 = \frac{k}{4}$

$$\frac{k}{3} - \frac{k}{4} = 3$$

$$\frac{4k - 3k}{12} = 3$$

$$k = 36$$

$$\therefore y = \frac{36}{\sqrt{x}}$$

Answer $y = \frac{36}{\sqrt{x}}$ [3]

(b) the exact value of x when $y = 5$.

$$5 = \frac{36}{\sqrt{x}}$$

$$x = \frac{1296}{25} = 51\frac{21}{25}$$

Answer $x = 51\frac{21}{25}$ [1]

For Examiner's Use

For Examiner's Use

8 Jon travels at 20 km/h for 50% of the journey and 25 km/h for the rest of the journey.
Calculate Jon's average speed, in km/h for the whole journey.

Let x km : the distance

$$\frac{x}{\frac{0.5x}{20}+\frac{0.5x}{25}}$$

$$=\frac{x}{\frac{2.5x+2x}{100}}$$

$$=\frac{100}{4.5}=22.2$$

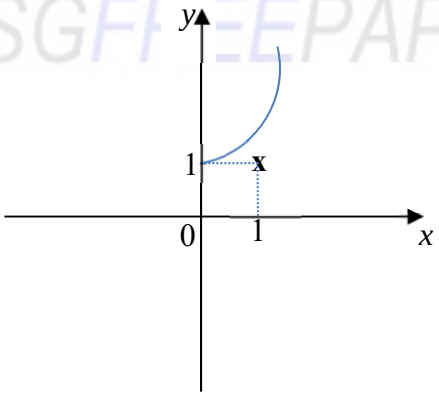
Answer 22.2 km/h [3]

---

9 The point (1,1) is marked on each diagram in the answer space below.
Sketch, for $x>0$, the graph of

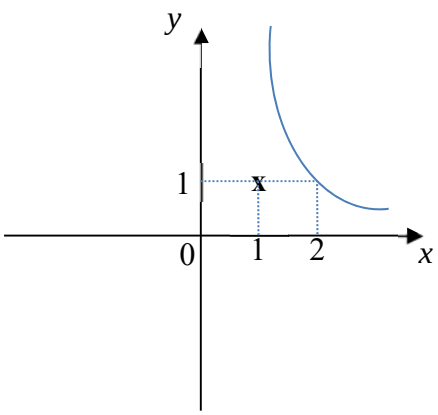
(a) $y=3^x$,

Answer [1]



(b) $y=\frac{2}{x}$.

Answer [1]



For Examiner's Use

For Examiner's Use

10 Vehicles approaching a crossroad may go in one of these directions: left, right or straight ahead.

If the probability of a vehicle turning left is $\frac{7}{20}$, the probability of it turning right is $\frac{9}{20}$ and the probability of it going straight ahead is $\frac{1}{5}$, calculate the probability that for any three vehicles approaching the crossroads,

(a) all will go straight,

$$\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=\frac{1}{125}$$

Answer $\frac{1}{125}$ [1]

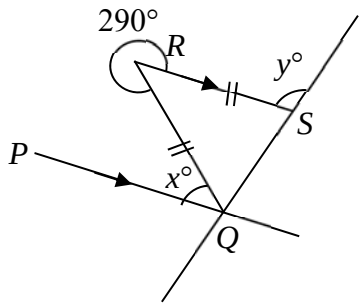
(b) at least one will turn right.

$$1-(\frac{11}{20})(\frac{11}{20})(\frac{11}{20})$$
$$=0.83362$$

Answer 0.834 [2]

---

11 In the diagram, PQ is parallel to RS, $RQ = RS$ and reflex $\angle QRS = 290°$.
Calculate the value of



(a) x,

Acute $\angle QRS = 360° - 290°$ ($\angle$s at a point)

$= 70°$

$\therefore \angle x = 70°$ (alt. $\angle$s, RS parallel to PQ)

Answer $x =$ 70 [1]

(b) y.

$\angle RSQ = (180° - 70°) \div 2$ (base $\angle$ of isos.Δ)

$=55°$

$\angle y = 180° - 55°$ (adj. $\angle$s on a straight line)

$=125°$

Answer $y =$ 125 [2]

For Examiner's Use

For Examiner's Use

12 The cash price of a computer is \$2 750. Mr Ong bought the computer by paying a 30% downpayment and monthly instalments of \$90 over 2 years.

(a) How much is the downpayment?

$$\text{Downpayment} = \frac{30}{100} \times 2750$$
$$= \$825$$

Answer \$ 825 [1]

(b) Calculate the total amount Mr Ong paid for the computer.

$$\text{Total amount paid} = 825 + 90(12)(2)$$
$$= 825 + 2160$$
$$= \$2\ 985$$

Answer \$ 2 985 [2]

(c) Find the flat rate of interest per annum for the instalments.

$$\text{Loan} = \$ (2750 - 825) = \$1\ 925$$
$$\text{Interest} = \$ (2160 - 1925) = \$235$$
$$\text{S. I.} = \frac{1925 \times R \times 2}{100}$$
$$R = \frac{235 \times 100}{1925 \times 2}$$
$$= 6.1038$$
$$= 6.10 \text{ (3sf)}$$

Answer 6.10 % [3]

---

13 On a certain day, the exchange rate between the pound (£) and the Singapore dollar (S\$) was S\$1.95 = £1.

(a) Calculate the amount of pounds which May can buy with S\$1170.

\$1.95 = £1

\$1170 = £$\frac{1170}{1.95}$ = £600

Answer £ 600 [1]

(b) After four weeks, she realised she has too much pounds and she now wants to change £200 back to Singapore dollars.
If the loss by this transaction is \$6, what is the current exchange rate?

Old rate : £200 = \$200 ×1.95= \$ 390

If new rate is £1 =\$$x$, £200 = \$200x

$390 - 200x = 6$

x =1.92

Answer £1= S\$ 1.92 [2]

For Examiner's Use

For Examiner's Use

14 **(a)** The perimeter of a rectangular lawn is 72 m and its area is 323 m^2. Calculate the length of a diagonal of the lawn, without solving for its length and breadth.

$2(L + B) = 72$
$L + B = 36$
$LB = 323$
$(L+B)^2 = L^2 + B^2 + 2(LB)$
$36^2 = L^2 + B^2 + 2(323)$
$L^2 + B^2 = 1296 - 646$
$= 650$
Length of diagonal $= \sqrt{650} = 25.495\,\text{m}$

Answer 25.5 m [4]

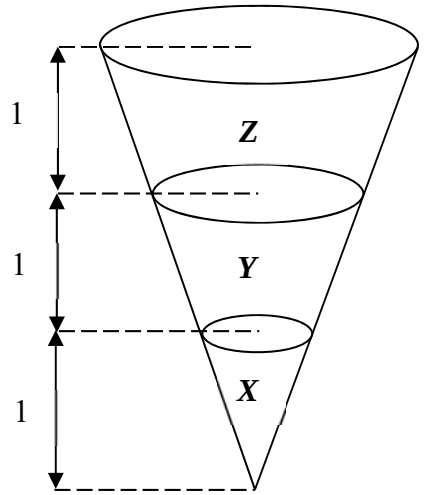
(b) Another rectangular lawn is 18 m long and 7 m wide, correct to the nearest metre. Find the greatest possible area of this lawn.

$18.5 \times 7.5 = 138.75$

Answer 138.75 m^2 [1]

---

15 A right circular cone is divided into 3 portions, X, Y and Z, as shown in the diagram below. The height of each portion is 1 unit.
Calculate volume of Y : volume of Z.



$$\frac{\text{volume of } X}{\text{volume of } X + Y} = (\frac{1}{2})^3 = \frac{1}{8}$$

$$\frac{\text{volume of } X}{\text{volume of } X + Y + Z} = (\frac{1}{3})^3 = \frac{1}{27}$$

$$\frac{\text{volume of } Y}{\text{volume of } Z} = \frac{7}{19}$$

Answer 7 : 19 [3]

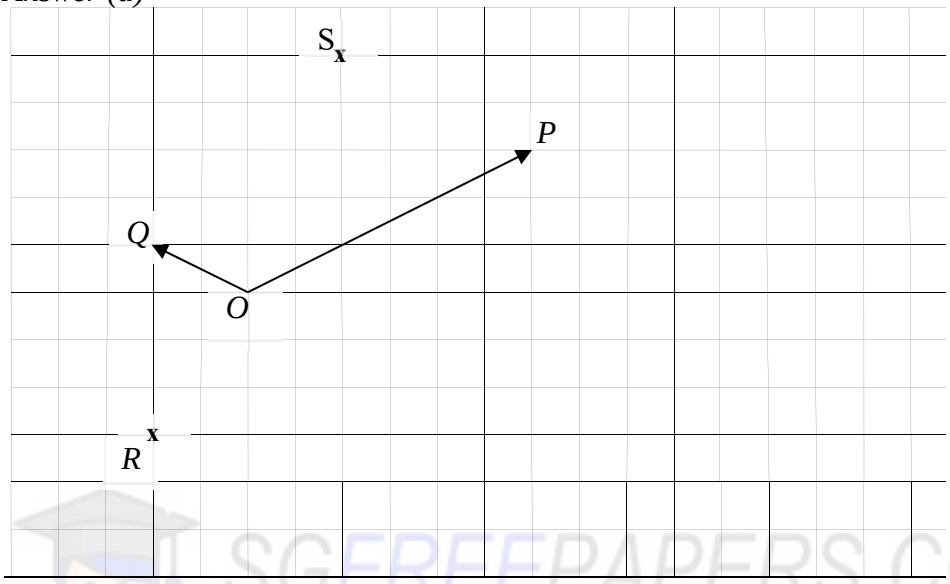
For Examiner's Use

For Examiner's Use

16 On the grid in the answer space, $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OQ} = \mathbf{q}$.
The point *R* is also marked.

(a) Draw and label clearly on the grids below, the vector, $\overrightarrow{OS}$, such that $\overrightarrow{OS} = \mathbf{p} + 2\,\mathbf{q}$.

Answer (a) [1]



(b) Express $\overrightarrow{OR}$ in terms of **p** and / or **q**.

Answer $-\frac{2}{3}\mathbf{p} - \mathbf{q}$ [1]

(c) If the length of each small square is 1 unit, calculate $\left|\overrightarrow{OS}\right|$.

$\left|\overrightarrow{OS}\right| = \sqrt{2^2 + 5^2}$
$= 5.3851$
$= 5.39$ unit

Answer 5.39 unit [2]

For Examiner's Use

For 2xaminer's Use

17 (a) Calculate the size of an exterior angle of a regular polygon with 12 sides.

$1 \text{ exterior angle} = \frac{360}{12} = 30^{o}$

Answer 30^{o} [1]

(b) Tile *P* is in the shape of a regular 12-sided polygon.
Explain, showing your working clearly, whether tiles *P* will fit together on the floor without gaps.

Answer
$1 \text{ interior angle} = 180 - 30 = 150^{o}$
$\frac{360}{150} = 2.4$ which is not a positive integer so the tiles will not fit together on the floor without gaps. [2]

---

18 On a map, a straight road measuring 600 m is represented by a line segment of length 7.5 cm.

(a) Express the scale of the map in the ratio $1 : r$.

7.5 cm : 60 000cm
1 : 8000

Answer 1 : 8000 [1]

(b) The length of a canal on the map is 25 cm. Find the actual length in km.

1 cm : 8000 cm
25 cm : 200 000 cm

Answer 2 km [1]

(c) The actual area of a school is 22 400 m^2. Find the area of the school on the map.

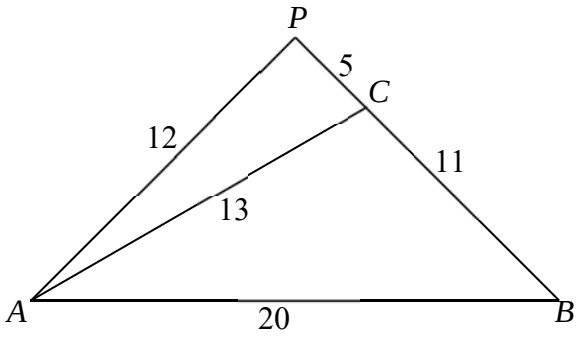
80 m : 1 cm
6400 m^2 : 1 cm^2
22 400 m^2 : 22400/6400 = 3.5 cm^2

Answer 3.5 cm^2 [2]

For Examiner's Use

For Examiner's Use

19 ABC is a triangle in which $AC = 13$ cm, $BC = 11$ cm and $AB = 20$ cm. P is a point on BC produced, where $CP = 5$ cm and $AP = 12$ cm.



(a) Explain why $\angle APB = 90°$.

Answer

$AP^2 + PC^2 = 12^2 + 5^2$

$= 13^2$

$\therefore\ AP^2 + PC^2 = AC^2$

By converse of Pythagoras' Theorem, ΔAPC is a right-angled Δ. So $\angle APB = 90^o$ [2]

(b) Expressing your answer as a fraction in its simplest form, find $\dfrac{\tan \angle ABC}{\cos \angle ACB}$.

$\dfrac{\tan \angle ABC}{\cos \angle ACB}$

$= \dfrac{12/16}{-5/13}$ B2

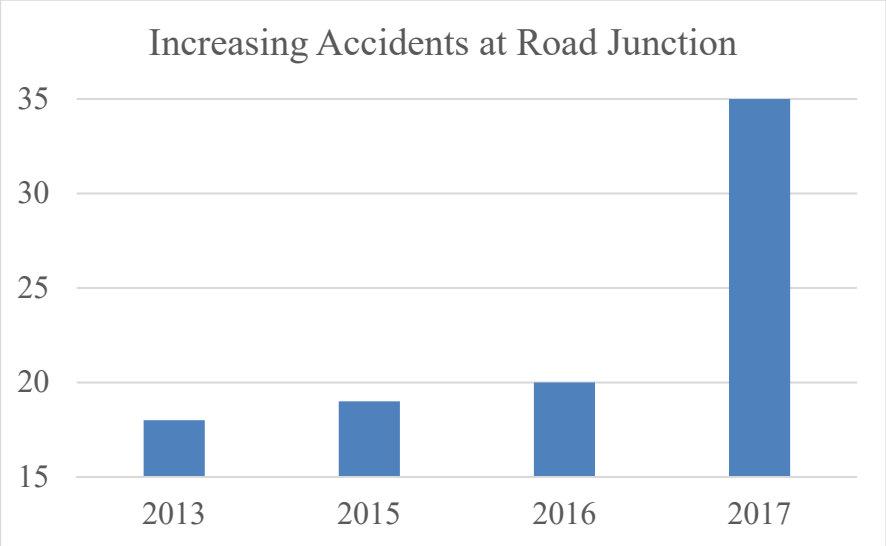
$= -\dfrac{39}{20}$ A1

Answer $-\dfrac{39}{20}$ [3]

For Examiner's Use

For Examiner's Use

20 The graph shows the number of accidents occurring at a road junction over a number of years.
State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.



Answer

Title is biased and misleading. It does not allow the reader to form their own opinion. [2]

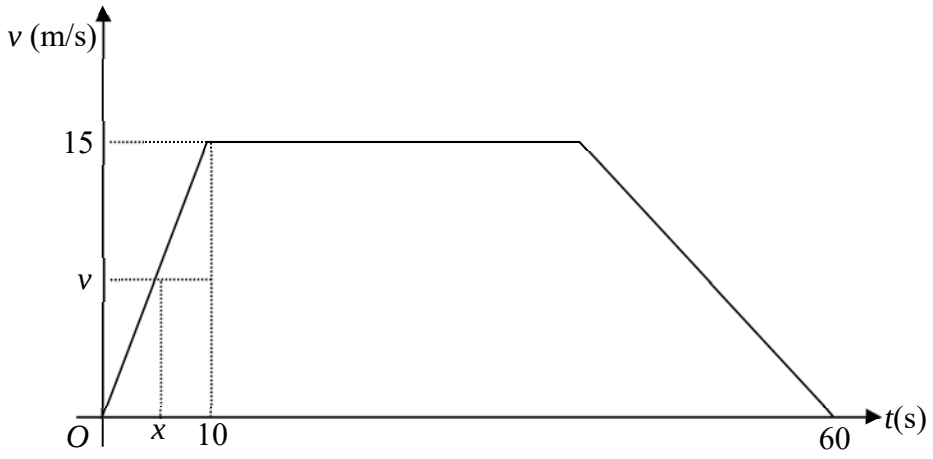
The year 2014 is left out. The number of accidents may be higher or lower.

The vertical axis did not start from 0, giving an exaggerated difference between the bars.

For Examiner's Use

For Examiner's Use

21 The diagram shows the speed-time graph of a lorry. The lorry starts from rest and accelerates uniformly to a speed of 15m/s in 10 seconds. The lorry then travels at a constant speed for some time before it decelerates uniformly at 0.75 m/s^2 until it comes to rest.
The whole journey takes one minute.



(a) Given that the speed after x seconds is v m/s, express v in terms of x.

$$\frac{v}{x} = \frac{15}{10}$$
$$v = 1.5x$$

Answer $v =$ $1.5x$ [1]

(b) For how long does the lorry travel at the maximum speed ?

$$\frac{15-0}{x-60} = -\frac{3}{4}$$
$$60 = 3x + 180$$
$$3x = 120$$
$$x = 40$$

It travels for $40 - 10 = 30$ s

Answer 30 s [2]

(c) Calculate the total distance travelled by the lorry during this 1 minute journey.

Total distance $= \frac{1}{2}(30+60)15$ M1
$= 675$ m A1

Answer 675 m [2]

End of Paper

CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

MATHEMATICS 4048/02

Paper 2 **16 August 2018**

2 hours 30 minutes

Additional Materials: Answer Paper (10 sheets)
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $\dfrac{24c^3d^2}{(3de^2)^3} \div \dfrac{5c^{-2}}{10df}$. [2]

(b) Express as a single fraction $\dfrac{7}{(6-5p)^2} - \dfrac{2p-1}{10p-12}$. [3]

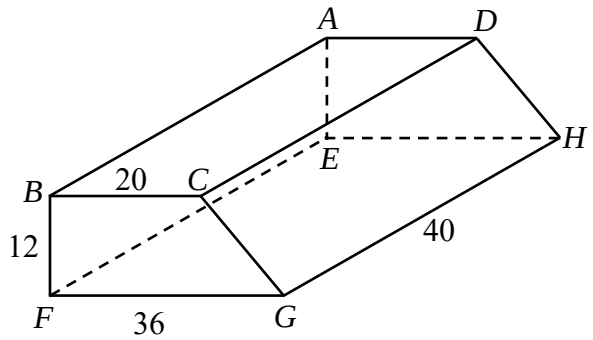
(c) Simplify $\dfrac{6x^2-17x+5}{18x^2-2} \times \dfrac{15x+5}{10-4x}$. [3]

(d) It is given that $1-\dfrac{a-b}{b+2c} = \dfrac{2a-1}{2}$.

Express b in terms of a and c. [3]

---

2 The diagram shows a solid prism $ABCDEFGH$ with a horizontal rectangular base $EFGH$ and a horizontal rectangular top $ABCD$.
B is vertically above F and A is vertically above E.
BC = 20 cm, FG = 36 cm, BF = 12 cm and GH = 40 cm.



(a) Find the length of BH. [2]

(b) Find the **total surface area** of the prism. [3]

(c) The prism is melted and recast into a right pyramid with a square base.
The height of the pyramid is 24 cm.
Find the length of each side of the square base. [3]

---

3 Answer the whole of this question on a single sheet of graph paper.

A bakery makes a profit of y thousand dollars for selling x thousand pieces of blueberry tarts.

The variables x and y are connected by the equation

$$y = 5x - x^2 - 2 .$$

Some corresponding values of x and y are given in the table below.

x	0	0.5	1	1.5	2	3	4
y	–2	0.25	2	3.25	4	4	2

(a) Using a scale of 4 cm to represent 1 unit, draw a horizontal x-axis for $0 \le x < 4$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $-3 \le y \le 5$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to find

(i) the maximum profit obtained from selling the blueberry tarts, [1]

(ii) the minimum number of blueberry tarts the bakery must sell in order to cover the costs of baking the tarts, [1]

(iii) the range of values of x for which the profit is more than \$2750. [1]

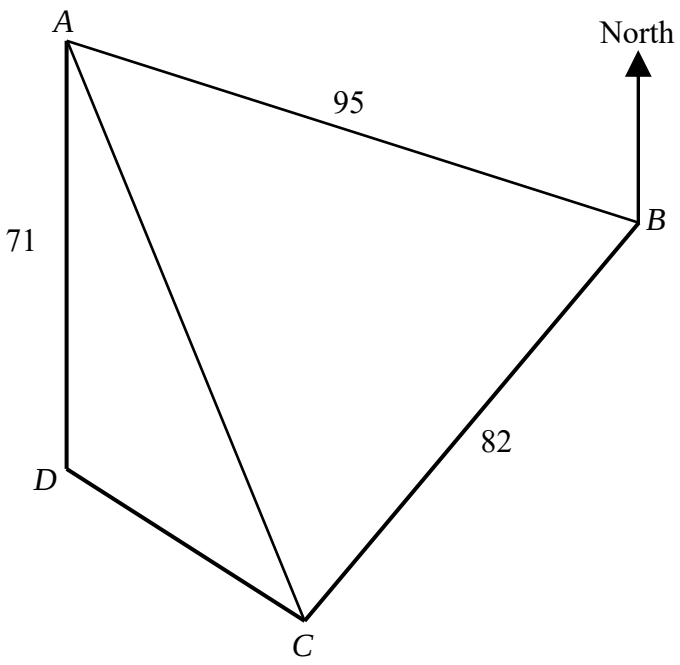
(c) **(i)** On the same axes, draw the graph of $\frac{y}{x} = \frac{1}{4}$. [1]

(ii) Write down the x-coordinate of the point where the two graphs intersect. [1]

(iii) State briefly what the value of this x-coordinate represents. [1]

(iv) The value of x in **(c)(ii)** is the solution of the equation $4x + \frac{A}{x} + B = 0$.
Find each of the value of integers A and B. [2]

4



Quadrilateral *ABCD* is a field on horizontal ground.
A is 95 m from *B* on a bearing of 280°.
B is 82 m from *C* on a bearing of 025°.
D is due south and 71 m from *A*.

Calculate

(a) area of triangle *ABC*, [2]

(b) *AC*, [2]

(c) angle *ACB*. [2]

A vertical tower of 67 m stands at *D*.
A man walked along *AC*.
He stopped at *E* to take a picture of the tower where the angle of elevation of the top of the tower was the greatest.

(d) Calculate the angle of elevation of the top of the tower at *E*. [3]

5 A route up a mountain is 25 km long.
Hwee Ling walked along this route up the mountain at an average speed of x km/h.

(a) Write down an expression, in terms of x, the number of hours she took to walk up the mountain. [1]

She walked down the mountain by a different route.
The length of this route is 30 km long.
Her average speed walking down the mountain was 3 km/h greater than her average speed walking up the mountain.

(b) Write down an expression, in terms of x, the number of hours she took to walk down the mountain. [1]

(c) It took Hwee Ling $1\frac{1}{4}$ hours less to walk down the mountain than to walk up the the mountain.

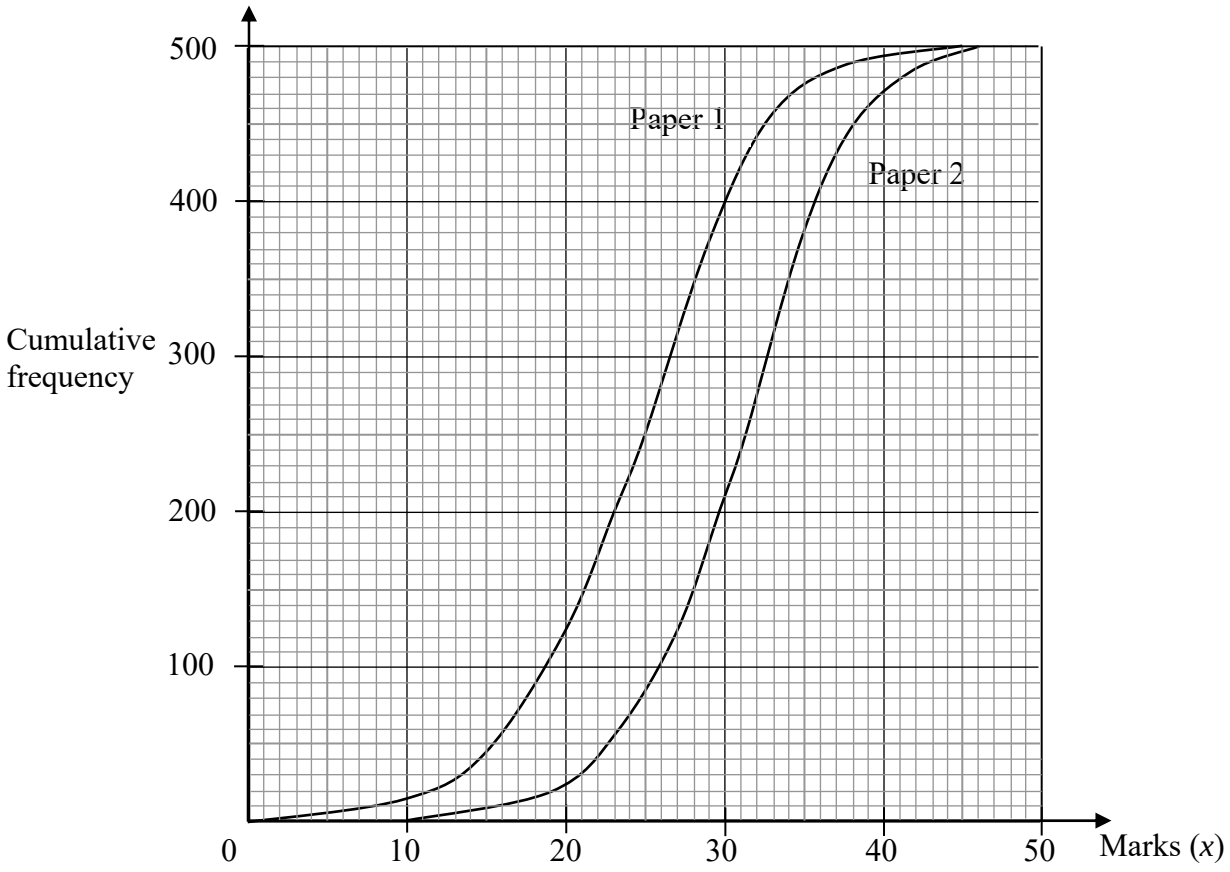
Write down an equation to represent this information and show that it simplifies to

$$x^2 + 7x - 60 = 0.$$ [3]

(d) Solve the equation $x^2 + 7x - 60 = 0$. [2]

(e) Find the time Hwee Ling took to walk down the mountain. [1]

6 In a Mathematics examination, 500 students each took two papers.
Both papers were marked out of 50.
The cumulative frequency curves show the distribution of the marks for the two papers.



(a) For **Paper 1**, estimate the interquartile range. [2]

(b) The grouped frequency table of the marks obtained by the students for **Paper 2** is shown below.

Marks (x)	$10 \le x < 19$	$19 \le x < 28$	$28 \le x < 37$	$37 \le x < 46$
Frequency	p	q	r	s

(i) Find each of the values of p, q, r and s. [2]

(ii) Calculate an estimate of the mean mark obtained for Paper 2. [2]

(iii) Calculate an estimate of the standard deviation for Paper 2. [2]

(c) An additional student obtained 28 marks in Paper 1, but was absent for Paper 2.
Estimate the mark she would have obtained if she had taken Paper 2. [1]

(d) Which was the more difficult paper? Justify your answer. [2]

7 **(a)** The first four terms in a sequence of numbers, $T_1, T_2, T_3,$ are given below.

$$T_1 = 3^0 - 2 = -1$$
$$T_2 = 3^1 - 5 = -2$$
$$T_3 = 3^2 - 8 = 1$$
$$T_4 = 3^3 - 11 = 16$$

(i) Write down an expression for T_5. [1]

(ii) Find an expression, in terms of n, for T_n. [3]

(iii) Evaluate T_{15}. [1]

(b) The table shows the numbers of English and Chinese copies of a book sold on a typical weekday and a weekend.

	English	Chinese
Weekday	6	8
Weekend	12	14

The cost price of an English copy and a Chinese copy of the book is \$12 and \$15 respectively.
The selling price of an English copy and a Chinese copy of the book is \$$p$ and \$$q$ respectively.

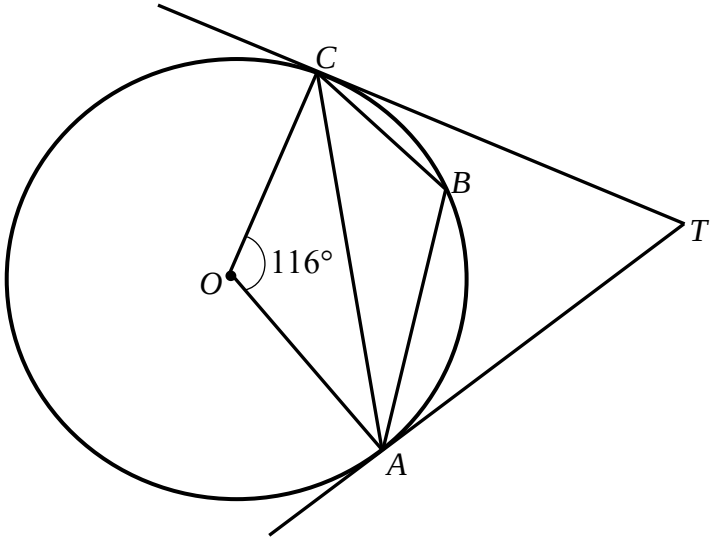
All the information can be represented by the matrices

$$\mathbf{A} = \begin{pmatrix} 6 & 8 \\ 12 & 14 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 12 \\ 15 \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} p \\ q \end{pmatrix}.$$

(i) Let $\mathbf{B} = \mathbf{S} - \mathbf{P}$.
Evaluate $\mathbf{C} = \mathbf{AB}$. [2]

(ii) The total profits gained from selling the English and Chinese copies of the book on a typical weekday and a weekend are \$92 and \$170 respectively.
Write down a 2×1 matrix $\mathbf{D}$ to represent the information and hence find the value of p and of q. [4]

8 **(a)** In the diagram, A, B and C lie on a circle, centre O.
The tangents at A and C meet at T.
Angle $COA = 116°$.



Find, stating your reasons clearly,

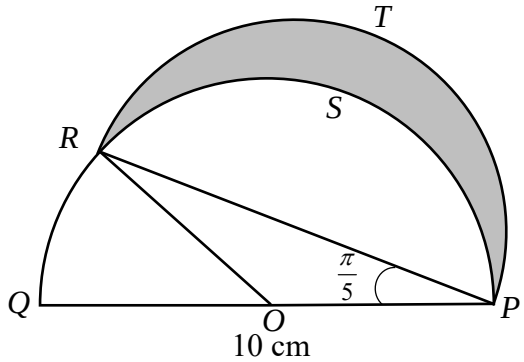
(i) obtuse angle ABC, [1]

(ii) angle CAT, [1]

(iii) angle CTA. [1]

(b) The figure shows a semicircle PQS with centre O with diameter PQ and a semicircle PRT with diameter PR.

PQ =10 cm and angle $RPO = \frac{\pi}{5}$ radians.

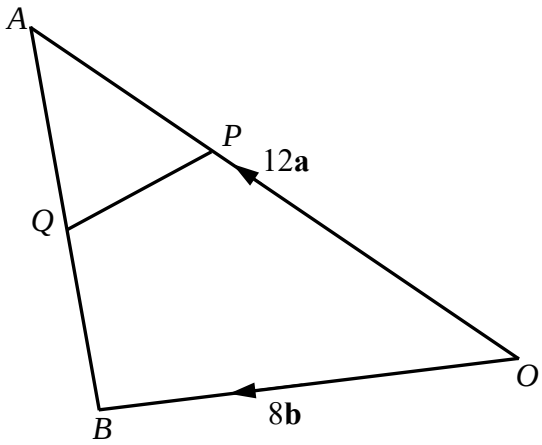


(i) Show that $PR = 8.0902$ cm, correct to 5 significant figures. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

9



The position vectors of A and B, relative to O, are 12**a** and 8**b** respectively.
$\overrightarrow{OP} = 2\overrightarrow{PA}$ and $\overrightarrow{AQ} = \overrightarrow{QB}$.

(a) Express each of the following in terms of **a** and **b**

(i) $\overrightarrow{AQ}$, [1]

(ii) $\overrightarrow{BP}$, [1]

(iii) $\overrightarrow{QP}$. [1]

(b) Find the position vector of R such that $\overrightarrow{PR} = 4\overrightarrow{PQ}$. [1]

(c) Make two statements about the points O, B and R. [2]

(d) Find the position vector of S such that $PQBS$ is a parallelogram. [1]

(e) Find $\dfrac{\text{Area of } \Delta OBP}{\text{Area of } \Delta ORA}$. [1]

(f) Given that $\mathbf{a} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$, find $\left|\overrightarrow{AB}\right|$. [2]

10 The electricity tariff is the cost of electricity per kilowatt hour (kWh) and is revised every quarter of the year to reflect the actual cost of electricity.

(a) In June 2018, Mr Lim paid $148.13, inclusive of 7% GST, for an electricity consumption of 625 kWh in his household.
Calculate the electricity tariff in June 2018. [2]

From July 2018, Mr Lim decided to purchase electricity from a new supplier for a duration of 12 months.
Below are the available plans.

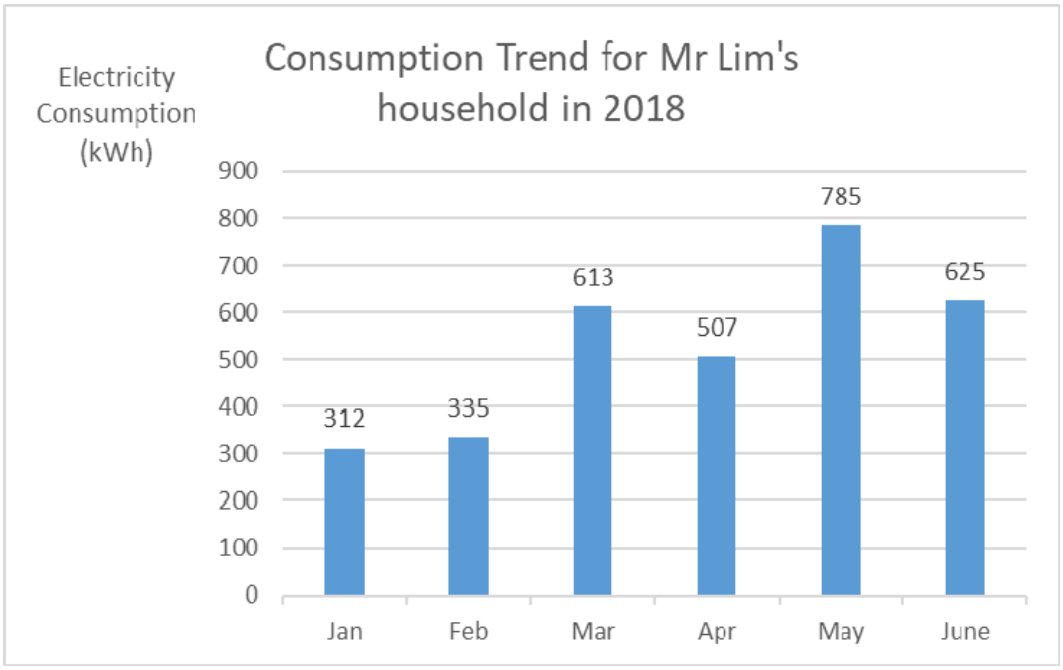
Company	**Type of Plan**	**Details of Plan**
Best Power	Fixed rate	• 1 year contract • $0.1667/kWh
Marco Energy	Fixed rate	• 1 year contract • $10 per month + $0.1535/kWh
Infinity Power	Fixed rate	• 6 months contract (one time 5% discount for renewal of another 6 months) • $0.1730/kWh (for first 6 months)
Rainbow Energy	Discount off tariff *	• 1 year contract • 20% off tariff
Unicorn Supply	Peak and off-peak	• Peak: $0.1685/kWh (7am to 10.59pm) • Off-peak: $0.1438/kWh (11pm to 6.59am)

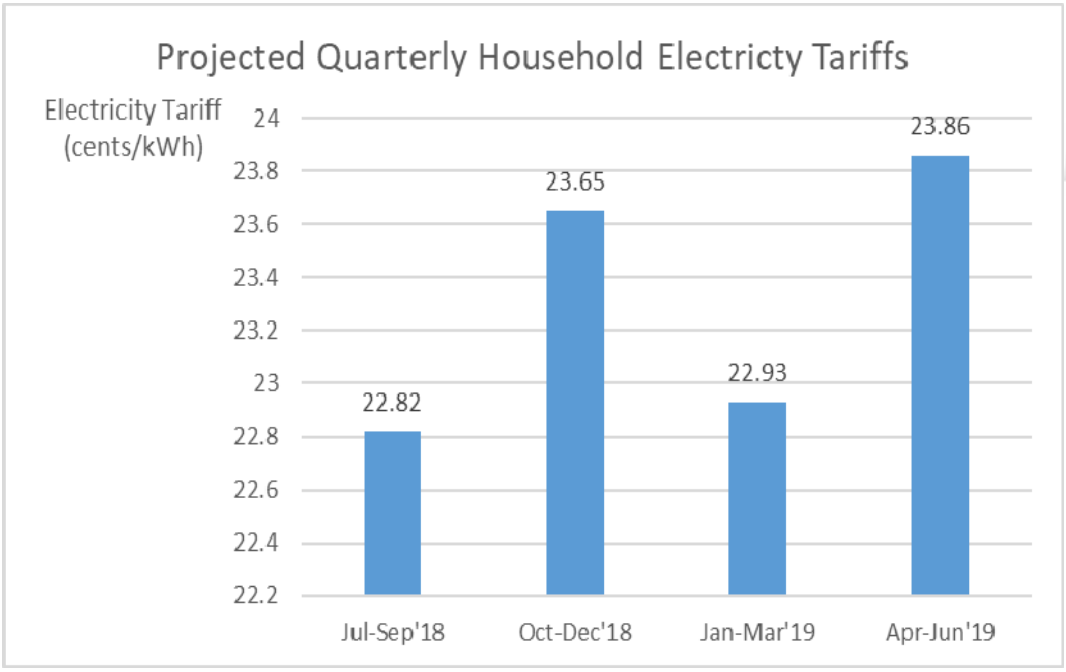
** Based on the prevailing electricity tariff for that quarter*

The consumption trend for Mr Lim's household and the projected quarterly household electricity tariffs for the next 12 months are on the next page.
The electricity consumption of his household during peak hours is estimated to be 3 times that during off-peak hours.

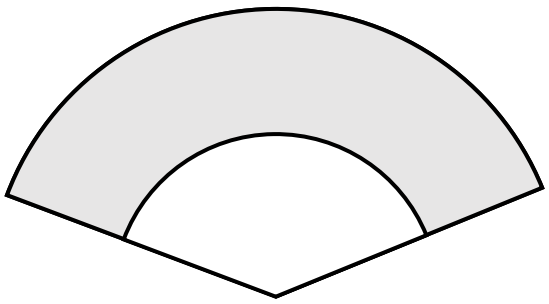
(b) Find the average monthly electricity consumption of Mr Lim's household from January 2018 to June 2018. [1]

(c) Determine which electricity supplier should Mr Lim choose.
Justify your answer with relevant working and state an assumption made. [7]


Consumption Trend for Mr Lim's household in 2018
Electricity Consumption (kWh)
900
800
700
600
500
400
300
200
100
0
312
335
613
507
785
625
Jan
Feb
Mar
Apr
May
June


Projected Quarterly Household Electricty Tariffs
Electricity Tariff (cents/kWh)
24
23.8
23.6
23.4
23.2
23
22.8
22.6
22.4
22.2
22.82
23.65
22.93
23.86
Jul-Sep'18
Oct-Dec'18
Jan-Mar'19
Apr-Jun'19

End of Paper



CEDAR GIRLS' SECONDARY SCHOOL

SECONDARY 4 MATHEMATICS

Answer Key for 2018 Preliminary Examination

PAPER 4048/2

1(a)	$\frac{16c^5 f}{9e^6}$	**6(a)**	9 marks
		6(bi)	$p = 20,\ q = 130,\ r = 280,\ s = 70$
1(b)	$\frac{-10p^2 + 17p + 8}{2(6-5p)^2}$	**6(bii)**	30.7 marks
		6(biii)	6.49 marks (3 s.f)
1(c)	$-\frac{5}{4}$	**6(c)**	34 marks
1(d)	$b = \frac{4ac - 6c + 2a}{5 - 2a}$	**6(d)**	Paper 1 The median mark for Paper 1 is lower than the median mark for Paper 2.
2(a)	55.1 cm (3 s.f)	**7(ai)**	$T_5 = 3^4 - 14 = 67$
2(b)	4192 cm^2	**7(aii)**	$T_n = 3^{n-1} - 3n + 1$
2(c)	41.0 cm	**7(aiii)**	4 782 925
3(bi)	\$4250 (accept \$4200 to \$4300)	**7(b)(i)**	$\mathbf{C} = \begin{pmatrix} 6p + 8q - 192 \\ 12p + 14q - 354 \end{pmatrix}$
3(bii)	425 tarts (accept 400 to 450)	**7(b)(ii)**	$q = 22,\ p = 18$
3(biii)	$1.28 \le x \le 3.72$ (accept lower limit: 1.25 to 1.35 (accept upper limit: 3.65 to 3.75)	**8 (ai)**	122° **8(aii)** 58° **8(aiii)** 64°
		8(bii)	22.1 cm (3 s.f) **8(biii)** 14.0 cm^2 (3 s.f)
		9(ai)	**4b** − **6a** **9(aii)** − **8b** + **8a**
3(cii)	0.45 ± 0.05	**9(aiii)**	**2a** − **4b**
		9(b)	$\overrightarrow{OR} = 16\mathbf{b}$
		9(c)	*O*, *B* and *R* are collinear. *OR* = 2*OB* or *B* is the midpoint of *OR*.
3(ciii)	The number of thousand pieces of blueberry tarts to be sold such that the profit made per piece is \$0.25.	**9(d)**	$\overrightarrow{OS} = 2\mathbf{a} + 4\mathbf{b}$ **9(e)** $\frac{1}{3}$ **9(f)** 20.4 units (3 sf)
		10(a)	\$0.2215 / kWh
3(civ)	$A = 8,\ B = -19$	**10(b)**	529.5 kWh
4(a)	3760 cm^2 (3 s.f)	**10(c)**	<u>Best Power (all for 12 months)</u> Total cost $= 12 \times 529.5 \times 0.1667 = \$1059.21$ <u>Marco Energy</u> Total cost $= 12(10 + 0.1535 \times 529.5) = \$1095.34$ <u>Infinity Power</u> Total cost $= (6 \times 529.5 \times 0.1730) + (6 \times 0.95 \times 0.1730 \times 529.5)$ $= \$1071.76$ <u>Rainbow Energy</u> Total cost $= 3 \times 0.8 \times 529.5(0.2282 + 0.2365 + 0.2293 + 0.2386)$ $= \$1185.15$ <u>Unicorn Supply</u> Total cost $= 12 \times (0.75 \times 529.5 \times 0.1685 + 0.25 \times 529.5 \times 0.1438)$ $= \$1031.41$ Mr Lim should choose Unicorn Supply as it has the lowest total costs for the next 12 months. The electricity consumption in Mr Lim's household is consistent throughout the year OR The projected quarterly household electricity tariffs are fairly accurate.
4(b)	108 m (3 s.f)		
4(c)	58.0°		
4(d)	60.0° (1 dp)		
5(a)	$\left(\frac{25}{x}\right)$ h		
5(b)	$\left(\frac{30}{x+3}\right)$ h		
5(d)	$x = -12$ or $x = 5$		
5(e)	3.75 h		

2018 Sec 4 Prelim
Mathematics 4048 P2
Solutions

Qn	Working
1a	$\frac{24c^3d^2}{(3de^2)^3} \div \frac{5c^{-2}}{10df}$ $= \frac{24c^3d^2}{27d^3e^6} \times \frac{10df}{5c^{-2}}$ $= \frac{16c^5 f}{9e^6}$
1b	$\frac{7}{(6-5p)^2} - \frac{2p-1}{10p-12}$ $= \frac{7}{(6-5p)^2} + \frac{2p-1}{2(6-5p)}$ $= \frac{14+(2p-1)(6-5p)}{2(6-5p)^2}$ $= \frac{14+(2p-1)(6-5p)}{2(6-5p)^2}$ $= \frac{14+12p-10p^2-6+5p}{2(6-5p)^2}$ $= \frac{-10p^2+17p+8}{2(6-5p)^2}$
1c	$\frac{6x^2-17x+15}{18x^2-2} \times \frac{15x+5}{10-4x}$ $= \frac{(3x-1)(2x-5)}{2(3x-1)(3x+1)} \times \frac{5(3x+1)}{-2(2x-5)}$ $= -\frac{5}{4}$
1d	$1-\frac{a-b}{b+2c} = \frac{2a-1}{2}$ $\frac{b+2c-a+b}{b+2c} = \frac{2a-1}{2}$ $\frac{2b+2c-a}{b+2c} = \frac{2a-1}{2}$ $4b+4c-2a = 2ab+4ac-b-2c$ $5b-2ab = 4ac-6c+2a$ $b(5-2a) = 4ac-6c+2a$ $b = \frac{4ac-6c+2a}{5-2a}$

Qn	Working
2a	$FH^2 = 36^2 + 40^2$ $FH = \sqrt{2896}$ or 53.814 cm $BH = \sqrt{12^2 + 2896}$ $BH = 55.1$ cm (3 s.f)
2b	$CG = \sqrt{12^2 + 16^2} = 20$ cm Perimeter of the cross-sectional area $= 20 + 12 + 36 + 20 = 88$ cm Lateral surface area $= 88 \times 40 = 3520$ cm^2 Total Surface area $= 3520 + 2\left(\frac{1}{2} \times (20 + 36) \times 12\right)$ $= 4192$ cm^2
2c	Volume of prism $= 336 \times 40$ m^3 $= 13440$ m^3 Let the side of the square base be x. $\frac{1}{3} \times x^2 \times 24 = 13440$ $x = 41.0$ cm (3 s.f)

Qn	Working
3(a)	Correct plotting of points Correct scale and axes Smoothness of curve
(b)(i)	\$4250 (accept \$4200 to \$4300)
(b)(ii)	425 tarts (accept 400 to 450)
(b)(iii)	$1.275 < x < 3.675$ (accept lower limit: 1.25 to 1.35 (accept upper limit: 3.65 to 3.75)
(c)(i)	Draw line $\frac{y}{x} = \frac{1}{4}$
(c)(ii)	0.45 (accept 0.4 to 0.5)
(c)(iii)	The number of pieces of the blueberry tarts to be sold such that the profit made per piece is \$0.25.
(c)(iv)	$5x - x^2 - 2 = \frac{1}{4}x$ $x^2 - 4\frac{3}{4}x + 2 = 0$ $4x^2 - 19x + 8 = 0$ $4x - 19 + \frac{8}{x} = 0$ $A = 8, \quad B = -19$

Qn	Working
4a	Area of triangle ABC $= \frac{1}{2}(95)(82)\sin 75°$ $= 3760 \text{ cm}^2 (3 \text{ s.f})$
4b	$AC^2 = 95^2 + 82^2 - 2(95)(82)\cos 75°$ $AC = 108.24$ $= 108 \text{ m} (3 \text{ s.f})$
4c	$\frac{\sin \angle ACB}{95} = \frac{\sin 75°}{108.24}$ $\angle ACB = 57.970°$ $= 58.0°$
4d	$\angle DAC = 57.970° - 25°$ $= 32.97°$ (alt $\angle$s, parallel lines) $\sin 32.97° = \frac{DE}{71}$ $DE = 38.638 \text{ m}$ Let the greatest angle of elevation be θ. $\tan\theta = \frac{67}{38.638}$ $\theta = 60.0°$ (1 dp)

Qn	Working
5a	$\left(\frac{25}{x}\right)$ h
5b	$\left(\frac{30}{x+3}\right)$ h
5c	$\frac{25}{x}-\frac{30}{x+3}=\frac{5}{4}$ $\frac{25(x+3)-30x}{x(x+3)}=\frac{5}{4}$ $\frac{75-5x}{x(x+3)}=\frac{5}{4}$ $300-20x=5x^2+15x$ $5x^2+35x-300=0$ $x^2+7x-60=0$ (shown)
5d	$x^2+7x-60=0$ $(x+12)(x-5)=0$ $x=-12$ or $x=5$
5e	$\frac{30}{5+3}=3.75$ h

Qn	Working
6a	Interquartile range = $29 - 20 = 9$ marks
6bi	$p = 20,\ q = 130,\ r = 280,\ s = 70$
6bii	Mean $= \dfrac{15350}{500}$ $= 30.7$ marks
6biii	SD $= \sqrt{\dfrac{492305}{500} - \left(\dfrac{15350}{500}\right)^2}$ $= 6.49$ marks (3 s.f)
6c	34 marks
6d	Median mark for Paper 1 = 25 Median mark for Paper 2 = 31.5 (between 31 and 32) Since the median mark for Paper 1 is lower than the median mark for Paper 2, Paper 1 is the more difficult paper.

Qn	Working
7ai	$T_5 = 3^4 - 14 = 67$
7aii	$T_n = 3^{n-1} - \left(2 + 3(n-1)\right)$ $= 3^{n-1} - (3n-1)$ $= 3^{n-1} - 3n + 1$
7aiii	$T_{15} = 3^{14} - 3(15) + 1 = 4\ 782\ 925$
7bi	$\mathbf{B} = \begin{pmatrix} p-12 \\ q-15 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 6 & 8 \\ 12 & 14 \end{pmatrix}\begin{pmatrix} p-12 \\ q-15 \end{pmatrix}$ $= \begin{pmatrix} 6p-72+8q-120 \\ 12p-144+14q-210 \end{pmatrix}$ $= \begin{pmatrix} 6p+8q-192 \\ 12p+14q-354 \end{pmatrix}$
7bii	$\mathbf{D} = \begin{pmatrix} 92 \\ 170 \end{pmatrix}$ $\begin{pmatrix} 6p+8q-192 \\ 12p+14q-354 \end{pmatrix} = \begin{pmatrix} 92 \\ 170 \end{pmatrix}$ $6p+8q-192 = 92$ $6p+8q = 284 ----(1)$ $12p+14q-354 = 170$ $12p+14q = 524 ----(2)$ $(1)\times 2 : 12p+16q = 568 ----(3)$ $(3)-(2) : 2q = 44$ $q = 22, p = 18$

Qn	Working
8ai	Reflex angle $COA = 360° - 116°$ $= 244°$ Angle $CBA = \frac{244°}{2}$ $= 122°$($\angle$ at centre = 2$\angle$ at circumference)
8aii	Angle $OAC = \frac{180° - 116°}{2}$(base $\angle$s of isos. Δ) $= 32°$ Angle $CAT = 90° - 32°$(tan $\perp$ rad) $=58°$
8aiii	Angle ACT = angle CAT $= 58°$(tangents from ext. pt) Angle $CTA = 180° - 2(58°)$ ($\angle$ sum of Δ) $= 64°$
8bi	Triangle QRP is a right angle triangle ($\angle$ in semicircle) $\cos\frac{\pi}{5} = \frac{RP}{10}$ $RP = 10\cos\left(\frac{\pi}{5}\right)$ $PR = 8.0902$ cm (to 5 s.f) OR Angle $ROP = \pi - 2\left(\frac{\pi}{5}\right) = \frac{3\pi}{5}$(base $\angle$s of isos Δ, $\angle$sum of Δ) $PR^2 = 5^2 + 5^2 - 2(5)(5)\cos\frac{3\pi}{5}$ $PR = 8.0902$ cm (to 5 s.f)
8bii	Arc length $QR = 5\left(\frac{2\pi}{5}\right) = 2\pi$ Arc length $RSP = 5\pi - 2\pi = 3\pi$ Semicircle $PRT = \frac{8.0902}{2}\pi = 4.0451\pi$ Perimeter of shaded region = $3\pi + 4.0451\pi$ $= 22.1$ cm (3 s.f)

8biii	Area of sector $ORSP = \frac{1}{2}(5)^2\left(\frac{3\pi}{5}\right) = 7.5\pi \text{ cm}^2$ Area of triangle $ORP = \frac{1}{2}(5)(5)\sin\left(\frac{3\pi}{5}\right) = 11.888 \text{ cm}^2$ Area of segment $RSP = 7.5\pi - 11.888 = 11.674 \text{ cm}^2$ Area of shaded region $= \frac{1}{2}\pi\left(\frac{8.0902}{2}\right)^2 - 11.674 = 14.0 \text{ cm}^2$ (to 3 sf)

Qn	Working
9ai	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= 8\mathbf{b} - 12\mathbf{a}$ $\overrightarrow{AQ} = 4\mathbf{b} - 6\mathbf{a}$
9aii	$\overrightarrow{BP} = \overrightarrow{BO} + \overrightarrow{OP}$ $= -8\mathbf{b} + 8\mathbf{a}$
9aiii	$\overrightarrow{QP} = \overrightarrow{QA} + \overrightarrow{AP}$ $= -4\mathbf{b} + 6\mathbf{a} - 4\mathbf{a}$ $= 2\mathbf{a} - 4\mathbf{b}$
9b	$\overrightarrow{PR} = 4\overrightarrow{PQ}$ $\overrightarrow{OR} - \overrightarrow{OP} = 4(4\mathbf{b} - 2\mathbf{a})$ $\overrightarrow{OR} - 8\mathbf{a} = 16\mathbf{b} - 8\mathbf{a}$ $\overrightarrow{OR} = 16\mathbf{b}$
9c	*OB* is parallel to *OR* and since *O* is a common point, *O, B* and *R* are collinear.
9d	*OR* = 2*OB* or *B* is the midpoint of *OR*. Since *PQBS* is a parallelogram, $\overrightarrow{QP} = \overrightarrow{BS}$ $2\mathbf{a} - 4\mathbf{b} = \overrightarrow{OS} - 8\mathbf{b}$ $\overrightarrow{OS} = 2\mathbf{a} + 4\mathbf{b}$
9e	$\frac{\text{Area of } \Delta OBP}{\text{Area of } \Delta ORA} = \frac{\text{Area of } \Delta OBP}{\text{Area of } \Delta OBA} \times \frac{\text{Area of } \Delta OBA}{\text{Area of } \Delta ORA}$ $= \frac{2}{3} \times \frac{1}{2}$ $= \frac{1}{3}$
9f	$\overrightarrow{AB} = 8\begin{pmatrix} -1 \\ -1 \end{pmatrix} - 12\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 4 \\ -20 \end{pmatrix}$ $\left\lvert \overrightarrow{AB} \right\rvert = \sqrt{4^2 + (-20)^2}$ = 20.4 units (3 sf)

Qn	Working
10a	Electricity tariff $= \dfrac{\frac{100}{107}\times 148.13}{625}$ $= \$0.2215 / \text{kWh}$
10b	Average consumption per month = 529.5 kWh
10c	<u>Best Power</u> Total cost for 12 months $= 12\times 529.5\times 0.1667$ $= \$1059.21$ <u>Marco Energy</u> Total cost for 12 months $= 12(10+0.1535\times 529.5)$ $= \$1095.34$ <u>Infinity Power</u> Total cost for 12 months $= (6\times 529.5\times 0.1730)+(6\times 0.95\times 0.1730\times 529.5)$ $= \$1071.76$ <u>Rainbow Energy</u> Total cost for 12 months $= 3\times 0.8\times 529.5(0.2282+0.2365+0.2293+0.2386)$ $= \$1185.15$ <u>Unicorn Supply</u> Total cost for 12 months $= 12\times(0.75\times 529.5\times 0.1685 + 0.25\times 529.5\times 0.1438)$ $= \$1031.41$ Mr Lim should choose Unicorn Supply as it has the lowest total costs. Assumptions: The electricity consumption in Mr Lim's household is consistent throughout the year OR The projected quarterly household electricity tariff is fairly accurate.

Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

MATHEMATICS 4048/01

Paper 1 **14 Aug 2018**

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use										
Qn No.	1	2	3	4	5	6	7	8	9	10
Marks	2	2	2	2	2	2	2	3	4	3
Qn No.	11	12	13	14	15	16	17	18	19	20
Marks	2	3	3	3	4	3	6	7	4	4
Qn No.	21	22	Total No. of Marks							
Marks	9	8	80							

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 By approximating each number to 2 significant figures, estimate the value of $\frac{12.1\times\sqrt{48.8}}{\sqrt[3]{27.3}}$. Show your working and give your answer to a reasonable degree of accuracy.

Answer ..... [2]

---

2 **(a)** Express the ratio of 1 minute and 30 seconds to 1 hour in its simplest form.

(b) If $5a = 4b$ and $8b = 3c$, find $a : b : c$.

Answer (a) ..... [1]

(b) .... [1]

---

3 Twelve workers are hired to build a wall in 9 days, assuming that they all work at the same rate. After 3 days, two workers left. How many days would the remaining workers take to finish building the wall ?

Answer .....days [2]

4 The value of a new house depreciated 10% each year for 3 years in a row. Then, for the next 3 years, the value of the house increased 10% each year. Did the value of the house increase or decrease after 6 years? Explain your answer.

Answer: .....

.....

.....

..... [2]

---

5 The exterior angles of a hexagon are in the ratio 2 : 3 : 3 : 4 : 4 : 8. Find the smallest interior angle of the hexagon.

Answer .....^{o} [2]

---

6 Express 0.00952 nanoseconds in megaseconds, giving your answer in standard form.
(1 nano unit = 1×10^{-9} unit ; 1 mega unit = 1×10^{6} unit)

Answer: ..... megaseconds [2]

7 On a particular day at noon, the temperature 15 m above the sea level is 4^o C. The temperature 30 m below the sea level is -23^o C. Calculate

(a) the difference between these temperatures,

Answer .....^{o} *C* [1]

(b) the temperature at sea level at noon, assuming that the temperature changes uniformly with height.

Answer .....^{o} *C* [1]

8 A tablet is sold at \$1450 after a discount of 20%.

(a) Find the marked price of the tablet.

Answer \$..... [1]

(b) A customer bought the tablet at the discounted price and he paid for it using a hire purchase scheme according to the following terms: a down-payment of 60% and the remaining to be paid in monthly instalments over 16 months at a simple interest rate of x % per annum. Given that the total interest he paid is \$34.80, find x .

Answer x =..... [2]

9 **(a)** Factorise completely $4+6ab-9a^2-b^2$.

Answer ..... [2]

(b) Without using a calculator, show that $2^{17}-2^{14}$ is divisible by 7.

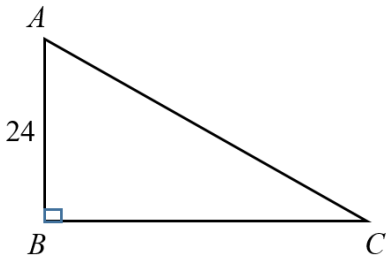
Answer ..... [2]

---

10 In triangle ABC, $\angle ABC = 90^o$, $AB = 24$ cm and $\sin\angle ACB = \frac{3}{5}$. Without the use of calculator, find the value of

(i) AC

(ii) $5\cos\left(180^o - \angle ACB\right)$



Answer (i) ..... [1]

(ii) ..... [2]

11 Solve the following inequality $6 < 2x + \frac{3x-5}{4} \leq 4x + 2$.

Answer ……..…… [2]

---

12 **(a)** A designer has to design a box in the shape of a cube of length l cm so as to store rectangular bricks of dimensions 45 cm by 21 cm by 15 cm. To save cost, he must ensure that the bricks fit exactly into the box, leaving no gaps in between. What is the smallest possible value of l?

Answer ..... [2]

(b) How many bricks can fit into the box?

Answer ..... [1]

13 A map is drawn to a scale of 1 : 20 000

(a) The distance from one end of a grassland to the other end is represented by a line of length 8 cm on the map. Calculate the actual distance, giving your answer in kilometres.

Answer ..... km [1]

(b) The actual area of the grassland is 2.5 hectares. Calculate the area on the map, giving your answer in square centimetres. [1 hectare = 10 000 m^2]

Answer ……..… cm^2 [2]

---

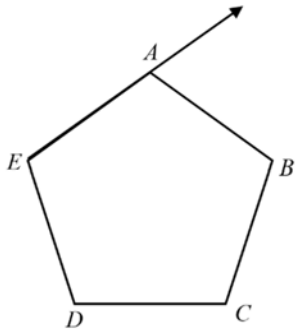
14 In the diagram, *ABCDE* is a regular pentagon where *A* is due north of *E*.

Find the bearing of

(i) *B* from *A*,

(ii) *D* from *A*,

(iii) *C* from *E*.



Answer *(i)*…..…… ^{o} [1]

(ii)……..…….. ^{o} [1]

(iii) .....…. ^{o} [1]

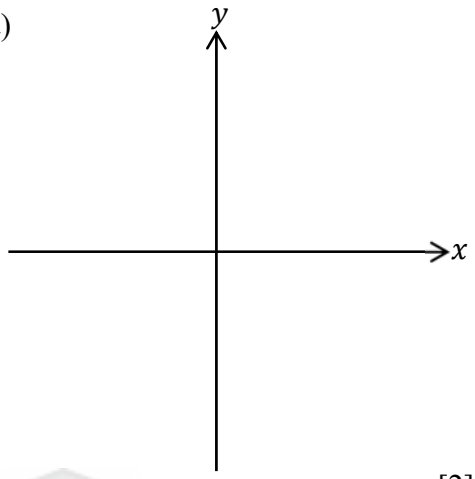
15 In the axes provided below, sketch the graphs of the following. State intercept(s) if any.

(a) $y=(x-1)^3$,

(b) $y=\dfrac{x+1}{x}$.

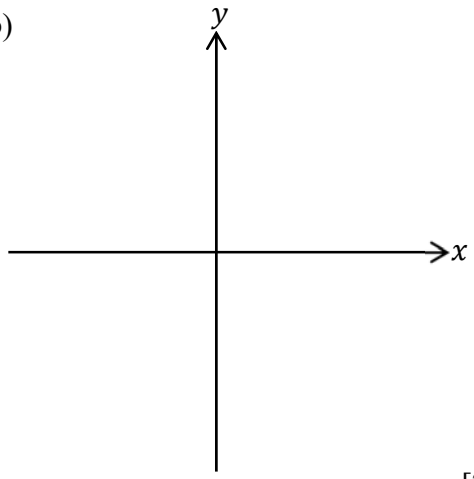
Answer

(a)



[2]

(b)



[2]

16 **(a)** Consider the sequence $\frac{2}{3}, \frac{5}{8}, \frac{13}{21}, \frac{34}{55}, ...$

Write down the next two terms of the sequence.

Answer .....[1]

(b) Write down an expression, in terms of n, for the nth term of the sequence

3 , 6 , 10 , 15 , 21 , ...

Answer ……..……..…… [2].

17 The volume of cylinder A of radius r cm and height h cm is 240 cm^3.

(a) Find the volume of cylinder B of radius $2r$ cm and height $\frac{1}{3}h$ cm.

Answer ..... cm^3 [2]

(b) Cylinder C is similar to cylinder A. If the radius of cylinder C is $\frac{1}{2}r$ cm, find its volume.

Answer ..... cm^3 [2]

(c) A cone D has the same volume as cylinder A. If the height of cone D is h cm, find the ratio of the radius of cone D to that of cylinder A.

Answer ..... [2]

18 Given that $\overrightarrow{AB} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} -6 \\ 12 \end{pmatrix}$ and C is the point on OB such that $OC : CB = 1 : 2$.

(a) Find

(i) $\left|\overrightarrow{AB}\right|$, [1]

(ii) the position vector of C , [1]

(iii) $\overrightarrow{AC}$ [2]

(b) Given that $\begin{pmatrix} 2 \\ m \end{pmatrix}$ and $\overrightarrow{AB}$ are parallel vectors. Find the value of m. [1]

(c) Find the coordinates of D, such that $ABCD$ is a parallelogram. [2]

Answer *(a)(i)* .... units [1]

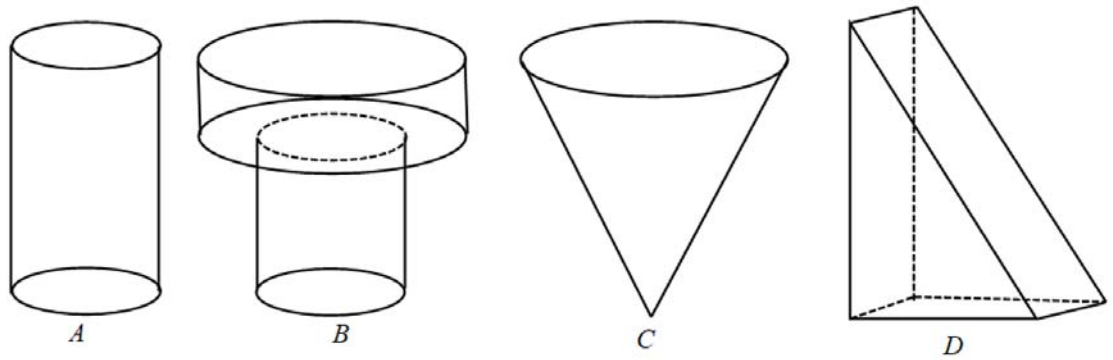
(ii) .... …………… ……….. [1]

(iii) $\overrightarrow{AC}$ = ……………………………[2]

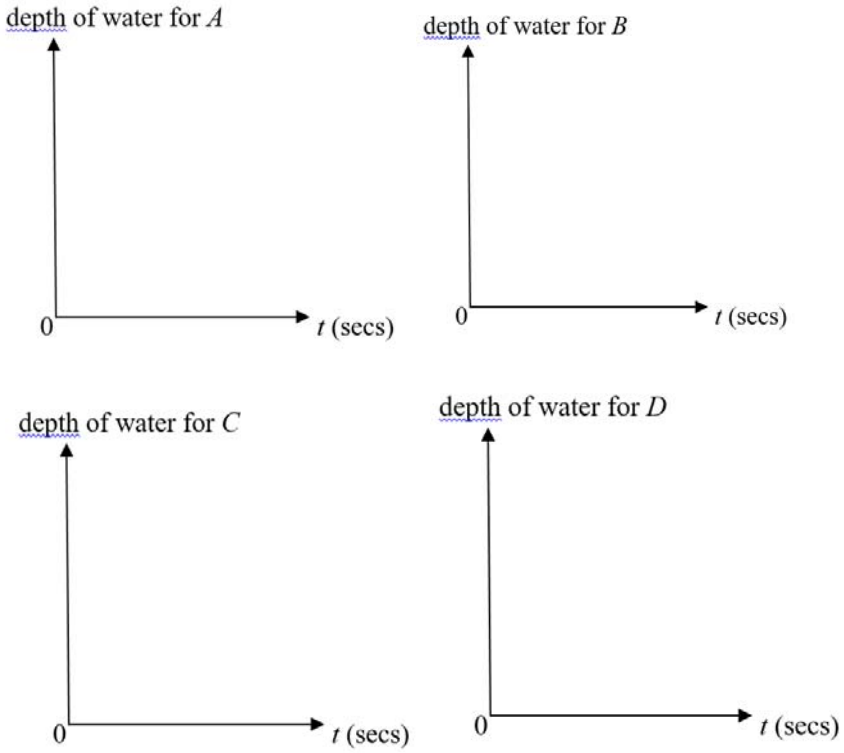
(b) m =……… …………………… [1]

(c) D is ……………………………. [2]

19 The diagrams below show four containers (not drawn to scale), *A*, *B*, *C* and *D* each with a height of *h* cm. The containers are initially empty. It takes *t* seconds to fill each container with water at a constant rate.



(a) On the axes in the answer space below, sketch the graph of the depth of the water against time for each of the four containers.

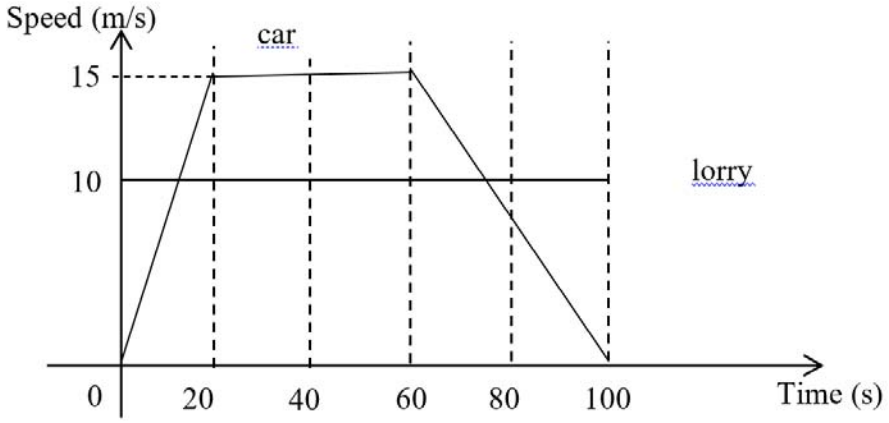


[2]

(b) It takes 12 seconds to fill container *D* to the brim. Find the time it takes to fill container *D* to half its height.

Answer ..... secs [2]

20 The diagram shows the speed-time graphs of a car and a lorry travelling on the road for a period of 100 seconds. After accelerating, the car travels at a constant speed of 15 m/s for the next 40 seconds before decelerating to a stop, while the lorry travels at a constant speed of 10 m/s throughout.



(a) Find the time when the speed of the car is 10 m/s.

Answer .....sec [1]

(b) Find the speed of the car 10 seconds before it comes to rest.

Answer …….. m/s [1]

(c) Find the time when the car overtakes the lorry.

Answer…… sec [2]

21 **(a)** A box contains five slips of paper. Each slip has one of the numbers 4 , 6 , 7 , 8 or 9 written on it. There are two players for the game. The first player reaches into the box and draws two slips and adds the two numbers. If the sum is even, the player wins. If the sum is odd, the player loses. What is the probability that the first player wins.

Answer: ..... [2]

(b) A game is such that a fair die is rolled respectively until a '6' is obtained.
Find the probability that the game ends by the fourth roll.

Answer ..... [2]

Suppose now that the game is such that the same die is rolled repeatedly until two '6's are obtained. Find the probability that

(i) the game ends on the third roll,

(ii) the game ends on the third roll and the sum of the scores is odd.

Answer (i) .....[1]

(ii)..... [1]

(c) Of the 33 students in a class, 25 own tablet PC and 9 own desktop computers.
It is given that $\xi = \{$ students in the class $\}$,

$A = \{$ students who own a tablet PC $\}$,

$B = \{$ students who own a desktop computer $\}$ and $n(A \cap B) = x$

(i) Express $n(A' \cap B')$ in terms of x.

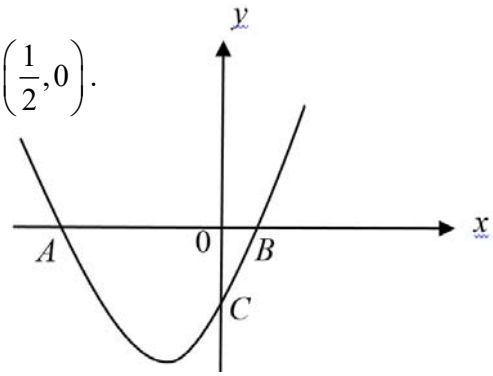
(ii) Express in set notation { students who own tablet PC but not desktop computer}.

Answer (i) ……..[2]

(ii) …..…….. [1]

22 The diagram shows part of the graph of $5(y+3)=ax^2+bx$, where a and b are constants.

The graph cuts the x-axis at $A\left(-2\frac{1}{2},0\right)$ and $B\left(\frac{1}{2},0\right)$.



The graph meets the y-axis at the point C.
Find

(i) the value of a and of b,

(ii) the coordinates of C,

(iii) the coordinates of the minimum point,

(iv) the equation of the line of symmetry,

(v) the area of triangle ABC.

Answer (i) a = …… b = …… [2]

(ii) C…… [1]

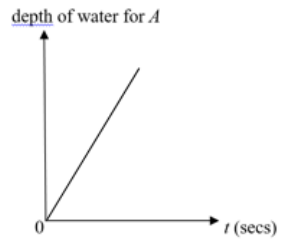
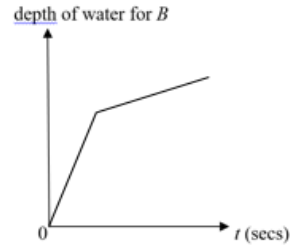
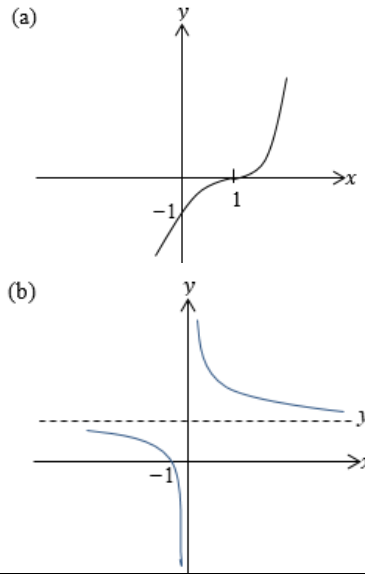
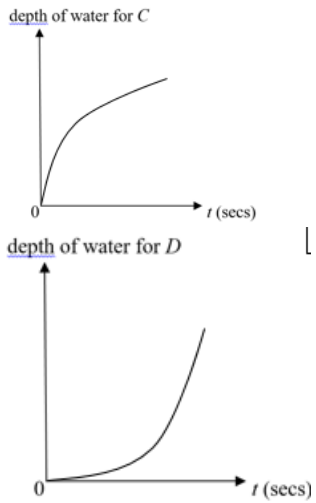
(iii) …… [2]

(iv) …… [1]

(v) …… [2]

END OF PAPER

Answer Key

Qn No.		Qn No.	
1	28 or 30	17	(a) $320\ \text{cm}^3$ (b) $30\ \text{cm}^3$ (c) $\sqrt{3}:1$
2	(a) 1 : 40 (b) 12 : 15 : 40		
3	7.2 days	18	(a) (i) 10 units (ii) $\begin{pmatrix}-2\\4\end{pmatrix}$ (iii) $\begin{pmatrix}12\\-14\end{pmatrix}$
4	$0.9703P < P$, decreased		
5	60^o		
6	9.52×10^{-18} megaseconds		
7	(a) $27^o\ C$ (b) -5^oC		
8	(a) \$1812.50 (b) 4.5		(b) $-1\frac{1}{2}$
9	(a) $(2-3a+b)(2+3a-b)$ (b) $2^{14}\times7$ is divisible by 7		(c) $D(-10,10)$
10	(i) 40 (ii) -4	19(a)	
11	$x > 2\frac{7}{11}$		
12	(a) 315 (b) 2205		
13	(a) 1.6 km (b) $0.625\ \text{cm}^2$		
14	(i) 072^o (ii) 144^o (iii) 072^o		
15			

16	(a) $\frac{89}{144}, \frac{233}{377}$ (b) $T_n = \frac{1}{2}(n+1)(n+2)$	19(b)	9 secs
20	(a) 13.3 sec , 73.3 sec (b) 3.75 m/s (c) 30 sec	21	(a) $\frac{2}{5}$ (b) $\frac{671}{1296}$ (i) $\frac{5}{108}$ (ii) $\frac{1}{36}$ (c) (i) $x-1$ (ii) $A \cap B'$
22	(i) $a=12$, $b=24$ (ii) $C(0,-3)$ (iii) $\left(-1,-5\frac{2}{5}\right)$ (iv) $x=-1$ (v) 4.5 units^2		

Name: MARK SCHEME	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

MATHEMATICS
Paper 1

4048/01
14 Aug 2018
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use										
Qn No.	1	2	3	4	5	6	7	8	9	10
Marks	2	2	2	2	2	2	2	3	4	3
Qn No.	11	12	13	14	15	16	17	18	19	20
Marks	2	3	3	3	4	3	6	7	4	4
Qn No.	21	22	Total No. of Marks							
Marks	9	8	80							

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 By approximating each number to 2 significant figures, estimate the value of $\frac{12.1\times\sqrt{48.8}}{\sqrt[3]{27.3}}$. Show your working and give your answer to a reasonable degree of accuracy.

$$\frac{12.1\times\sqrt{48.8}}{\sqrt[3]{27.3}} \approx \frac{12\times\sqrt{49}}{\sqrt[3]{27}}$$
$$= \frac{12\times 7}{3}$$
$$= 28$$
$$\approx 30$$

B1 for estimating all values to 2 s.f.

Answer .....28 or 30..... [2]

---

2 (a) Express the ratio of 1 minute and 30 seconds to 1 hour in its simplest form.

(b) If $5a = 4b$ and $8b = 3c$, find $a : b : c$.

(a) 1 min 30 s : 1 h = 90 s : 3600 s
= 1 : 40

(b) $a : b : c$
4 x 3 : 5 x 3
3 x 5: 8 x 5

Answer *(a)* .....1 : 40..... [1]
(b) .....12 : 15 : 40 ..... [1]

---

3 Twelve workers are hired to build a wall in 9 days, assuming that they all work at the same rate. After 3 days, two workers left. How many days would the remaining workers take to finish building the wall ?

After 3 days, number of man-day needed to complete the job = 6×12
$= 72$

Hence number of days needed for remaining workers = $\frac{72}{10}$ M1A1
$= 7.2$

Answer .....7.2.....days [2]

4 The value of a new house depreciated 10% each year for 3 years in a row. Then, for the next 3 years, the value of the house increased 10% each year. Did the value of the house increase or decrease after 6 years? Explain your answer. [2]

Answer: Let original value of house be *P*. Price after 3 years is $P(0.9)^3$. After another 3 years the value of the house will be $P(0.9)^3(1.1)^3$, which equals approximately $0.9703P$. Since $0.9703P < P$, the value of the house decreased after 6 years, by approximately 3%

B1 for $P(0.9)^3$; B1 for $0.9703P$

---

5 The exterior angles of a hexagon are in the ratio 2 : 3 : 3 : 4 : 4 : 8. Find the smallest interior angle of the hexagon.

Largest exterior angle = $\frac{8}{24}\times 360^o = 120^o$ B1

Hence smallest interior angle = $180^o - 120^o = 60^o$ B1

Answer ..60°..... [2]

6 Express 0.00952 nanoseconds in megaseconds, giving your answer in standard form.
(1 nano unit = 1×10^{-9} unit ; 1 mega unit = 1×10^{6} unit)

0.00952×10^{-9} seconds = $k\times10^{6}$ seconds

$k = 0.00952\times10^{-15}$

$= \frac{9.52}{1000}\times10^{-15}$

$= 9.52\times10^{-18}$

M1A1

Or 0.00952×10^{-9} seconds

$= 0.00952\times10^{-9-6}$ megaseconds

$= 9.52\times10^{-3}\times10^{-15}$

$= 9.52\times10^{-18}$

Answer: ..... 9.52×10^{-18} ... megaseconds [2]

7 On a particular day at noon, the temperature 15 m above the sea level is 4^o C. The temperature 30 m below the sea level is -23^o C. Calculate

(a) the difference between these temperatures,

$$\text{difference} = 4-(-23)$$
$$= 27^o \text{ C}$$

Answer .....27..... ^{o} *C* [1]

(b) the temperature at sea level at noon, assuming that the temperature changes uniformly with height.

$$\text{Rate of temperature increase} = \frac{27}{45} = 0.6^o C$$
$$\text{Hence temperature at sea level} = -23+30\times 0.6$$
$$= -5 \ ^o C$$

Answer .....−5..... ^{o} *C* [1]

8 A tablet is sold at \$1450 after a discount of 20%.

(a) Find the marked price of the tablet.

$$\frac{1450}{80}\times 100 = 1812.50$$

Answer \$.....1812.50..... [1]

(b) A customer bought the tablet at the discounted price and he paid for it using a hire purchase scheme according to the following terms: a down-payment of 60% and the remaining to be paid in monthly instalments over 16 months at a simple interest rate of x % per annum. Given that the total interest he paid is \$34.80, find x .

$$(40\%\times 1450)\times\frac{x}{100}\times\frac{16}{12} = 34.80 \qquad \text{M1A1}$$
$$x = \frac{34.80}{580}\times 100\times\frac{12}{16}$$
$$= 4.5$$

Answer x =.....4.5..... [2]

9 **(a)** Factorise completely $4+6ab-9a^2-b^2$.

$4+6ab-9a^2-b^2$
$=4-\left(9a^2-6ab+b^2\right)$
$=4-\left(3a-b\right)^2$
$=\left(2-3a+b\right)\left(2+3a-b\right)$

M1 for $\left(3a-b\right)^2$

Answer $\left(2-3a+b\right)\left(2+3a-b\right)$ ..... [2]

(b) Without using a calculator, show that $2^{17}-2^{14}$ is divisible by 7.

$2^{17}-2^{14}$
$=2^{14}.2^3-2^{14}$
$=2^{14}\left(8-1\right)$
$=2^{14}\times 7$

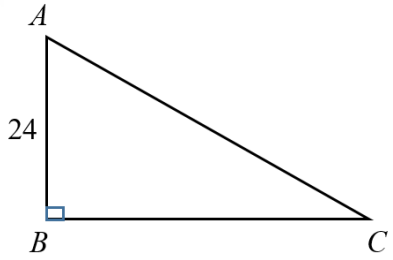
M1A1

Answer ..As 7 is a factor of $2^{17}-2^{14}$, $2^{17}-2^{14}$ is divisible by 7 ..... [2]

10 In triangle ABC, $\angle ABC=90^o$, $AB=24$ cm and $\sin\angle ACB=\frac{3}{5}$. Without the use of calculator, find the value of

(i) AC

(ii) $5\cos\left(180^o-\angle ACB\right)$



(i) $\sin\angle ACB=\frac{24}{AC}$

$AC=24\times\frac{5}{3}$

$=40$

(ii) $BC^2=40^2-24^2$

$BC=32$

$5\cos\left(180^o-\angle ACB\right)=-5\cos\angle ACB$ M1

$=-5\times\frac{32}{40}$

$=-5\times\frac{32}{40}$ A1

Answer (i) .....40..... [1]

(ii) ……..… –4 …… [2]

11 Solve the following inequality $6 < 2x + \frac{3x-5}{4} \le 4x + 2$.

$6 < 2x + \frac{3x-5}{4}$ and $2x + \frac{3x-5}{4} \le 4x + 2$

$24 < 8x + 3x - 5$ $\quad 8x + 3x - 5 \le 16x + 8$

$29 < 11x$ $\quad -5x \le 13$

$x > \frac{29}{11}$ $\quad x \ge -\frac{13}{5}$

$x > 2\frac{7}{11}$ $\quad x \ge -2\frac{3}{5}$

deduct 1 mark from full mark of 2, if 'and' is missing

B1 for any one inequality correct

Answer …… $x > 2\frac{7}{11}$ …… [2]

12 **(a)** A designer has to design a box in the shape of a cube of length *l* cm so as to store rectangular bricks of dimensions 45 cm by 21 cm by 15 cm. To save cost, he must ensure that the bricks fit exactly into the box, leaving no gaps in between. What is the smallest possible value of *l*?

$45 = 3^2 \times 5$

$21 = 3 \times 7$ M1 for prime factorisation

$15 = 3 \times 5$

LCM = $3^2 \times 5 \times 7$

= 315 A1

Answer .....315..... [2]

(b) How many bricks can fit into the box?

Number of bricks = $\frac{315^3}{45 \times 21 \times 15}$

= 2205

Answer .....2205..... [1]

13 A map is drawn to a scale of 1 : 20 000

(a) The distance from one end of a grassland to the other end is represented by a line of length 8 cm on the map. Calculate the actual distance, giving your answer in kilometres.

1 cm : 0.2 km
8 cm : 1.6 km

Answer .....1.6..... km [1]

(b) The actual area of the grassland is 2.5 hectares. Calculate the area on the map, giving your answer in square centimetres. [1 hectare = 10 000 m^2]

2.5 hectares = 25 000 m^2

$$= \frac{25000}{10^6}$$

$= 0.025$ km^2 B1

Area scale is 1 cm^2 : 0.04 km^2

$$\text{Area on the map} = \frac{0.025}{0.04}$$

$= 0.625$ cm^2 B1

Answer ……0.625… cm^2 [2]

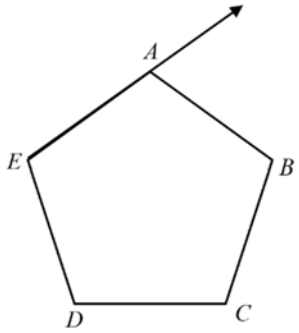
14 In the diagram, *ABCDE* is a regular pentagon where *A* is due north of *E*.

Find the bearing of

(i) *B* from *A*,

(ii) *D* from *A*,

(iii) *C* from *E*.



(i) Exterior angle = $\frac{360}{5} = 72^o$

Bearing = 072°

(ii) $\angle BAD = 72^o$

Bearing = 144°

(iii) Bearing = 072°

Answer *(i)*……072°…… [1]
(ii)……144°…….. [1]
(iii) .....072°……. [1]

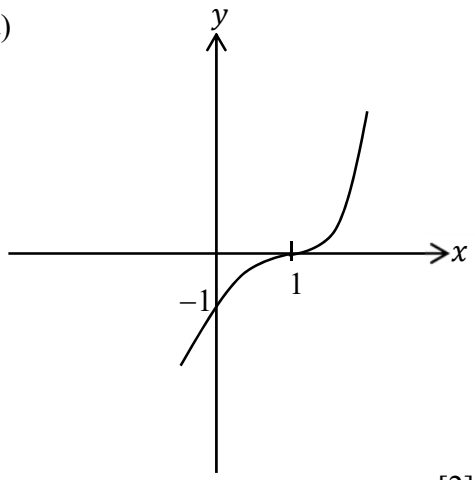
15 In the axes provided below, sketch the graphs of the following. State intercept(s) if any.

(a) $y=(x-1)^3$,

(b) $y=\dfrac{x+1}{x}$. $y=1+\dfrac{1}{x}$

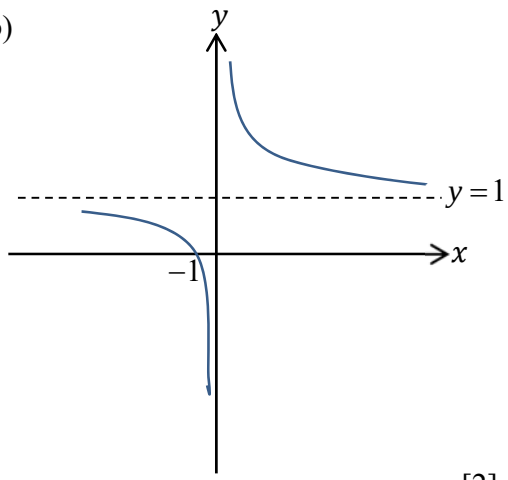
Answer

(a)



[2]

(b)



[2]

(a) Shape with point of inflexion on *x*-axis i.e. check that graph curve towards origin B1 ; *y*-intercept and *x*-intercept B1

(b) Shape and dash asymptote B1 ; *x*-intercept and $y=1$ B1

16 **(a)** Consider the sequence $\frac{2}{3}, \frac{5}{8}, \frac{13}{21}, \frac{34}{55}, ...$

Write down the next two terms of the sequence.

$\dfrac{34+55}{55+89}=\dfrac{89}{144}$, $\dfrac{233}{377}$

Answer ..... $\dfrac{89}{144}$, $\dfrac{233}{377}$ ..... [1]

(b) Write down an expression, in terms of *n*, for the *n*th term of the sequence

3 , 6 , 10 , 15 , 21 , ...

1+2 , 1+2+3 , 1+2+3+4 , 1+2+3+4+5 , ... $T_n=(n+2)\dfrac{n+1}{2}$ M1A1

Answer …. $T_n = (n+2)\frac{n+1}{2}$ …… [2].

17 The volume of cylinder *A* of radius *r* cm and height *h* cm is 240 cm^3.

(a) Find the volume of cylinder *B* of radius 2*r* cm and height $\frac{1}{3}h$ cm.

Volume of cylinder $B = \pi(2r)^2\frac{1}{3}h$ M1A1

$= \frac{4}{3}\pi r^2 h$

$= \frac{4}{3}\times 240$

$= 320$

Answer: ……320…… cm^3 [2]

(b) Cylinder *C* is similar to cylinder *A*. If the radius of cylinder *C* is $\frac{1}{2}r$ cm, find its volume.

$\frac{\text{volume of } C}{240} = \left(\frac{1}{2}\right)^3$

Volume of $C = \frac{1}{8}\times 240$ M1A1

$= 30$

Answer: ……30…… cm^3 [2]

(c) A cone *D* has the same volume as cylinder *A*. If the height of cone *D* is *h* cm, find the ratio of the radius of cone *D* to that of cylinder *A*.

$\frac{1}{3}\pi(r_1)^2 h = \pi r^2 h$ M1A1

$\left(\frac{r_1}{r}\right)^2 = 3$

$\frac{r_1}{r} = \sqrt{3}$

Accept 1.73 : 1

Answer ……$\sqrt{3}:1$…… [2]

18 Given that $\overrightarrow{AB} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} -6 \\ 12 \end{pmatrix}$ and C is the point on OB such that $OC : CB = 1 : 2$.

(a) Find

(i) $\left|\overrightarrow{AB}\right|$, [1]

(ii) the position vector of C, [1]

(iii) $\overrightarrow{AC}$ [2]

(b) Given that $\begin{pmatrix} 2 \\ m \end{pmatrix}$ and $\overrightarrow{AB}$ are parallel vectors. Find the value of m. [1]

(c) Find the coordinates of D, such that $ABCD$ is a parallelogram. [2]

(a) (i) $\left|\overrightarrow{AB}\right| = 10$ units

(ii) $\overrightarrow{OC} = \frac{1}{3}\overrightarrow{OB}$

$= \begin{pmatrix} \frac{-6}{3} \\ \frac{12}{3} \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

(iii) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BO} + \overrightarrow{OC}$

$= \begin{pmatrix} 8 \\ -6 \end{pmatrix} - \begin{pmatrix} -6 \\ 12 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 12 \\ -14 \end{pmatrix}$

M1A1

(b) $\frac{m}{2} = \frac{-6}{8}$

$= -1\frac{1}{2}$

(c) $\overrightarrow{CD} = \overrightarrow{BA} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$

Hence D is $(-2-8, 4+6)$

$= (-10, 10)$

M1A1

Answer *(a)(i)* .....10..... units [1]

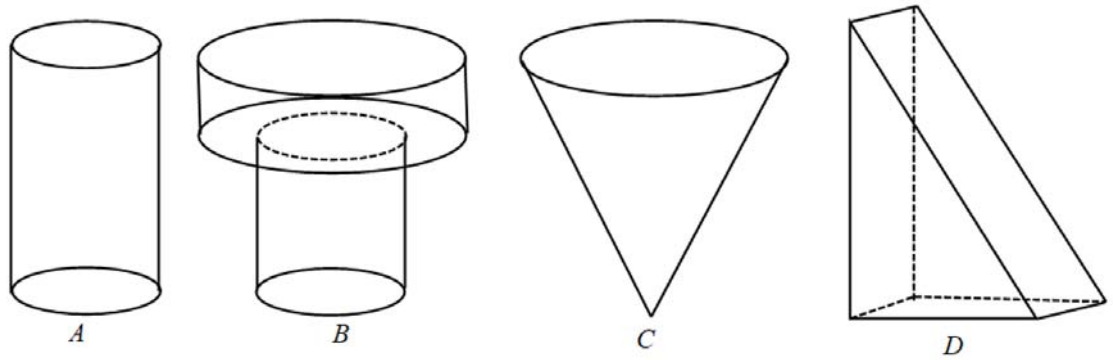
(ii) ..... $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ .... [1]

(iii) $\overrightarrow{AC} = \begin{pmatrix} 12 \\ -14 \end{pmatrix}$ .....[2]

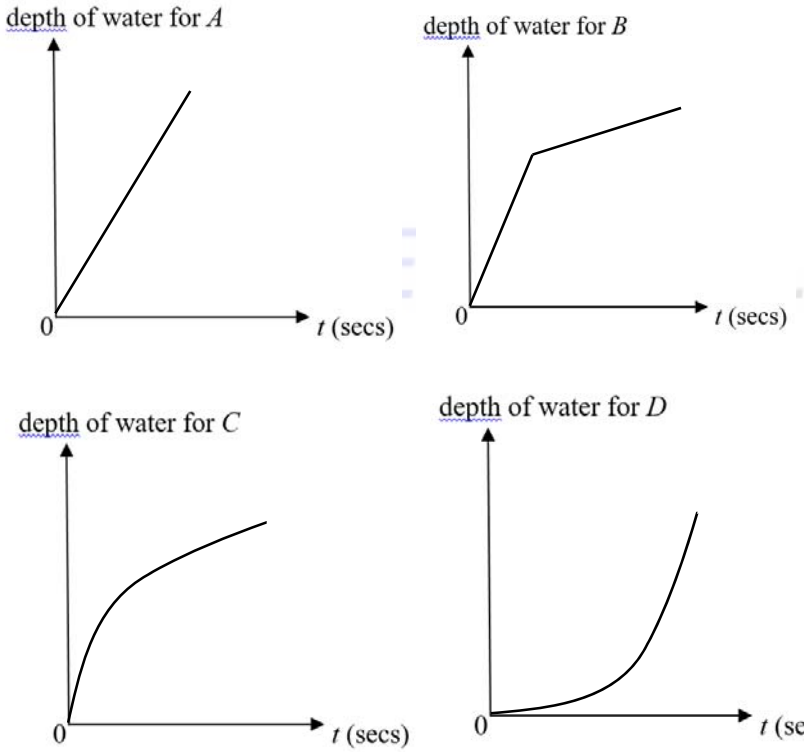
(b) $m =$ $-1\frac{1}{2}$ ..... [1]

(c) D is (-10 , 10)..... [2]

19 The diagrams below show four containers (not drawn to scale), *A*, *B*, *C* and *D* each with a height of *h* cm. The containers are initially empty. It takes *t* seconds to fill each container with water at a constant rate.



(a) On the axes in the answer space below, sketch the graph of the depth of the water against time for each of the four containers.



For *B*: The depth for bottom line must be approximately double that of top line

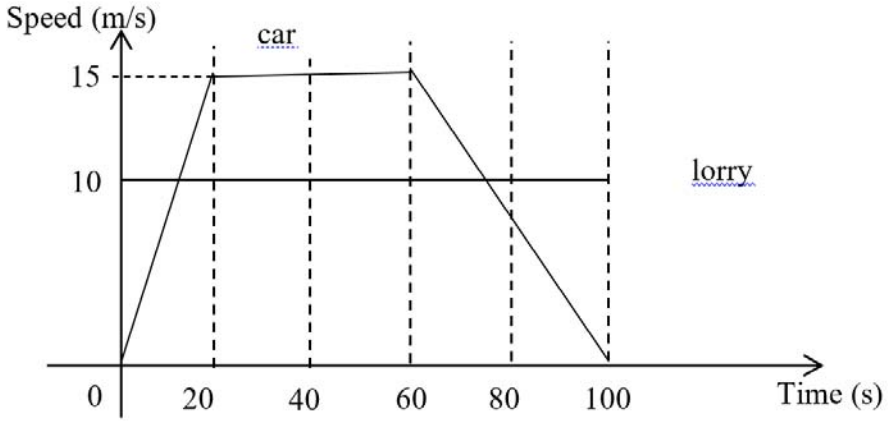
B2 or B1(any two correct) [2]

(b) It takes 12 seconds to fill container *D* to the brim. Find the time it takes to fill container *D* to half its height.

Ratio of volume of prisms = $\frac{1}{4}$. Required time = $\frac{3}{4}\times 12 = 9$ seconds M1A1

Answer .....9..... secs [2]

20 The diagram shows the speed-time graphs of a car and a lorry travelling on the road for a period of 100 seconds. After accelerating, the car travels at a constant speed of 15 m/s for the next 40 seconds before decelerating to a stop, while the lorry travels at a constant speed of 10 m/s throughout.



(a) Find the time when the speed of the car is 10 m/s.

Acceleration = $\frac{15}{20} = 0.75$ m/s^2 Deceleration = 0.375 m/s in 1 sec

Hence time = $\frac{10}{0.75} = 13.3$ Hence time = $100 - \frac{10}{0.375}$

$= 73.3$

Answer .....13.3 , 73.3.....sec [1]

(b) Find the speed of the car 10 seconds before it comes to rest.

Deceleration = $\frac{15}{40}$ m/s^2

Required speed = $\frac{15}{40} \times 10 = 3.75$

Answer ……3.75…… m/s [1]

(c) Find the time when the car overtakes the lorry.

$\frac{1}{2}(t + t - 20) \times 15 = 10t$

$(t - 10) = \frac{2}{3}t$

$t = 30$ M1A1

Answer……30…… sec [2]

21 **(a)** A box contains five slips of paper. Each slip has one of the numbers 4 , 6 , 7 , 8 or 9 written on it. There are two players for the game. The first player reaches into the box and draws two slips and adds the two numbers. If the sum is even, the player wins. If the sum is odd, the player loses. What is the probability that the first player wins.

P(even and even) + P(odd and odd) $= \left(\frac{3}{5}\times\frac{2}{4}\right)+\left(\frac{2}{5}\times\frac{1}{4}\right)$

$= \frac{2}{5}$ M1A1

Answer: ..... $\frac{2}{5}$ ..... [2]

(b) A game is such that a fair die is rolled respectively until a '6' is obtained.
Find the probability that the game ends by the fourth roll.

$1-\left(\frac{5}{6}\right)^4 = \frac{671}{1296}$

M1A1

Answer ..... $\frac{671}{1296}$ ..... [2]

Suppose now that the game is such that the same die is rolled repeatedly until two '6's are obtained. Find the probability that

(i) the game ends on the third roll,

(ii) the game ends on the third roll and the sum of the scores is odd.

(i) $2\left(\frac{1}{6}\times\frac{5}{6}\times\frac{1}{6}\right)=\frac{5}{108}$

(ii) $2\left(\frac{1}{6}\times\frac{1}{2}\times\frac{1}{6}\right)=\frac{1}{36}$

Answer(i) ..... $\frac{5}{108}$ ..... [1]

(ii) ..... $\frac{1}{36}$ .....[1]

(c) Of the 33 students in a class, 25 own tablet PC and 9 own desktop computers.
It is given that

$\xi = \{$ students in the class $\}$,

$A = \{$ students who own a tablet PC $\}$,

$B = \{$ students who own a desktop computer $\}$ and $n(A\cap B) = x$

(i) Express $n(A'\cap B')$ in terms of x.

(ii) Express in set notation { students who own tablet PC but not desktop computer}.

(i) $n(A'\cap B') = n(A\cup B)'$

$= 33 - (25 - x) - 9$ M1A1

$= x - 1$

Answer (i).....x – 1 ..... [2]

(ii) ..... $A\cap B'$ ..... [1]

22 **(a)** The diagram shows part of the graph of $5(y+3)=ax^2+bx$, where a and b are constants. The graph cuts the x-axis at $A\left(-2\frac{1}{2},0\right)$ and $B\left(\frac{1}{2},0\right)$. The graph meets the y-axis at the point C.

Find

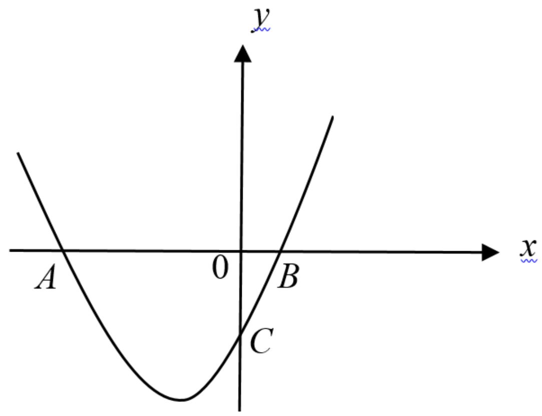
(i) the value of a and of b,

(ii) the coordinates of C,

(iii) the coordinates of the minimum point,

(iv) the equation of the line of symmetry,

(v) the area of triangle ABC.



(i) $a=60-2b$ ----- (1)

$12=5a-2b$ ----- (2)

Sub (1) into (2)

$b=24$

$a=12$ M1A1

(ii) $C(0,-3)$

(iii) $x=\dfrac{-2\frac{1}{2}+\frac{1}{2}}{2}$ M1A1

$=-1$

Hence minimum point is $\left(-1,-5\frac{2}{5}\right)$

(iv) $x=-1$

(v) Area of triangle ABC = $\frac{1}{2}\times3\times3=4.5$ units^2 M1A1

Answer (i) …a = 12…. , …b = 24… [2]

(ii) ……….$C(0,-3)$……… [1]

(iii) ……..…$\left(-1,-5\frac{2}{5}\right)$…….. [2]

(iv)…….. $x=-1$ …….….. [1]

(v) ……..….4.5…… units^2 [2]

END OF PAPER

BLANK PAGE

Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION 2018

MATHEMATICS **4048/02**
Paper 2 **17 August 2018**
2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper (1 sheet)
Mark Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
The use of a scientific calculator is expected, where appropriate.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

For Examiner's Use
100

This document consists of **13** printed pages.

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Solve the equation $\frac{x}{x^2-3x-4}-\frac{x+5}{4-x}=1$. [3]

(b) Make t the subject in the formula $x=2t\sqrt{\frac{k^2}{2k^2+3t^2}}$. [3]

(c) Simplify the expression $\sqrt[3]{\frac{y}{x^2}}\times\frac{y}{x}\div\sqrt{\frac{x^3}{9y^{-2}}}$. [2]

---

2 A shop sells two flavours of ice-cream, Rum Raisin and Super Chunkies.
Each flavour is sold in cups of three different sizes, small, medium and large, and of different prices. The sales in two successive days are given in the table below.

	Saturday			Sunday		
Size	Small	Medium	Large	Small	Medium	Large
Cost of ice-cream per cup	\$2.50	\$3.20	\$4.50	\$2.50	\$3.20	\$4.50
Number of cups of Rum Raisin sold	12	17	8	14	12	10
Number of cups of Super Chunkies sold	18	15	11	13	21	16

The information for Saturday's sales can be represented by the matrix,
$\mathbf{M}=\begin{pmatrix}12 & 17 & 8\\ 18 & 15 & 11\end{pmatrix}$ and the cost of each flavour for each size can be represented by the matrix
$\mathbf{C}=\begin{pmatrix}2.5\\ 3.2\\ 4.5\end{pmatrix}$. The information for the Sunday's sales can be represented by a 2 × 3 matrix **N**.

(a) Write down the matrix **N**. [1]

(b) Calculate **P** = (**M** + **N**). [1]

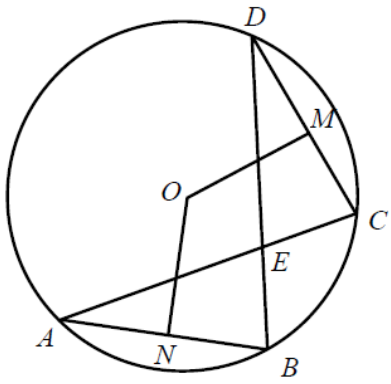
(c) Describe what is represented by the elements in **P**. [1]

(d) Calculate $\mathbf{Q}=\frac{1}{2}\mathbf{PC}$. [2]

(e) Describe what is represented by the elements of **Q**. [1]

(f) Calculate and describe what is represented by the elements of $\mathbf{R}=\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{PC}$. [2]

3



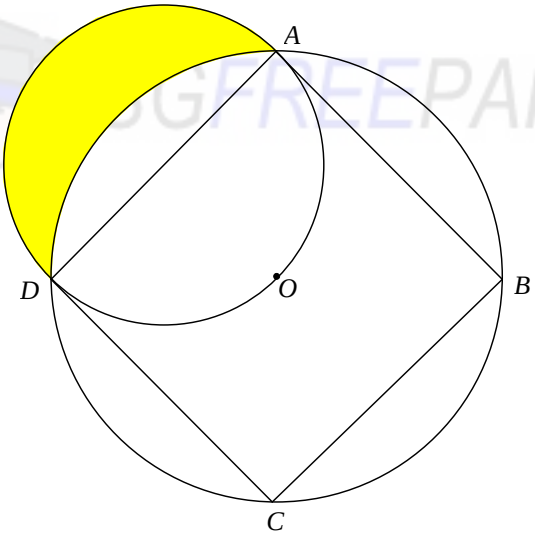
The points *A, B, C, D* lie on a circle, centre *O*. *N* and *M* are midpoints of *AB* and *CD* respectively. It is given that *ON* = *OM*.

(a) Show that the triangles *ABE* and *DCE* are congruent. [3]

(b) It is given that *AB* = 6 cm and *ON* = $(r-1)$ cm, where *r* is the radius of the circle. Find the value of *r*. [2]

---

4 In the diagram, *ABCD* is a square whose diagonals are each 2 cm long. Given that *O* is the centre of the bigger circle and *BD* and *AD* are the diameters of the bigger and smaller circle respectively.
Find the area of the shaded region. [4]



---

5 The first four terms in a sequence of numbers, p_1, p_2, p_3, p_4, …, are given below.

$p_1 = 1^2 + 2^2 + 2^2 = 3^2$

$p_2 = 2^2 + 3^2 + 6^2 = 7^2$

$p_3 = 3^2 + 4^2 + 12^2 = 13^2$

$p_4 = 4^2 + 5^2 + 20^2 = 21^2$

(a) Write down an expression for p_5 and show that $p_5 = 961$. [1]

(b) Given that $p_{10} = 10^2 + 11^2 + s^2 = k$, express *k* as a perfect square in terms of *s*. [1]

(c) Given that $p_w = w^2 + (w+1)^2 + r^2 = 5257^2$, find the value of *r* and of *w*. [2]

(d) Show that $p_n = n^4 + 2n^3 + 3n^2 + 2n + 1$. [3]

6 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = x + \frac{7}{x} - 6$.

The table below gives some values of x and the corresponding values of y.

x	0.5	1	2	3.5	5	6	7	8
y	p	2	–0.5	q	0.4	1.17	2	2.88

(a) Find the values of p and q. [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw the graph of $y = x + \frac{7}{x} - 6$ for the values of x in the range $0 < x \leq 8$. [3]

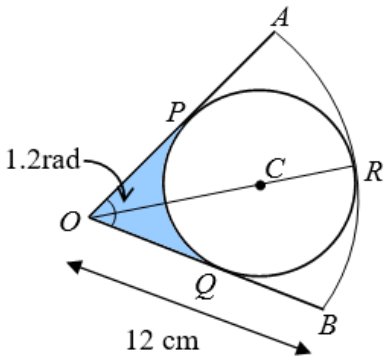
(c) Draw the line $y = -\frac{1}{2}x + 3$. [1]

(d) Use your graph to find the x-coordinate of a point on the curve $y = x + \frac{7}{x} - 6$ at which the gradient of the tangent is equal to -0.5. [2]

(e) Find the range of values of x for which $x^2 - 6x + 7 \leq 2.5x$. [2]

(f) By drawing a suitable line on your graph, find the solutions of the equation $2x^2 - 13x + 14 = 0$. [3]

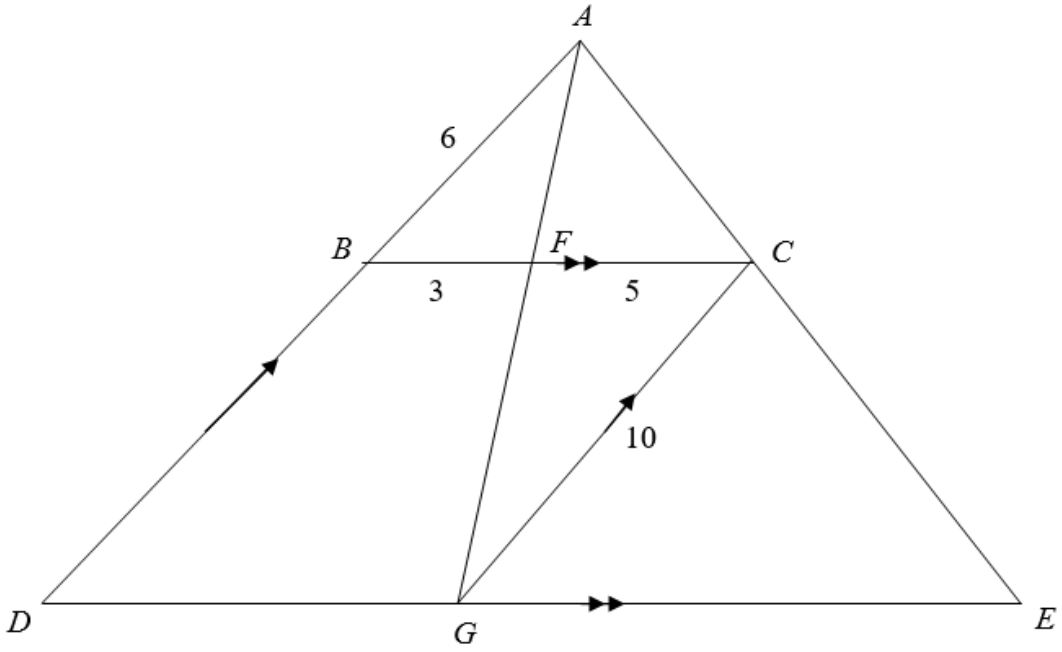
7 **(a)** In the diagram, *OARB* is a sector of a circle with centre *O*, radius 12 cm and angle *AOB* = 1.2 radians. *C* is the centre of the circle enclosed inside the sector, *OCR* is a straight line and the circle touches the sector at *P*, *Q* and *R*.



(i) Show that the radius of the enclosed circle is 4.3305 cm, correct to 4 decimal places. [3]

(ii) Calculate the perimeter of the shaded region *POQ*. [2]

(b) In the diagram below, *ABD, AFG, ACE, BFC* and *DGE* are straight lines.
BFC is parallel to *DGE* and *DBA* is parallel to *GC*.
AB = 6 cm, *BF* = 3 cm, *FC* = 5 cm and *GC* = 10 cm.



(i) Prove that triangle *BFA* is similar to triangle *CFG*. [2]

(ii) Calculate *GE*. [2]

(iii) Given the area of triangle *ABF* is 6.4 cm^2, find the area of trapezium *ACGD*. [3]

8 Amanda ran the 21km of a half-marathon race at an average speed of x km/h.

(a) Write down, in terms of x, an expression for the number of hours it took her to complete the race. [1]

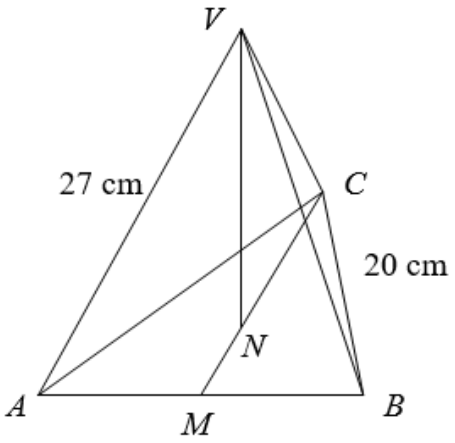
(b) Deborah ran the same race at an average speed which is 3km/h faster than Amanda's speed. Write down, in terms of x, an expression for the number of hours which Deborah took. [1]

(c) Given that the difference between the two times was 20 minutes, write down an equation in x and show that it reduces to $x^2 + 3x - 189 = 0$. [3]

(d) Solve the equation $x^2 + 3x - 189 = 0$, giving your answers correct to 2 decimal places. [2]

(e) Find, in hours and minutes, the time it took Amanda to complete the race. [1]

9



[The volume of tetrahedron $= \frac{1}{3} \times$ base area $\times$ height]

The diagram shows a tetrahedron, *VABC*, which has a horizontal equilateral triangular base *ABC* of side 20 cm.
The slant edge of the tetrahedron (*VA*, *VB* and *VC*) are each of length 27 cm.
M is the mid-point of *AB* and the vertical line *VN* meets the plane *ABC* at *N* where $MN : NC = 1 : 2$. Calculate

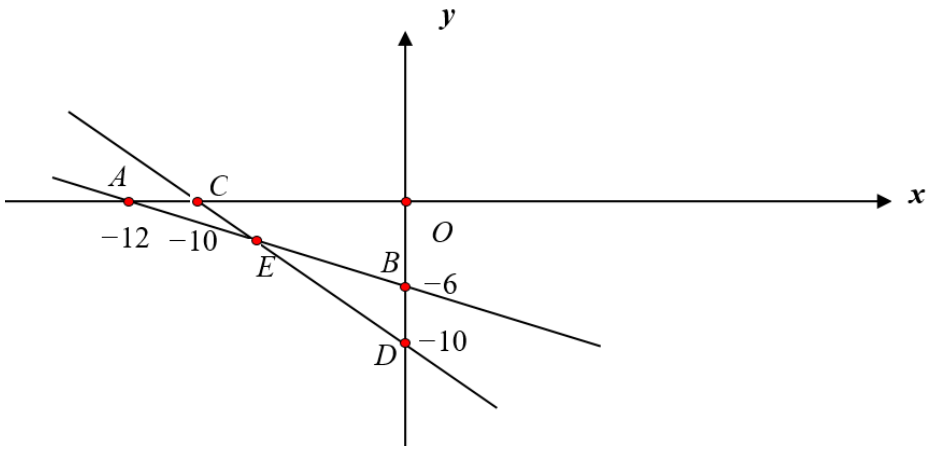
(a) *CM*, [2]

(b) the angle of elevation of *V* from *A*, [3]

(c) the volume of the tetrahedron. [3]

---

10 In the figure below, the x-intercept and y-intercept of the line AB are -12 and -6 respectively. Both the x-intercept and y-intercept of the line CD are -10.



Find

(a) the equation of the line AB and CD, [2]

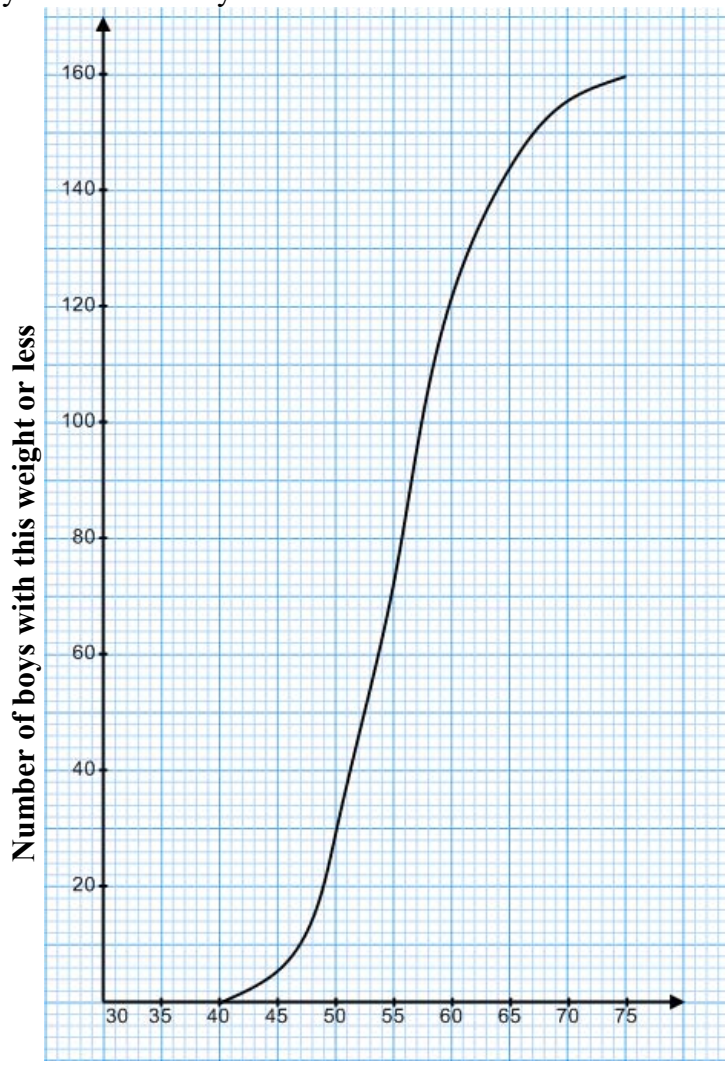
(b) the coordinates of E, [2]

(c) the area of $OCEB$, [2]

(d) the coordinates of F given that point F lies on AB produced such that $AF : FB = 5 : 3$, [2]

(e) find the coordinates of point G where G is the point on the x – axis such that OE is parallel to GD. [2]

11 The cumulative frequency curve below shows the weights of a sample of 160 boys from a school when they enter Secondary One.



(a) Use the graph to find an estimate for

(i) the number of boys whose weight is more than 60kg, [1]

(ii) the median weight, [1]

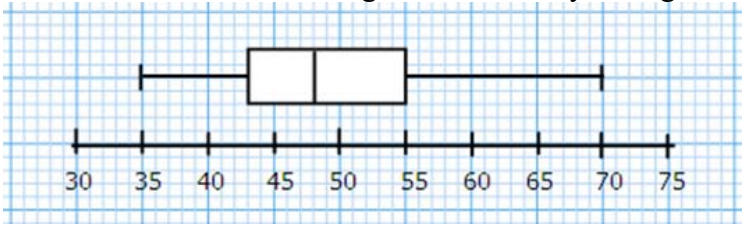
(iii) the percentage of boys whose weight is less than or equal to 52 kg. [1]

(b) The lightest 25% of the boys are classified as "slim".
The heaviest 25% of the boys are classified as "big-sized".
The remaining boys are classified as "medium".
Find an estimate for

(i) the greatest possible difference in weight between 2 boys classified as "big-sized", [2]

(ii) the least possible difference between the weight of a boy classified as "slim" and the weight of a boy classified as "big-sized". [2]

(c) The following box-and-whisker plot shows the distribution of weights of 160 Secondary One girls from the same school. Compare the data provided and record 2 observations about the distribution of weights between boys and girls. [2]



12 Mrs Tan, a mother of two children, saw a newspaper article as shown.

Price of formula milk in Singapore has soared

The average price of a 900g tin of formula milk has increased sharply over the last 5 years, outstripping the price increases of other dairy products and household staples.

On Monday, the Government announced it is tightening rules to encourage greater price competition.

The table below shows the price (in SGD) per 100 grams of different brands of formula milk in year 2012 and 2017.

Brand / Date	Similac	Friso	Nan	S26	Mamil
Dec 2012	5.71	5.22	5.20	5.13	4.96
Mar 2017	7.05	6.56	7.45	6.36	6.41
% increase	[illegible]		43.3	24.0	29.2

Coffee stain was found on the newspaper article covering some of the information.

(a) **(i)** Calculate the percentage increase in the price of the Similac and Friso formula milk covered by the coffee stain.. [2]

(ii) Hence, do you agree with the headline of the newspaper article? Support your answer with a reason. [2]

A few days later, Mrs Tan saw another article regarding the price of similar brands of formula milk sold in Singapore, Malaysia and China.

Brand / Country	Similac	Friso	Nan	S26	Mamil
Singapore	7.05	6.56	7.45	6.36	6.41
Malaysia	3.92	3.54	4.29	4.13	3.51
China	4.79	5.58	9.06	4.25	4.00

Price (in SGD) per 100 grams of formula milk in Singapore, Malaysia and China.

Upon seeing the article, Mrs Tan intends to purchase some cans of formula milk in China and ship them back during her holidays.

Mrs Tan did an online research and found the following shipping rate from China to Singapore by SHIPPER Company.

Weight of parcel, x (kg)	**Shipping Rate** 1st kg 150 *RMB* follow by
$x \leq 10$	75 *RMB*/ kg
$10 < x \leq 20$	35 *RMB*/ kg
$20 < x \leq 50$	31 *RMB*/ kg
$50 < x \leq 75$	27 *RMB*/ kg
$75 < x \leq 100$	25 *RMB*/ kg
$100 < x \leq 200$	24 *RMB*/ kg
$x > 200$	22 *RMB*/ kg

The information below shows a can of the 900 g formula milk that Mrs Tan intends to purchase in China and the online currency conversion.

1 Singapore Dollar equals

4.89 Chinese Yuan Renminbi (RMB)

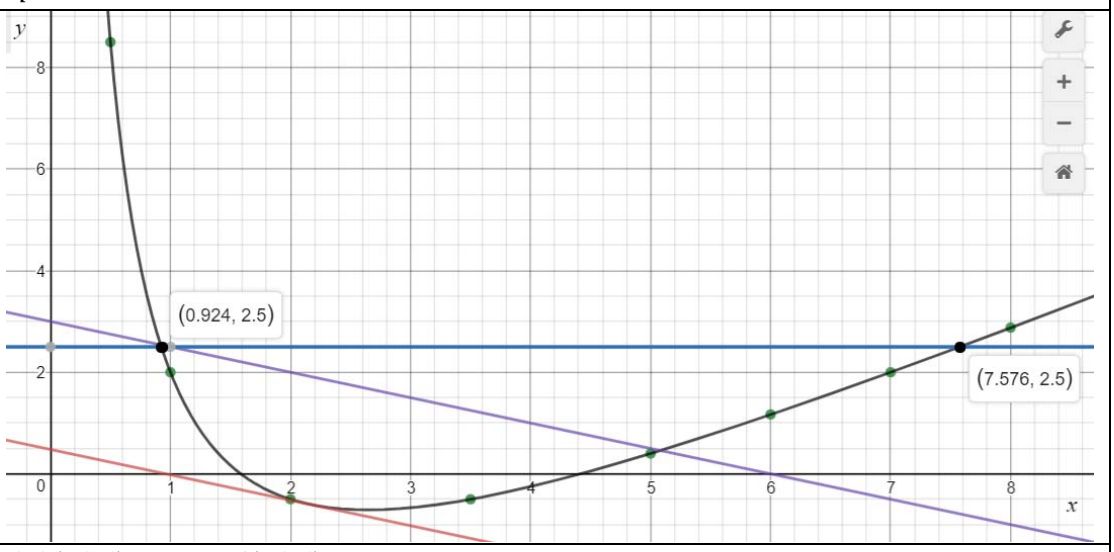
1	Singapore Dollar
4.89	Chinese Yuan Renminbi

Mrs Tan intends to spend at most S$650 for both the formula milk and shipping fee.

(b) Calculate the maximum number of cans of formula milk that Mrs Tan can buy. [5]

END OF PAPER

ANSWER KEY

1	**a**	$x = -\frac{9}{10}$
	b	$t = \pm\sqrt{\frac{-2k^2x^2}{3x^2 - 4k^2}}$ or $t = \pm\sqrt{\frac{2k^2x^2}{4k^2 - 3x^2}}$
	c	$\frac{3y^{\frac{1}{3}}}{x^{\frac{19}{6}}}$
2	**a**	$\mathbf{N} = \begin{pmatrix} 14 & 12 & 10 \\ 13 & 21 & 16 \end{pmatrix}$
	b	$\mathbf{P} = \begin{pmatrix} 26 & 29 & 18 \\ 31 & 36 & 27 \end{pmatrix}$
	d	$\begin{pmatrix} 119.4 \\ 157.1 \end{pmatrix}$
	f	(553)
3	**b**	$r = 5\text{cm}$
4		$\frac{1}{2}$ or $0.500\ \text{cm}^2$
5	**a**	961
	b	$k = (s+1)^2$
	c	$r = 5256$ $w = 72$
6	**a**	$p = 8.5$ $q = -0.5$
	b	
	e	$0.9(\pm 0.1) \le x \le 7.6(\pm 0.1)$
	f	$x = 0.6 \pm 0.1$ and $= 5.9 \pm 0.1$

7	**aii**	21.1 cm
	bii	$13\frac{1}{3}$ cm
	biii	74.0 cm^2
8	**a**	$\frac{21}{x}$hr
	b	$\frac{21}{x+3}$hr
	d	$x = 12.33$ (2dp) or $x = -15.33$ (2dp)
	e	1hr 42 mins
9	**a**	17.3 cm
	b	64.7°
	c	1410 cm^3
10	**a**	$y = -x - 10$
	b	$(-8, -2)$
	c	34 $units^2$
	d	$(18, -15)$
	e	$(40, 0)$
11	**ai**	40
	aii	56 kg
	aiii	27.5%
	bi	15 kg
	bii	8.5 kg
12	**ai**	% increase of Similac = 23.5% % increase of the Friso = 25.7%
	b	13 tins

Name: MARK SCHEME	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION 2018

MATHEMATICS **4048/02**
Paper 2 **17 August 2018**
2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper (1 sheet)
Mark Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
The use of a scientific calculator is expected, where appropriate.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

For Examiner's Use
100

This document consists of **21** printed pages.

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Solve the equation $\frac{x}{x^2-3x-4}-\frac{x+5}{4-x}=1$. [3]

(b) Make t the subject in the formula $x=2t\sqrt{\frac{k^2}{2k^2+3t^2}}$. [3]

(c) Simplify the expression $\sqrt[3]{\frac{y}{x^2}}\times\frac{y}{x}\div\sqrt{\frac{x^3}{9y^{-2}}}$. [2]

---

1 **(a)**

$\frac{x}{x^2-3x-4}-\frac{x+5}{4-x}=1$

$\frac{x}{(x-4)(x+1)}-\frac{x+5}{4-x}=1$

$\frac{x}{(x-4)(x+1)}+\frac{x+5}{x-4}=1$ M1

Multiply throughout by $(x-4)(x+1)$

$x+(x+5)(x+1)=(x-4)(x+1)$

$x+x^2+6x+5=x^2-3x-4$ M1

$10x+9=0$

$x=-\frac{9}{10}$ A1

(b)

$x=2t\sqrt{\frac{k^2}{2k^2+3t^2}}$

$\frac{x}{2t}=\sqrt{\frac{k^2}{2k^2+3t^2}}$

$\frac{x^2}{4t^2}=\frac{k^2}{2k^2+3t^2}$ M1

$x^2(2k^2+3t^2)=4k^2t^2$

$2k^2x^2+3t^2x^2=4k^2t^2$ M1

$3t^2x^2-4k^2t^2=-2k^2x^2$

$t^2(3x^2-4k^2)=-2k^2x^2$

$t=\pm\sqrt{\frac{-2k^2x^2}{3x^2-4k^2}}$ or $t=\pm\sqrt{\frac{2k^2x^2}{4k^2-3x^2}}$ A1 (No marks if $\pm$ is not shown)

(c)

$$\sqrt[3]{\frac{y}{x^2}} \times \frac{y}{x} \div \sqrt{\frac{x^3}{9y^{-2}}}$$

$$= \frac{y^{\frac{1}{3}}}{x^{\frac{2}{3}}} \times \frac{y}{x} \div \frac{x^{\frac{3}{2}}}{3y^{-1}}$$

$$= \frac{y^{\frac{1}{3}}}{x^{\frac{2}{3}}} \times \frac{y}{x} \times \frac{3y^{-1}}{x^{\frac{3}{2}}}$$

M1 – indices of variables correct

$$= \frac{3y^{\frac{1}{3}}}{x^{\frac{19}{6}}}$$

A1

2 A shop sells two flavours of ice-cream, Rum Raisin and Super Chunkies.
Each flavour is sold in cups of three different sizes, small, medium and large, and of different prices. The sales in two successive days are given in the table below.

	Saturday			Sunday		
Size	Small	Medium	Large	Small	Medium	Large
Cost of ice-cream per cup	\$2.50	\$3.20	\$4.50	\$2.50	\$3.20	\$4.50
Number of cups of Rum Raisin sold	12	17	8	14	12	10
Number of cups of Super Chunkies sold	18	15	11	13	21	16

The information for Saturday's sales can be represented by the matrix,

$\mathbf{M} = \begin{pmatrix} 12 & 17 & 8 \\ 18 & 15 & 11 \end{pmatrix}$ and the cost of each flavour for each size can be represented by the matrix

$\mathbf{C} = \begin{pmatrix} 2.5 \\ 3.2 \\ 4.5 \end{pmatrix}$. The information for the Sunday's sales can be represented by a 2×3 matrix $\mathbf{N}$.

(a) Write down the matrix $\mathbf{N}$. [1]

(b) Calculate $\mathbf{P} = (\mathbf{M} + \mathbf{N})$. [1]

(c) Describe what is represented by the elements in $\mathbf{P}$. [1]

(d) Calculate $\mathbf{Q} = \frac{1}{2}\mathbf{PC}$. [2]

(e) Describe what is represented by the elements of $\mathbf{Q}$. [1]

(f) Calculate and describe what is represented by the elements of $\mathbf{R} = \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{PC}$. [2]

(a) $\mathbf{N} = \begin{pmatrix} 14 & 12 & 10 \\ 13 & 21 & 16 \end{pmatrix}$ Sizes ↓ B1

(b) $\mathbf{P} = \begin{pmatrix} 26 & 29 & 18 \\ 31 & 36 & 27 \end{pmatrix}$ ←Flavours B1

(c) Total number of cups of Rum Raisin and Super Chunkies ice-cream of different sizes sold on Saturday and Sunday. B1

(d) $\mathbf{Q} = \frac{1}{2}\mathbf{PC}$

$= \frac{1}{2}\begin{pmatrix} 238.8 \\ 314.2 \end{pmatrix}$ M1

$= \begin{pmatrix} 119.4 \\ 157.1 \end{pmatrix}$ ←Flavours A1

(e) The **average** sum received from the sales of each flavour of ice-cream on Saturday and Sunday. B1

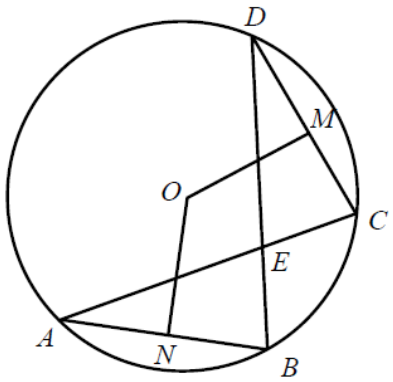
(f) $\mathbf{R} = \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}238.8\\314.2\end{pmatrix} = \begin{pmatrix}553\end{pmatrix}$

B1

It represents the total amt. collected from the sales of all flavours and cup sizes of ice-cream on Saturday and Sunday.

B1

3



The points *A, B, C, D* lie on a circle, centre *O*. *N* and *M* are midpoints of *AB* and *CD* respectively. It is given that *ON* = *OM*.

(a) Show that the triangles *ABE* and *DCE* are congruent. [3]

(b) It is given that *AB* = 6 cm and *ON* = $(r-1)$ cm, where *r* is the radius of the circle. Find the value of *r*. [2]

(a) $\angle ABE = \angle DCE$ (angles in the same segment)

$\angle BAE = \angle CDE$ (angles in the same segment)

$ON = OM$ (*given*)

$\therefore AB = CD$ (equal chords, equidistant from centre)

$\therefore \Delta ABE \equiv \Delta DCE$ (ASA)

M2 – All 3 reasons are correctly given
M1 – Any 2 correct reasons are given

A1 – congruency statement

(b) $OB^2 = ON^2 + NB^2$

$r^2 = 3^2 + (r-1)^2$

M1

$r^2 = 9 + r^2 - 2r + 1$

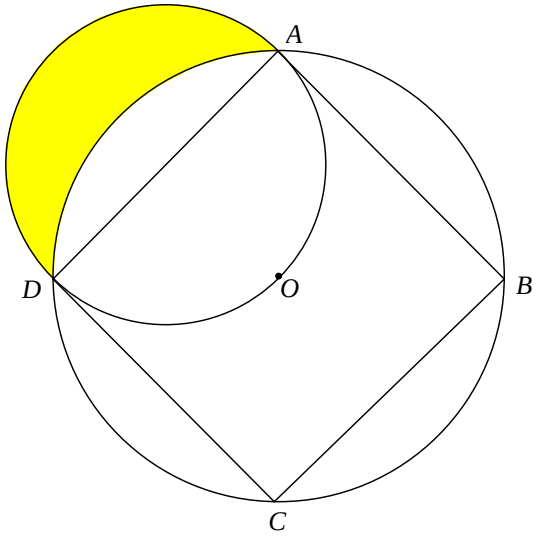
$2r = 10$

$r = 5\text{cm}$

A1

---

4 In the diagram, *ABCD* is a square whose diagonals are each 2 cm long. Given that *O* is the centre of the bigger circle and *BD* and *AD* are the diameters of the bigger and smaller circle respectively.
Find the area of the shaded region. [4]



By Pythagoras' theorem

$AD = \sqrt{2} \Rightarrow$ Radius of small circle $= \frac{\sqrt{2}}{2}$ M1

Area of semi-circle $= \frac{1}{2}\pi\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{\pi}{4}$ or 0.78539 cm^2 M1

Area of segment $AD = \frac{1}{2}(1)^2[\frac{\pi}{2} - \sin\frac{\pi}{2}] = \frac{\pi}{4} - \frac{1}{2}$ or 0.28539 cm^2 M1

Area of shaded region $= \frac{\pi}{4} - 0.28539 = \frac{1}{2}$ or 0.500 cm^2 (3sf) A1

---

5 The first four terms in a sequence of numbers, $p_1, p_2, p_3, p_4, \ldots$, are given below.

$p_1 = 1^2 + 2^2 + 2^2 = 3^2$
$p_2 = 2^2 + 3^2 + 6^2 = 7^2$
$p_3 = 3^2 + 4^2 + 12^2 = 13^2$
$p_4 = 4^2 + 5^2 + 20^2 = 21^2$

(a) Write down an expression for p_5 and show that $p_5 = 961$. [1]

(b) Given that $p_{10} = 10^2 + 11^2 + s^2 = k$, express k as a perfect square in terms of s. [1]

(c) Given that $p_w = w^2 + (w+1)^2 + r^2 = 5257^2$, find the value of r and of w. [2]

(d) Show that $p_n = n^4 + 2n^3 + 3n^2 + 2n + 1$. [3]

(a) $p_5 = 5^2 + 6^2 + 30^2$
$= 25 + 36 + 900$
$= 961$ B1

(b) $k = (s + 1)^2$ B1

(c) $r = 5256$ B1
$w(w+1) = 5256$
$= 72 \times 73$
$w = 72$ A1

(d) $p_n = n^2 + (n + 1)^2 + [n(n + 1)]^2$ M1
$= n^2 + n^2 + 2n + 1 + (n^2)(n+1)^2$
$= 2n^2 + 2n + 1 + n^2(n^2 + 2n + 1)$ M1
$= 2n^2 + 2n + 1 + n^4 + 2n^3 + n^2$
$= n^4 + 2n^3 + 3n^2 + 2n + 1$ A1

---

6 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = x + \frac{7}{x} - 6$.

The table below gives some values of x and the corresponding values of y.

x	0.5	1	2	3.5	5	6	7	8
y	p	2	–0.5	q	0.4	1.17	2	2.88

(a) Find the values of p and q. [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw the graph of $y = x + \frac{7}{x} - 6$ for the values of x in the range $0 < x \leq 8$. [3]

(c) Draw the line $y = -\frac{1}{2}x + 3$. [1]

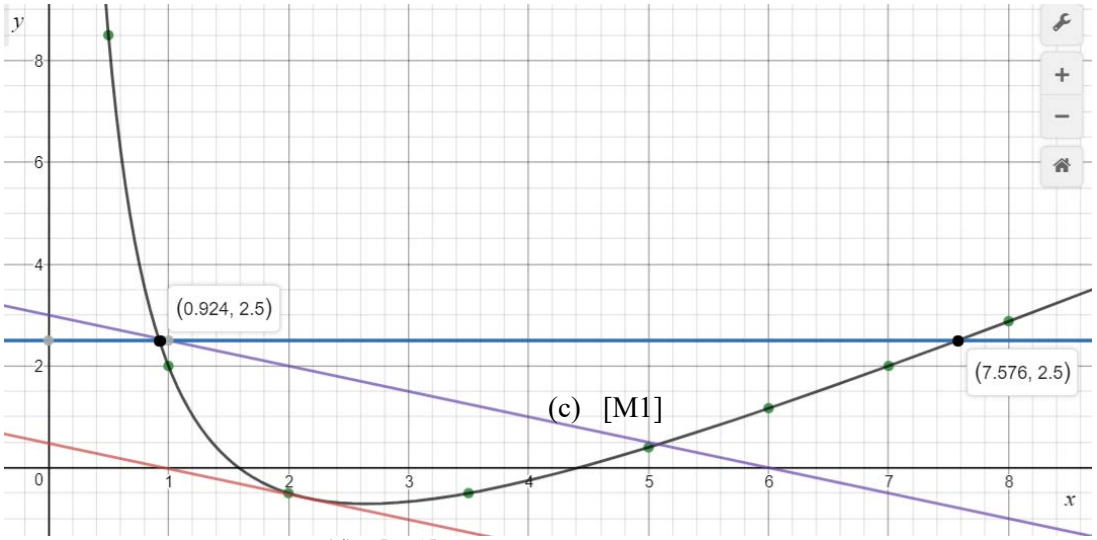
(d) Use your graph to find the x-coordinate of a point on the curve $y = x + \frac{7}{x} - 6$ at which the gradient of the tangent is equal to -0.5. [2]

(e) Find the range of values of x for which $x^2 - 6x + 7 \leq 2.5x$. [2]

(f) By drawing a suitable line on your graph, find the solutions of the equation $2x^2 - 13x + 14 = 0$. [3]

(a) $p = 8.5$
$q = -0.5$ A1 – both correct answer

(b)



(c) [M1]

(d) [M1]

P1 – correctly plotted points
M1 – smooth, neat + labelled curve
M1 – labelled axis and correct scale

(d) $x = 2.15 \pm 0.2$ M1

(e) $x^2 - 6x + 7 \le 2.5x$

$x - 6 + \frac{7}{x} \le 2.5$

$y \le 2.5$ M1

$0.9(\pm 0.1) \le x \le 7.6(\pm 0.1)$ A1 – both correct answer

(f) $2x^2 - 13x + 14 = 0$

$x + \frac{7}{x} - 6.5 = 0$

$x + \frac{7}{x} - 6 = 0.5$

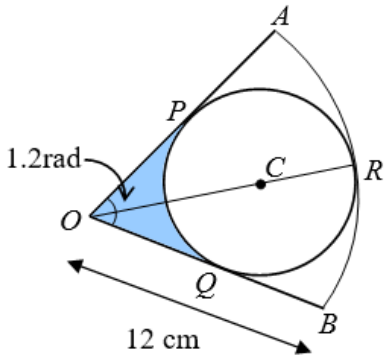
$y = 0.5$ M1

Draw $y = 0.5$ on the graph M1

$x = 0.6 \pm 0.1$

$= 5.9 \pm 0.1$ A1 – both correct answer

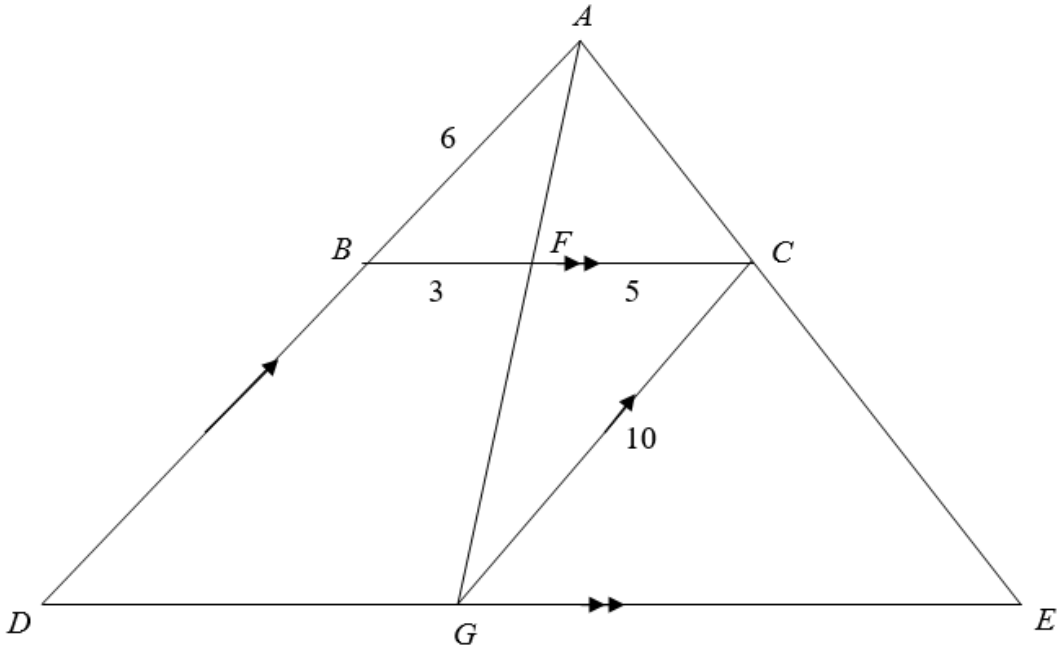
7 **(a)** In the diagram, *OARB* is a sector of a circle with centre *O*, radius 12 cm and angle *AOB* = 1.2 radians. *C* is the centre of the circle enclosed inside the sector, *OCR* is a straight line and the circle touches the sector at *P*, *Q* and *R*.



(i) Show that the radius of the enclosed circle is 4.3305 cm, correct to 4 decimal places. [3]

(ii) Calculate the perimeter of the shaded region *POQ*. [2]

(b) In the diagram below, *ABD, AFG, ACE, BFC* and *DGE* are straight lines.
BFC is parallel to *DGE* and *DBA* is parallel to *GC*.
AB = 6 cm, *BF* = 3 cm, *FC* = 5 cm and *GC* = 10 cm.



(i) Prove that triangle *BFA* is similar to triangle *CFG*. [2]

(ii) Calculate *GE*. [2]

(iii) Given the area of triangle *ABF* is 6.4 cm^2, find the area of trapezium *ACGD*. [3]

(i) Let *r* be the radius of the enclosed circle.
In ΔCOP,

$$\sin 0.6 = \frac{CP}{OC}$$

$$= \frac{r}{12-r}$$ M1

$$\sin 0.6(12-r) = r$$

$$r\sin 0.6 + r = 12\sin 0.6$$

$$r = \frac{12\sin 0.6}{\sin 0.6 + 1}$$ M1

$\therefore\ r =$ 4.3305 cm (4 dp) (shown) A1

(ii) $\angle PCQ = 2\pi - \frac{\pi}{2} - \frac{\pi}{2} - 1.2 = \pi - 1.2 = 1.9415$ rad M1

In ΔCOP, $\tan 0.6 = \frac{4.3305}{OP}$

$OP =$ 6.3298 cm $= OQ$

Perimeter of shaded region $= 2\ (6.3298) + (4.3305)(1.9415)$

$= 21.0672$

$\approx$ 21.1 cm (3sf) A1

(b)(i)

$\angle BFA = \angle CFG$ (vertically opposite angles)

$\angle ABF = \angle GCF$ (alternate angles)

M1 – both reasons are correct

$\therefore \Delta BFA$ is similar to ΔCFG A1

(b)(ii)

$\frac{AB}{AD} = \frac{BC}{DE}$ (ΔABC is similar to ΔADE)

$$\frac{6}{6+10} = \frac{8}{8+GE}$$ M1

$$\frac{6}{16} = \frac{8}{8+GE}$$

$$8 + GE = \frac{8\times 16}{6}$$

$$GE = 13\frac{1}{3} \text{ cm}$$ A1

OR

$\frac{GE}{BC} = \frac{BC}{AB}$ (ΔABC is similar to ΔCGE)

$\frac{GE}{10} = \frac{8}{6}$ M1

$GE = 13\frac{1}{3}$ cm A1

(b)(iii)

$$\text{Area of } \Delta ABC = \frac{8}{3} \times \text{Area of } \Delta ABF$$

$$= \frac{8}{3} \times 6.4$$

$$= 17\frac{1}{15} \text{ cm}^2 \quad \text{M1}$$

$$\text{Area of trapezium } ACGD = \frac{1}{2}(CG + AD) \times \text{perpendicular distance from } CG \text{ to } AD$$

$$\text{Area of triangle } ABC = \frac{1}{2}(AB) \times \text{perpendicular distance from } CG \text{ to } AD$$

$$\frac{\text{Area of trapezium } ACGD}{\text{Area of triangle } ABC} = \frac{CG + AD}{AB}$$

$$\text{Area of trapezium } ACGD = \frac{16+10}{6} \times 17\frac{1}{15} \quad \text{M1}$$

$$= 74.0 \text{ cm}^2 \text{ (3sf)} \quad \text{A1}$$

8 Amanda ran the 21km of a half-marathon race at an average speed of x km/h.

(a) Write down, in terms of x, an expression for the number of hours it took her to complete the race. [1]

(b) Deborah ran the same race at an average speed which is 3km/h faster than Amanda's speed. Write down, in terms of x, an expression for the number of hours which Deborah took. [1]

(c) Given that the difference between the two times was 20 minutes, write down an equation in x and show that it reduces to $x^2 + 3x - 189 = 0$. [3]

(d) Solve the equation $x^2 + 3x - 189 = 0$, giving your answers correct to 2 decimal places. [2]

(e) Find, in hours and minutes, the time it took Amanda to complete the race. [1]

(a) $T_1 = \frac{21}{x}\text{hr}$ A1

(b) $T_2 = \frac{21}{x+3}\text{hr}$ A1

(c) $T_1 - T_2 = \frac{20}{60}$

$\frac{21}{x} - \frac{21}{x+3} = \frac{1}{3}$ M1

$\frac{21(x+3)-21x}{x(x+3)} = \frac{1}{3}$ M1

$x^2 + 3x = 3 \times 63$

$x^2 + 3x - 189 = 0$ (shown) A1

(d) $x^2 + 3x - 189 = 0$

$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-189)}}{2}$ M1

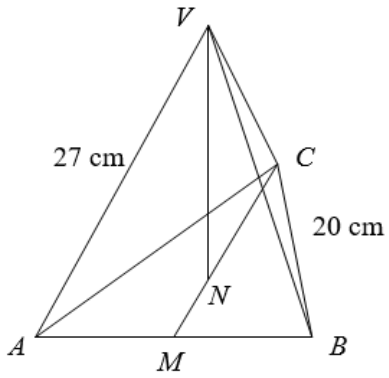
$x = 12.33$ (2dp) or $x = -15.33$ (2dp) A1

(e) $x = -15.33$ (rejected)

Time taken $= \frac{21}{12.33}$

$= 1$ hr 42 mins A1

9



[The volume of tetrahedron $= \frac{1}{3} \times$ base area $\times$ height]

The diagram shows a tetrahedron, *VABC*, which has a horizontal equilateral triangular base *ABC* of side 20 cm.
The slant edge of the tetrahedron (*VA*, *VB* and *VC*) are each of length 27 cm.
M is the mid-point of *AB* and the vertical line *VN* meets the plane *ABC* at *N* where $MN : NC = 1 : 2$. Calculate

(a) *CM*, [2]
(b) the angle of elevation of *V* from *A*, [3]
(c) the volume of the tetrahedron. [3]

(a) $CM^2 = CB^2 - MB^2$ (Pythagoras Theorem)

$= 20^2 - 10^2$ M1

$= 300$

$CM = 17.3$ cm (3sf) A1

(b) $AN^2 = AM^2 + MN^2$ (Pythagoras Theorem)

$= 10^2 + \left[\frac{1}{3}(17.320)\right]^2$

$= 11.546$ cm M1

$\cos \angle VAN = \frac{11.546}{27}$ M1

$\angle VAN = 64.7°$ (1dp) A1

(c) $VN = VA \sin \angle VAN$

$= 27 \sin 64.68$

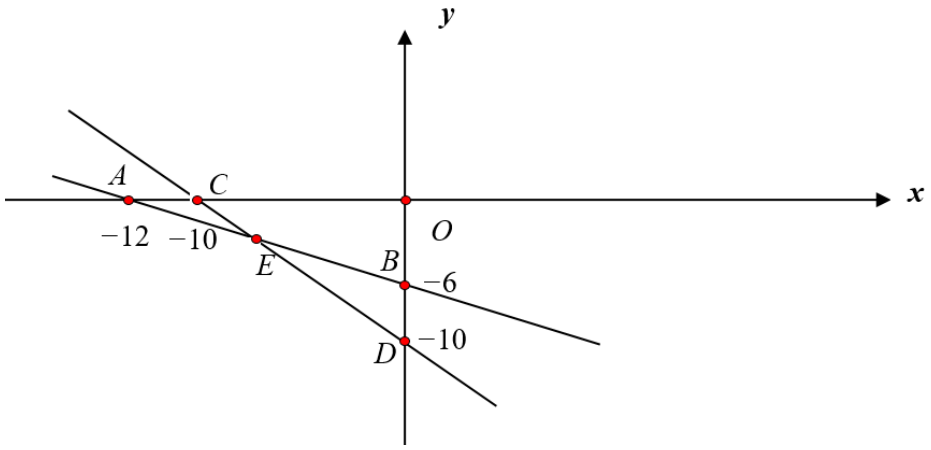
$= 24.4$ cm M1

Volume of tetrahedron $= \frac{1}{3} \times$ Area of base $\times VN$

$= \frac{1}{3} \times \left(\frac{1}{2} \times 20 \times 20 \sin 60°\right) \times 24.406$ M1

$= 1410$ cm^3 A1

10 In the figure below, the x-intercept and y-intercept of the line AB are -12 and -6 respectively. Both the x-intercept and y-intercept of the line CD are -10.



Find

(a) the equation of the line AB and CD, [2]

(b) the coordinates of E, [2]

(c) the area of $OCEB$, [2]

(d) the coordinates of F given that point F lies on AB produced such that $AF : FB = 5 : 3$, [2]

(e) find the coordinates of point G where G is the point on the x – axis such that OE is parallel to GD. [2]

(a) Gradient of the line $AB = \dfrac{-6-0}{0-(-12)} = -\dfrac{1}{2}$

Equation of the line AB: $y = -\dfrac{1}{2}x - 6$ A1

Gradient of the line $CD = \dfrac{-10-0}{0-(-10)} = -1$

Equation of the line CD: $y = -x - 10$ A1

(b) $y = -\dfrac{1}{2}x - 6 \quad (1)$

$y = -x - 10 \quad (2)$

$(1) - (2)$

$x = -8$ M1

$y = -2$

Coordinates of $E = (-8, -2)$ A1

(c) Area of $OCEB$ = Area of ΔCDO – Area of ΔEBD

$= \frac{1}{2} \times 10 \times 10 - \frac{1}{2} \times 4 \times 8$ M1

$= 34 \text{ units}^2$ A1

(d) $\frac{AF}{FB} = \frac{5}{3}$

Using similar triangles,

$\frac{-12 - x}{0 - x} = \frac{5}{3}$

$x = 18$

$\frac{0 - y}{-6 - y} = \frac{5}{3}$

$y = -15$ A1

Coordinates of $F = (18, -15)$

A1

(e) Let the coordinates of G be (k , 0)

$\frac{0-(-2)}{0-(-8)} = \frac{0-(-10)}{k-0}$ M1

$\frac{1}{4} = \frac{10}{k}$

$k = 40$

Coordinates of $G = (40, 0)$ A1

OR

$\overrightarrow{OE} = m\overrightarrow{GD}$

$\begin{pmatrix} -8 \\ -2 \end{pmatrix} = m \begin{pmatrix} -k \\ -10 \end{pmatrix}$ M1

$-2 = -10m$

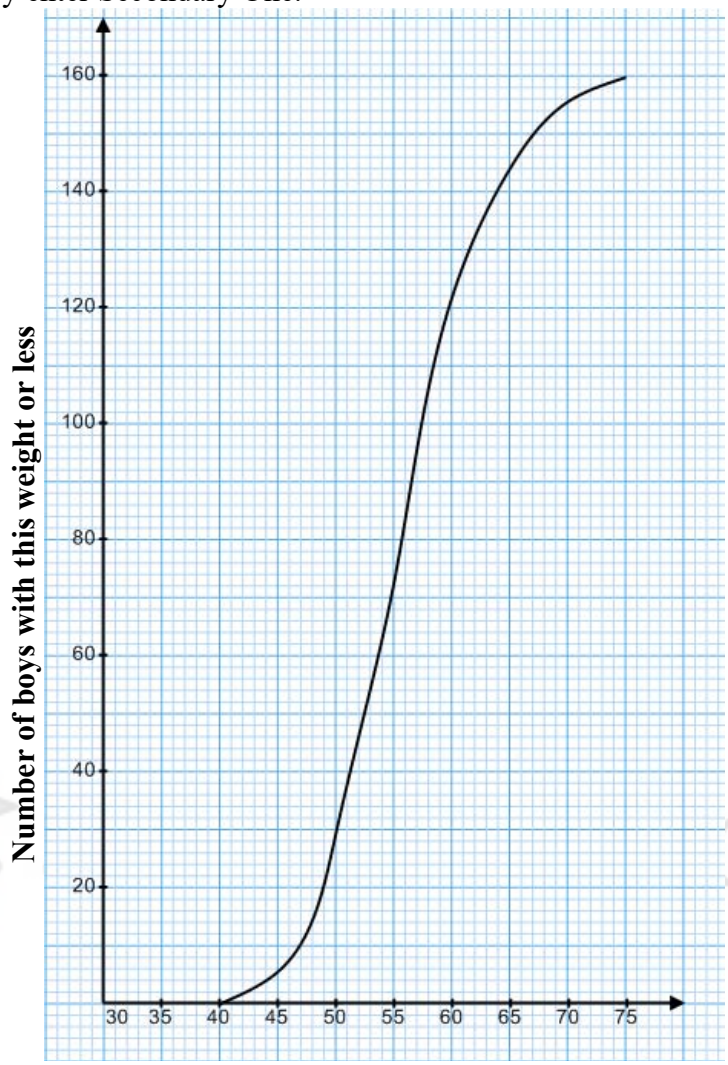
$m = \frac{1}{5}$

$-8 = -mk$

$k = 40$

Coordinates of $G = (40, 0)$ A1

11 The cumulative frequency curve below shows the weights of a sample of 160 boys from a school when they enter Secondary One.



(a) Use the graph to find an estimate for

(i) the number of boys whose weight is more than 60kg, [1]

(ii) the median weight, [1]

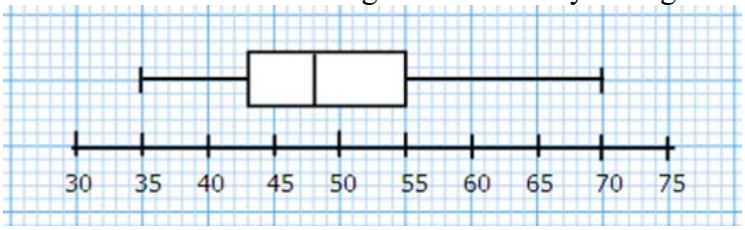
(iii) the percentage of boys whose weight is less than or equal to 52 kg. [1]

(b) The lightest 25% of the boys are classified as "slim".
The heaviest 25% of the boys are classified as "big-sized".
The remaining boys are classified as "medium".
Find an estimate for

(i) the greatest possible difference in weight between 2 boys classified as "big-sized", [2]

(ii) the least possible difference between the weight of a boy classified as "slim" and the weight of a boy classified as "big-sized". [2]

(c) The following box-and-whisker plot shows the distribution of weights of 160 Secondary One girls from the same school. Compare the data provided and record 2 observations about the distribution of weights between boys and girls. [2]



11 **(a)** **(i)** Number of boys whose weight is more than 60kg = 40. B1

(ii) median weight = 56 kg. B1

(iii) Percentage of boys whose weight ≤ 52 kg $= \frac{44}{160} \times 100\%$

$= 27.5\%$ B1

(b) **(i)** Based on distribution of number of boys

Greatest possible difference $= 75 - 60$ M1

$= 15$ kg A1

(ii) Based on distribution of weight

Least possible difference $= 60 - 51.5$ M1

$= 8.5$ kg A1

(c) Median weight of girls lower than boys; girls are lighter ;
For boys, $Q_3 - Q_1 = 8.5$kg. For girls, $Q_3 - Q_1 = 12$kg; Girls' weight has more variation. B2

---

12 Mrs Tan, a mother of two children, saw a newspaper article as shown.

Price of formula milk in Singapore has soared

The average price of a 900g tin of formula milk has increased sharply over the last 5 years, outstripping the price increases of other dairy products and household staples.

On Monday, the Government announced it is tightening rules to encourage greater price competition.

The table below shows the price (in SGD) per 100 grams of different brands of formula milk in year 2012 and 2017.

Brand / Date	Similac	Friso	Nan	S26	Mamil
Dec 2012	5.71	5.22	5.20	5.13	4.96
Mar 2017	7.05	6.56	7.45	6.36	6.41
% increase	[illegible]	[illegible]	43.3	24.0	29.2

Coffee stain was found on the newspaper article covering some of the information.

(a) **(i)** Calculate the percentage increase in the price of the Similac and Friso formula milk covered by the coffee stain.. [2]

(ii) Hence, do you agree with the headline of the newspaper article? Support your answer with a reason. [2]

A few days later, Mrs Tan saw another article regarding the price of similar brands of formula milk sold in Singapore, Malaysia and China.

Brand / Country	Similac	Friso	Nan	S26	Mamil
Singapore	7.05	6.56	7.45	6.36	6.41
Malaysia	3.92	3.54	4.29	4.13	3.51
China	4.79	5.58	9.06	4.25	4.00

Price (in SGD) per 100 grams of formula milk in Singapore, Malaysia and China.

Upon seeing the article, Mrs Tan intends to purchase some cans of formula milk in China and ship them back during her holidays.

Mrs Tan did an online research and found the following shipping rate from China to Singapore by SHIPPER Company.

Weight of parcel, x (kg)	**Shipping Rate** 1st kg 150 *RMB* follow by
$x \leq 10$	75 *RMB*/ kg
$10 < x \leq 20$	35 *RMB*/ kg
$20 < x \leq 50$	31 *RMB*/ kg
$50 < x \leq 75$	27 *RMB*/ kg
$75 < x \leq 100$	25 *RMB*/ kg
$100 < x \leq 200$	24 *RMB*/ kg
$x > 200$	22 *RMB*/ kg

The information below shows a can of the 900 g formula milk that Mrs Tan intends to purchase in China and the online currency conversion.

1 Singapore Dollar equals
4.89 Chinese Yuan Renminbi (RMB)

1	Singapore Dollar
4.89	Chinese Yuan Renminbi

Mrs Tan intends to spend at most S$650 for both the formula milk and shipping fee.

(b) Calculate the maximum number of cans of formula milk that Mrs Tan can buy. [5]

12 (a) (i) Percentage increase of Similac $= \frac{7.05-5.71}{5.71} \times 100\%$

$= 23.5\%$ B1

Percentage increase of the Friso $= \frac{6.56-5.22}{5.22} \times 100\%$

$= 25.7\%$ B1

(ii) Mean of percentage increase $= \frac{23.5+25.7+43.3+24.0+29.2}{5}$

$= 29.14\%$

Agree. B1

as the mean of percentage increase is greater than 29% which is much higher than price increase of general food consumption items. B1

(accept any logical answer)

(b) S\$650 $= 4.89 \times 650$

$= 3178.50$ RMB M1

Let x be the number of can of milk powder.

$\underbrace{4.25 \times 4.89 \times 9 \times x}_{\text{M1}} + 150 + 35\left(\frac{900x}{1000}\right) \le 3178.50$ M1-accept if students use equation

$187.0425x + 31.5x \le 3028.50$

$218.5425x \le 3078.50$ M1

$x \le 13.857$

$x = 13$ tins of formula milk A1

Or working in term of Singapore Dollar

Shipping rate of
150 RMB = S\$30.675
35 RMB = S\$7.157 M1

Let x be the number of can of milk powder.

$\underbrace{4.25 \times 9 \times x}_{\text{M1}} + 30.675 + 7.157\left(\frac{900x}{1000}\right) \le 650$ M1-accept if students use equation

$44.6913x \le 619.325$ M1

$x \le 13.857$

$x = 13$ tins of formula milk A1

END OF PAPER

Name _____ ( ) Class:_____

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS 4048/01
PAPER 1 2 hours

Classes: 403, 404, 405, 406, 501, 502

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, hand in **separately**:
1. Section A with exam cover sheet
2. Section B

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

FOR EXAMINER'S USE	
Total marks	**/80**

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Name _____ ( ) Class:_____

Answer **all** the questions.

Section A

1 Calculate $\dfrac{11.27^{\frac{1}{4}}}{30.67-5.23}$.

Write your answer correct to 3 significant figures.

Answer ……..……..…. [1]

---

2 Without using a calculator, show that $7^{103}-7^{101}$ is a multiple of 2.

Answer

[1]

---

3 The following stem-and-leaf diagram shows the masses of 10 parcels that arrive at the post office.

0	1	2 6
1	3	7
2	2	5
3	1	4 9

Key: 1 | 3 represents 13 kg

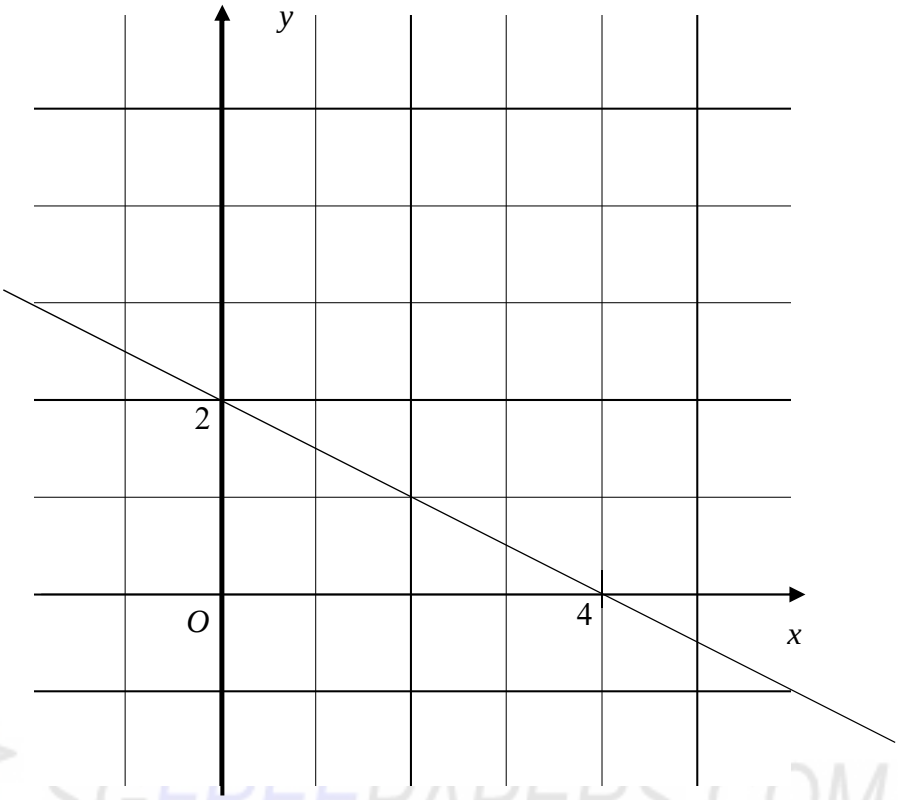
A parcel is chosen at random.

Find, as a fraction in its simplest form, the probability that the parcel has a mass between 12 kg and 32 kg.

Answer ……..……..…. [1]

---

4 The diagram shows the line $y = -\frac{1}{2}x + 2$.



The line $y = -\frac{1}{2}x + 2$ undergoes a translation represented by the vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Draw the line after translation, on the diagram above. [1]

---

5 Use prime factorisation to explain why 72 is **not** a perfect cube.

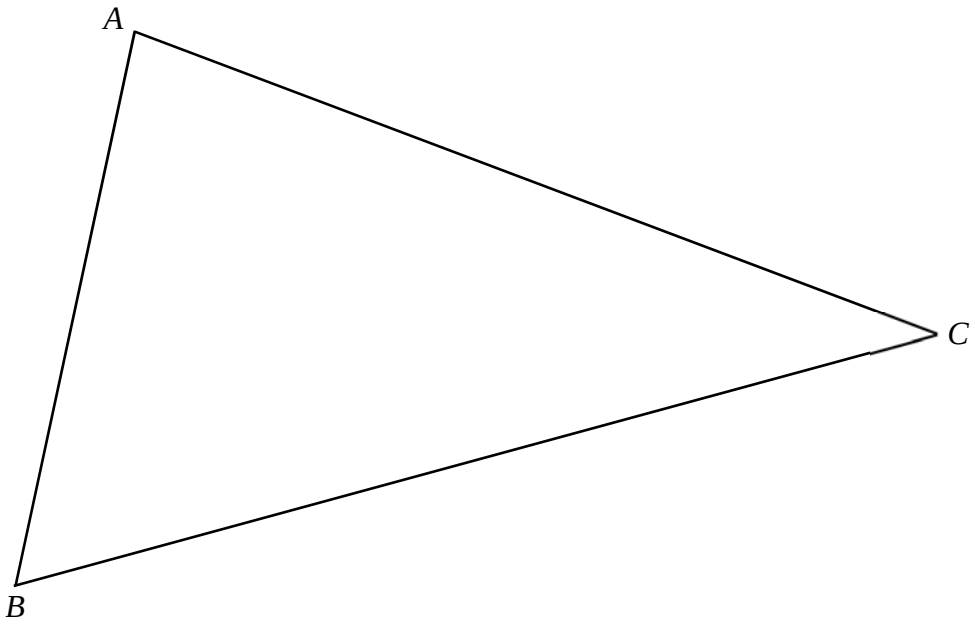
Answer ……

…… [2]

---

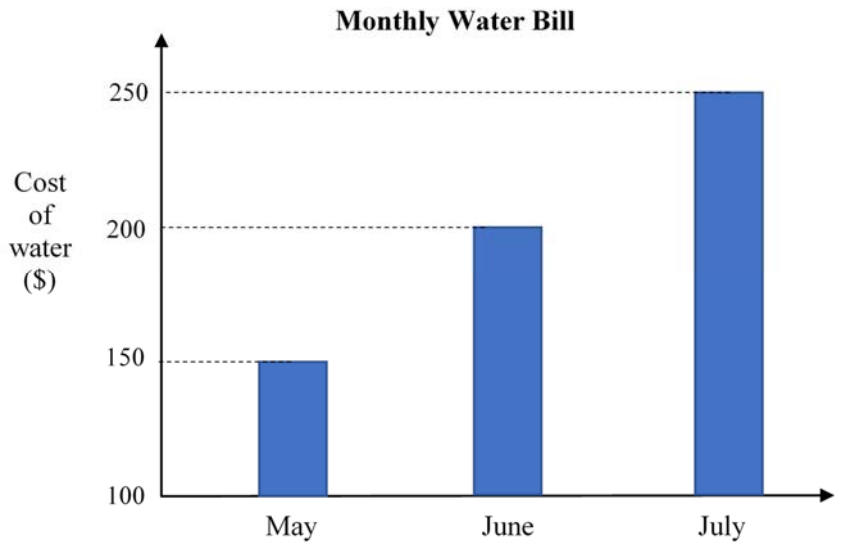
6 The diagram shows a triangle *ABC*.

Label the point *O* that is equidistant from *B* and *C*, and also equidistant from *AB* and *BC*.



[2]

7 Wally draws this graph to show his monthly water bill for each of the last three months.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer ……..……...

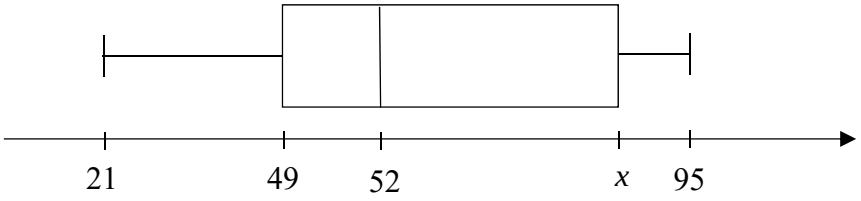
……..…….……..

……..…… [2]

8 The table shows the scores of 10 students in a Mathematics test.

Test score	Frequency
21	2
49	3
55	1
65	1
80	1
95	2

The test scores are also represented in the box-and-whisker plot below.



(a) Find the value of x.

Answer $x =$ ……..……..…. [1]

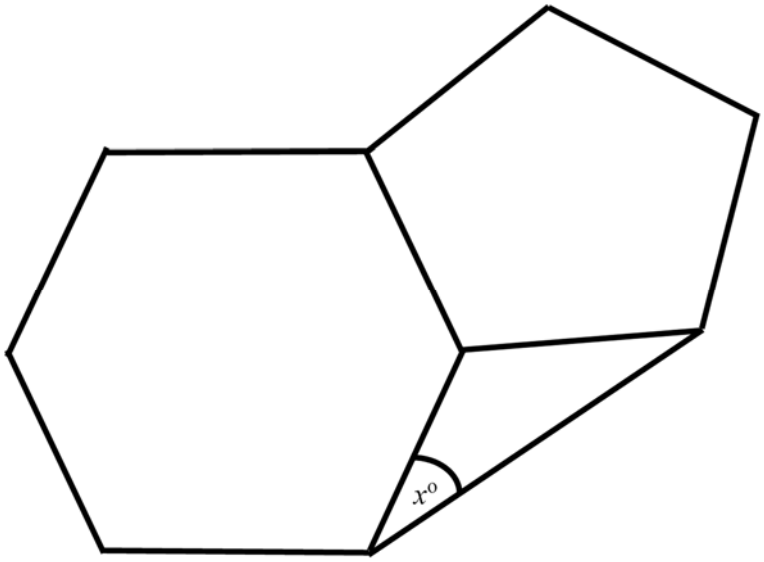
(b) Calculate the standard deviation of the test scores.

Answer ……..……..…. [1]

9 Solve $3x^2 - 2x - 11 = 0$.

Answer x = ……..……..…. or x = ……..……..…. [3]

10 The diagram shows a regular hexagon and a regular pentagon.



Find x.

Answer x = ……..……..…. [3]

11 **(a)** Simplify $2(3x+5)-2(1-2x)$.

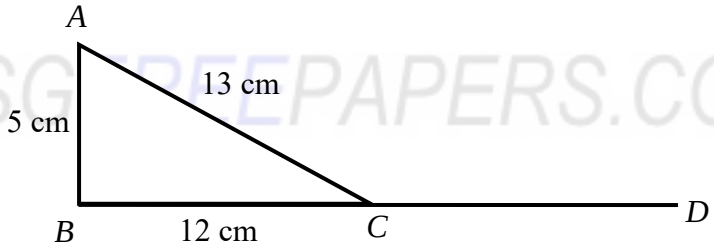
Answer ……..……..…. [1]

(b) Factorise completely $18-24x+8x^2$.

Answer ……..……..…. [2]

12 ABC is a triangle.

AB = 5 cm, BC = 12 cm and AC = 13 cm.



(a) Show that ABC is a right-angled triangle.

Answer

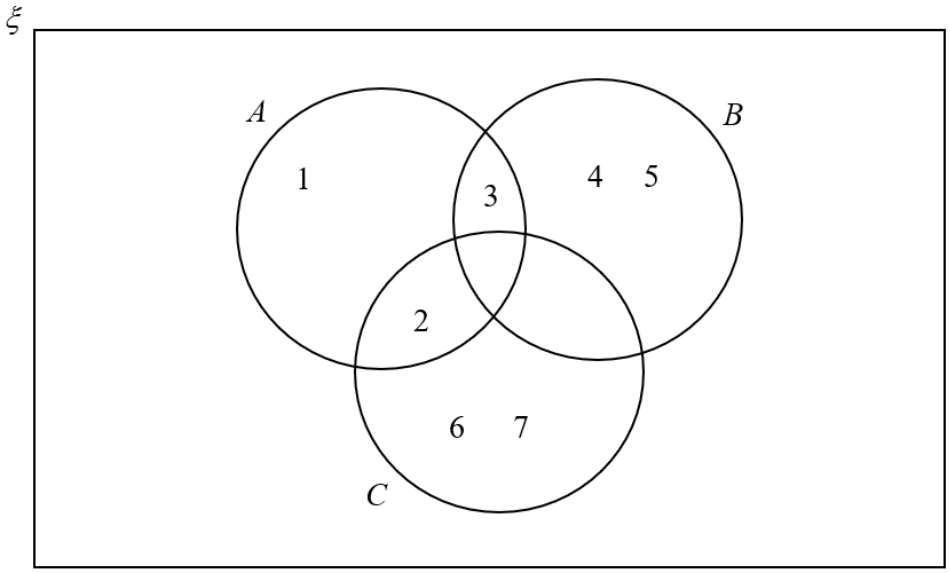
[2]

(b) Find $\sin A\hat{C}D$.

Answer ……..……..…. [1]

13 $\xi = \{\text{integers } x \ : \ 1 \le x \le 7\}$

The Venn diagram shows the elements of ξ and three sets A, B and C.



Use one of the symbols below to complete each statement.

$$\varnothing \quad \subset \quad \not\subset \quad \notin \quad \in \quad \xi$$

(a) $\{4,5\}$ ……..……..…. B [1]

(b) 2 ……..……..…. $A \cup B$ [1]

(c) $B \cap C =$ ……..……..…. [1]

---

14 The body mass index (BMI) of a person is defined as

$$\text{BMI} = \frac{\text{mass}}{\text{height}^2}.$$

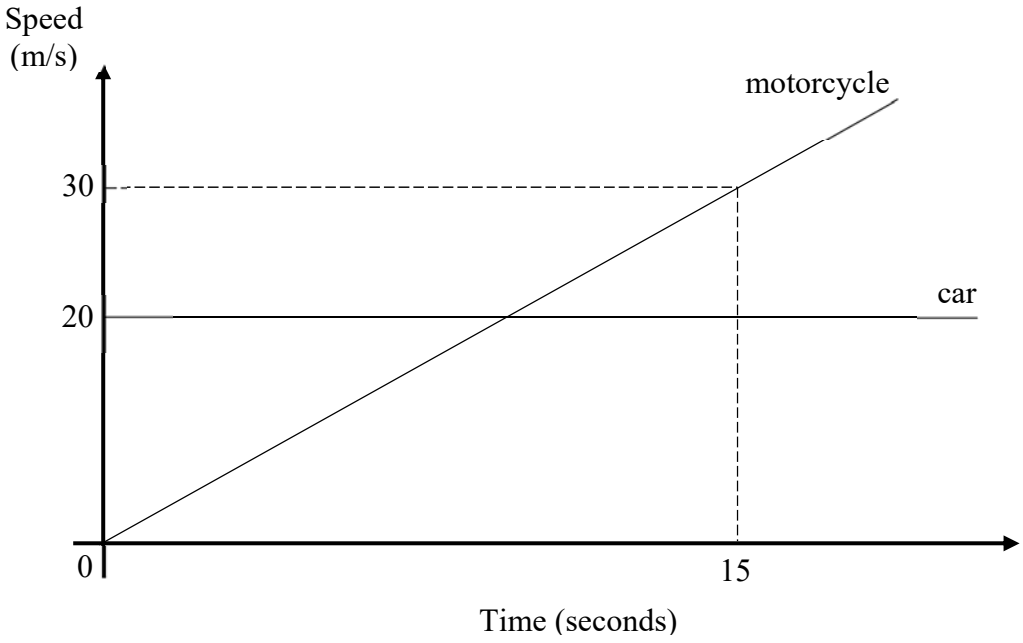
Sam's mass was 62 kg. One year later, Sam's mass increased by 10%, while his BMI increased by 2.01 kg/m^2. Sam's height remained the same.

Find Sam's height.

Answer ……..……..…. m [3]

---

15 The diagram shows the speed–time graphs for a car and a motorcycle travelling along a straight road.



(a) Calculate the acceleration of the motorcycle.

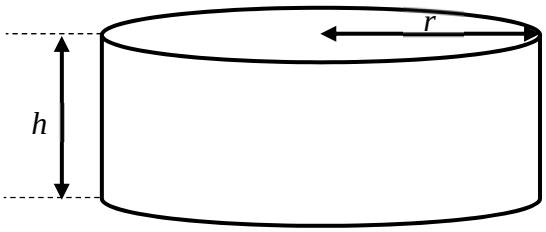
Answer ……..…….…. m/s^2 [1]

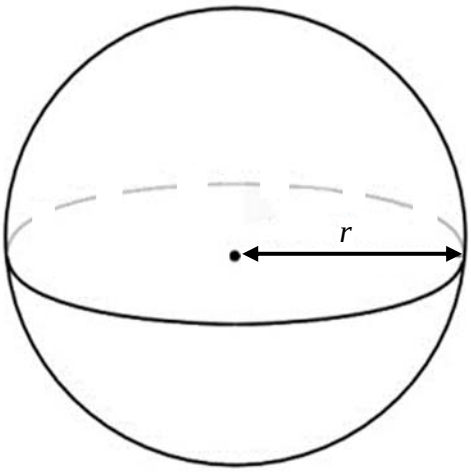
(b) Both the motorcycle and the car were beside each other at the start.
At t seconds, the motorcycle overtook the car.

Find the value of t.

Answer t = ……..…….…. [2]

16 A solid cylinder has radius r cm and height h cm.
A solid sphere has radius r cm.
The total surface areas of the solid cylinder and the sphere are equal.

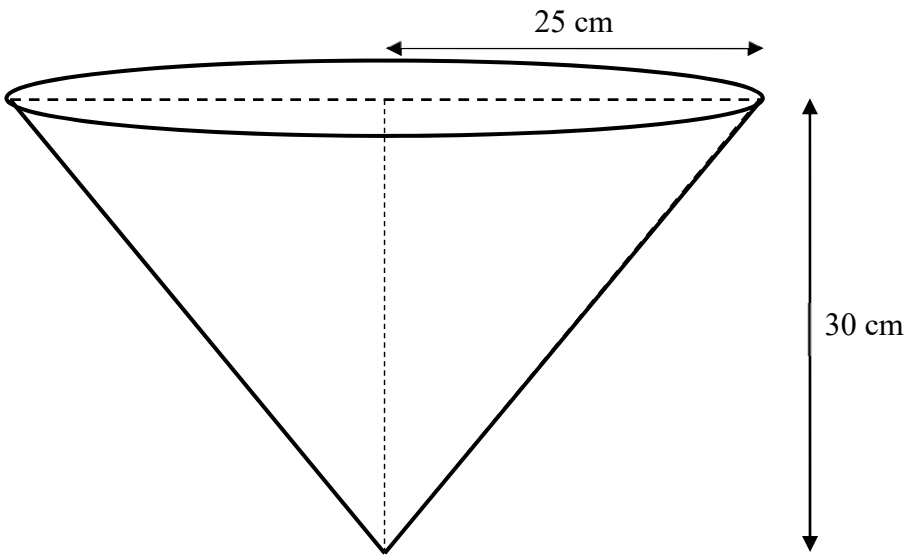




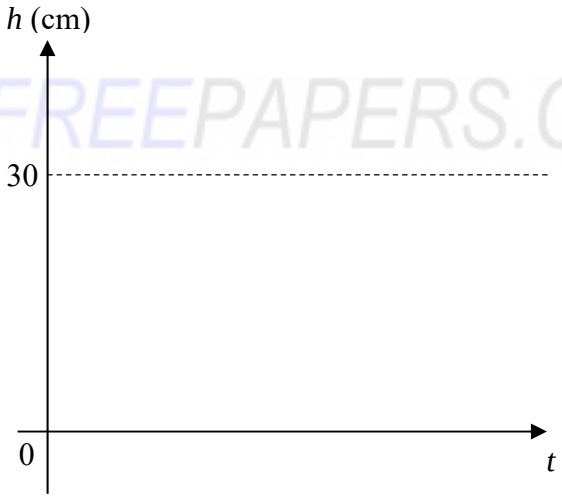
Work out, in terms of r, the total volume of the cylinder.

Answer ……..……..…. [3]

17 Water is being poured into the cone below at a constant rate. The cone is initially empty.



(a) Sketch a graph on the axes below to show how the height, h of the water level in the cone increases with time, t.



[1]

(b) Calculate the volume of water in the cone when $h = 10$ cm.

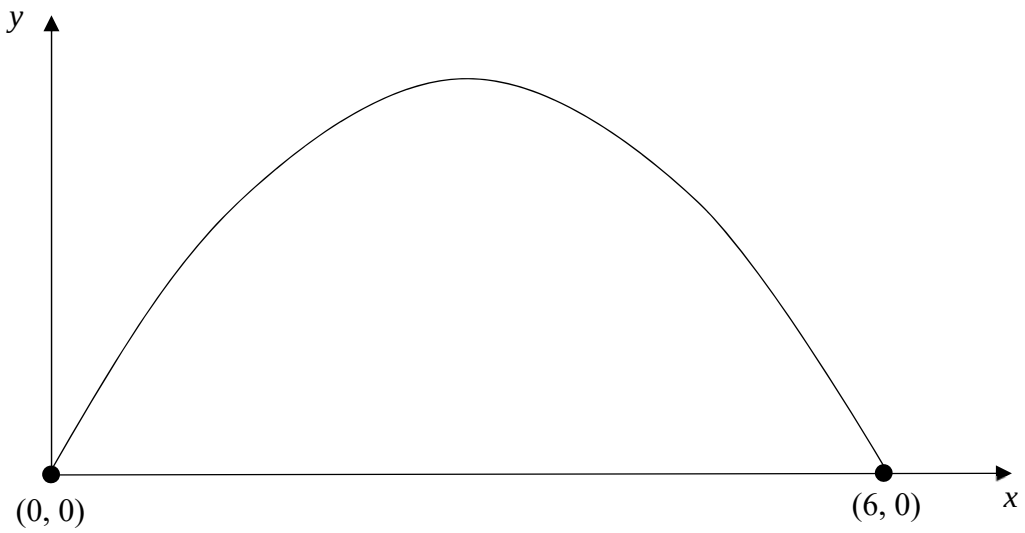
Answer ……..……..…. cm^3 [3]

Name _____ ( ) Class:_____

Answer **all** the questions.

Section B

18 The sketch shows the graph of $y = -x^2 + ax + b$.
The points (0, 0) and (6, 0) lie on the graph.



(a) Show that $a = 6$ and $b = 0$.

Answer

[2]

(b) Find the coordinates of the maximum point of the graph.

Answer (…… , ……) [2]

19 A map of Singapore has a scale of 1: 80 000.

(a) The actual length of the Singapore River is 3.2 km.

Calculate the length, in centimetres, of the river on the map.

Answer ……..……..…. cm [2]

(b) The actual area of the Bishan-Ang Mo Kio Park is 620 000 m^2.

Calculate the area, in square centimetres, of the park on the map.

Answer ……..……..…. cm^2 [2]

20 The point L is $(1,\ 2)$ and the point M is $(16,\ 16)$.

(a) Find $\left|\overrightarrow{LM}\right|$.

Answer $\left|\overrightarrow{LM}\right|$ = ……..……..…. units [2]

20 **(b)** The point N is such that $\overrightarrow{LN} = \frac{1}{3}\overrightarrow{NM}$.

Find the position vector $\overrightarrow{ON}$.

Answer $\overrightarrow{ON}$ = ……..……..…. [2]

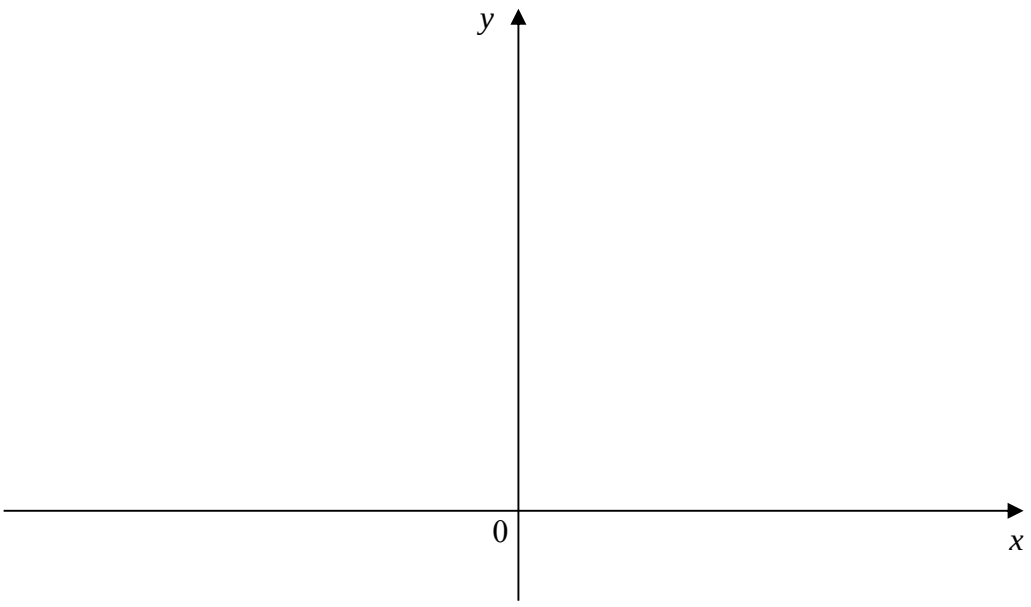
---

21 The point (–2, 1) lies on the graph $y = \frac{a}{x^2}$.

(a) Find the value of a.

Answer a = ……..……..…. [1]

(b) Hence, sketch the graph of $y = \frac{a}{x^2}$ on the axes below.



[1]

---

21 **(c)** Explain how you can tell from the graph, the number of solutions to the equation $\frac{a}{x^2} = k$ for positive values of k.

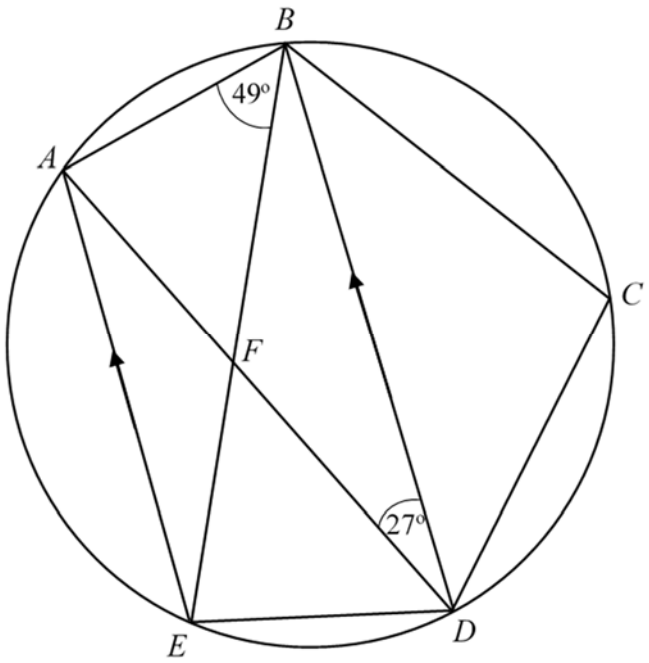
Answer ……..

……..

……... [2]

---

22 The diagram shows a circle that passes through A, B, C, D and E.
The lines AE and BD are parallel.
Angle $ADB = 27^\circ$ and angle $ABE = 49^\circ$.



(a) Find the angle AFE.

Show your working and give reasons.

Answer ……..……..…. ° [3]

---

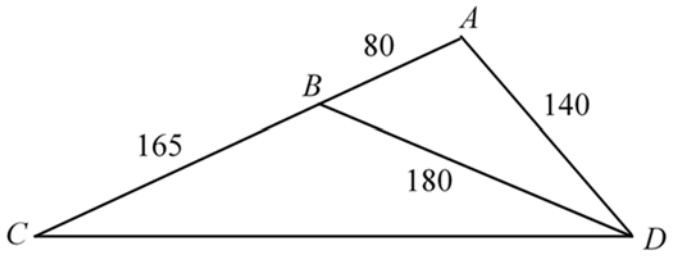
22 **(b)** Find angle *BCD* .

Show your working and give reasons.

Answer ……..……..…. ° [2]

---

23 *ABD* is a triangle where *AB* = *80* cm, *AD* = 140 cm, and *BD* = 180 cm.
AB is produced to *C* and *BC* = 165 cm.



(a) Show that triangle *ACD* is similar to triangle *ADB*.

Answer

[2]

(b) Calculate the length *CD*.

Answer ……..……..…. cm [1]

---

23 **(c)** Calculate the perpendicular distance from *A* to *BD*.

Answer ……..……..…. cm [3]

---

24 Singapore adopts a progressive tax structure.
The table shows the tax rates for various annual income brackets.

Assessable annual Income	Chargeable income	Income tax rate (%)	Gross tax payable
≤ $20 000	0	0	0
$20 001 - $30 000	First $20 000 Next $10 000	0 2	0 $200
$30 001 - $40 000	First $30 000 Next $10 000	– 3.50	$200 $350
$40 001 - $80 000	First $40 000 Next $40 000	– 7	$550 $2800

(Adapted from www.iras.gov.sg.)

Note: For "–" under Income tax rate, refer to Gross tax payable for the amount of tax.

(a) Calculate the amount of tax a manager has to pay if his assessable annual income is $55 000.

Answer $ ……..……..…. [2]

---

24 **(b)** To reduce the amount of tax payable, the manager makes use of the Supplementary Retirement Scheme (SRS). Each dollar deposited into the SRS reduces the assessable annual income by a dollar.

Calculate the amount of tax savings the manager enjoys if he deposits $15 300 into the SRS.

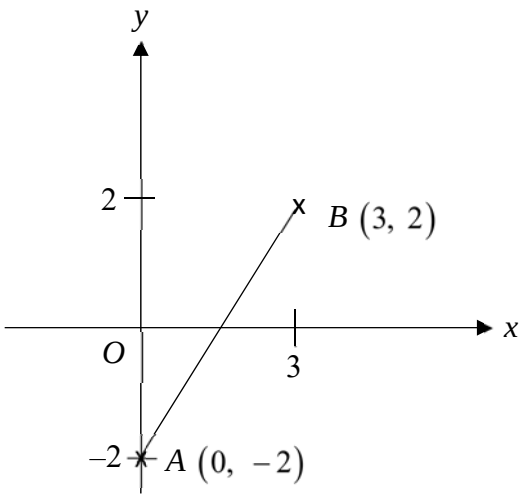
Answer $ ……..……..…. [2]

(c) Suppose the manager further invests the $15 300 he had deposited in the SRS in a savings bond which provides a compound interest of 2.63% per year for 10 years.

Calculate the amount of money he has in the SRS after 10 years.
Give your answer correct to the nearest dollar.

Answer $ ……..……..…. [2]

25 Point A has coordinates $(0, -2)$. Point B has coordinates $(3, 2)$.



(a) Find the equation of the line AB.

Answer ……..……..…. [2]

(b) Find the length of the line AB.

Answer ……..……..…. units [2]

(c) The point D is $(0, 4)$.

Write down the coordinates of the point C such that $ABCD$ is a parallelogram.

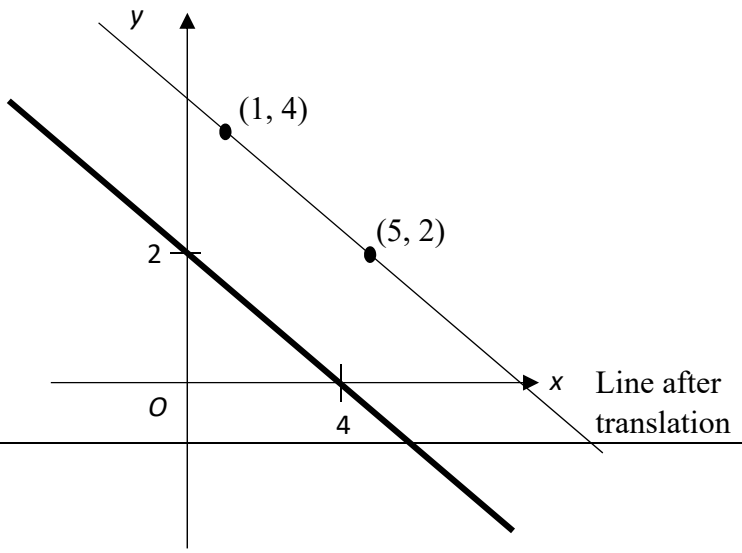
Answer (…… , ……) [1]

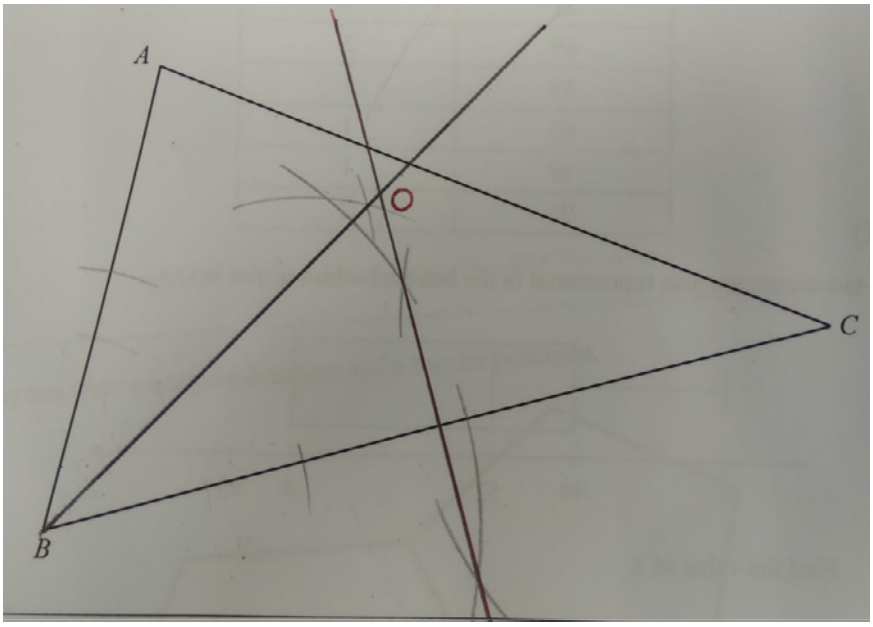
(d) Find the area of the parallelogram $ABCD$.

Answer ……..……..…. units^2 [2]

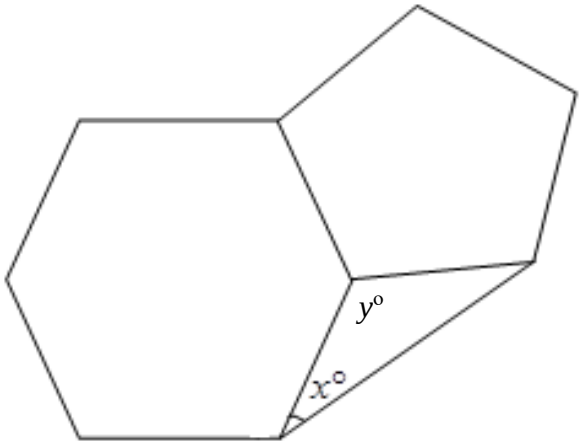
End of Paper

CHIJ KC
2018 S4E/5N E Math Preliminary Examination
Paper 1 Solutions

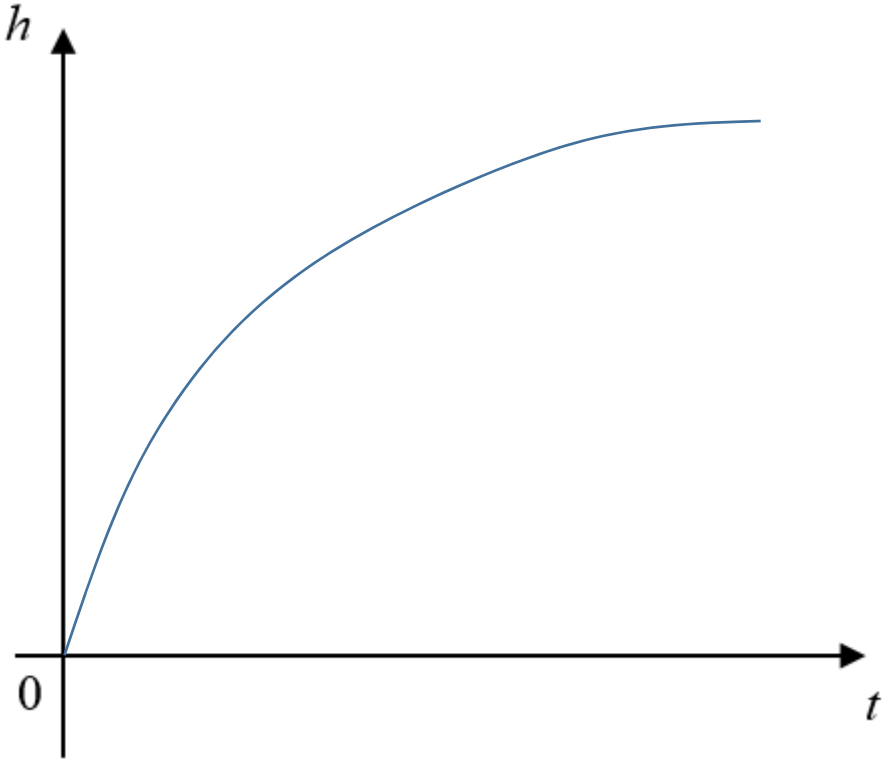
Question No.	Solution
1	0.0720
2	$7^{103} - 7^{101}$ $= 7^2\left(7^{101}\right) - 7^{101}$ $= 7^{101}\left(7^2 - 1\right)$ $= 7^{101}(48)$ $= 7^{101}\left(2^4 \times 3\right)$ • Common factor is 701^{101} • Not the same as $7^{103-101} = 7^{103} - 7^{101}$ • 49 or 7 are NOT multiples of 2 • $7^2 \neq 14$ *Alternative Solution:* 7 raised to any index is odd. Hence 7^{103} and 7^{101} are both odd. The difference of two odd numbers is an even number.
3	No. of parcels between 12 kg and 32: 5 parcels $P = \frac{5}{10} = \frac{1}{2}$ Given answers: • $\frac{5}{10} = \frac{1}{5}$ • $\frac{5}{10} = \frac{2}{5}$
4	The point (0, 2) after translation will become (1, 4). The point (4, 0) after translation will become (5,2) Award 1 mark for a straight line // to original line and passing through the above points. Do not penalise if students do not label points (1, 4) and/ or (5,2)
Issues with vector translation NOT moving to coordinate (1, 2)	

5	$72 = 2^3 \times 3^2$ $\sqrt[3]{72} = \sqrt[3]{2^3 \times 3^2}$ $= 2 \times 3^{\frac{2}{3}}$ Since $3^{\frac{2}{3}}$ is not an integer, 72 is not a perfect cube. (A) Index of factor 3 is not a multiple of 3. / 3^2 is not a perfect cube Perfect cube – cube of an integer (any index in **multiples of 3**) - "3^2 is not a multiple of 3" o $= 3 \times 3$ which is a multiple of 3 VS - Index of factor 3 (means differently)
6	1 mark for angle bisector of angle *ABC* AND perpendicular bisector of *BC*; 1 mark for marking out the intersection of the two bisectors as *O*. 
7	The bar charts do not start from \$0. A student may infer the water bill directly from the height of the bar charts without looking at the axes and conclude that the water bill in June is twice that of May, and the water bill for July was thrice that of May. (A) maybe a base amount of \$100 even with no usage Perceive water bill to be **lower** than true value (lower height of graph)

	- Question is not about reading of values (already indicated on y-axis) - Many unable to state the misinterpretation (usually gave very vague responses)
8	**(a)** $x = 80$ i.e. the upper quartile, position no. 8. [A1] **(b)** $\text{S.D.} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$ $= \sqrt{\frac{39785}{10} - \left(\frac{579}{10}\right)^2}$ $= \sqrt{\frac{39785}{10} - \left(\frac{579}{10}\right)^2}$ $= 25.0$ Many error arises from manual calculation of $\sum fx^2$, etc Do not penalise if student directly cites value for S.D. from the calculator.
9	$3x^2 - 2x - 11 = 0$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-11)}}{6}$ $= \frac{2 \pm 11.66190379}{6}$ $= 2.276983 \text{ or } -1.610317298$ $x = 2.28 \text{ or } x = -1.61$

10	Interior angle of hexagon $= \dfrac{(6-2)180^\circ}{6} = 120^\circ$ Interior angle of pentagon $= \dfrac{(5-2)180^\circ}{5} = 108^\circ$ angle $y = 360^\circ - 120^\circ - 108^\circ = 132^\circ$ $x = \dfrac{180^\circ - 132^\circ}{2}$ (isoceles triangle) $= 24^\circ$ 
11(a)	$2(3x+5)-2(1-2x)$ $= 6x+10-2+4x$ $= 10x+8$ $= 2(5x+4)$
11(b)	$18-24x+8x^2$ $= 2(4x^2-12x+9)$ $= 2(2x-3)^2$ Accept: $2(3-2x)^2$ Reject: $2(3-2x)(3-2x)$
12(a)	$AC^2 = 13^2 = 169$ $AB^2 + BC^2 = 12^2 + 5^2 = 169$ Since $AC^2 = AB^2 + BC^2$, by the converse of Phythagoras theorem, ABC is a right-angle triangle. Many did not conclude. Just apply PT.

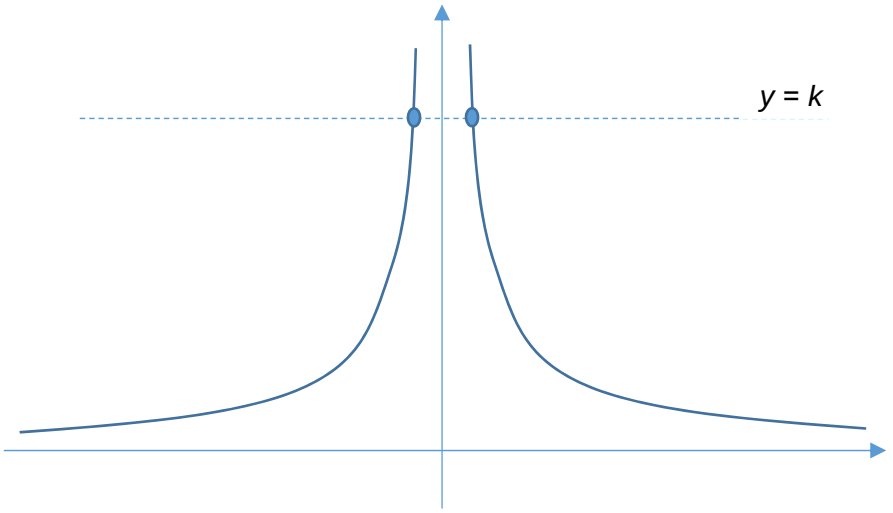
12(b)	Using $\sin(180-\theta)=\sin\theta$ $\text{Sin } A\hat{C}D = \text{Sin } A\hat{C}B$ $= \frac{5}{13}$ Accept: 0.385 B1
13	**(a)** $\{4,5\} \subset B$ **(b)** $2 \in A \cup B$ **(c)** $B \cap C = \varnothing$
14	$\frac{62\times 1.1}{h^2} - \frac{62}{h^2} = 2.01$ $\frac{68.2}{h^2} - \frac{62}{h^2} = 2.01$ $68.2 - 62 = 2.01h^2$ $h^2 = 3.084577$ $h = 1.756$ $= 1.76$ m, negative answer rejected Alternative Method: Since the mass increased by 10%, the BMI must have also increased by 10%. This is because BMI and mass are directly proportional. 10% of old BMI = 2.01 Old BMI = 20.1 $\frac{62}{h^2} = 20.1$ $h = 1.756$ $= 1.76$ m
15(a)	$a = \frac{30-0}{15-0}$ $= 2$ m/s^2

15(b)	let the time at which the motorcycle overtakes the car be t : Distance travelled by car $= 20t$ Distance travelled by motorcycle $= \frac{1}{2}(t)(2t)$ $20t = \frac{1}{2}(t)(2t)$ $20t = t^2$ $t^2 - 20t = 0$ $t(t-20) = 0$ $t = 0$ or $\underline{t = 20}$ - Overtaking is NOT intersection of two lines (D-T graph)
16	S.A. of cylinder = S.A. of sphere $2\pi r^2 + 2\pi rh = 4\pi r^2$ $2\pi rh = 2\pi r^2$ $r = h$ Vol. of cylinder $= \pi r^2 h$ $= \pi r^3$ Accept: $3.14\,r^3$
17(a)	

17(b)	$\frac{V_{10\text{ cm}}}{V_{30\text{ cm}}} = \left(\frac{h_{10\text{ cm}}}{h_{30\text{ cm}}}\right)^3$ $\frac{V_{10\text{ cm}}}{\frac{1}{3}\pi(25)^2(30)} = \left(\frac{1}{3}\right)^3$ $V_{10\text{ cm}} = \left(\frac{1}{27}\right)\frac{1}{3}\pi(25)^2(30)$ $= 727.22$ $= 727\text{ cm}^3$ When height changes, radius will change. - Need to find radius at
18	(a) $y = -(x-0)(x-6)$ $= -x(x-6)$ $= -x^2 + 6x + 0$ $\therefore a = 6,\ b = 0$ Alternatively solution: Subst in (0, 0) $0 = b$ Subst in (6, 0) $0 = -36 + 6a$ $a = 6$ (b) $y = -x^2 + 6x$ $= -\left(x^2 - 6x\right)$ $= -\left[(x-3)^2 - 9\right]$ $= -(x-3)^2 + 9$ $\therefore$ the max. point is (3, 9) Alternative method: The quadratic curve is symmetrical about $x = 3$ The max point is thus at $x = 3$ $y = -(3)^2 + 6(3) = \underline{9}$

	Common Errors: • Students substituted a = 6, b = 0 immediately, and then showed the provided coordinates were correct. Since the question asked students to prove a = 6, b = 0, students cannot substitute the values for a and b immediately. Students should substitute the provided coordinates to show a = 6, b = 0. Some students had the misconception that b = y-intercept. This is not true, this is not a straight-line graph.
19	(a) $\text{Length of Singapore River on Map} = \dfrac{3.2\times1000\times100}{80000}$ $= 4.0 \text{ cm}$ (b) $\text{Acutal size of Bishan-Ang Mo Kio Park} = 620\ 000 \text{ m}^2$ $= 620\ 000\ (10^2 \text{ cm})^2$ $= 620\ 000\times10^4 \text{ cm}^2$ $= 6.2\times10^9 \text{ cm}^2$ map:Actual 1 cm : 80 000 cm $1 \text{ cm}^2 : 6.4\times10^9 \text{ cm}^2$ $\text{Size of park on Map} = \dfrac{6.2\times10^9}{6.4\times10^9}$ $= 0.96875 \text{ cm}^2$ **Common Errors:** • 1 cm on map : 800 m on actual ground. Students commonly wrote 1 cm : 80 m, or 1 cm : 80 km. • 1 cm^2 on map : 800^2 m^2 on actual ground i.e. 1 cm^2 on map : 640 000 m^2. Students commonly wrote 1 cm : 800 m^2, or 1 cm : 6400 m^2. • Students also could not convert m^2 to cm^2 e.g. students commonly wrote 620 000 m^2 = 6.2 x 10^7 cm^2 when it should be 6.2 x 10^9 cm^2.
20(a)	$\left\lvert\overrightarrow{LM}\right\rvert = \sqrt{(16-2)^2+(16-1)^2}$ $= 20.518$ $= 20.5 \text{ units}$

	Common Errors: • Some students determined the vector $\overrightarrow{LM}$ incorrectly. Of these students, a handful incorrectly determined $\overrightarrow{LM}$ as $\overrightarrow{OL}-\overrightarrow{OM}$ which is $\overrightarrow{ML}$ instead. • Students who got this question wrong often could not recall the correct formula for the length of a vector.
20(b)	$\overrightarrow{LN}=\frac{1}{3}\overrightarrow{NM}$ $\overrightarrow{ON}-\overrightarrow{OL}=\frac{1}{3}\left(\overrightarrow{OM}-\overrightarrow{ON}\right)$ $\frac{4}{3}\overrightarrow{ON}=\overrightarrow{OL}+\frac{1}{3}\overrightarrow{OM}$ $\overrightarrow{ON}=\frac{3}{4}\overrightarrow{OL}+\frac{1}{4}\overrightarrow{OM}$ $=\frac{3}{4}\begin{pmatrix}1\\2\end{pmatrix}+\frac{1}{4}\begin{pmatrix}16\\16\end{pmatrix}$ $=\begin{pmatrix}4.75\\5.5\end{pmatrix}$ Accept: $\begin{pmatrix}4\frac{3}{4}\\5\frac{1}{2}\end{pmatrix}$ **Marker's Comments** M1 awarded only if student has expressed the vectors in the position vectors, $\overrightarrow{LO}$, $\overrightarrow{NO}$, must be converted to $\overrightarrow{OL}$ and $\overrightarrow{ON}$.
21	**(a)** $y=\frac{a}{x^2}$ Subst $(-2,\ 1)$ into the equation $1=\frac{a}{4}$ $a=4$ **Common Errors** • a = - 4. Careless mistake arose because students did not put brackets – poor book keeping i.e. if students had put brackets a = $(-2)^2$, they would be less likely to make the careless mistake.

	(b) 1 mark for correct shape, and curve drawn in the 1st and 2nd quadrant.  **Common Errors** Some graphs resembled the solution provided above but was not given credit because: (1) the curve deflected backwards, or (2) the curve intersected the y or x axis, or (3) the curve was skewed/ unsymmetrical, or (4) only one side of the graph was provided. **(c)** There will be <u>two solutions</u>. Any **<u>horizontal</u>** <u>straight line above $y = 0$ will intersect the graph twice</u> i.e. once in the 2nd quadrant, and another time in the first quadrant.
22	**(a)** $\angle AEB = \angle ADB$ (angles in the same segment) $= 27^o$ $\angle EAD = \angle ADB$ (alternate angles since $AE \,//\, BD$) $= 27^o$ $\angle AFE = 180^o - \angle AEB - \angle EAD$ $= 126^o$

	(b) $\angle AFE = \angle DAB + \angle ABE$ (Exterior angle = Sum of Interior Opposite angles) $126^\circ = \angle DAB + 49^\circ$ $\angle DAB = 126^\circ - 49^\circ$ $= 77^\circ$ $\angle BCD = 180^\circ - \angle DAB$ (Opposite angles of a cyclic quadrilateral add up to 180°) $= 180^\circ - 77^\circ$ $= 103^\circ$ **Common Mistakes** Students (incorrectly) assume F is the centre of the circle, or triangle AEB is a right angle triangle, or triangles AFE and BFD are isosceles triangles.
23	**(a)** The two triangles share $\angle CAD$ $\frac{AB}{AD} = \frac{80}{140} = \frac{4}{7}$ $\frac{AD}{AC} = \frac{140}{80+165} = \frac{4}{7}$ $\therefore \frac{AB}{AD} = \frac{AD}{AC} = \frac{4}{7}$ [must show the two fractions $= \frac{4}{7}$] $\therefore$ triangle ACD is similar to triangle ADB because the corresponding sides are of the same proportion, and the included angle is the same Accept: If students suggest triangle ACD is similar to triangle ADB because of SAS. **(b)** $\frac{AD}{AC} = \frac{DB}{CD}$ $\frac{140}{245} = \frac{180}{CD}$ $CD = 315$ cm

(c)

$$\cos(\angle ABD) = \frac{80^2 + 180^2 - 140^2}{2(80)(180)}$$

$$\angle BAD = 48.18968^\circ$$

$$\text{Area of triangle } ABD = \frac{1}{2}(BD)(\text{perpendicular distance}) = \frac{1}{2}(AB)(BD)\sin(\angle ABD)$$

$$\text{perpendicular distance} = (80)\sin(48.18968^\circ)$$

$$\text{P. distance} = 59.6284$$

$$= 59.6 \text{ cm}$$

Common Errors

For part (a), many students wrote the following, which resulted in a penalty for bad presentation. Of course 4/7 is equal to 4/7.

$$\frac{AB}{AD} = \frac{AD}{AC}$$

$$\frac{80}{140} = \frac{140}{80+165}$$

$$\frac{4}{7} = \frac{4}{7}$$

Part (C) was poorly done. For e.c.f., there will be no A1 marks.

24

(a)

First \$40 000, tax = \$550

Next \$15 000, tax $= \frac{7}{100} \times 15\,000$

$= \$1050$

Total Tax $= \$550 + \$1050$

$= \$1600$

(b)

If manager deposits \$15 300 into SRS,

his assessable annual income becomes \$39 700

First \$30 000, tax = \$200

Next \$9700, tax = \$339.50

Total tax = \$539.50

Amount saved $= \$1600 - \$539.50 = \underline{\$1060.50}$

(c)

$A = P\left(1 + \frac{R}{100}\right)^n$

$= 15300\left(1 + \frac{2.63}{100}\right)^{10}$

$= \$19835.11$

$= \$19835$

Common Errors

- For part (a), most students knew that they had to refer to the last row of the table. Those who got this wrong usually calculated total tax = \$550 + 2800 or \$550 + \$2800 + 7% of \$15 000. The answer is just \$550 + 7% of the remainder i.e. \$15 000.
- Students who were unable to do part (a) usually could not do part (b) as well.
- For part (c), many students did not heed the question's instruction to leave the answer to the nearest dollar, resulting in the loss of 1 mark.

25

(a)

gradient, $m = \dfrac{y_1 - y_2}{x_1 - x_1}$

$= \dfrac{2-(-2)}{3-0}$

$= \dfrac{4}{3}$

$y = \dfrac{4}{3}x - 2$

(b)

length $AB = \sqrt{4^2 + 3^2}$

$= 5$

Accept if student uses $\sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$

(c)

$C : (3,8)$ i.e. 6 units above (3, 2)

(d)

Area of parallelogram = base × height

$= BC \times (\text{distance from } AD \text{ to } BC)$

$= 6 \times 3$

$= 18 \text{ units}^2$

Common Errors

- For part (c), a common mistake was (-3, 0). Although students would get a parallelogram, the letters would not be in running order i.e. ABDC.
- When calculating the parallelogram's, a handful of students took the product of the length of the sides which is incorrect. It should be base x perpendicular height.

Name: _____ ( ) Class: _____

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS 4048/02
PAPER 2
2 hours 30 minutes

Classes: 401, 402, 403, 404, 405, 406, 501, 502

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, hand in **separately**:
1. Section A with cover page
2. Section B

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

Section A

1 **(a)** Simplify $\left(\frac{ab}{c}\right)^3 \times ab^{-3}c^2$. [2]

(b) Express as a single fraction in its simplest form $\frac{x}{(1-5x)^2} + \frac{3}{1-5x}$. [2]

(c) Factorise completely $3rx - 3y + x - 9ry$. [2]

(d) Solve the inequality $-2 \le \frac{4x-10}{5} < 2$. [2]

(e) It is given that $\frac{x}{4-hy} = \frac{1}{4y+h}$.

Express y in terms of x and h. [3]

---

2 Every Thursday, Sana jogs a distance of j km and then walks a distance of w km.
She jogged at 9 km/h and walked at 5 km/h.

(a) Write down an expression, in terms of j, for the length of time that Sana jogged. [1]

(b) Sana travelled a total distance of 8 km.
She jogged half an hour more than she walked.

Write down two simultaneous equations in j and w to represent this information. [2]

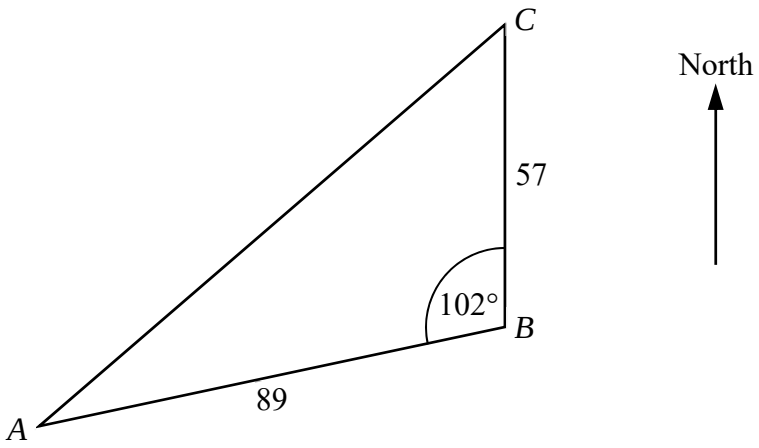
(c) Solve your simultaneous equations to find j and w. [3]

(d) Find Sana's average speed for the total distance. [2]

(e) One Thursday, Sana increases her speed by 120% for the distance of w km.

Find the percentage decrease in the time that she takes to travel the w km. [2]

3



ABC is a triangular plot of land.
$AB = 89$ m, $BC = 57$ m and angle $ABC = 102°$.
B is due south of C.

(a) Calculate AC. [2]

(b) Calculate the bearing of C from A. [2]

(c) P is a point vertically above B.
The height BP is 14 m.

(i) Calculate the angle of elevation of P from A. [2]

(ii) $PABC$ is a pyramid with vertex P and base ABC.

Calculate the volume of the pyramid $PABC$.
Give your answer correct to the nearest 10 m^3. [3]

4 The first four terms in a sequence of numbers are given below.

$$T_1 = 6 + (1-2)^2 - 2 = 5$$

$$T_2 = 6 + (2-2)^2 - 4 = 2$$

$$T_3 = 6 + (3-2)^2 - 6 = 1$$

$$T_4 = 6 + (4-2)^2 - 8 = 2$$

(a) Find T_5. [1]

(b) Show that the nth term of the sequence, T_n, is given by $n^2 - 6n + 10$. [2]

(c) T_k and T_{3k} are terms in the sequence.

It is given that $\frac{T_{3k}}{T_k} = 17$.

Show that this equation simplifies to

$$2k^2 - 21k + 40 = 0.$$ [3]

(d) Solve the equation $2k^2 - 21k + 40 = 0$. [3]

(e) Explain why one of the solutions in part **(d)** must be rejected as the position of T_k in the sequence. [1]

Section B

Start Section B on a new sheet of writing paper.

5 A café serves cappuccinos (C) and lattes (L).
Each cup of cappuccino contains 60 ml of espresso, 60 ml of milk and 60 ml of foam.
Each cup of latte contains 60 ml of espresso, 300 ml of milk and no foam.

(a) This information can be represented in the 3×2 matrix, $\mathbf{V}$.

$$\mathbf{V} = \begin{matrix} \\ \\ \\ \\ \end{matrix} \begin{matrix} \text{C} & \text{L} \\ \end{matrix} \begin{pmatrix} & \\ & \\ & \end{pmatrix} \begin{matrix} \\ \text{Espresso} \\ \text{Milk} \\ \text{Foam} \end{matrix}$$

Copy and complete the matrix $\mathbf{V}$. [1]

(b) In one day, the café sold 26 cups of cappuccino and 45 cups of latte.

Evaluate the matrix $\mathbf{S} = \mathbf{V}\begin{pmatrix} 26 \\ 45 \end{pmatrix}$. [2]

(c) Explain what each element in matrix $\mathbf{S}$ represents. [1]

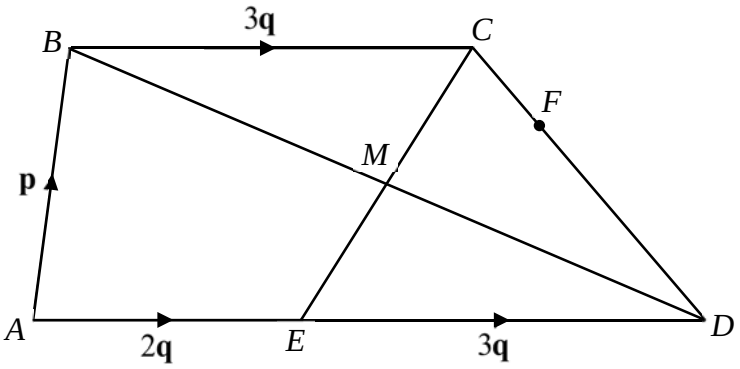
(d) 60 ml of espresso costs 30 cents.
The elements of the matrix $\mathbf{E}$, where $\mathbf{E} = \mathbf{UV}$, represent the costs, in cents, of the espresso contained in each cup of cappuccino and each cup of latte.

Write down the matrix $\mathbf{U}$. [1]

(e) Foam is made from milk.
100 ml of milk makes 350 ml of foam.

Calculate the largest number of cups of cappuccino that 2 litres of milk can make. [3]

6



$ABCD$ is a quadrilateral and E is a point on AD.
M is the point of intersection of BD and CE.
$\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AE} = 2\mathbf{q}$ and $\overrightarrow{BC} = \overrightarrow{ED} = 3\mathbf{q}$.

(a) Show that triangles BMC and DME are congruent.
Give a reason for each statement you make. [3]

(b) Express, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$,

(i) $\overrightarrow{AC}$, [1]

(ii) $\overrightarrow{BD}$, [1]

(iii) $\overrightarrow{AM}$. [1]

(c) F is the point on CD such that $CF : FD = 2 : 5$.

(i) Explain why A, M and F lie in a straight line. [3]

(ii) Find the ratio area of triangle AME : area of triangle FMD. [1]

7 Answer the whole of this question on a sheet of graph paper.

A ball is thrown upwards.
The height of the ball, y metres, t seconds after it is thrown is given by the formula

$$y = t^3 - 10t^2 + (22.6)t \ .$$

The table shows some corresponding values of t and y, correct to 1 decimal place.

t	0	0.5	1	1.5	2	2.5	3	3.25
y	0.0	8.9	13.6	14.8	13.2	9.6	4.8	p

(a) Find the value of p. [1]

(b) Using a scale of 4 cm to represent 1 second, draw a horizontal t-axis for $0 \le t \le 4$.
Using a scale of 1 cm to represent 1 metre, draw a vertical y-axis for $0 \le y \le 16$.

On your axes, draw a graph to show the height of the ball for $0 \le t \le 3.25$. [3]

(c) Use your graph to find the height of the ball 0.8 seconds after it is thrown. [1]

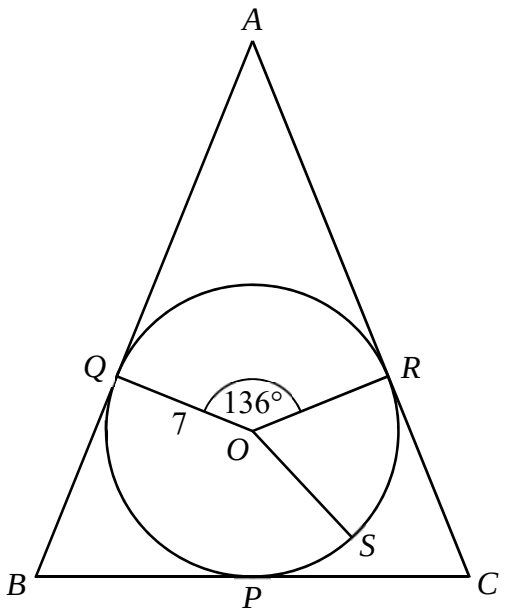
(d) By drawing a tangent, find the gradient of the curve at $(2, 13.2)$.
State the units of your answer. [3]

(e) When the ball is thrown, a feather is dropped at a height of 9 metres.
The feather falls vertically downwards at a constant speed.
4 seconds after the ball is thrown, the feather is at a height of 5 metres.

(i) On the same axes, draw a line to show the height of the feather for $0 \le t \le 4$. [1]

(ii) Use your line to find when the ball first falls below the feather. [1]

8



The diagram shows a circle with centre O and radius 7 cm.
P, Q, R and S are points on the circle.
The tangents to the circle at P, Q and R form the triangle ABC.
Triangle ABC is isosceles with $AB = AC$.
Angle $QOR = 136°$.

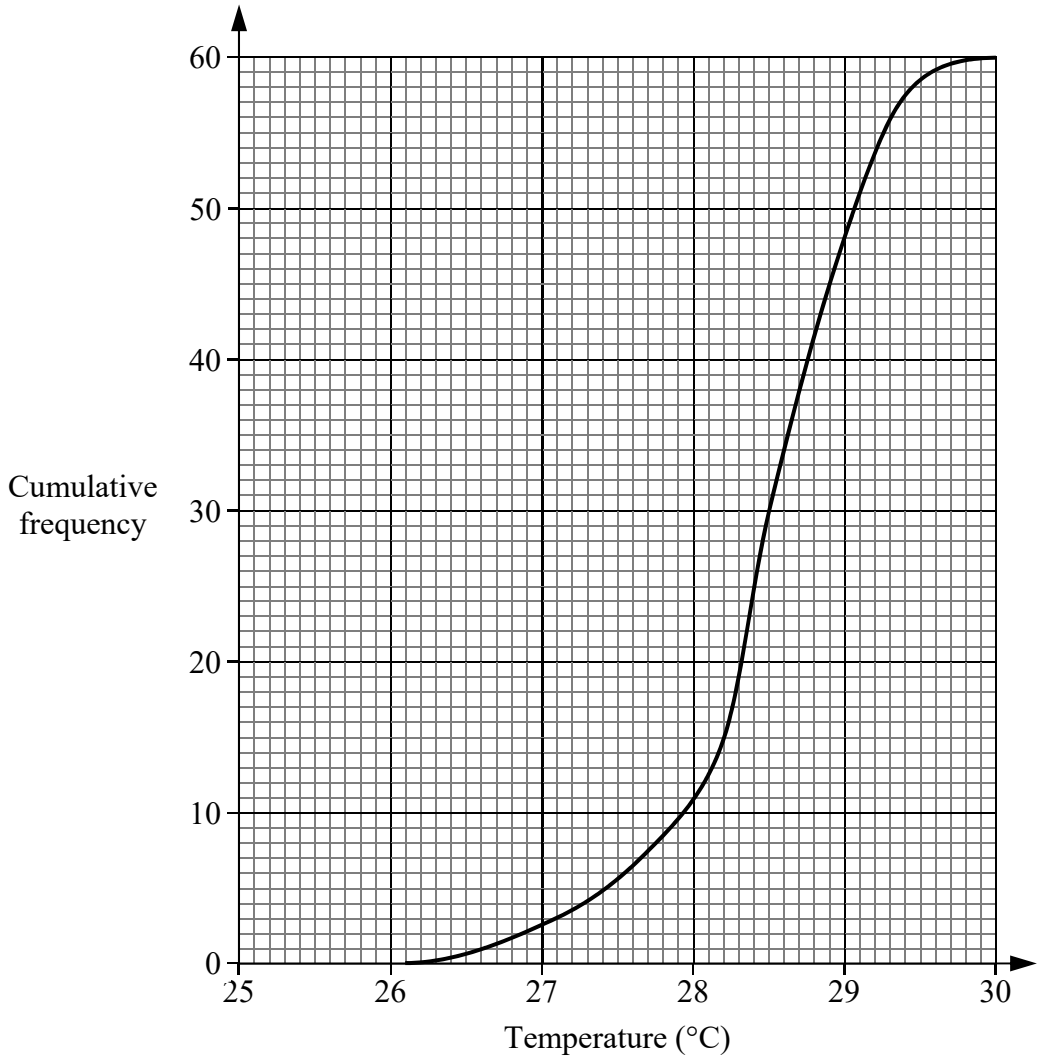
(a) Show that angle $OAR = 22°$.
Give a reason for each step of your working. [3]

(b) Calculate the area of the triangle ABC. [4]

(c) Angle $ROS = \theta$ radians.
The perimeter of the sector ORS is $2(\theta + 10)$ cm.

Calculate the length of the arc RS. [3]

9 **(a)** The temperature at Simei was recorded every day for 60 days.
The cumulative frequency curve below shows the distribution of the temperatures.



(i) Use the curve to estimate

(a) the median temperature, [1]

(b) the interquartile range of the temperatures. [2]

(ii) Estimate the number of days that had temperatures above 29°C. [1]

The temperature at Jurong was recorded every day for the same period.
The interquartile range of the temperatures at Jurong is 1.5°C.

(iii) Make a comment comparing the temperatures at Simei and at Jurong. [1]

(iv) The temperatures at Jurong are converted to degrees Fahrenheit (°F) using the formula

$$\text{temperature in °F} = 1.8 \times (\text{temperature in °C}) + 32.$$

Find the interquartile range, in °F, of the converted temperatures. [1]

(b) A drawer contains 2 blue socks and 6 white socks.
Two socks are taken from the drawer at random without replacement.
If the two socks are different colours, then a third sock is taken from the drawer.
Otherwise, no third sock is taken.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [3]

(ii) Find, as a fraction in its simplest form, the probability that

(a) the first two socks taken are white, [1]

(b) a third sock is taken and it is the same colour as the first sock. [2]

10 Meg would like to buy an air conditioner.

(a) Meg writes down how long she would use the air conditioner in the following table.

Monday to Thursday	6 hours each day
Friday	7 hours 15 minutes
Saturday and Sunday	8 hours each day

Find the mean length of time that she would use the air conditioner each day. [2]

Meg is deciding between two models of air conditioner.

The next page shows information that she needs, including the electricity consumptions of the two models.

(b) Based on her usage, Meg estimates that the electricity consumptions in 1 year will be 1755 kWh for Model S and 1066.5 kWh for Model E.

Explain how she found these estimates. [1]

(c) The total cost of an air conditioner includes its price, the cost of the electricity it consumes and the cost of servicing it.

Electricity costs 25.3 cents per kWh, including GST.
Meg would like the air conditioner to be serviced once every 4 months.

Based on her usage, which model will have a lower total cost after 7 years of use? Justify your decision with calculations. [7]
(You should assume that the costs of electricity and servicing remain the same.)

Residential Air Conditioners

	Model S (Standard)	**Model E** (Energy efficient)
Price of air conditioner	$650	$1300
Electricity consumption in one year	2080 kWh	1264 kWh

Notes:
- Prices include GST
- Electricity consumptions are based on 8 hours of use each day

Service Contracts

Frequency	**Price per service before 7% GST**
1 service every 2 months	$25
1 service every 3 months	$30
1 service every 4 months	$35

Offer: **40% discount on service contract** with purchase of Model S

End of paper

Penalties:
Presentation / Rounding off [P/Ro] – 1m
Units [U] – 1m

Solution

1 (a) $\left(\frac{ab}{c}\right)^3 \times ab^{-3}c^2$

$= \frac{(ab)^3}{c^3} \times ab^{-3}c^2$

$= \frac{a^3b^3}{c^3} \times \frac{ac^2}{b^3}$

$= \frac{a^4}{c}$

1 (b) $\frac{x}{(1-5x)^2} + \frac{3}{1-5x}$

$= \frac{x+3(1-5x)}{(1-5x)^2}$

$= \frac{3-14x}{(1-5x)^2}$

1 (c) $3rx - 3y + x - 9ry$

$= 3rx - 9ry - 3y + x$

$= 3r(x-3y) + (x-3y)$

$= (3r+1)(x-3y)$

1 (d) $-2 \le \frac{4x-10}{5}$ and $\frac{4x-10}{5} < 2$

$-10 \le 4x - 10$ and $4x - 10 < 10$

$0 \le 4x$ and $4x < 20$

$0 \le x$ and $x < 5$

$0 \le x < 5$

1 (e) $\dfrac{x}{4-hy}=\dfrac{1}{4y+h}$

$x(4y+h)=4-hy$

$4xy+hx=4-hy$

$4xy+hy=4-hx$

$(4x+h)y=4-hx$

$y=\dfrac{4-hx}{4x+h}$

2 (a) $\dfrac{j}{9}$ hours

2 (b) $j+w=8$

$\dfrac{j}{9}=\dfrac{w}{5}+\dfrac{1}{2}$

2 (c) $j=8-w$

Substituting into $\dfrac{j}{9}=\dfrac{w}{5}+\dfrac{1}{2}$,

$\dfrac{8-w}{9}=\dfrac{w}{5}+\dfrac{1}{2}$

$5(8-w)=9w+\dfrac{45}{2}$

$40-5w=9w+\dfrac{45}{2}$

$14w=17.5$

$w=1.25$

Substituting into $j=8-w$,

$j=8-1.25$

$=6.75$

$\therefore j=6.75,\ w=1.25$

2 (d) Total distance $= 8$

$$\text{Total time} = \frac{6.75}{9} + \frac{1.25}{5}$$
$$= 1$$
$$\text{Average speed} = \frac{8}{1}$$
$$= 8 \text{ km/h}$$

2 (e) $\text{New speed} = 5 \times 220\%$
$$= 11$$

$$\text{New time} = \frac{1.25}{11}$$

Percentage decrease
$$= \frac{\frac{1.25}{5} - \frac{1.25}{11}}{\left(\frac{1.25}{5}\right)} \times 100\%$$
$$= 54.5\% \text{ (3 s.f.)}$$

Alternative method:

Ratio of old speed : new speed
$$= 100 : (100 + 120)$$
$$= 5 : 11$$

Length of time is inversely proportional to speed,
so the ratio of old time to new time is $11 : 5$.

Percentage decrease
$$= \frac{11 - 5}{11} \times 100\%$$
$$= 54.5\% \text{ (3 s.f.)}$$

3 (a) $AC^2 = 89^2 + 57^2 - 2(89)(57)\cos 102°$

$AC = 115.24$

$= 115 \text{ m}$ (3 s.f.)

3 (b) $\dfrac{\sin \angle ACB}{89} = \dfrac{\sin 102°}{115.24}$

$\angle ACB = \sin^{-1}\left(\dfrac{\sin 102°}{115.24} \times 89\right)$

$= 49.062°$

Bearing of C from $A = 049.1°$ (1 d.p.)

3 (c) (i) Angle of elevation of P from A

$= \tan^{-1}\left(\dfrac{14}{89}\right)$

$= 8.9°$ (1 d.p.)

3 (c) (ii) Area of triangle ABC

$= \dfrac{1}{2}(89)(57)\sin 102°$

$= 2481.1$

Volume of pyramid $PABC$

$= \dfrac{1}{3}(2481.1)(14)$

$= 11578$

$= 11580 \text{ m}^3$ (to nearest 10 m^3)

4 (a) $T_5 = 6 + (5-2)^2 - 10$

$= 5$

4 (b) $T_n = 6 + (n-2)^2 - 2n$

$= 6 + n^2 - 4n + 4 - 2n$

$= n^2 - 6n + 10$

4 (c) $\dfrac{(3k)^2-6(3k)+10}{k^2-6k+10}=17$

$\dfrac{9k^2-18k+10}{k^2-6k+10}=17$

$9k^2-18k+10=17k^2-102k+170$

$8k^2-84k+160=0$

$2k^2-21k+40=0$ (shown)

4 (d) $(2k-5)(k-8)=0$

$2k-5=0$ or $k-8=0$

$k=2.5$ or $k=8$

4 (e) $k=2.5$ must be rejected
because it is not a positive integer

5 (a) $\mathbf{V}=\begin{pmatrix}60 & 60\\ 60 & 300\\ 60 & 0\end{pmatrix}$

5 (b) $\mathbf{S}=\begin{pmatrix}60 & 60\\ 60 & 300\\ 60 & 0\end{pmatrix}\begin{pmatrix}26\\ 45\end{pmatrix}$

$=\begin{pmatrix}60\times26+60\times45\\ 60\times26+300\times45\\ 60\times26+0\times45\end{pmatrix}$

$=\begin{pmatrix}4260\\ 15060\\ 1560\end{pmatrix}$

5 (c) The elements in **S** represent the total volumes, in ml,
of espresso, milk and foam in the drinks sold.

Alternative answer:

In the drinks sold, there was a total of
4260 ml of espresso,
15060 ml of milk, and
1560 ml of foam.
These are the elements in **S**.

5 (d) From $\mathbf{E} = \begin{pmatrix} 30 & 30 \end{pmatrix}$,

$$\mathbf{E} = \mathbf{U}\begin{pmatrix} 60 & 60 \\ 60 & 300 \\ 60 & 0 \end{pmatrix}$$

$$\therefore \mathbf{U} = \begin{pmatrix} \frac{30}{60} & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$$

5 (e) Volume of milk to make 60 ml of foam

$$= 60 \times \frac{100}{350}$$

$$= 17.143$$

Volume of milk to make one cup of cappuccino

$$= 60 + 17.143$$

$$= 77.143$$

Largest number of cups of cappuccino

$$= \frac{2 \times 1000}{77.143}$$

$$= 25.926$$

$= 25$ (rounded down to nearest integer)

6 (a) $\overrightarrow{BC} = \overrightarrow{ED}$, so BC and ED are parallel.

$\angle BCM = \angle DEM$ (alternate angles, $BC \parallel ED$)

$BC = DE$ (since $\overrightarrow{BC} = \overrightarrow{ED}$)

$\angle CBM = \angle EDM$ (alternate angles, $BC \parallel ED$)

Therefore, triangles BMC and DME are congruent (ASA).

Alternative method:

$\overrightarrow{BC} = \overrightarrow{ED}$, so BC and ED are parallel.

$\angle BMC = \angle DME$ (vertically opposite angles)

$\angle BCM = \angle DEM$ (alternate angles, $BC \parallel ED$)

$BC = DE$ (since $\overrightarrow{BC} = \overrightarrow{ED}$)

Therefore, triangles BMC and DME are congruent (ASA).

6 (b) (i) From triangle ABC,

$\overrightarrow{AC} = \mathbf{p} + 3\mathbf{q}$

6 (b) (ii) From triangle ABD,

$\overrightarrow{BD} = 5\mathbf{q} - \mathbf{p}$

6 (b) (iii) Since triangles BMC and DME are congruent,

$BM = DM$

$\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BD}$

$$\begin{aligned}\overrightarrow{AM} &= \overrightarrow{AB} + \overrightarrow{BM} \\ &= \mathbf{p} + \frac{1}{2}\overrightarrow{BD} \\ &= \mathbf{p} + \frac{1}{2}(5\mathbf{q} - \mathbf{p}) \\ &= \frac{1}{2}\mathbf{p} + \frac{5}{2}\mathbf{q}\end{aligned}$$

6 (c) (i) $\overrightarrow{CD} = \overrightarrow{AD} - \overrightarrow{AC}$

$= 5\mathbf{q} - (\mathbf{p} + 3\mathbf{q})$

$= 2\mathbf{q} - \mathbf{p}$

$\overrightarrow{CF} = \frac{2}{7}\overrightarrow{CD}$

$= \frac{2}{7}(2\mathbf{q} - \mathbf{p})$

$\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF}$

$= \mathbf{p} + 3\mathbf{q} + \frac{2}{7}(2\mathbf{q} - \mathbf{p})$

$= \frac{5}{7}\mathbf{p} + \frac{25}{7}\mathbf{q}$

$= \frac{10}{7}\left(\frac{1}{2}\mathbf{p} + \frac{5}{2}\mathbf{q}\right)$

$= \frac{10}{7}\overrightarrow{AM}$

This shows that $\overrightarrow{AF}$ and $\overrightarrow{AM}$ are parallel.
Also, $\overrightarrow{AF}$ and $\overrightarrow{AM}$ have the common point A.
Therefore, the points A, M and F lie on a straight line.

6 (c) (ii) area of triangle AME : area of triangle $EMD = 2:3$

area of triangle EMD : area of triangle $CMD = 1:1$

area of triangle CMD : area of triangle $FMD = 7:5$

area of triangle AME : area of triangle $FMD = 14:15$

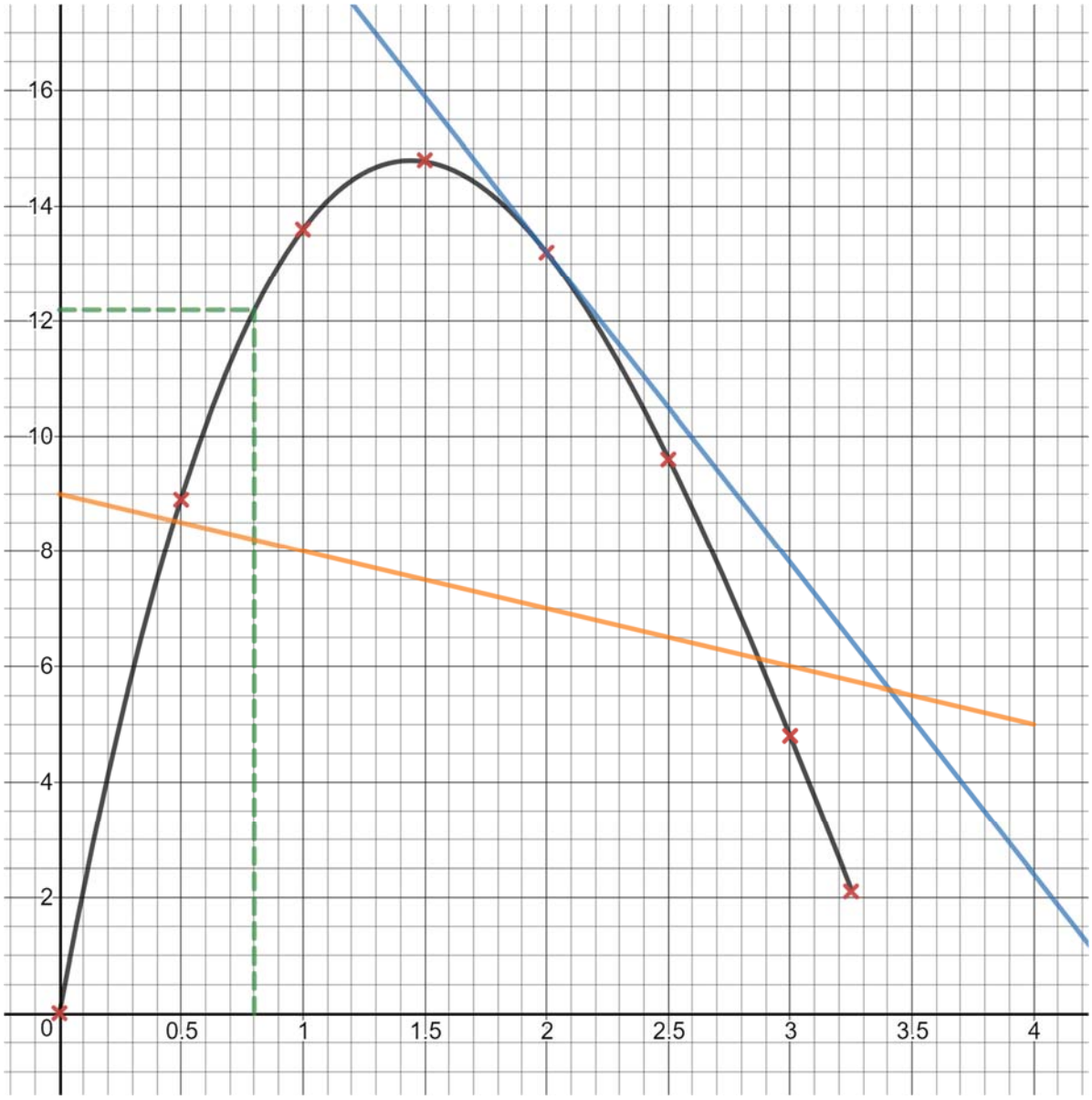
7 (a) $p = 2.153125$

7 (b) Horizontal axis drawn covering $0 \le t \le 4$ with correct scale

Vertical axis drawn covering $0 \le y \le 16$ with correct scale

All 8 points plotted

Smooth curve drawn through plotted points



7 (c) 12.2 m (or from your graph)

7 (d) Tangent drawn at $t = 2$ and estimated (change in y)/(change in x)
Gradient = –4.8 to –6.1 (exact answer: –5.4)
Units are m/s

7 (e) (i) Line drawn from (0, 9) to (4, 5)

7 (e) (ii) 2.85s, 2.875s, 2.9s (or from your graph)

8 (a) $\angle ARO = 90°$ (tangent perpendicular to radius)

$$\angle AOR = \frac{136°}{2}$$ (tangents from external point)

$$= 68°$$

$\angle OAR = 180° - 90° - 68°$ (angles in a triangle)

$= 22°$ (shown)

Alternative method:

$\angle AQO = \angle ARO = 90°$ (tangent perpendicular to radius)

$\angle QAR = 360° - 136° - 90° - 90°$ (angles in a quadrilateral)

$= 44°$

$$\angle OAR = \frac{44°}{2}$$ (tangents from external point)

$= 22°$ (shown)

8 (b) $\sin 22° = \dfrac{7}{AO}$

$$AO = \frac{7}{\sin 22°}$$
$$= 18.686$$

$$\angle ACB = \frac{180° - 44°}{2}$$
$$= 68°$$
$$\angle OCP = \frac{68°}{2}$$
$$= 34°$$

$$\tan 34° = \frac{7}{PC}$$
$$PC = \frac{7}{\tan 34°}$$
$$= 10.378$$

Area of triangle ABC

$$= \frac{1}{2}(BC)(AP)$$
$$= \frac{1}{2}(2 \times PC)(AO + 7)$$
$$= \frac{1}{2}(2 \times 10.378)(18.686 + 7)$$
$$= 267 \text{ cm}^2 \text{ (3 s.f.)}$$

Alternative methods:

- Find AR and RC, then area is $\frac{1}{2}(AC)^2 \sin \angle BAC$
- Find AR and RC, then area is $2 \times \frac{1}{2}(AR)(7) + 4 \times \frac{1}{2}(RC)(7)$

8 (c) Perimeter of sector ORS

$= 7 + 7 + r\theta$

$= 14 + 7\theta$

$14 + 7\theta = 2(\theta + 10)$

$5\theta = 6$

$\theta = 1.2$

Length of arc RS

$= 7\theta$

$= 7(1.2)$

$= 8.4$ cm

9 (a) (i) (a) Median $= 28.5°C$

9 (a) (i) (b) Interquartile range $= 28.9 - 28.2$

$= 0.7°C$

9 (a) (ii) $60 - 48 = 12$ days

9 (a) (iii) The temperatures at Jurong have a larger spread than the temperatures at Simei.

Alternative answer:
The temperatures at Jurong were less consistent than the temperatures at Simei.

9 (a) (iv) After every temperature is multiplied by 1.8,

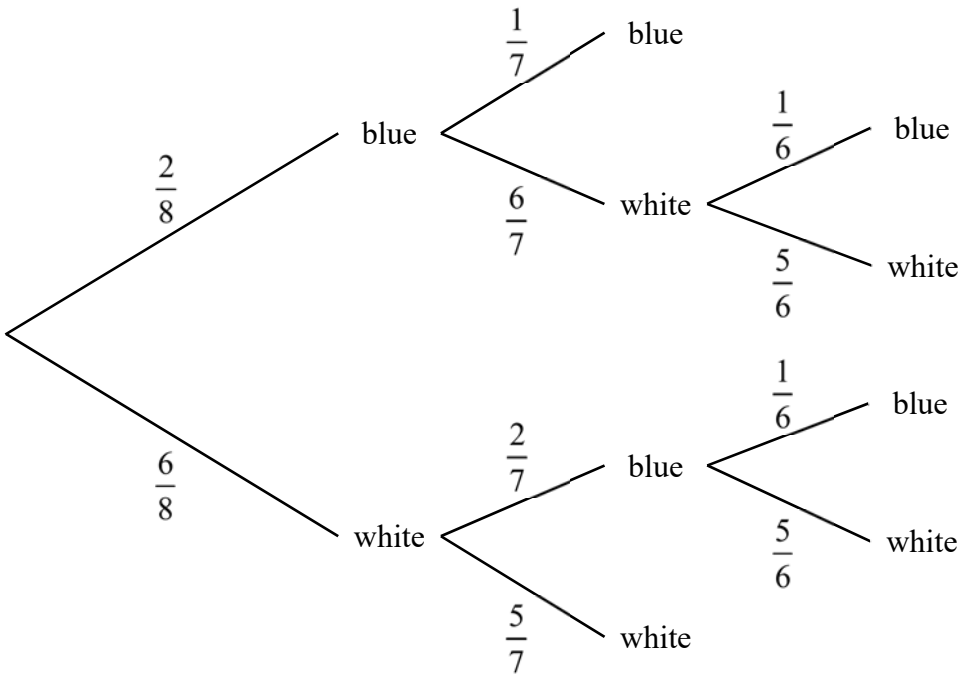
Interquartile range $= 1.8 \times 1.5$

$= 2.7$

After 32 is added to every temperature,

Interquartile range $= 2.7°F$

9 (b) (i)



9 (b) (ii) (a) $\frac{6}{8}\times\frac{5}{7}=\frac{15}{28}$

9 (b) (ii) (b) $\left(\frac{2}{8}\times\frac{6}{7}\times\frac{1}{6}\right)+\left(\frac{6}{8}\times\frac{2}{7}\times\frac{5}{6}\right)$

$=\frac{3}{14}$

10 (a) $\frac{6\times4+7.25+8\times2}{7}$

$=6.75$ hours

10 (b) Meg multiplied the given annual electricity consumptions by $\frac{6.75}{8}$.

10 (c) Model S:

Cost of electricity per year

$= 25.3 \times 1755$

$= 44401.5 \text{ cents}$

$= \$444.02$

Cost of servicing per year before discount

$= \$35 \times \frac{12}{4} \times \frac{107}{100}$

$= \$112.35$

Total cost of servicing per year

$= \$112.35 \times \frac{100 - 40}{100}$

$= \$67.41$

Total cost of Model S

$= \$650 + 7 \times (\$444.02 + \$67.41)$

$= \$4230.01$

Model E:

Cost of electricity per year

$= 25.3 \times 1066.5$

$= 26982.45 \text{ cents}$

$= \$269.82$

Total cost of Model E

$= \$1300 + 7 \times (\$269.82 + \$112.35)$

$= \$3975.19$

Since $\$3975.19$ is less than $\$4230.01$, Model E has a lower total cost.

COMMONWEALTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2018

MATHEMATICS
PAPER 1

Name: _____ ( ) Class: _____

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL
SECONDARY FOUR NORMAL (O)

Monday 20 August 2018
08 00 – 10 00
2 h

4048/1

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Name of setter: Mrs Tan HP

For Examiner's Use	
Presentation	
Accuracy	
Total	**80**

Parent's Signature: _____

This paper consists of **17** printed pages including the cover page.

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 Given that $\frac{1}{343} = 7^k$, find k.

Answer $k =$ ..... [1]

2(a) Simplify $\frac{3a}{8} - \frac{a+2}{4}$.

Answer ..... [2]

(b) Hence solve $\frac{3a}{8} - \frac{a+2}{4} = 0$.

Answer $a =$ ..... [1]

3(a) Solve the inequalities $-6 < 3 - 2x \leq 9$.

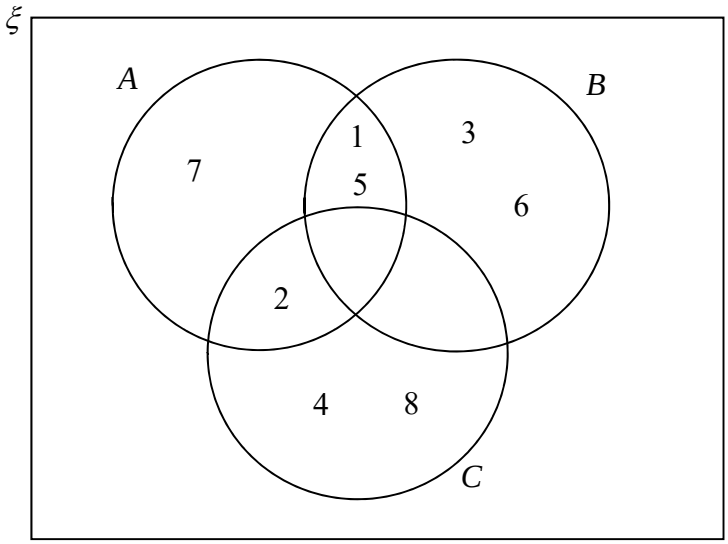
Answer ..... [2]

(b) Write down all the prime numbers that satisfy $-6 < 3 - 2x \leq 9$.

Answer $x =$ …….. [1]

4 $\xi = \{\text{ integers } x : 1 \leq x \leq 8\}$

The Venn diagram shows the elements of ξ and three sets, A, B and C.



Use one of the symbols below to complete each statement.

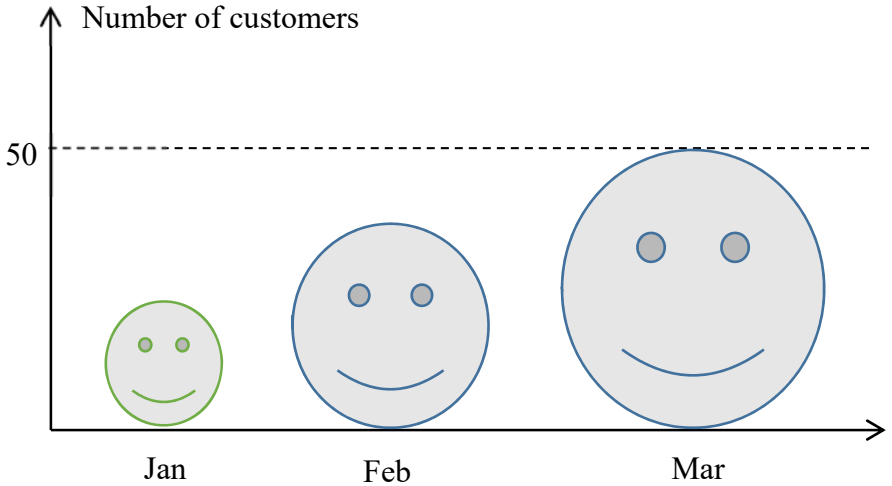
$\varnothing \quad \subset \quad \not\subset \quad \in \quad \notin \quad \xi$

(a) 3 ..... C [1]

(b) $\{1\}$ ..... $(A \cap B)$ [1]

(c) $(A \cup B \cup C)' =$ ..... [1]

5 Anil draws this graph to show the number of customers who visited his store for each of the first three months since it opened.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer .....

..... [2]

6 A man normally takes 1 hour 45 minutes to cycle at a constant speed from Town A to Town B. One day, he increases his speed by 5 km/h and finds that the journey takes 25 minutes less than the normal time. Find his normal speed.

Answer ..... km/h [2]

7 A shopkeeper bought a pair of shoes for $56. He still made a percentage profit of 15% even though he offered a 20% discount to his customer. Calculate the normal selling price of this pair of shoes.

Answer $ ..... [2]

8 Two similar cans of beans, *A* and *B*, have total surface areas of 40 cm^2 and 90 cm^2 respectively. The smaller can contains 400 g of beans.
Jane worked out the amount of beans in the bigger can in the following way.

Amount of beans in smaller can = $40 \times 10 = 400$g
So amount of beans in bigger can = $90 \times 10 = 900$g

Is Jane's working correct? Support your answer with clear workings.

Answer ……..

……... [2]

9 Alec has written down six numbers. The mean of these numbers is 4.5, the median is 5.5 and the mode is 6. The smallest number appears twice. Find the six numbers.

Answer ……. , ……. , ……. , ……. , ……. , ……. [3]

10(a) Factorise $x^2 - 169$.

Answer ..... [1]

(b) Use your answer to part (a) to find two factors of 731 other than 1 and 731.

Answer ..... , ... [2]

11 One day, Tom used this formula to estimate the air temperature at different heights above sea level.

$$T = 21 - \frac{h}{120},$$

where T is the temperature in degrees Celsius, and h is the height, in metres, above sea level.

(a) An aircraft records the air temperature as $-5°C$. Use the formula to estimate the height of the aircraft above sea level.

Answer ..... m [1]

(b) Two aircrafts are flying at different heights. The difference in temperatures at their two heights is $10°C$. Estimate the difference between the heights of the two aircraft.

Answer ..... m [2]

12 A packet of sweets is divided among Ahmad, Benny, Carl and Dexter. The number of sweets that Ahmad, Benny and Carl have are in the ratio 5 : 9 : 7. If Carl gets 12 sweets more than Ahmad and Benny gets twice as many sweets as Dexter, find

(a) the total number of sweets,

Answer ..... sweets [2]

(b) the number of sweets that Dexter gets.

Answer ..... sweets [1]

13 Jane can varnish 3 jars in 5 hours. Jim can varnish 2 jars in 3 hours.

Jane and Jim work together to varnish a total of 20 jars.

If they continue to varnish at the same rate, how long will it take them to varnish the 20 jars? Give your answer in hours and minutes, to the nearest minute.

Answer ..... hr ..... mins [3]

14 A club has between 50 to 60 members. During a donation drive, each member contributes \$12. The total proceeds is shared equally amongst 14 children in an orphanage, with each child getting \$$x$, where x is an integer.

(a) How many members are there in the club?

Answer ..... members [2]

(b) Find the value of x.

Answer ..... [1]

15 A driving theory test is set every month. John takes the test each month until he passes. Each time he takes the test, the probability he passes is 0.9.
Find the probability that John

(a) passes on his second attempt,

Answer ..... [1]

(b) takes at least two attempts to pass the test,

Answer ..... [1]

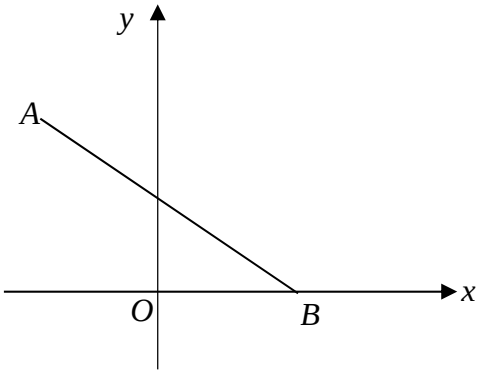
(c) fails the first n test,

Answer ..... [1]

(d) passes the test in one of the first n months.

Answer ..... [1]

16 In the diagram, the point $A = (-4, 6)$ and B lies on the x-axis. The midpoint of AB lies on the y-axis.



(a) Find the coordinates of the midpoint of AB.

Answer (..... , ...) [2]

(b) The point $C(6, k)$ lies on a line parallel to AB and passing through the origin. Find the value of k.

Answer $k =$ ..... [2]

17(a) The ratio of an interior angle to an exterior angle of a regular polygon is 5 : 1. Find
(i) the size of each exterior angle,

Answer ..... ° [1]

(ii) the number of sides of the regular polygon.

Answer ..... sides [1]

(b) A decagon has 6 interior angles of 125°. The remaining interior angles are all equal. Find the size of the each of the remaining interior angles.

Answer ..... ° [2]

18(a) Convert 482 cm^3 per second to litres per hour.

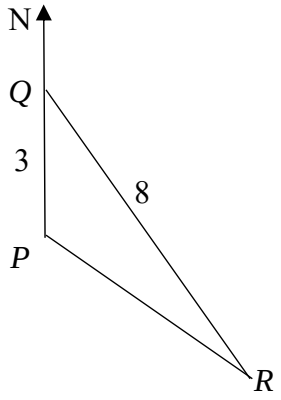
Answer ..... litres / hour [2]

(b) The average volume of water flowing over a waterfall is 7.79×10^3 litres per second. After a rainstorm the volume of water increased to 2.38×10^4 litres per second.

Calculate the percentage increase in the volume of water flowing over the waterfall.

Answer ..... % [2]

19 The diagram shows the positions of three points P, Q and R on level ground. $PQ = 3$ m, $QR = 8$ m and $\sin Q\hat{P}R = \frac{7}{8}$. Q is due north of P.



(a) Calculate the bearing of R from P.

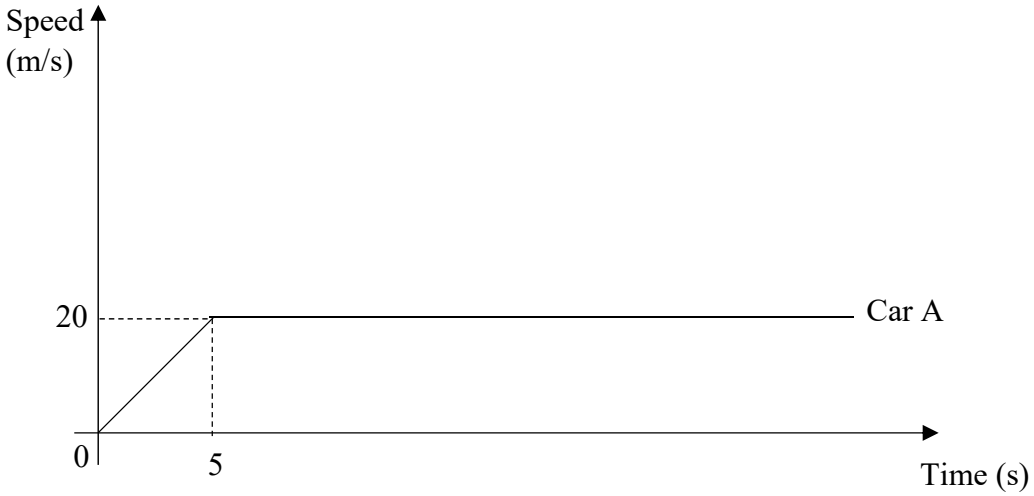
Answer ..... ° [1]

(b) A particle starts from R and moves in the direction of RP. Find the distance travelled by the particle such that it is nearest to Q.

Answer ..... m [3]

20 Car A and Car B travel along the same straight route. Car A accelerates from rest to a speed of 20 m/s in a time of 5 seconds. It then continues at this speed. Car B starts 5 seconds later and accelerates to a speed of 67.5 m/s in 30 seconds after which it continues at this speed.

(a) The speed-time graph for Car A is shown in the diagram. Sketch the speed-time graph for Car B in the same axes.



[1]

(b) Explain, with relevant workings, whether Car B will overtake Car A within the first 20 seconds after Car A starts its journey.

Answer ……...….. [3]

21(a) L is the point $(4,-2)$. The point M is the result of the translation of point L by $\begin{pmatrix}-8\\-6\end{pmatrix}$.

(i) Find the coordinates of point M.

Answer M = (..... , ...) [1]

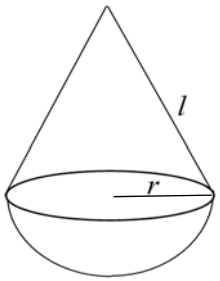
(ii) Find the equation of line LM.

Answer ..... [2]

(b) Explain whether or not $\begin{pmatrix}-3\\8\end{pmatrix}$ and $\begin{pmatrix}9\\-24\end{pmatrix}$ are parallel.

Answer ..... [2]

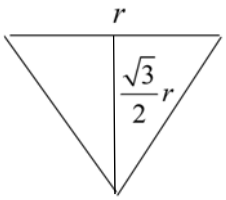
22(a) A solid is made from a cone and a hemisphere. The cone has radius r cm and slant height l cm. The hemisphere has radius r. Write down the total surface area of the solid in terms of r and l.



Answer ..... cm^2 [1]

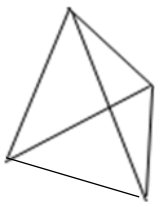
(b) The height and base an equilateral triangle are $\frac{\sqrt{3}}{2}r$ cm and r cm respectively.

(i) Find the area of the equilateral triangle.



Answer ..... cm^2 [1]

(ii) 4 of the equilateral triangles in (i) are used to make a tetrahedron (a right triangular pyramid) shown in the diagram. Find the total surface area of the tetrahedron.



Answer ..... cm^2 [1]

(c) The total surface area of the solid in (a) is equal to the total surface area of the tetrahedron in (b). Find l in terms of r.

Answer $l =$ ..... [2]

23 In supermarket A, water costs \$1.50 per litre, milk costs \$2.40 per litre and cola costs \$1.40 per litre. In supermarket B, water costs \$0.20 more per litre, milk costs \$0.40 less per litre and cola costs \$0.10 less per litre.

This information can be represented by the matrix $\mathbf{P} = \begin{matrix} \\ \text{A} \\ \text{B} \end{matrix} \begin{matrix} \begin{matrix} \text{W} & \text{M} & \text{C} \end{matrix} \\ \begin{pmatrix} 1.5 & 2.4 & 1.4 \\ 0.2 & -0.4 & -0.1 \end{pmatrix} \end{matrix}$.

(a) Andy and John go shopping.
Andy buys 4 litres of water, 2 litres of milk and 3 litres of cola.
John buys 3 litres of water and 4 litres of cola.
Represent their purchases in a (3×2) matrix **Q**.

Answer (a) $\mathbf{Q} = \begin{pmatrix} & \\ & \\ & \end{pmatrix}$ [1]

(b) Evaluate the matrix $\mathbf{R} = \mathbf{PQ}$.

Answer (b) $\mathbf{R} = \begin{pmatrix} & \\ & \\ & \end{pmatrix}$ [2]

(c) From your answer in (b), write down how much money John would save by shopping in supermarket A.

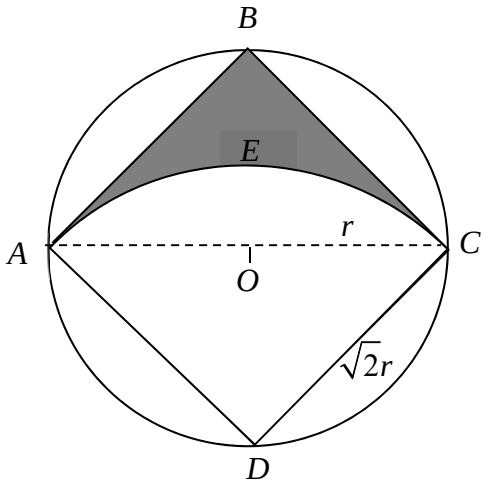
Answer (c) \$..... [1]

(d) Use your answer in (b) to explain whether it is better for Andy to shop at supermarket A or supermarket B.

Answer ..... [1]

24 *ABCD* is a square of sides $\sqrt{2}r$ cm. Its vertices lie on the circumference of a circle, with centre *O* and radius *r*. Arc *AEC* has centre *D*.

What fraction of the circle *ABCD* is **not** shaded? Give your answer in terms of π.



Answer ..... [5]

END OF PAPER

Answer Key

1	-3		16a	(0,3)
2a	$\frac{a-4}{8}$		16b	$-\frac{9}{2}$
2b	4		17ai	30°
3a	$-3 \le x < 4.5$		ii	12
3b	2, 3		17b	172.5°
4a	$\notin$		18a	1.7352×10^{-3}
4b	$\subset$		18b	206
4c	$\varnothing$		19a	119.0°
5	The scale on the vertical axis is not defined so it is not possible to determine the number of customers for Jan and Feb.		19b	7.56
6	16		21ai	(-4, -8)
7	\$80.50		ii	$y = \frac{3}{4}x - 5$
9	2, 2, 5, 6, 6, 6		22a	$\pi rl + 2\pi r^2$
10a	$(x+13)(x-13)$		22bi	$\frac{\sqrt{3}}{4}r^2$
10b	17, 43		ii	$\sqrt{3}r^2$
11a	3120		22c	$\frac{r(\sqrt{3}-2\pi)}{\pi}$
11b	1200		23a	$\mathbf{Q} = \begin{pmatrix} 4 & 3 \\ 2 & 0 \\ 3 & 4 \end{pmatrix}$
12a	153		23b	$\begin{pmatrix} 15 & 10.1 \\ -0.3 & 0.2 \end{pmatrix}$
12b	27		23c	\$0.20
13	15 h 47 min		23d	At supermarket B, Andy incurs an additional -\$0.30, which means he pays 30 cents less at supermarket B. So it is better for Andy to shop at supermarket B.
14a	56		24	$\frac{3\pi-4}{2\pi}$
14b	48			
15a	0.09			
15b	0.1			
15c	0.1^n			
15d	$1-0.1^n$			

2018 Prelim Exam MAP1 Answers

1 $\frac{1}{343} = 7^k$

$\frac{1}{7^3} = 7^k$

$k = -3$

2(a) $\frac{3a}{8} - \frac{a+2}{4}$

$= \frac{3a - 2a - 4}{8}$

$= \frac{a-4}{8}$

(b) $\frac{3a}{8} - \frac{a+2}{4} = 0$

$\frac{a-4}{8} = 0$

$a = 4$

3(a) $-6 < 3 - 2x \leq 9$

$-9 < -2x \leq 6$

$-6 \leq 2x < 9$

$-3 \leq x < 4.5$

(b) $x = 2,\ 3$

4(a) $3 \notin C$ [1]

(b) $\{ 1 \} \subset (A \cap B)$ [1]

(c) $(A \cup B \cup C)' = \varnothing$ [1]

5 The scale on the vertical axis is not defined so it is not possible to determine the number of customers for Jan and Feb.
A comparison of the number of customers over the three months may lead to a misinterpretation.

6 Let his normal speed by x km/h.

$\frac{105}{60}x = \frac{80}{60}(x+5)$

$\frac{7}{4}x = \frac{4}{3}x + \frac{20}{3}$

$x = 16$

7 Let the normal selling price be \$$x$.

Discounted price = 115% of cost price

$$\frac{80}{100}x = \frac{115}{100} \times 56$$

$$x = 80.50$$

8

Let the height/radius of base of each can be h cm.

$$\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2$$

$$\frac{40}{90} = \left(\frac{h_1}{h_2}\right)^2$$

$$\frac{h_1}{h_2} = \frac{2}{3}$$

$$\frac{M_1}{M_2} = \left(\frac{2}{3}\right)^3$$

$$M_2 = 1350$$

Since the amount of beans in the bigger can should be 1350g, Jane's workings is incorrect.

9 Let the smallest number be x.

The numbers are x, x, 5, 6, 6, 6.

$$\frac{x + x + 5 + 6 + 6 + 6}{6} = 4.5$$

$$x = 2$$

The numbers are 2, 2, 5, 6, 6, 6.

10(a) $x^2 - 169$

$= x^2 - 13^2$

$= (x + 13)(x - 13)$

(b) $731 = 900 - 169$

$= 30^2 - 13^2$

$= (30 + 13)(30 - 13)$

$= 43 \times 17$

The factors are 17 and 43.

11(a)

$$-5 = 21 - \frac{h}{120}$$

$$h = 3120 \text{ m}$$

(b)

Let the heights of the two aircrafts be h_1 and h_2.

$T_1 = 21 - \frac{h_1}{120}$

$T_2 = 21 - \frac{h_2}{120}$

$T_1 - T_2 = 21 - \frac{h_1}{120} - 21 + \frac{h_2}{120}$

$10 = \frac{h_2 - h_1}{120}$

$h_2 - h_1 = 1200$

The difference is heights is 1200 m.

12(a) $A : B : C : D$

$5 : 9 : 7 : \frac{9}{2}$

C has 2 parts more than A, which is 12 sweets,

25.5 parts is equivalent to 153 sweets.

(b) 4.5 parts is equivalent to 27 sweets.
Dexter gets 27 sweets

13 In 1 hr,

Jane varnishes $\frac{3}{5}$ jar,

Jim varnishes $\frac{2}{3}$ jar;

together they varnish $\frac{3}{5} + \frac{2}{3} = \frac{19}{15}$ jar.

For 20 jars, they need $20 \times \frac{15}{19} = 15\frac{15}{19}$ hr

$= 15\text{hr } 47\text{min}$

14(a) Let the number of members be y.
$12y$ is divisible by 14 children.

$2^2 \times 3 \times y$ is divisible by 2×7.

y must have 7 as its factor.

Since $50 \le y \le 60$, $\therefore y = 56$

There are 56 members in the club.

(b)

$x = \frac{56 \times 12}{14} = 48$

15(a) P(1st : fail and 2nd : pass)
$= 0.1 \times 0.9$
$= 0.09$

(b) P(at least 2 attempts)
$=1 - $ P(1 attempt)
$= 1 - 0.9$
$= 0.1$

(c) P(1st : fail and 2nd : fail and 3rd : fail .....and nth : fail)
$= (0.1) \times (0.1) \times (0.1) \times ..... \times (0.1)$
$= (0.1)^n$

(d) P(passes the test in one of the first n months)
$=1 - (0.1)^n$

16(a) Midpoint of $AB = (0,3)$

(b) Gradient of $AB = -\frac{3}{4}$

Sub $(6,k)$ into $y = -\frac{3}{4}x$

$k = -\frac{3}{4} \times 6$

$k = -\frac{9}{2}$

17(a)(i) 1 int.$\angle$ +1 ext.$\angle$=180°
6 parts =180°
Each exterior angle = 30°

(ii) No. of sides $= \frac{360}{30} = 12$

(b) Let each of the remaining angles be x.
Sum of interior angles $= 8 \times 180° = 1440°$
$6 \times 125° + 4x = 1440$
$x = 172.5°$

18(a) 482 cm^3 per second $= \frac{0.482 \text{ litres}}{(1 \div 3600) \text{ hr}}$
$= 1735.2$ litres/hr
$= 1.7352 \times 10^{-3}$ litres/hr

(b) $\%\text{ increase} = \dfrac{2.38\times10^4 - 7.79\times10^3}{7.79\times10^3}\times100\%$

$= 205.52\%$

$= 206\%$ (3 sig fig)

19(a) $\sin Q\hat{P}R = \dfrac{7}{8}$

$Q\hat{P}R = 119.0°$ (to 1 dec pl)

(b) Let the particle travel to a point X, nearest to Q.

$Q\hat{X}P = 90°$

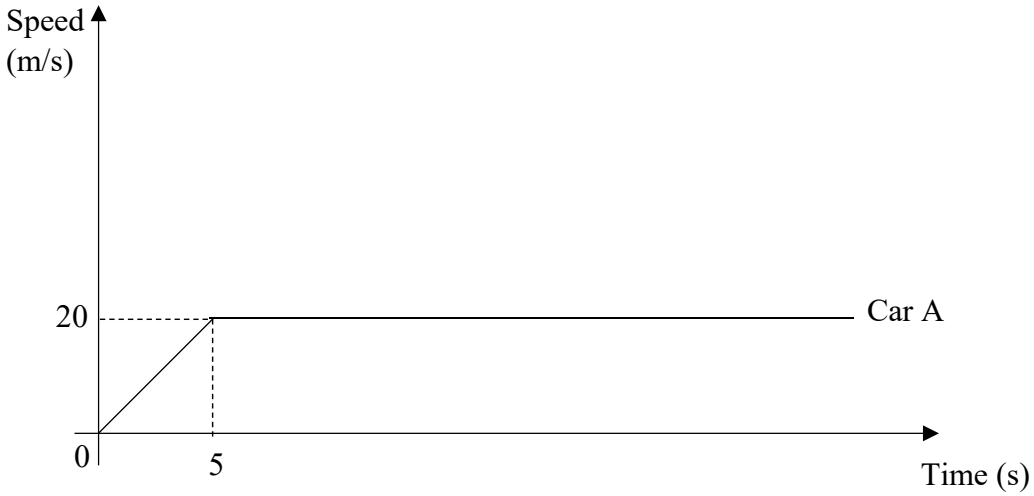
$\sin Q\hat{P}X = \dfrac{7}{8}$

$\dfrac{QX}{3} = \dfrac{7}{8}$

$QX = \dfrac{21}{8}$

$XR = \sqrt{8^2 - \left(\dfrac{21}{8}\right)^2} = 7.56\text{m}$

20(a)



(b) 20 s after A has started,

A has travelled $\dfrac{1}{2}(5)(20)+(15)(20) = 350$ m

For speed of B, v : $\dfrac{v}{(20-5)} = \dfrac{67.5}{30}$

$v = 33.75$

B has travelled $\dfrac{1}{2}(15)(33.75) = 253$ m

So Car B will not overtake Car A within the first 20 s.

21(a)(i) $M = (-4, -8)$

(ii) Gradient $= \frac{3}{4}$

Eqn is $y + 2 = \frac{3}{4}(x - 4)$

$y = \frac{3}{4}x - 5$

(b) $\begin{pmatrix} 9 \\ -24 \end{pmatrix} = -3\begin{pmatrix} -3 \\ 8 \end{pmatrix}$

Since $\begin{pmatrix} 9 \\ -24 \end{pmatrix} = k\begin{pmatrix} -3 \\ 8 \end{pmatrix}$ where k is a constant,

they are parallel.

22(a) Total surface area $= \pi rl + 2\pi r^2$ cm^2

(b)(i) Area of 1 triangle $= \frac{\sqrt{3}}{4}r^2$ cm^2

(ii) Total surface area $= \sqrt{3}r^2$ cm^2

(c) $\pi rl + 2\pi r^2 = \sqrt{3}r^2$

$l = \frac{r^2\left(\sqrt{3} - 2\pi\right)}{\pi r}$

$= \frac{r\left(\sqrt{3} - 2\pi\right)}{\pi}$ cm

23(a) $\mathbf{Q} = \begin{pmatrix} 4 & 3 \\ 2 & 0 \\ 3 & 4 \end{pmatrix}$

(b) $\mathbf{R} = \begin{pmatrix} 1.5 & 2.4 & 1.4 \\ 0.2 & -0.4 & -0.1 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 2 & 0 \\ 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 15 & 10.1 \\ -0.3 & 0.2 \end{pmatrix}$

(c) \$0.20

(d) At supermarket B, Andy incurs an additional -\$0.30,
which means he pays 30 cents less at supermarket B.
So it is better for Andy to shop at supermarket B.

24 Area of shaded region $= \left(\sqrt{2}r\right)^2 - \frac{1}{4}\pi\left(\sqrt{2}r\right)^2$

$= 2r^2 - \frac{\pi}{2}r^2$

Area of circle that is unshaded $= \pi r^2 - 2r^2 + \frac{\pi}{2}r^2$

$= \frac{3\pi}{2}r^2 - 2r^2$

Fraction that is unshaded $= \frac{\frac{3\pi}{2}r^2 - 2r^2}{\pi r^2}$

$= \frac{3\pi - 4}{2\pi}$

COMMONWEALTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2018

MATHEMATICS
PAPER 2

Name: _____ ( ) Class: _____

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC
SECONDARY FOUR NORMAL ACADEMIC (EXPRESS)
4048/2

Tuesday 21 August 2018
08 00 – 10 30
2h 30min

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Name of setter: Mrs Philip

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Factorise completely $x^3 - x - 1 + x^2$. [2]

(b) Express as a single fraction in its simplest form

(i) $\dfrac{y}{x^2 + y^2 - 2xy} + \dfrac{x}{xy - x^2}$, [2]

(ii) $\dfrac{m^2}{4n^4} \div \dfrac{m^5}{\left(6n^3\right)^2}$. [1]

(c) Solve the equation $\dfrac{5}{3 - 2x} = x - 7$. [3]

(d) **(i)** Express $y^2 + 7y + 5$ in the form $\left(y + a\right)^2 + b$. [1]

(ii) **Hence** solve the equation $y^2 + 7y + 5 = 0$, giving your answers correct to two decimal places. [2]

---

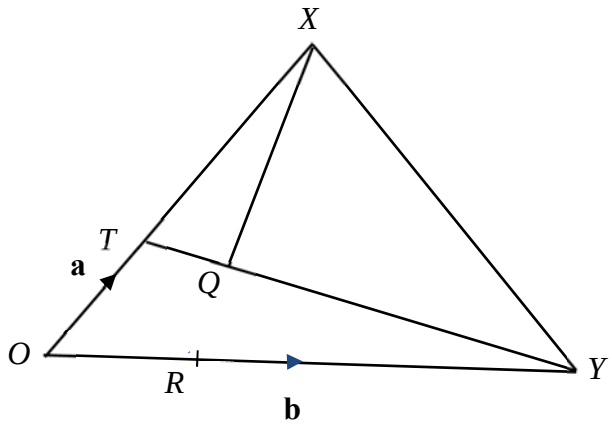
2 The lengths of the sides of a triangle are $(x - 1)$ cm, $(x + 1)$ cm and $(x + 3)$ cm. The largest angle is 120°.

(i) Write down an equation in x to represent this information. [3]

(ii) Solve the equation and find the sides of the triangle. [3]

(iii) Find the **sine** of the smallest angle. [2]

3



In the diagram, $\overrightarrow{OT} = \mathbf{a}$ and $\overrightarrow{OY} = \mathbf{b}$.

X is a point on OT produced such that $OT : TX = 2 : 3$ and Q is a point on TY such that $TQ : QY = 1 : 5$.

(a) Express as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{XY}$, [1]

(ii) $\overrightarrow{TY}$, [1]

(iii) $\overrightarrow{QY}$, [1]

(iv) $\overrightarrow{XQ}$. [2]

R lies on OY such that $\overrightarrow{OR} = \frac{1}{4}\overrightarrow{OY}$.

(b) Express the vector $\overrightarrow{XR}$ in terms of **a** and of **b**. [2]

(c) Show that $\overrightarrow{XQ} = h\overrightarrow{XR}$ where h is a constant. [1]

(d) Write down two facts about X, Q and R. [2]

(e) Find the ratio of the area of triangle XQY to the area of triangle OXY. [2]

4 **(a)** Jack wants to buy a toy from a Japanese web-site for 4650 yen. The Japanese customs charges a 16% VAT for all overseas mail orders. The freight charge for product delivery to Singapore is 2800 yen.

(i) Use the information in the conversion table to calculate the total cost in Singapore dollars, if Jack buys the toy by mail order. [2]

Singapore dollars to 100 units of foreign currency.	Buying	Selling
Japanese Yen	1.2660	1.2798

(ii) The same toy, normally priced locally at S$160.50, exclusive of GST, was sold at a discount of 20% during the Great Singapore Sale. Calculate the selling price of the toy, purchased during the sale inclusive of 7% GST. [2]

(iii) Calculate the percentage Jack saved when buying the toy by mail order. [2]

(b) Amanda invested $ 8000 over a period of 2 years into two different investment plans.
Plan A offers 9.25% per annum of compound interest compounded annually,
Plan B offers 9 % per annum compound interest compounded monthly.

Which plan is a better choice? And why? [3]

(c) The cash price of a Television set is $ 4000.
John purchases the Television set on a hire purchase scheme making n monthly instalments of $90.

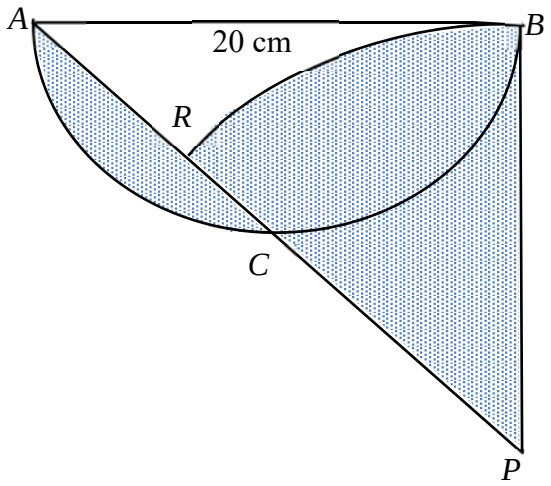
(i) Express, in terms of n,
(a) the total amount payable by hire purchase, [1]
(b) the interest incurred. [1]

(ii) The flat rate for the hire purchase loan is 3% per annum. Express the interest payable in terms of n. [1]

(iii) Form an equation in n and solve it. [1]

(iv) Hence, state the duration of the hire purchase loan in years and months. [1]

5



The diagram shows a semi-circle ACB and a sector PBR of a circle with centre P. It is given that AB is perpendicular to BP and $AB=RP = 20$ cm.

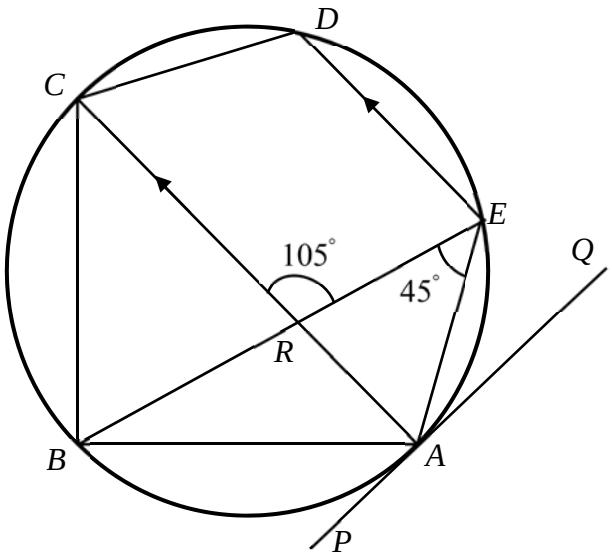
(i) Find, in radians, the angle BPA. [1]

(ii) For the shaded region $ACBRA$, find, correct to one decimal place,

(a) the area, [3]

(b) the perimeter. [2]

6 In the diagram, A, B, C, D and E are points on a circle.
DE is parallel to CA, PQ is a tangent to the circle at A.
$AB = BC$, angle $BEA = 45^\circ$ and angle $CRE = 105^\circ$.



(a) **(i)** Show that AC is the diameter of the circle. [2]

Find, stating reasons clearly,

(ii) angle EAQ, [2]

(iii) angle CDE. [2]

(b) Given that $AR = \frac{1}{4}AC$ and the area of triangle ARE is 90 cm^2, calculate the area of triangle BRC.

[2]

7 The variables x and y are connected by the equation

$$y = \frac{8}{x} - 3x + 8 .$$

Some corresponding values of x and y, correct to one decimal places, are given by the table below.

x	1	2	3	4	5	6	7	8
y	13	6	1.7	–2	–5.4	–8.7	p	–15

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for the values of x in the range $1 \leq x \leq 8$. Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for the values of y in the range $-16 \leq y \leq 14$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve at $x = 1.5$. [2]

(d) By drawing a suitable straight line, solve the equation $2x^2 - 6.5x = 4$. [2]

8 A hollow glass container, shown in Diagram 1, is formed by joining a hemispherical base to a cone.
The hemisphere has a radius of 6 cm and the height of the cone is h cm.
The volume of the cone is 980 cm^3.

.

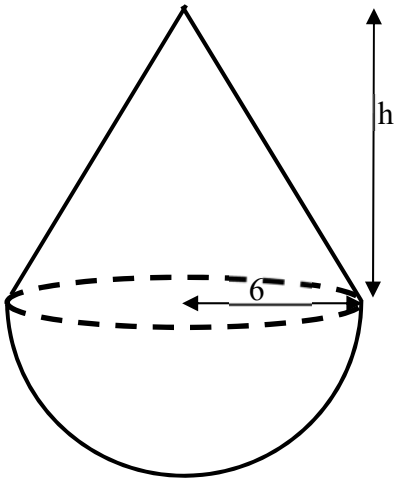


Diagram 1

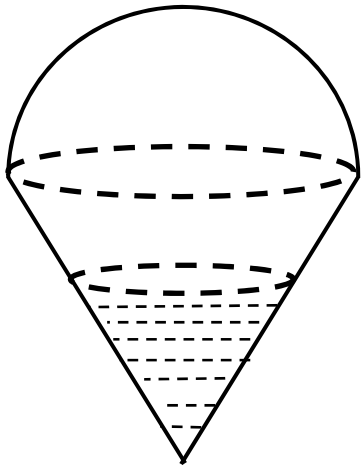


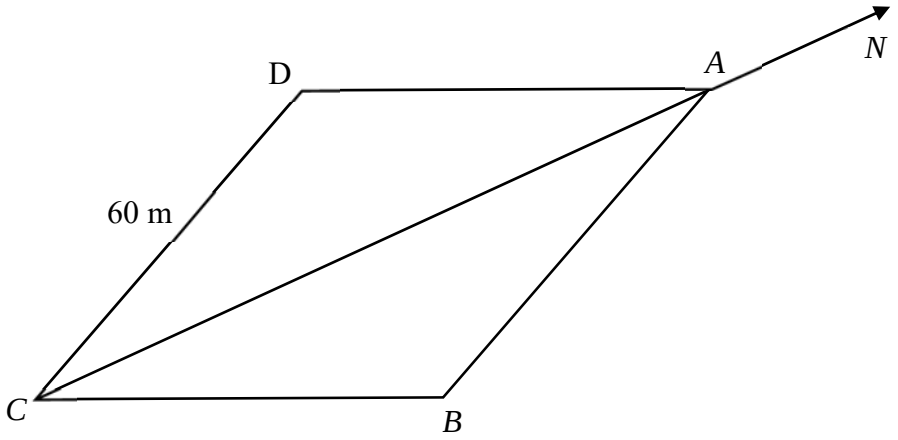
Diagram 2

(i) Show that $h = 26.0$ cm. [2]

(ii) Find the surface area, in square metres, of the exterior of container. [3]

(b) The container was half filled with water and then inverted as shown in diagram 2.
Find the height of water level in Diagram 2. [4]

9



In the diagram, A, B, C and D are points on a level field forming a shape of a rhombus with $CD = 60$ m and A is due north of C. The bearing of D from C is 332°.

(a) Find the bearing of D from A. [1]

(b) State the bearing of B from D. [1]

(c) Calculate the distance BD. [1]

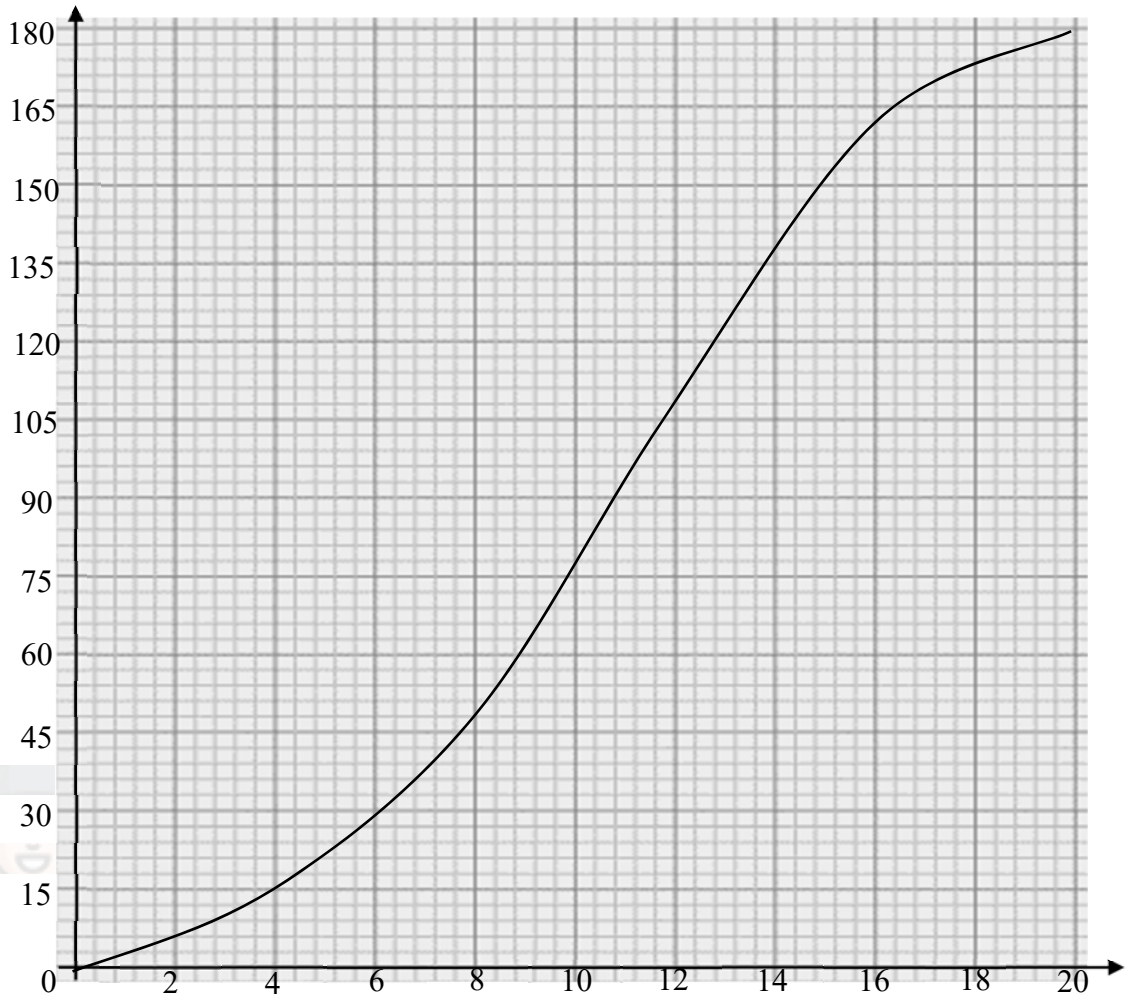
A bird flies horizontally from C to A at the fixed height of 30 m.

(d) Find the greatest angle of elevation of the bird from B as the bird flies above CA. [2]

The field is drawn on a map with scale 1 : 2000.

(e) Find the area of the field $ABCD$ on the map in cm^2. [2]

10 **(a)** 180 alkaline batteries were tested to find out how long they would last.
The cumulative frequency curve below shows the distribution of their life in hours.



(i) Use the graph to estimate

(a) the median time, [1]

(b) the interquatile range of the battery life, [2]

(c) the percentage of the batteries which lasted at least 12 hours. [1]

(ii) The life in hours of 180 lithium batteries has the same interquartile range as the alkaline batteries but a higher median.
Describe how the cumulative frequency curve for the life of lithium batteries differ from the curve for the alkaline batteries. [1]

(b) The table shows the life in hours of another set of 300 batteries that were tested.

Life (x hours)		$2 \leq x < 4$	$4 \leq x < 6$	$6 \leq x < 8$	$8 \leq x < 10$	$10 \leq x < 12$
Frequency	Alkaline	4	22	30	62	10
	Lithium	3	41	52	71	5

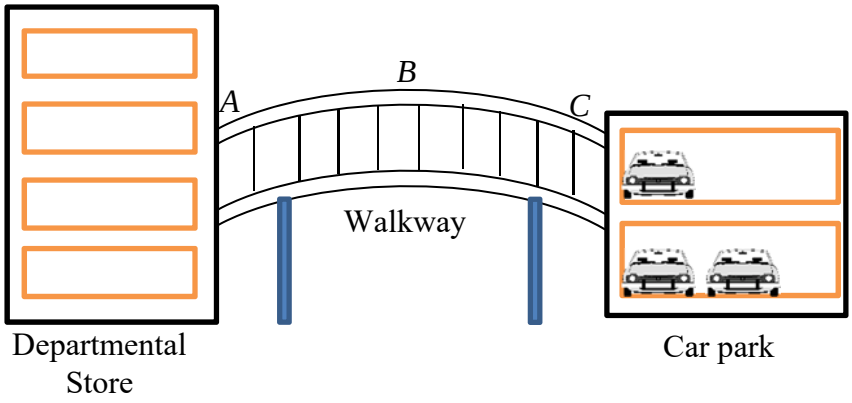
(i) Tom randomly chose one of these batteries.
Find, as a fraction in its lowest terms, the probability that the battery has

(a) a life of 10 hours or more **,** [1]

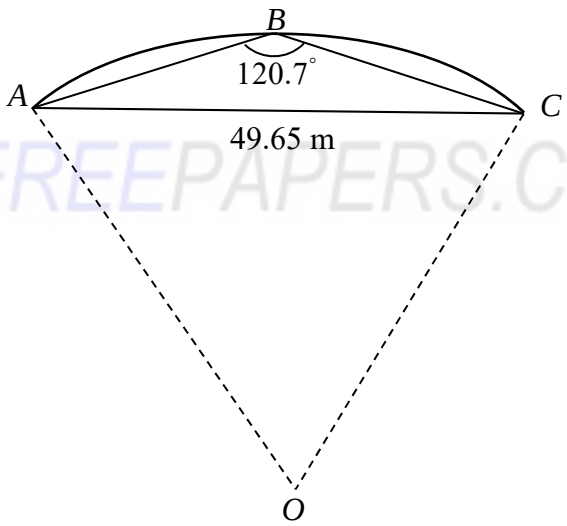
(b) life of under 6 hours. [1]

(ii) Tom randomly chose two of these batteries.
Find the probability that both the batteries chosen are alkaline and both have a battery life of under 8 hours. [2]

11 The figure shows a pedestrian walkway joining a multi-storey car park and a Departmental Store.



To estimate its length the walkway is modelled by the arc ABC as shown in the figure below, where A is the entrance to the department store and C is the exit to the car park.
The arc ABC is part of a sector with centre O.



Given $AC = 49.65$ m and angle $ABC = 120.7^\circ$,

(a) show that $AO = 29$ m. [3]

(b) show that the length of arc ABC is 60 m. [1]

(c) As a safety measure , John is required to conduct an emergency evacuation drill. He planned to conduct the drill on a Thursday and record the time taken by visitors to evacuate the walkway. The table below shows the average walking speed of visitors along the walkway to the Departmental Store at various timings in a day.

Time	Average walking speed in km/h	
	Weekends	Week days
11 00	4.5	5.0
14 00	3.5	4.5
17 00	3.5	3.5

At 12 noon, John will make an announcement for everyone to evacuate the department store via exit *A*.

John predicts each visitor will be able to evacuate the walkway from *A* to *C* in less than a minute.

(i) With clear mathematical working, determine whether John's prediction is accurate. [3]

(ii) State 1 assumption you made in the calculation [1]

End of Paper

Answers to Sec4 Preliminary Examination Paper 2

1	(a)	$(x+1)^2(x-1)$	5 (i)	$\frac{\pi}{4}$
	(b)(i)	$\frac{2y-x}{(x-y)^2}$	(ii)(a)	$114.2cm^2$
	(ii)	$\frac{9n^2}{m^3}$	(b)	55.4 cm
	(c)	$x = 6.5$ or $x = 2$		
	(d)(i)	$\left(y+3.5\right)^2 - \frac{29}{4}$	6(a)(ii)	30^0
	(ii)	y = -0.81 or -6.91	(iii)	120^0
			(b)	540 cm^2
2	(i)	$x^2 - 3x - 4 = 0$		
	(ii)	$x = 4 \quad or \quad x = -1$ The sides are 3 cm, 5 cm and 7 cm.	7(a)	P = -11.9 (3 s f)
	(iii)	0.371	(c)	Gradient of tangent ≈ 6.55
			(d)	≈ 3.78
3	(a)(i)	$\underset{\sim}{b} - \frac{5}{2}\underset{\sim}{a}$		
	(ii)	$\underset{\sim}{b} - \underset{\sim}{a}$	8 (ii)	$0.0729\,m^2$
	(iii)	$\frac{5}{6}(\underset{\sim}{b} - \underset{\sim}{a})$	(b)	23.4
	(iv)	$\frac{1}{6}(\underset{\sim}{b} - 10\underset{\sim}{a})$		
	(b)	$\frac{1}{4}(\underset{\sim}{b} - 10\underset{\sim}{a})$	9(a)	208^0 (b) 090^0 (c) 56.3 m
	(c)	$\overrightarrow{XQ} = \frac{2}{3}\overrightarrow{XR}$	(d)	46.8^0 (e) 7.46 cm^2
	(d)	i) X , Q and R are collinear points. ii) $XQ : XR$ = 2 : 3		
	(e)	1 : w	10(i)	(a)10.8hrs (b) 6 hours (c) 40%
			(ii)	The curve shifts to the right of the given curve.
4	(a)(i)	\$104.87	b(i)	(a) 1/20 (b) 7/30 (c) 154/4485
	(ii)	\$137.39 (2dp)		
	(iii)	23.7%		
			4(ii)	\$10n
	(b)	Plan B is better as the plan yields better compound interest for the amount \$ 8000 invested.	4(iii)	n=50
	(c)(i)(a)	\$ (90 n)	4(iv)	4 yrs 2 months
	(b)	\$(90n – 4000)		

Commonwealth Secondary School
Preliminary Examination 2018 (Sec 4E/4NA/5NA)
Solutions to E Math Paper 2

1	(a)	$x^3 - x - 1 + x^2$ $= x^3 - x + x^2 - 1$ $= x(x^2 - 1) + (x^2 - 1)$ $= (x^2 - 1)(x + 1)$ $= (x + 1)(x - 1)(x + 1)$ $= (x + 1)^2(x - 1)$	
	(b)(i)	$\frac{y}{x^2 + y^2 - 2xy} + \frac{x}{xy - x^2}$ $= \frac{y}{(x - y)^2} + \frac{x}{x(y - x)}$ $= \frac{y}{(x - y)^2} - \frac{x}{x(x - y)}$ $= \frac{y}{(x - y)^2} - \frac{(x - y)}{(x - y)^2}$ $= \frac{2y - x}{(x - y)^2}$	
	(b)(ii)	$\frac{m^2}{4n^4} \div \frac{m^5}{\left(6n^3\right)^2}$ $\frac{m^2}{4n^4} \times \frac{(6n^3)^2}{m^5}$ $= = \frac{m^2}{4n^4} \times \frac{36n^6}{m^5}$ $= \frac{9n^2}{m^3}.$	
	(c)	$\frac{5}{3 - 2x} = x - 7$ $5 = (x - 7)(3 - 2x)$ $2x^2 - 17x + 26 = 0$ $(2x - 13)(x - 2) = 0$ $x = 6.5 \quad or \quad x = 2$	

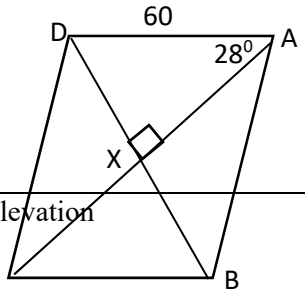
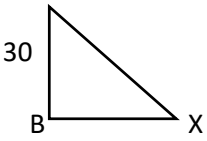
	(d)(i)	Express y^2+7y+5 in the form $(y+a)^2+b$. y^2+7y+5 $=(y+3.5)^2+5-(3.5)^2$ $=(y+3.5)^2-\frac{29}{4}$	
	(ii)	$y^2+7y+5=0$ $(y+3.5)^2-\frac{29}{4}=0$ $(y+3.5)^2=\frac{29}{4}$ $(y+3.5)=\pm\sqrt{\frac{29}{4}}$ $y=-3.5\pm\sqrt{\frac{29}{4}}$ $y=-0.81$ *or* -6.91 (2 *decimal places*)	
2	(i)	Using the cosine Rule , $\cos 120^\circ=\frac{(x-1)^2+(x+1)^2-(x+3)^2}{2(x-1)(x+1)}$ $-\frac{1}{2}=\frac{x^2-6x-7}{2(x^2-1)}$ $x^2-3x-4=0$	
	(ii)	$x^2-3x-4=0$ $(x-4)(x+1)=0$ $x=4$ *or* $x=-1$ For x = 4, the sides of the triangle are 3 cm, 5 cm and 7 cm.	
	(iii)	**sine** of the smallest angle Let α be the smallest angle. $\frac{\sin\alpha}{3}=\frac{\sin 120^\circ}{7}$ $\sin\alpha=\frac{3\sin 120^\circ}{7}$ $=0.371$ (3 s f)	
3	(a)(i)	$\overrightarrow{XY}=\overrightarrow{OY}-\overrightarrow{OX}$ $=\underset{\sim}{b}-\frac{5}{2}\underset{\sim}{a}$	

	(ii)	$\overrightarrow{TY} = \overrightarrow{OY} - \overrightarrow{OT}$ $= \underset{\sim}{b} - \underset{\sim}{a}$	
	(iii)	$\overrightarrow{QY} = \frac{5}{6}\overrightarrow{TY}$ $= \frac{5}{6}(\underset{\sim}{b} - \underset{\sim}{a})$	
	(iv)	$\overrightarrow{XQ} = \overrightarrow{XY} + \overrightarrow{YQ}$ $= \frac{1}{6}(\underset{\sim}{b} - 10\underset{\sim}{a})$	
	(b)	$\overrightarrow{XR} = \overrightarrow{OR} - \overrightarrow{OX}$ $= \frac{1}{4}\overrightarrow{OY} - \frac{5}{2}\underset{\sim}{a}$ $= \frac{1}{4}\underset{\sim}{b} - \frac{5}{2}\underset{\sim}{a}$ $= \frac{1}{4}(\underset{\sim}{b} - 10\underset{\sim}{a})$	
	(c)	$\overrightarrow{XQ} = \frac{1}{6}(4\overrightarrow{XR}) = \frac{2}{3}\overrightarrow{XR}$	
	(d)	i) X , Q and R are collinear points. ii) $XQ : XR = 2 : 3$	
	(e)	Area of triangle XQY : Area of triangle $OXY = 1 : 2$	
4	(a)(i)	Total cost in sing dollars $= (1.16 \times 4650 + 2800) \times \frac{1.2798}{100}$ $= \$104.87$	
	(ii)	Selling price of toy locally = $1.07 \times 0.8 \times 160.50 = \$137.388$ $= \$137.39 \quad (2dp)$	
	(iii)	Percentage Jack saved = $\frac{(137.388 - 104.87)}{137.388} \times 100$ $= 23.7\%$	
	(b)	Plan A $\quad Amount = 8000\left(1 + \frac{9.25}{100}\right)^2$ $= \$9548.45$	

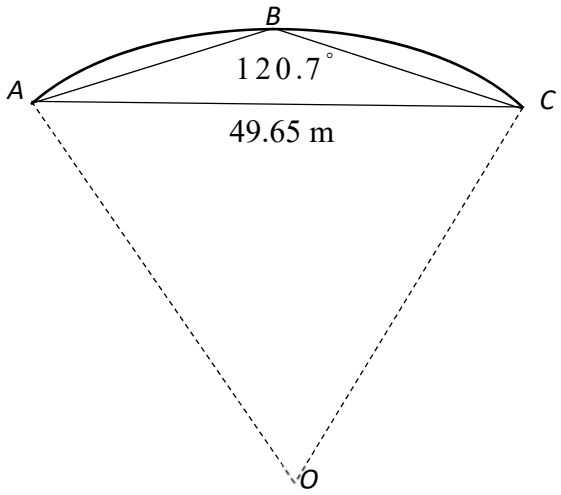
		Plan B $Amount = 8000\left(1+\frac{\frac{9}{12}}{100}\right)^{24}$ $= \$9571.31$ Plan B is better as the plan yields better compound interest for the amount \$ 8000 invested.	
	(c)(i)a	\$ (90 n)	
	(b)	\$(90n – 4000)	
	(ii)	$Interest\ payable = \frac{4000 \times 3 \times \frac{n}{12}}{100}$ $= \$10n$ Note the interest calculated should be simple interest unless otherwise stated.	
	(iii)	90 n - 4000 = 10 n 80 n = 4000 n = 50 months	
	(iv)	50 months = 4 years 2 months.	
5	(i)	*AB=RP* = BP=20 cm as P is the centre of circle. $\Rightarrow$ ABR is an isosceles right angled triangle. Angle BPA = $\frac{\pi - \frac{\pi}{2}}{2} = \frac{\pi}{4}$	
	(ii)(a)	Area of region *ACBRA* = Area of semicircle *ACB* – Area of unshaded region *ARB* $= \frac{1}{2}\pi(10)^2 -$ (Area of triangle ABP – Area of sector PRB) $= 50\pi - \left(\frac{1}{2}(20)(20) - \frac{1}{2}(20)^2\frac{\pi}{4}\right)$ $= 50\pi - 200 + 50\pi$ $= 114.2\,cm^2\ (1dp)$	
	(b)	Perimeter of *ACBRA* = Circumfrence of semi-circle ACB + length of arc RB + length of line segment AR. $= \pi(10) + 20\left(\frac{\pi}{4}\right) + (AP - PR)$	

		$= \pi(10) + 20\left(\frac{\pi}{4}\right) + \left(\sqrt{(20)^2 + (20)^2} - 20\right)$ = 55.4 cm (1 *dp*)	
6	(a) (i)	Angle *BCA* = Angle *BEA* = 45^0 *(Angles in the same segment)* Angle *BCA* = Angle *BAC* (given AB = BC) Therefore Angle *BCA* = Angle *BAC* = 45^0 Angle *CBA* = $180^0 - 45^0 - 45^0 = 90^0$. As angle in a semi-circle is a right angle and angle CBA = 90^0, Implies AC is a diameter.	
	(ii)	Angle *RAE* = $105^0 - 45^0 = 60^0$. *(In triangle RAE, exterior angle is the sum of interior opposite angles)* Angle EAQ = $90^0 - 60^0 = 30^0$ (AC is the diameter, radius is perpendicular to tangent at A)	
	(iii)	Angle CDE = 180^0 – angle CAE = $180^0 - 60^0 = 120^0$ (In a cyclic quadrilateral CAED the opposite angles are supplementary.)	
	(b)	Note that sides *AR* and *BR* are the corresponding sides of similar triangle *BRC* and triangle *ARC*. By Sine Rule $\frac{BR}{\sin 45^\circ} = \frac{3}{\sin 60^\circ}$ $BR = \frac{3\sin 45^\circ}{\sin 60^\circ}$ $\frac{Area\ of\ triangle\ ARE}{Area\ of\ triangle\ BRC} = \left(\frac{AR}{BR}\right)^2 = \frac{1}{6}$ Therefore Area of triangle *BRC* = 6 × 90 = 540 cm^2	
7	(a)	P = -11.9 (3 s f)	
	(c)	Gradient of tangent ≈ 6.55 (3 sf)	
	(d)	$2x^2 - 6.5x = 4$ Dividing by *x*	

		$2x - 6.5 = \frac{4}{x}$ *Multiplying by* 2 $4x - 13 = \frac{8}{x}$ $\frac{8}{x} - 3x + 8 = x - 5$ Insert the line $y = x-5$. The point of intersection is the solution to the given equation within the given range. $x \approx 3.78$ (3 sf)	
8	(a) (i)	Given volume of cone = 980 cm^3 $\frac{1}{3}\pi r^2 h = 980$ $h = \frac{3 \times 980}{6^2 \times \pi}$ $h = 25.9848...$ $h = 26.0cm\,(shown)$	
	(ii)	$Surface\ Area = 2\pi r^2 + \pi rl$ $= 2\pi 6^2 + \pi(6)\sqrt{25.9848^2 + 6^2}$ $= 729.17\ cm^2$ $= 0.0729\,m^2$	
	(b)	Volume of water when the container is full $= \frac{2}{3}\pi(6)^3 + 980$ $= 1432.3892$ Volume of water in the container = 0.5 × 1432.3892= 716.1946 cm^3. Let x be the height of water level required $\frac{716.195}{980} = \left(\frac{x}{26}\right)^3$ x = 23.4 cm (3 sf)	
9	(a)	Bearing of D from A = $180^0 + 28^0 = 208^0$.	.
	(b)	Bearing of B from D = 090^0 .	
	(c)	$BD = 2\,DX$ (X is the midpoint of the diagonals AC and BD)	
		= 2 (60)(*sin* 28^0)	

		= 56.3 m (3 s f)	
		Note BX = XD CX =XA  DX=60 sin28^0 AX = 60 cos 28^0	
	(d)	Greatest angle of elevation $= \tan^{-1}\left(\frac{30}{BX}C\right)$ $= \tan^{-1}\left(\frac{30}{60\sin 28^\circ}\right)$ $= 46.8^0$ (1 d p)	Bird  BX=XD
	(e)	Area of the field = 4 (Area of triangle *ADX*) = 4 (0.5)(*AX*)(*DX*) = 4 (0.5)(60)(sin 62^0)(60 cos 62^0) =2984.535 m^2 Given the scale 1 : 2000 1 cm represents 20 m 1 cm^2 represents 400 m^2 Area on the map = 2984.535 ÷ 400 = 7.46 cm^2 (3 s f)	
10	(a)(i)a	Median time = 10.8 hours	
	(b)	IQR = $Q_3 - Q_1$ = 13.6 – 7.6 = 6 hours	
	(c)	% of battery with life ≥ 12 hours $= \frac{72}{180} \times 100$ $= 40\%$	
	(ii)	The curve shifts to the right of the given curve due to a higher median value.	
	(b)(i)a	P(life of 10 hours or more) = $\frac{15}{300} = \frac{1}{20}$	
	(b)	P(life under 6 hours) $= \frac{70}{300} = \frac{7}{30}$	
	(ii)	P (both batteries have a life under 8 hours) $= \left(\frac{56}{300}\right)\left(\frac{55}{299}\right)$ $= \left(\frac{154}{4485}\right)$	

11



(a) Angle AOC = $360^0 - 2(120.7^0)$ (Angle at centre is twice angle on circumference)
$= 118.6^0$ Note that AB $\neq$ BC

Angle OAC = Angle OCA (Triangle OAC is isosceles as OA =OC=R)

$$= \frac{180^\circ - 118.6^\circ}{2}$$

$$= 30.7^\circ$$

Let R be the radius of the sector AOBC.

By Sine Rule

$$\frac{R}{\sin 30.7^\circ} = \frac{49.65}{\sin 118.6^\circ}$$

$$R = \frac{49.65 \sin 30.7^\circ}{\sin 118.6^\circ}$$

$$R = 29.0m \ (3\ sf)$$

(*b*) $$Length\ of\ arc = \frac{118.6^\circ}{360^\circ} \times 2\pi R$$

$$= \frac{118.6^\circ}{360^\circ} \times 2\pi(28.871)$$

$$= 60m\ (shown)$$

(c) Speed = 5 km/h = 5000 m/h

$$Time = \frac{60}{5000} = \frac{3}{250}h = 44\sec$$

John's prediction is right as it takes less than a minute to evacuate the walkway.

(d) Assuming at 12 noon the walking speed is 5 km/h as it is at 1100 am.

Name : _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2018
Secondary 4

Wednesday
15 Aug 2018

MATHEMATICS
Paper 1

4048/01
2 h

INSTRUCTIONS TO CANDIDATES
Write your name, class and index number on the question paper.
Write in dark blue or black ink on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give your answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

INFORMATION FOR CANDIDATES
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Marks
80

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{\text{r}}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Write the following in order of size, **largest** first.

$$-\frac{7}{8} \quad , \quad -8 \times 10^{-1} \quad , \quad -0.\dot{8}\dot{1}$$

Answer : ……..... [1]

---

2 **(a)** Calculate $\dfrac{\sqrt{4.73}-3.28}{\sqrt[3]{99.7}\times 1.25}$.

Write down the first four digits on your calculator display.

(b) The number 102 999 corrected to ***n*** significant figures is 103 000.
Write down the largest possible value of ***n***.

Answer : **(a)** …..…….. [1]

(b) …..…….. [1]

---

3 One hundred identical drops of oil have a total mass of 4550 milligrams.
1 milligram = 1×10^{-3} grams.
It is given that 1 cm^3 of oil weighs 0.91 grams.
Calculate the volume of one drop, in m^3, giving your answer in standard form.

Answer : ……..……... m^3 [2]

4 The numbers 98, 784 and 1176, written as the products of their prime factors, are

$98 = 2 \times 7^2$, $784 = 2^4 \times 7^2$ and $1176 = 2^3 \times 3 \times 7^2$.

(a) Find the largest possible integer, w, which divides 98, 784 and 1176 exactly.

(b) Find the smallest integer, p, such that $\sqrt[3]{\frac{784 \times 1176}{p}}$ is a whole number.

Answer : **(a)** $w =$ ……..……..……... [1]

(b) $p =$ …...…..…… [1]

---

5 The map of The Singapore Zoo has a scale of 1 : 3000.

(a) If the length from the entrance to Reptile Garden is 4.5 cm on the map, calculate the actual distance, in kilometres.

(b) The actual area of the Baboon Enclosure is 0.0225 km^2. Calculate the area on the map that represents the enclosure, giving your answer in square centimetres.

Answer : **(a)** ...…...…..……..... km [1]

(b) ……..……..…….. cm^2 [1]

---

6 Factorise $p^2 + 5p + 6$.

Hence, explain why $p^2 + 5p + 6$ is always even for all positive integer values of p.

Answer : …..……..…….....

……..

……..

……..

..……[2]

7 A tank is filled at a constant rate with water by 7 taps in 45 seconds.

(a) If two additional taps are used, how long does it take to fill the same tank?

(b) If the volume of the tank is V cm^3, express the rate of flow of water from each tap in terms of V.

Answer : **(a)** ……..…… s [1]

(b) ……..…… cm^3/s [1]

---

8 Anne planned to travel to Japan for holidays. She wanted to change 2800 Singapore Dollars (SGD) into Japanese Yen (JPY).

One day she went to *XYZ* Shopping Mall and saw two money changer shops there. Should Anne change her money at Shop *A* or Shop *B*? Justify your answer with clear calculation.

Money Changer Shop *A*
Exchange rate :
1 SGD = 80.32 JPY

Money Changer Shop *B*
Exchange rate :
100 JPY = 1.25 SGD

Answer : …...……

……. [3]

9 Solve the equation $\frac{1}{4y} = 2^{-6}$.

Answer : $y =$ …..…….... [2]

---

10 The income tax payable in 2017 was calculated as follows:

Chargeable Income	Income Tax Rate (%)	Gross Tax Payable ($)
First $20,000	0	0
Next 10,000	2	200
First $30,000	–	200
Next 10,000	3.50	350
First $40,000	–	550
Next $40,000	7	2,800
First $80,000	–	3,350
Next $40,000	11.5	4,600

Mr Lim paid an income tax amount of $2545 for his income earned in 2017.
Calculate his amount of chargeable income in 2017.

Answer : $ …….……..…….... [3]

11 **(a)** Express $-x^2+4x-5$ in the form of $a(x+h)^2+k$.

Answer : **(a)** ……..……..…….... [2]

(b) **Hence**, sketch the graph of $y=-x^2+4x-5$ on the axes below. Indicate clearly the turning point, the values where the graph crosses the *x*- and *y*- axes (if any).

Answer : **(b)** [2]

(c) **Hence**, explain why the equation $-x^2+4x-5=0$ has no solution.

Answer : **(c)** ……..……

……..……. [1]

12 The following diagram shows 2 geometrically similar bottles of perfumes of the same brand.

Bottle ***A***
135 ml
$48

Bottle ***B***
320 ml
$95

(a) Is the cost of the perfume directly proportional to the quantity of the perfume? Explain with clear calculation.

Answer : **(a)** ……

..……

…….…… [3]

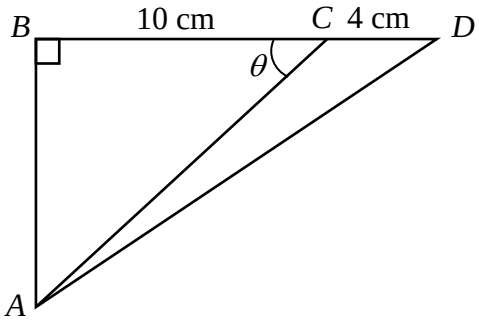
(b) Write down the ratio of the height of Bottle ***A*** to that of Bottle ***B***.

(c) It is given that the surface area of Bottle ***A*** is 270 cm^2. Calculate the surface area of Bottle ***B***.

Answer : **(b)** ……. : …….. [1]

(c) ..……...…… cm^2 [2]

13 In the diagram, not drawn to scale, BCD is a straight line. Given that BC = 10 cm, CD = 4 cm, and $\cos\theta = \frac{4}{5}$.



Find

(a) the length of AC,

(b) the value of $\tan\angle ACD$, giving your answer as a fraction in its simplest form,

(c) the exact value of AD^2,

(d) the shortest distance of C to AD.

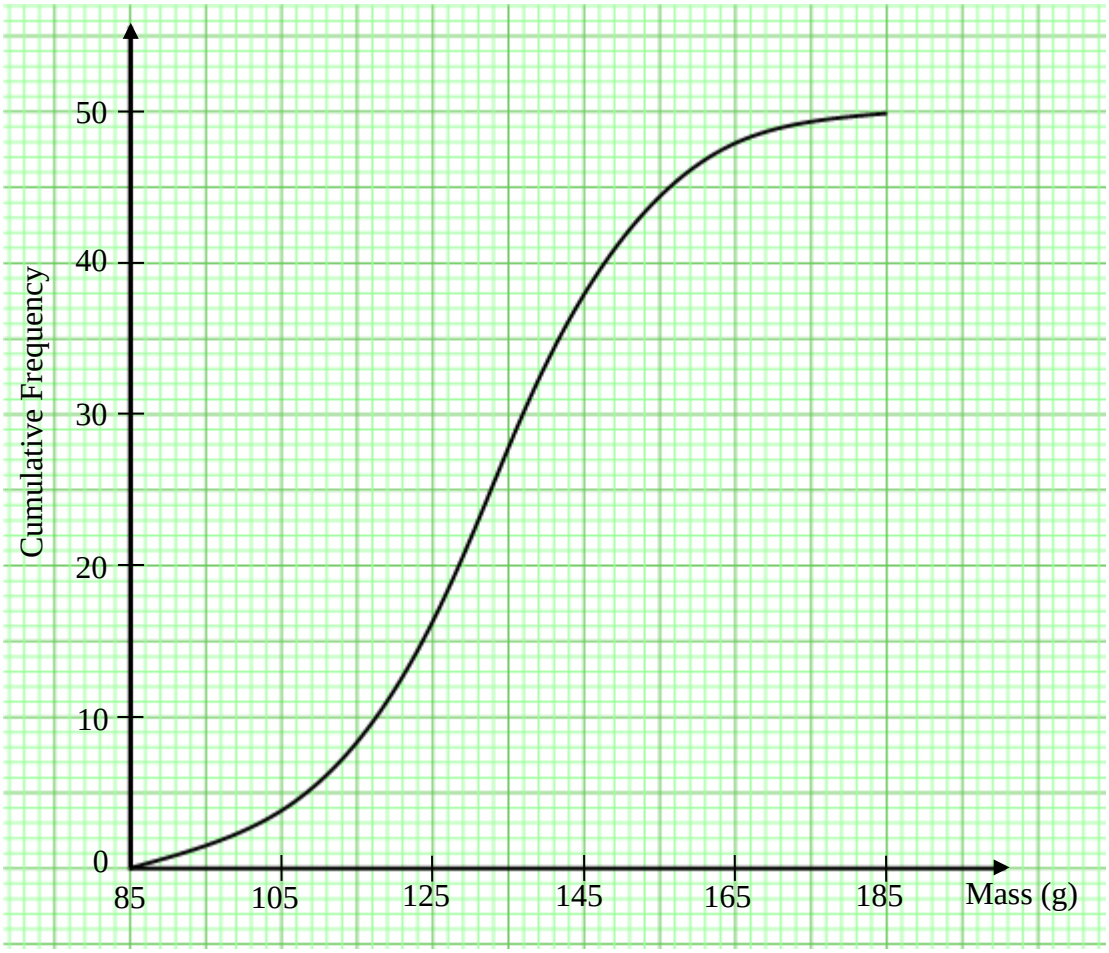
Answer : **(a)** ...…..…..……..... cm [1]

(b) ……..…….... [2]

(c) …...…….. [1]

(d) …...……... cm [2]

14 **(a)** The cumulative frequency curve below shows the distribution of the masses of 50 apples.



(i) The masses of these apples was tabulated into a grouped frequency table as follows. Find the values of p and of q.

Mass (x g)	Number of apples
$x < 85$	0
$85 \le x \le 105$	4
$105 \le x \le 125$	p
$125 \le x \le 145$	22
$145 \le x \le 165$	q
$165 \le x \le 185$	2

Answer : **(a)(i)** p = …..…….

q = ……. [1]

(ii) Using your grouped frequency table, calculate an estimate of

(a) the mean mass of the apples,

(b) the standard deviation.

Answer : **(a)(ii)(a)** ……..…… g [1]

(b) ……. g [1]

(b) Another set of 50 apples have the same median but a smaller standard deviation. Describe how the cumulative frequency curve will differ from the given curve.

Answer : **(b)** ……

……

…… [1]

---

15 The table below shows a number sequence.

Pattern	No. of terms (N)	Value (V)
1^2	1	1
$1^2 - 2^2$	2	–3
$1^2 - 2^2 + 3^2$	3	6
$1^2 - 2^2 + 3^2 - 4^2$	4	–10
$1^2 - 2^2 + 3^2 - 4^2 + 5^2$	5	a
$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2$	6	b
⋮		

(a) State the value of a and b.

(b) Form an equation for V in terms of N when

(i) N is an odd number.

(ii) N is an even number.

(c) Hence, find the value of $17^2 - 18^2 + 19^2 - 20^2 \ldots + 125^2$.

Answer : **(a)** a = …… , b = …… [2]

(b)(i) …… [1]

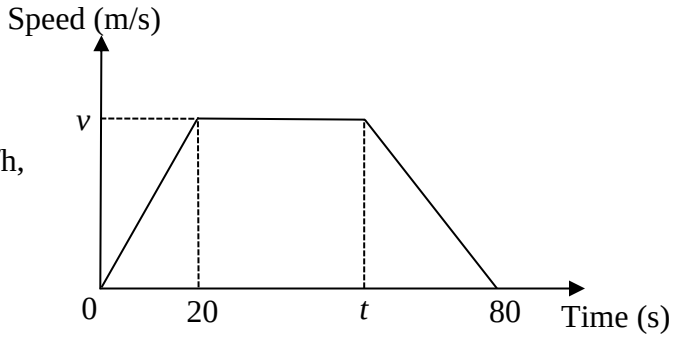
(ii) …… [1]

(c) …… [2]

16 The diagram below shows the speed-time graph of a bus journey.
The bus accelerated from rest at 1.25 m/s^2 to a speed of v m/s in 20 seconds, and travelled at this speed until t seconds before it came to a stop at t = 80 seconds. The total distance travelled for the whole journey was 1.45 km.

(a) Find the values of
  (i) v,
  (ii) t.

(b) Calculate the average speed, in km/h, of the bus for the whole journey.



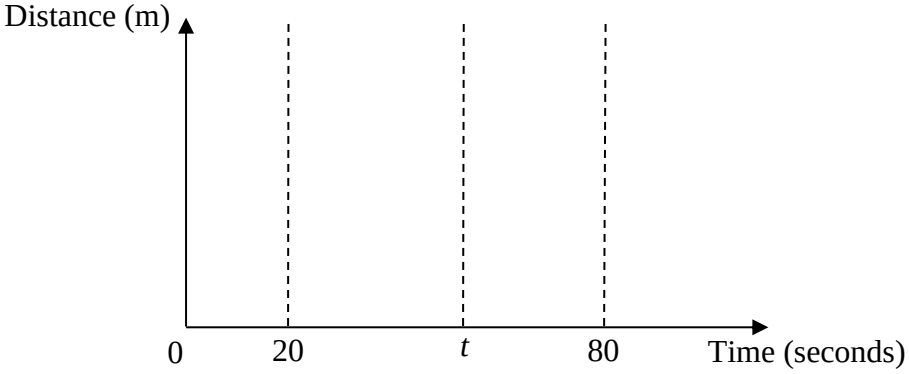
Answer : **(a)(i)** v = ……... [1]

(ii) t = ...……. [2]

(b) …… km/h [2]

(c) On the axes below, draw the distance-time graph of the bus journey, marking and stating the distance travelled for each time duration clearly on the vertical axis.

Answer : **(c)** [2]



17 Given that $\varepsilon = \{x : x \text{ is a positive integer and } x \leq 12\}$,

$A = \{x : x \text{ is an odd integer and } 3x + 5 > 11\}$,

$B = \{x : x \text{ is a factor of } 28\}$,

$C = \{x : x \text{ is a composite number}\}$.

(a) List the elements of

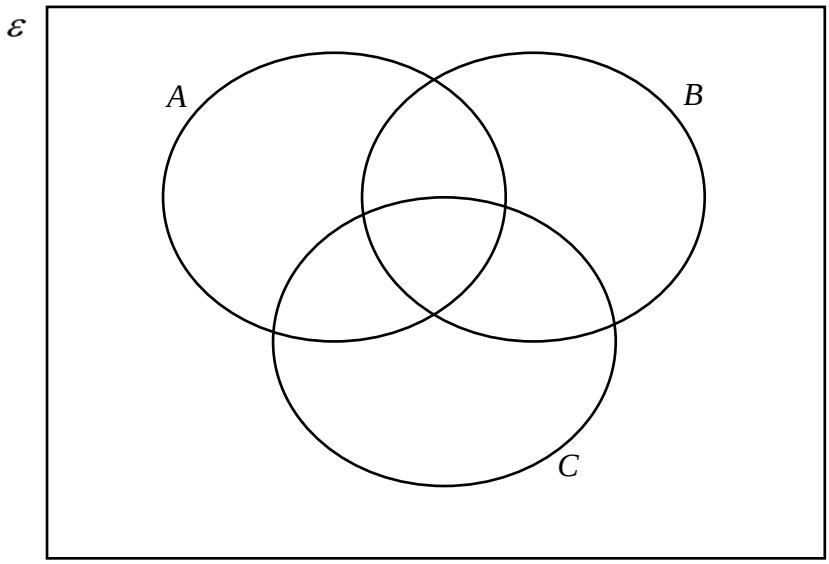
(i) $A \cup B$,

(ii) $B \cap C$,

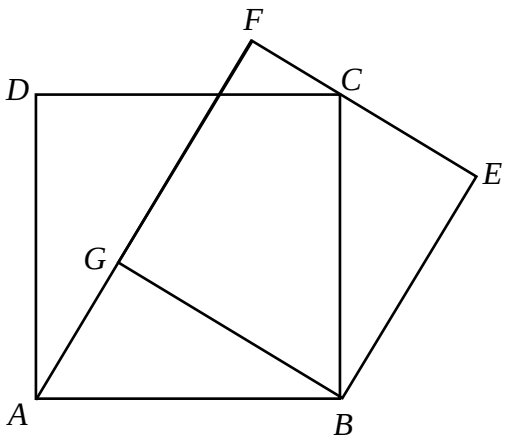
Answer : **(a)(i)** $A \cup B$ = ……[1]

(ii) $B \cap C$ =……[1]

(b) Represent the elements of ε and sets *A*, *B* and *C* in the Venn Diagram below. [1]



18 In the diagram below, *ABCD* is a square and *BEFG* is a rectangle.
AGF and *FCE* are straight lines.



(a) Show that ΔABG is congruent to ΔCBE.

Answer : **(a)** ……

……..

……..……

..…....……

…....……..

……...

…....……

…....……

…....……

…… [3]

(b) Alice commented that *BEFG* cannot be a square. Do you agree with her, giving your reasons?

Answer : **(b)** ……

…….

…....……

…....……

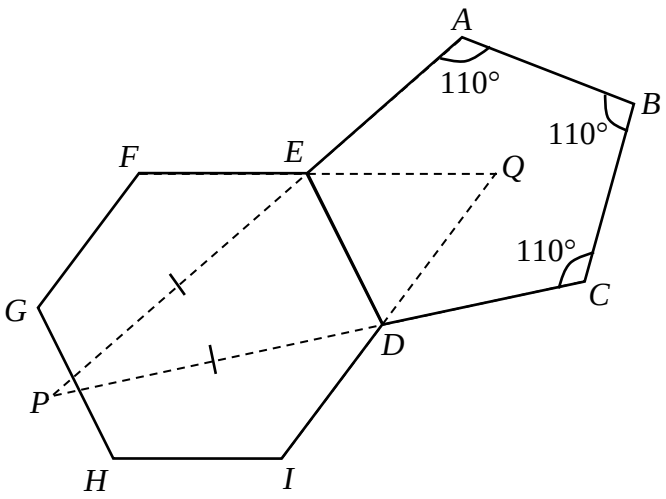
…....…… [1]

19 The diagram shows a pentagon *ABCDE* and a regular hexagon *DEFGHI* sharing a common side *DE*. *AE* and *CD* are produced to meet at *P* such that ΔDEP is an isosceles triangle. *FE* and *ID* are produced to meet at *Q*. $\angle EAB = \angle ABC = \angle BCD = 110°$.
Stating your reasons clearly, find

(a) $\angle AED$,

(b) $\angle DPE$,

(c) $\angle PDQ$.



Answer : **(a)** $\angle AED$ = …… ° [2]

(b) $\angle DPE$ = …… ° [2]

(c) $\angle PDQ$ = …… ° [2]

20 A box contains a number of blue, green and red balls. There are 27 blue and green balls altogether. A ball is selected at random from the box. The probability of drawing a green ball is twice the probability of drawing a blue ball. The probability of drawing a red ball is $\frac{4}{7}$.

(a) Show that the probability of drawing a **green** ball is $\frac{2}{7}$. [2]

(b) Find the number of red balls in the bag.

Answer : **(b)** ……... [1]

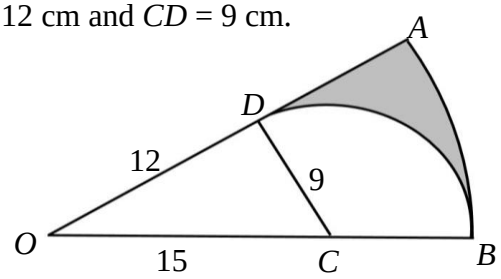
(c) Two balls are selected at random from the box, **with replacement**.
Find the probability that both balls are blue.

Answer : **(c)** ……... [1]

(d) Two balls are selected at random from the box, **without replacement**.
Find the probability that both balls are different colours.

Answer : **(d)** ……... [2]

21 The figure shows a sector OAB with centre O, and an arc BD of another circle with centre C. It is given that OC = 15 cm, OD = 12 cm and CD = 9 cm.



(a) Prove that ΔODC is a right-angled triangle.

……

……

……

……

…… [1]

(b) Find

(i) $\angle AOB$, in radian,

(ii) the perimeter of the shaded region,

(iii) the area of the shaded region.

Answer : **(b)(i)** ……. radian [1]

(ii) ……..…… cm [2]

(iii) ……..…… cm^2 [2]

~ END OF PAPER ~

Name : _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2018
Secondary 4

Wednesday
15 Aug 2018

MATHEMATICS
Paper 1 (Solution)

4048/01
2 h

INSTRUCTIONS TO CANDIDATES
Write your name, class and index number on the question paper.
Write in dark blue or black ink on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give your answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

INFORMATION FOR CANDIDATES
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Marks
80

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{\text{r}}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Write the following in order of size, **largest** first.

$-\frac{7}{8}$, -8×10^{-1} , $-0.\dot{8}\dot{1}$

-0.875 , -0.8 , $-0.8181\ldots$

Answer : -8×10^{-1} , $-0.\dot{8}\dot{1}$, $-\frac{7}{8}$ **[B1]** [1]

---

2 **(a)** Calculate $\frac{\sqrt{4.73}-3.28}{\sqrt[3]{99.7}\times 1.25}$ $= -0.190667\ldots$

Write down the first four digits on your calculator display.

(b) The number 102 999 corrected to ***n*** significant figures is 103 000.
Write down the largest possible value of ***n***.

Answer : **(a)** -0.190 **[B1]** [1]

(b) 5 **[B1]** [1]

---

3 One hundred identical drops of oil have a total mass of 4550 milligrams.
1 milligram = 1×10^{-3} grams.
It is given that 1 cm^3 of oil weighs 0.91 grams.
Calculate the volume of one drop, in m^3, giving your answer in standard form.

Volume of 1 drop of oil $= \frac{4550\times 10^{-3}}{0.91\times 100}$ **[M1]**

$= 0.05\ cm^3$

$= 0.05 \times 10^{-6}\ m^3$

$= 5 \times 10^{-8}\ m^3$

Answer : 5×10^{-8} m^3 **[A1]** [2]

4 The numbers 98, 784 and 1176, written as the products of their prime factors, are

$98 = 2 \times 7^2$, $784 = 2^4 \times 7^2$ and $1176 = 2^3 \times 3 \times 7^2$.

(a) Find the largest possible integer, w, which divides 98, 784 and 1176 exactly.

(b) Find the smallest integer, p, such that $\sqrt[3]{\frac{784 \times 1176}{p}}$ is a whole number.

(a) HCF, $w = 2 \times 7^2$

$= 98$

(b) $\frac{784 \times 1176}{p} = \frac{(2^4 \times 7^2) \times (2^3 \times 3 \times 7^2)}{p}$

$= \frac{2^7 \times 3 \times 7^4}{p}$

$= 2^6 \times 7^3$

$p = 2 \times 3 \times 7$

$= 42$

Answer : **(a)** $w =$ 98 **[B1]** [1]

(b) $p =$ 42 **[B1]** [1]

---

5 The map of The Singapore Zoo has a scale of 1 : 3000.

(a) If the length from the entrance to Reptile Garden is 4.5 cm on the map, calculate the actual distance, in kilometres.

(b) The actual area of the Baboon Enclosure is 0.0225 km^2. Calculate the area on the map that represents the enclosure, giving your answer in square centimetres.

(a) 1 cm : 3000 cm

1 cm : 0.03 km

4.5 cm : 0.135 km

(b) 1 cm : 0.03 km

1 cm^2 : 0.0009 km^2

25 cm^2 : 0.0225 km^2

Answer : **(a)** 0.135 **[B1]** km [1]

(b) 25 **[B1]** cm^2 [1]

---

6 Factorise $p^2 + 5p + 6$.

Hence, explain why $p^2 + 5p + 6$ is always even for all positive integer values of p.

Answer : $p^2 + 5p + 6 = (p + 2)(p + 3)$ **[M1]**

When p = odd integer , $p + 3$ = even.

When p = even integer , $p + 2$ = even. **[A1]**

Since one of the factors is always even, hence $p^2 + 5p + 6$ is always even.

[2]

7 A tank is filled at a constant rate with water by 7 taps in 45 seconds.

(a) If two additional taps are used, how long does it take to fill the same tank?

(b) If the volume of the tank is V cm^3, express the rate of flow of water from each tap in terms of V.

(a) Time taken $= \frac{7}{9} \times 45$

$= 35$ s

(b) Rate of flow of water per tap $= \frac{V}{7 \times 45}$ or $= \frac{V}{9 \times 35}$

$= \frac{V}{315}$ cm^3/s

Answer : **(a)** 35 **[A1]** s [1]

(b) $\frac{V}{315}$ **[A1]** cm^3/s [1]

---

8 Anne planned to travel to Japan for holidays. She wanted to change 2800 Singapore Dollars (SGD) into Japanese Yen (JPY).

One day she went to *XYZ* Shopping Mall and saw two money changer shops there. Should Anne change her money at Shop *A* or Shop *B*? Justify your answer with clear calculation.

Money Changer Shop *A*
Exchange rate :
1 SGD = 80.32 JPY

Money Changer Shop *B*
Exchange rate :
100 JPY = 1.25 SGD

At Shop *A*, 2800 SGD $=$ 2800×80.32 JPY **[M1]**

$= 224896$ JPY

At Shop *B*, 2800 SGD $= \frac{2800}{1.25} \times 100$ JPY **[M1]**

$= 224000$ JPY

Difference $=$ $224896 - 224000$

$= 896$ JPY

Answer : Anne should change her money at **Shop A** because she will **get** **[A1]**
extra 896 JPY. [3]

9 Solve the equation $\frac{1}{4y} = 2^{-6}$.

$\frac{1}{4y} = 2^{-6}$

$\frac{1}{4y} = \frac{1}{2^6}$

$4y = 2^6$ **[M1]**

$y = \frac{2^6}{2^2}$

$= 16$

Answer : $y =$ 35 **[A1]** [2]

---

10 The income tax payable in 2017 was calculated as follows:

Chargeable Income	Income Tax Rate (%)	Gross Tax Payable ($)
First $20,000	0	0
Next 10,000	2	200
First $30,000	–	200
Next 10,000	3.50	350
First $40,000	–	550
Next $40,000	7	2,800
First $80,000	–	3,350
Next $40,000	11.5	4,600

Mr Lim paid an income tax amount of $2545 for his income earned in 2017. Calculate his amount of chargeable income in 2017.

Let his amount of chargeable income be $\$x$.

$550 + (x - 40\,000) \times 0.07 = 2545$ **[M1]**

$x = \frac{2545 - 550}{0.07} + 40\,000$ **[M1]**

$= 68\,500$

Answer : $ 68 500 **[A1]** [3]

11 **(a)** Express $-x^2+4x-5$ in the form of $a(x+h)^2+k$.

$$-x^2+4x-5=-\left(x^2-4x+5\right)$$
$$=-\left[x^2-4x+\left(\frac{-4}{2}\right)^2-\left(\frac{-4}{2}\right)^2+5\right]$$
$$=-\left[(x-2)^2+1\right]$$
$$=-(x-2)^2-1$$

[M1]

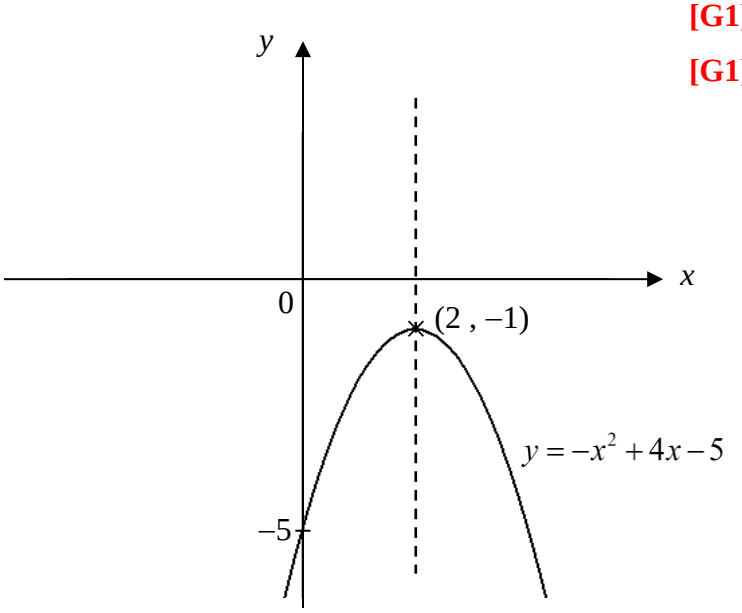
Answer : **(a)** $-(x-2)^2-1$ **[A1]** [2]

(b) **Hence**, sketch the graph of $y=-x^2+4x-5$ on the axes below. Indicate clearly the turning point, the values where the graph crosses the *x*- and *y*- axes (if any).

Answer : **(b)** [2]

[G1] – Correct shape & position

[G1] – Correct label turning point & *y*-intercept



(c) **Hence**, explain why the equation $-x^2+4x-5=0$ has no solution.

Answer : **(c)** The graph of $y=-x^2+4x-5$ will never cross the *x*-axis. **[B1]** [1]

12 The following diagram shows 2 geometrically similar bottles of perfumes of the same brand.

Bottle ***A***
135 ml
$48

Bottle ***B***
320 ml
$95

(a) Is the cost of the perfume directly proportional to the quantity of the perfume? Explain with clear calculation.

Costs per ml for Bottle $A = \frac{48}{135}$
$= \$0.36$ (2 d.p.) **[M1]**

Costs per ml for Bottle $B = \frac{95}{320}$
$= \$0.30$ (2 d.p.) **[M1]**

Answer : **(a)** Since **cost per ml not a constant**, the cost of the perfume **[A1]**
……
not directly proportional to the quantity of the perfume.
……
…… [3]

(b) Write down the ratio of the height of Bottle ***A*** to that of Bottle ***B***.

(c) It is given that the surface area of Bottle ***A*** is 270 cm^2. Calculate the surface area of Bottle ***B***.

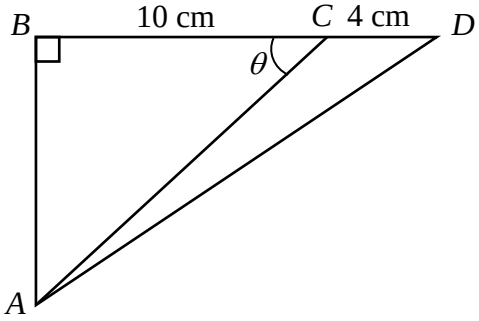
(b) $\frac{H_A}{H_B} = \sqrt[3]{\frac{135}{320}}$
$= \frac{3}{4}$

(c) $\frac{S_B}{270} = \left(\frac{4}{3}\right)^2$
Surface of Bottle $B = \left(\frac{4}{3}\right)^2 \times 270$ **[M1]**
$= 480$ cm^2

Answer : **(b)** ……3…… : ……4…….. **[A1]** [1]

(c) ……480…… cm^2 **[A1]** [2]

13 In the diagram, not drawn to scale, BCD is a straight line. Given that BC = 10 cm, CD = 4 cm, and $\cos\theta = \frac{4}{5}$.



Find

(a) the length of AC,

(b) the value of $\tan\angle ACD$, giving your answer as a fraction in its simplest form,

(c) the exact value of AD^2,

(d) the shortest distance of C to AD.

(a) $\cos\theta = \frac{10}{AC} = \frac{4}{5}$

$AC = \frac{10\times 5}{4} = 12.5$ cm

(b) $AB = \sqrt{12.5^2 - 10^2} = 7.5$ cm **[M1]**

$\tan\angle ACD = -\tan\angle ACB$

$= -\frac{7.5}{10}$

$= -\frac{3}{4}$

(c) $AD^2 = 7.5^2 + 14^2 = 252.25$

(d) Let the shortest distance be h cm.

Method 1

$\frac{1}{2}\times\sqrt{252.25}\times h = \frac{1}{2}\times 4\times 7.5$ **[M1]**

$h = \frac{4\times 7.5}{\sqrt{252.25}}$

=1.89 cm (3 s.f.)

Method 2

$\frac{1}{2}\times\sqrt{252.25}\times h = \frac{1}{2}\times 4\times 12.5\times\sin(180°-\theta)$ **[M1]**

$\frac{1}{2}\times\sqrt{252.25}\times h = \frac{1}{2}\times 4\times 12.5\times\sin\theta$

$\frac{1}{2}\times\sqrt{252.25}\times h = \frac{1}{2}\times 4\times 12.5\times\frac{7.5}{12.5}$

$h = \frac{4\times 7.5}{\sqrt{252.25}}$

=1.89 cm (3 s.f.)

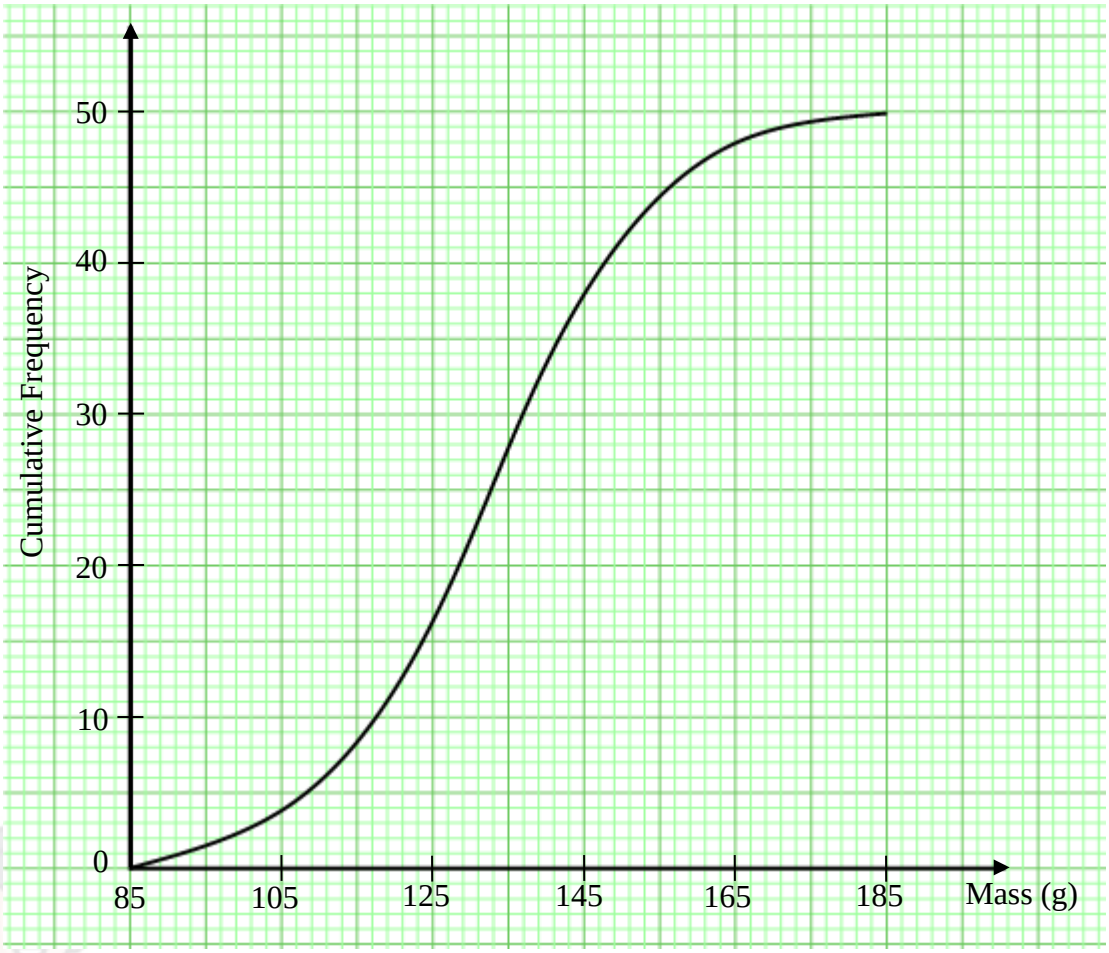
Answer : **(a)** 12.5 **[A1]** cm [1]

(b) $-\frac{3}{4}$ **[A1]** [2]

(c) 252.25 **[A1]** [1]

(d) 1.89 **[A1]** cm [2]

14 **(a)** The cumulative frequency curve below shows the distribution of the masses of 50 apples.



(i) The masses of these apples was tabulated into a grouped frequency table as follows. Find the values of p and of q.

Mass (x g)	Number of apples
$x < 85$	0
$85 \le x \le 105$	4
$105 \le x \le 125$	p
$125 \le x \le 145$	22
$145 \le x \le 165$	q
$165 \le x \le 185$	2

Answer : **(a)(i)** $p =$ 12 ..... **[B1 – Both must be correct]**

$q =$ 10 ..... [1]

(ii) Using your grouped frequency table, calculate an estimate of

(a) the mean mass of the apples,

(b) the standard deviation.

Answer : **(a)(ii)(a)** 132.6 ..... **[B1]** g [1]

(b) 19.0 ..... **[B1]** g [1]

(b) Another set of 50 apples have the same median but a smaller standard deviation. Describe how the cumulative frequency curve will differ from the given curve.

Answer : **(b)** The **second curve** will be **steeper** as smaller standard deviation shows that it has a narrower spread of data. **[B1]**

[1]

---

15 The table below shows a number sequence.

Pattern	No. of terms (N)	Value (V)
1^2	1	1
$1^2 - 2^2$	2	–3
$1^2 - 2^2 + 3^2$	3	6
$1^2 - 2^2 + 3^2 - 4^2$	4	–10
$1^2 - 2^2 + 3^2 - 4^2 + 5^2$	5	a
$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2$	6	b
⋮		

(a) State the values of a and b.

(b) Form an equation for V in terms of N when

(i) N is an odd number.

(ii) N is an even number.

(c) Hence, find the value of $17^2 - 18^2 + 19^2 - 20^2 \ldots + 125^2$.

(c) $V = \left[\frac{125\,(125+1)}{2}\right] - \left[-\frac{16\,(16+1)}{2}\right]$ **[M1]**

$= 7875 - (-136)$

$= 8011$

Answer : **(a)** $a =$ 15 **[B1]**, $b =$ –21 **[B1]** [2]

$V = \frac{1}{2}N^2 + \frac{1}{2}N$ **(b)(i)** $V = \frac{N(N+1)}{2}$ **[B1]** [1]

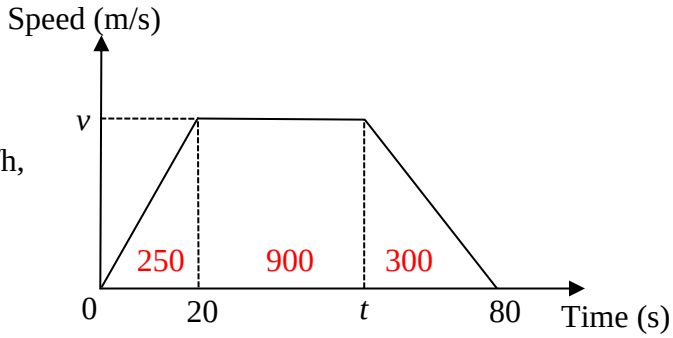
$V = -\frac{1}{2}N^2 - \frac{1}{2}N$ **(ii)** $V = -\frac{N(N+1)}{2}$ **[B1]** [1]

(c) 8011 **[A1]** [2]

16 The diagram below shows the speed-time graph of a bus journey.
The bus accelerated from rest at 1.25 m/s^2 to a speed of v m/s in 20 seconds, and travelled at this speed until t seconds before it came to a stop at t = 80 seconds. The total distance travelled for the whole journey was 1.45 km.

(a) Find the values of
(i) v,
(ii) t.

(b) Calculate the average speed, in km/h, of the bus for the whole journey.



(a) (i) $\frac{v}{20} = 1.25$

$v = 1.25 \times 20$

$= 25$

(a) (ii) $\frac{1}{2}(20)(25) + (t-20)(25) + \frac{1}{2}(80-t)(25) = 1450$ **[M1]**

$250 + 25t - 500 + 1000 - 12.5t = 1450$

$12.5t = 700$

$t = 56$

(b) Average speed $= \frac{1.45 \text{ km}}{\frac{80}{3600} \text{ h}}$ **[M1]**

$= 65.25$ km/h

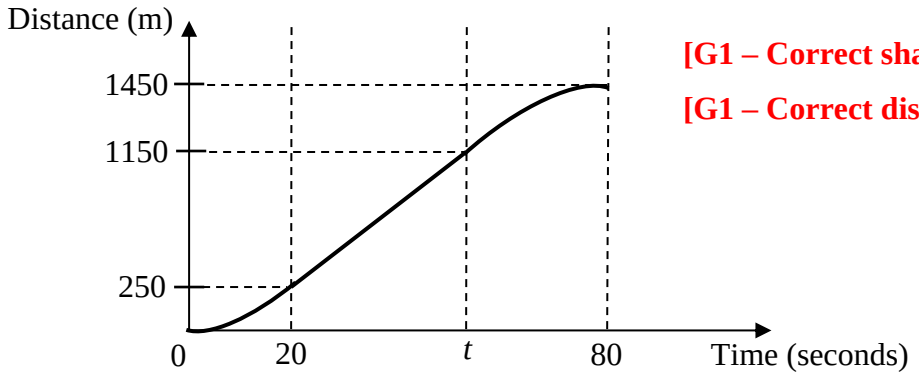
Answer : **(a)(i)** $v =$ 25 **[A1]** [1]

(ii) $t =$ 56 **[A1]** [2]

(b) 65.25 **[A1]** km/h [2]

(c) On the axes below, draw the distance-time graph of the bus journey, marking and stating the distance travelled for each time duration clearly on the vertical axis.

Answer : **(c)** [2]



[G1 – Correct shape]

[G1 – Correct distance]

17 Given that $\varepsilon = \{x : x \text{ is a positive integer and } x \leq 12\}$, **= { 1, 2, 3, ….. , 12 }**

$A = \{x : x \text{ is an odd integer and } 3x + 5 > 11\}$, **= { 3, 5, 7, 9, 11 }**

$B = \{x : x \text{ is a factor of } 28\}$, **= { 1, 2, 4, 7 }**

$C = \{x : x \text{ is a composite number}\}$. **= { 4, 6, 8, 9, 10, 12 }**

(a) List the elements of

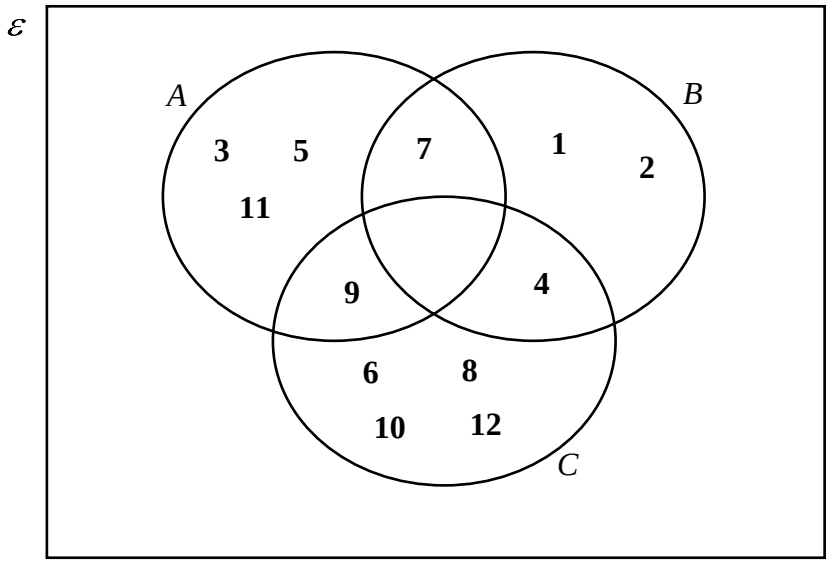
(i) $A \cup B$,

(ii) $B \cap C$,

Answer : **(a)(i)** $A \cup B$ = **{ 1, 2, 3, 4, 5, 7, 9, 11 }** **[B1]** [1]

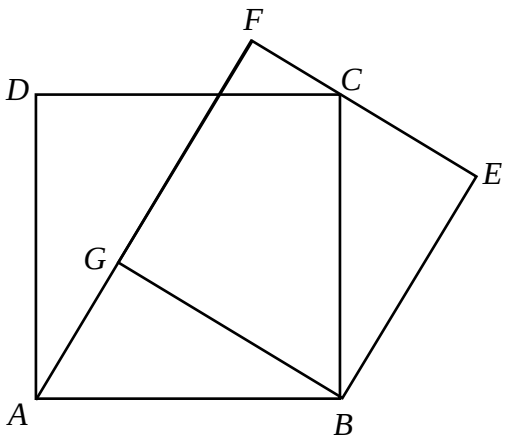
(ii) $B \cap C$ = **{ 4 }** **[B1]** [1]

(b) Represent the elements of ε and sets A, B and C in the Venn Diagram below. [1]



[B1 – All must be correct]

18 In the diagram below, *ABCD* is a square and *BEFG* is a rectangle.
AGF and *FCE* are straight lines.



(a) Show that ΔABG is congruent to ΔCBE.

Answer : **(a)** $\angle FGB = 90°$ ($\angle$s of rectangle *BEFG*)

$\angle AGB = 180° - 90° = 90°$ (Adjacent $\angle$s on a st line) **A**

$\angle BEC = 90°$ ($\angle$s of rectangle *BEFG*)

$\therefore \angle AGB = \angle BEC = 90°$ **[M1 – min 2 criteria given]**

Let $\angle GBC = x°$

$\angle GBA + x° = 90°$ ($\angle$s of square *ABCD*) **A** **[M2 – all 3 criteria given]**

$\angle EBC + x° = 90°$ ($\angle$s of rectangle *BEFG*)

$\therefore \angle GBA = \angle EBC$

AB = *BC* (Sides of square *ABCD*) **S**

$\therefore \Delta ABG \equiv \Delta CBE$ (AAS) **[A1 – Correct TEST]**

[3]

(b) Alice commented that *BEFG* cannot be a square. Do you agree with her, giving your reasons?

Answer : **(b)** **Disagreed.**

Since proven in part (a) $\Delta ABG \equiv \Delta CBE$,

GB = *BE* which are the sides of *BEFG*. **[B1]**

$\therefore$ *BEFG* is a square.

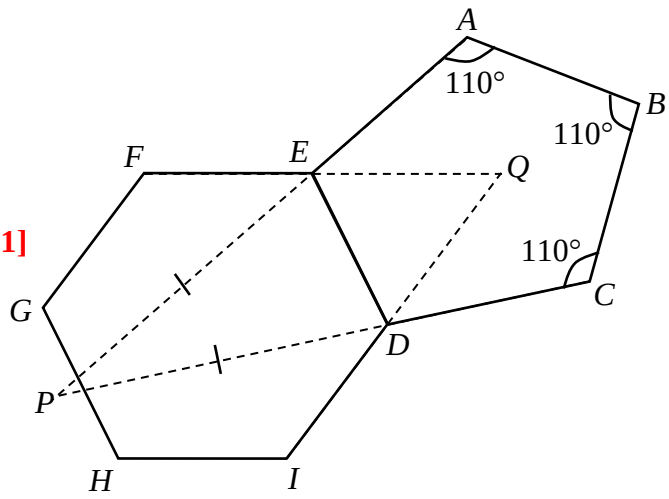
[1]

19 The diagram shows a pentagon *ABCDE* and a regular hexagon *DEFGHI* sharing a common side *DE*. *AE* and *CD* are produced to meet at *P* such that ΔDEP is an isosceles triangle. *FE* and *ID* are produced to meet at *Q*. $\angle EAB = \angle ABC = \angle BCD = 110°$.
Stating your reasons clearly, find

(a) $\angle AED$,

(b) $\angle DPE$,

(c) $\angle PDQ$.



(a) $\angle AED = \dfrac{(5-2)\times 180° - 3\times 110°}{2}$ **[M1]**

$= \dfrac{540° - 330°}{2}$

$= 105°$

(b) $\angle DEP = 180° - 105°$ (Adj. $\angle$s on a st. line) **[M1]**

$= 75°$

$\angle DPE = 180° - 2(75°)$ ($\angle$ sum of Δ)

$= 30°$

(c) $\angle EDI = \dfrac{(6-2)\times 180°}{6}$ **[M1]**

$= 120°$

$\angle EDQ = 180° - 120°$ (Adj. $\angle$s on a st. line)

$= 60°$

$\angle PDQ = 75° + 60°$

$= 135°$

Answer : **(a)** $\angle AED$ = 105 **[A1]** ° [2]

(b) $\angle DPE$ = 30 **[A1]** ° [2]

(c) $\angle PDQ$ = ... **[A1]** ° [2]

20 A box contains a number of blue, green and red balls. There are 27 blue and green balls altogether. A ball is selected at random from the box. The probability of drawing a green ball is twice the probability of drawing a blue ball. The probability of drawing a red ball is $\frac{4}{7}$.

(a) Show that the probability of drawing a **green** ball is $\frac{2}{7}$. [2]

(a) $P(G) + P(B) + P(R) = 1$

$P(G) + \frac{1}{2}P(G) + \frac{4}{7} = 1$ **[M1]**

$\frac{3}{2}P(G) = \frac{3}{7}$

$P(G) = \frac{3}{7} \times \frac{2}{3}$

$= \frac{2}{7}$ (shown) **[A1]**

(b) Find the number of red balls in the bag.

(b) G : B : R

2 : 1 : 4

No. of red balls $= \frac{27}{3} \times 4$

$= 36$

(2 + 1) units —— 27 balls

4 units —— ? balls

Answer : **(b)** 36 **[B1]** [1]

(c) Two balls are selected at random from the box, **with replacement**.
Find the probability that both balls are blue.

(c) G : B : R

18 : 9 : 36

$P(BB) = \left(\frac{9}{63}\right)\left(\frac{9}{63}\right)$ or $\left(\frac{1}{7}\right)\left(\frac{1}{7}\right)$

$= \frac{1}{49}$

Answer : **(c)** $\frac{1}{49}$ **[A1]** [1]

(d) Two balls are selected at random from the box, **without replacement**.
Find the probability that both balls are different colours.

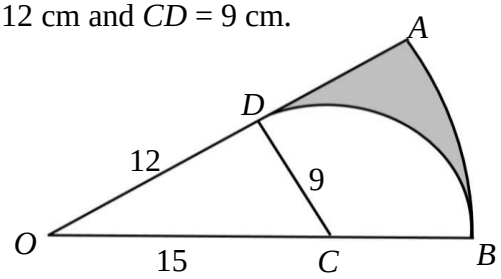
(d) P(different colours) = 1 – P(GG) – P(BB) – P(RR)

$= 1 - \left(\frac{18}{63}\right)\left(\frac{17}{62}\right) - \left(\frac{9}{63}\right)\left(\frac{8}{62}\right) - \left(\frac{36}{63}\right)\left(\frac{35}{62}\right)$ **[M1]**

$= \frac{18}{31}$

Answer : **(d)** $\frac{18}{31}$ **[A1]** [2]

21 The figure shows a sector OAB with centre O, and an arc BD of another circle with centre C. It is given that OC = 15 cm, OD = 12 cm and CD = 9 cm.



(a) Prove that ΔODC is a right-angled triangle.

$CD^2 + OD^2 = 9^2 + 12^2 = 225$

$OC^2 = 15^2 = 225$ **[A1]**

Since $CD^2 + OD^2 = OC^2$, by converse of Pythagoras' Theorem,

ΔODC is a right-angled triangle. [1]

(b) Find

(i) $\angle AOB$, in radian,

(ii) the perimeter of the shaded region,

(iii) the area of the shaded region.

(b) (i) $\tan \angle DOC = \frac{9}{12}$

$\angle AOB = \angle DOC = 0.643501$

$= 0.644$ radian (3 s.f.)

(b) (ii) $\angle DCB = \frac{\pi}{2} + 0.643501$ (Ext. $\angle$ of Δ)

$= 2.21429$ radians

Perimeter of shaded region

$= (15 + 9)(0.643501) + (24 - 12) + 9(2.21429)$ **[M1]**

$= 47.37261$

$= 47.4$ cm (3 s.f.)

(b) (iii) Area of shaded region

$= \frac{1}{2}(24)^2(0.643501) - \frac{1}{2}(9)(12) - \frac{1}{2}(9)^2(2.21429)$ **[M1]**

$= 41.649$

$= 41.6$ cm^2 (3 s.f.)

Answer : **(b)(i)** 0.644 **[A1]** radian [1]

(ii) 47.4 **[A1]** cm [2]

(iii) 41.6 **[A1]** cm^2 [2]

~ END OF PAPER ~

Name: _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2018
Secondary 4

Tuesday	**MATHEMATICS**	**4048/02**
14 August 2018	**Paper 2**	2 h 30 min

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to 3 significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together with the string provided.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{\text{r}}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** It is given that $V = 2\pi(r^2 - x^2)$.

(i) Evaluate V when $r = 4.5$ and $x = 0.9$. [1]

(ii) Express x in terms of V and r. [3]

(b) Simplify $\dfrac{4x^3 - 25xy^2}{2x^2 + 7xy - 30y^2}$. [3]

(c) **(i)** Solve the equation $\dfrac{1}{3-2x} = \dfrac{x}{9-12x+4x^2}$, $x \neq \dfrac{3}{2}$. [3]

(ii) Give a reason why x cannot be $\dfrac{3}{2}$. [1]

(d) Solve the inequalities $\dfrac{x+7}{3} < \dfrac{4}{5}x + 1 \leq -9 + x$. [3]

2 The following table shows the performance of three teams in the 2017 – 2018 English Football Premier League season. A draw is defined by a match ending with both teams having scored the same number of goals (e.g. 0 – 0, 2 – 2, etc).

Rank	Team	Wins	Draws	Losses
2nd	**Manchester Utd**	25	6	7
3rd	**Tottenham**	23	8	7
4th	**Liverpool**	21	12	5

(a) Write down a 3×3 matrix P to represent the information in the table above. [1]

A win is worth three points, a draw is worth one point and a loss results in no points scored. The greater the number of points accumulated, the higher the team's rank.

(b) **(i)** Write down a matrix Q, such that the product PQ, gives the total number of points the three teams accumulated respectively, at the end of the season. [1]

(ii) Evaluate the product PQ. [1]

(c) A Manchester Utd football fan made the following remark – "Your team will end the season with a higher rank than another team, as long as they have lost fewer matches than this other team."

Do you agree with his remark? Give a reason for your answer. [1]

The *Golden Boot* is awarded to the player who scored the most number of goals. The following players scored the most number of goals during the 2017 – 2018 season:
M. Salah – 32
H. Kane – 30
S. Aguero – 21

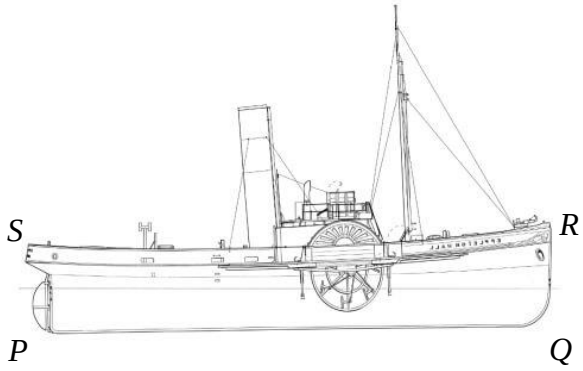
(d) **(i)** Write down a **row** matrix G to represent the above information. [1]

(ii) Given the matrix $T=\begin{pmatrix}1\\1\\1\end{pmatrix}$, evaluate the product GT. [1]

(iii) State what the product GT represents. [1]

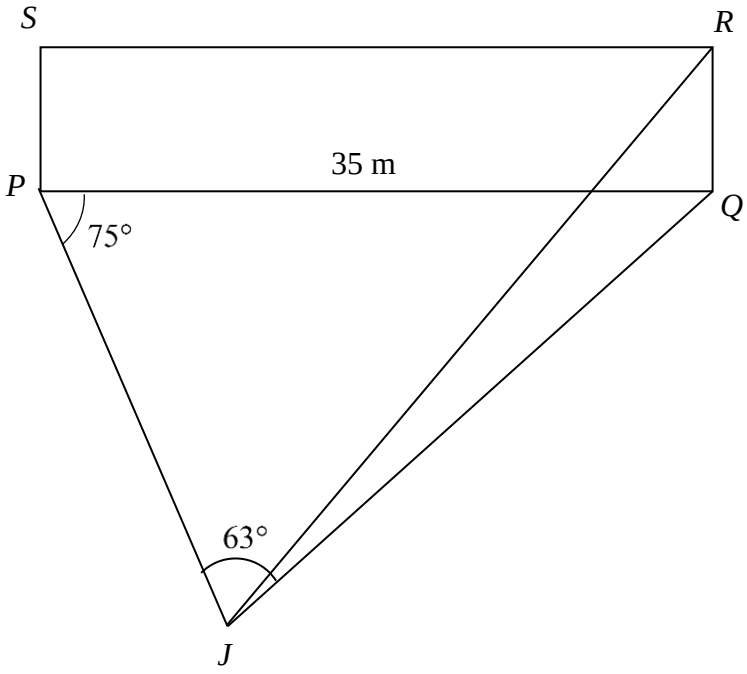
3

Figure 1



Source: www.shipmodell.com

Figure 2 (NOT TO SCALE)



The rectangle *PQRS* in **Figure 2** represents the vertical starboard (right-hand) side of a small fishing vessel shown in **Figure 1**. The bottom of the starboard side, *PQ*, runs from West to East and is at sea level. A jet ski is nearby at *J*.

Angle $QPJ = 75°$, angle $PJQ = 63°$ and $PQ = 35$m.

(a) Showing your reasons clearly, find the bearing of *J* from *Q*. [2]

(b) Show that $JQ = 37.9$m. [2]

(c) Calculate the area of triangle *JPQ*. [2]

(d) Find the shortest distance from the jet ski to the fishing vessel. [2]

(e) The angle of depression of the jet ski from a fisherman standing on the top deck of the fishing vessel at *R* is $18°$.

(i) Calculate the height of the top deck of the fishing vessel above sea level. [2]

(ii) Calculate the greatest possible value of the angle of elevation of the top deck of the fishing vessel when viewed from the jet ski. [2]

4 **(a)** The cash price of a new washer-dryer is \$2595. It is also available on hire purchase with a deposit of one quarter of the cash price followed by monthly instalments of \$60 for three years.

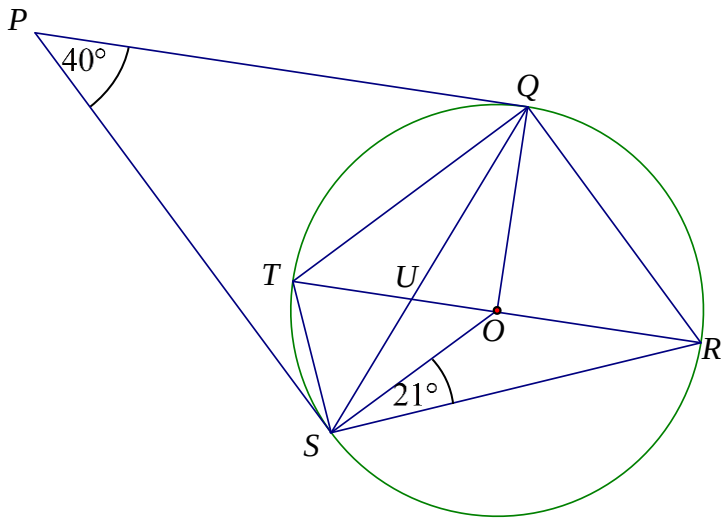
Pavithra buys this washer-dryer on hire purchase.

(i) Calculate the rate of simple interest charged per annum, correct to two decimal places. [4]

(ii) Find the extra cost of buying the washer-dryer on hire purchase as a percentage of the cash price. [2]

(b) A sum of money grows to \$5800.15 in 3 years at a compound interest of 2.75% per annum. Find the sum of money if the interest is compounded quarterly. [2]

5



The diagram shows a circle $QRST$, centre O. U is the point of intersection of QS and diameter RT. PQ and PS are tangents to the circle such that angle $QPS = 40°$. Angle $OSR = 21°$.

Find, with clearly stated reasons,

(a) obtuse angle QOS, [2]

(b) angle QTS, [2]

(c) angle OSU, [1]

(d) angle PQT. [3]

6 Answer the whole of this question on a sheet of graph paper.

During a Physics experiment conducted on gravity, a rubber ball was thrown from the top of a vertical building.

Its position during its flight is represented by the equation $y = 30 + 4x - x^2$, where y metres is the height of the ball above the ground and x metres is its horizontal distance from the foot of the building.

Some corresponding values of x and y are given in the table below.

x	0	1	2	3	4	5	6	7
y	30	33	34	33	30	25	p	9

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to represent 1 metre, draw a horizontal axis for $0 \le x \le 7$.
Using a scale of 2 cm to represent 5 metres, draw a vertical axis for $0 \le y \le 40$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the greatest height reached by the ball. [1]

(d) Find the estimated horizontal distances of the ball from the building when its height is 32 m above the ground. [2]

(e) **(i)** By drawing a tangent, find the gradient of the graph at the point (5, 25). [2]

(ii) Use your answer to **(e)(i)** to explain what was happening to the ball at $x = 5$. [1]

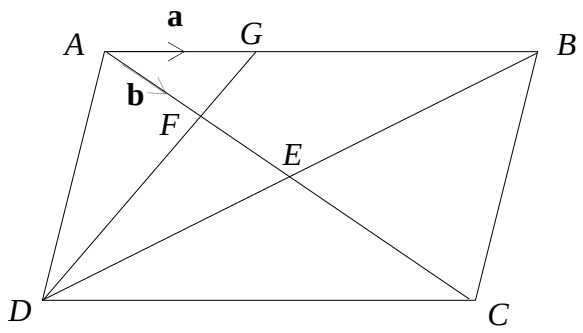
7 **(a)** X is the point (2, 3) and Y is the point (– 6, 10).

(i) Find the vector $\overrightarrow{XY}$. [1]

(ii) Find $|\overrightarrow{XY}|$. [1]

(iii) Given that $\overrightarrow{XY} = 2\overrightarrow{YZ}$, find the coordinates of the point Z. [2]

(b)



In the diagram above, $ABCD$ is a parallelogram. The diagonals AC and BD intersect at E and the point F is the mid-point of AE. G is a point on AB such that $2AG = GB$. $\overrightarrow{AG} = \mathbf{a}$ and $\overrightarrow{AF} = \mathbf{b}$.

(i) Express the following in terms of **a** and/or **b**.

(a) $\overrightarrow{FG}$, [1]

(b) $\overrightarrow{DF}$, [1]

(c) $\overrightarrow{GC}$. [2]

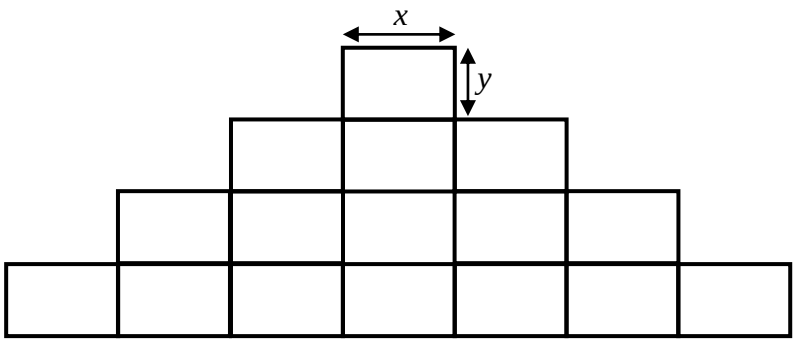
(ii) Show that D, F and G are collinear points. [1]

(iii) Find the ratio of

(a) $\dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta CFD}$, [1]

(b) $\dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta DEC}$. [2]

8 The diagram shows an exhibition area which has been divided into 16 identical rectangular plots with dimensions x m by y m, where $x > y$, which will be leased out as booth spaces.



345 m of tape was used by the planning committee to mark out the exhibition area.

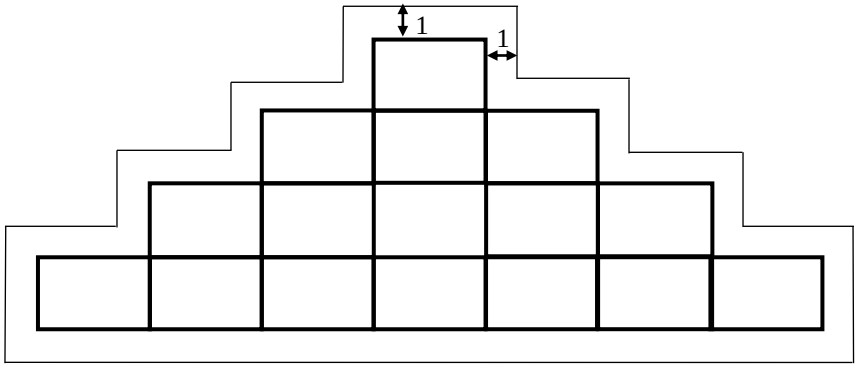
(a) Write down an equation connecting x and y. [1]

The total area enclosed by the tape is 644 m^2.

(b) Form an equation in x to represent this information and show that it reduces to
$$x^2 - 15x + 35 = 0.$$ [3]

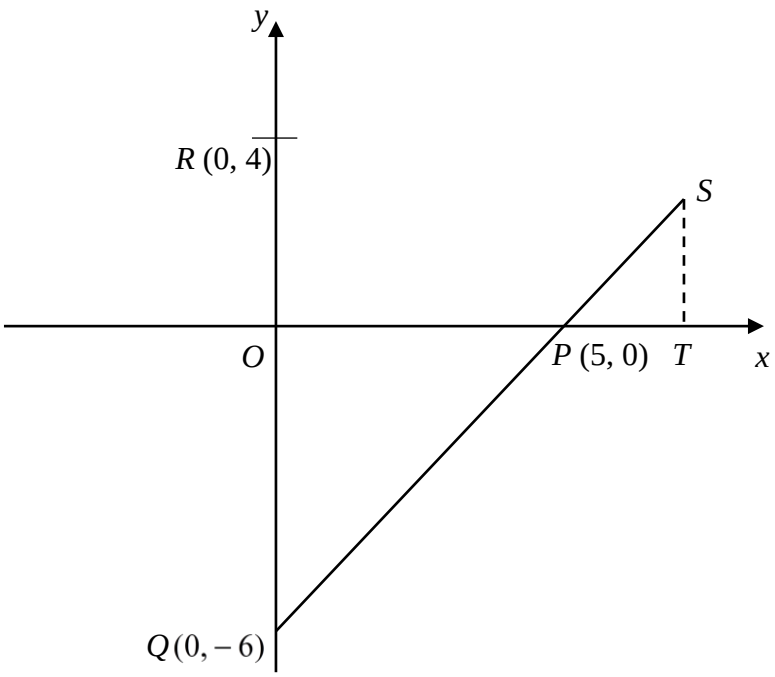
(c) Solve the equation $x^2 - 15x + 35 = 0$. [3]

(d) On the night before the commencement of the exhibition, the planning committee plans to cordon off a 1 metre–wide security barrier along the perimeter of the exhibition area.



Calculate the total area of the security barrier. [3]

9



In the diagram, P, Q and R are the points (5, 0), $(0, -6)$ and (0, 4) respectively.

(a) Find the equation of the line PQ. [1]

S is a point on QP produced such that the x-coordinate of S is k and T lies vertically below S on the x-axis.

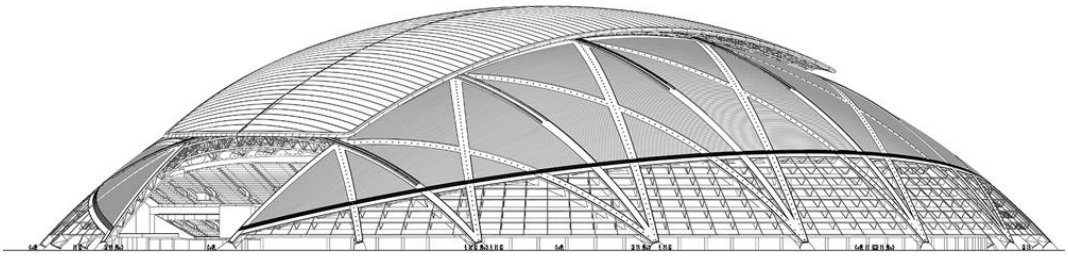
(b) Given that the area of triangle RQS is 40 units^2, show that $k = 8$. [2]

(c) **(i)** Calculate the lengths of PQ and PS respectively. [3]

(ii) Hence, using your answers in **(c)(i)** and stating your reasons clearly, show that triangle PQO is similar to triangle PST. [3]

10 **(a)** The National Day Parade will be held at the National Stadium in August this year. The stadium dome shown below, may be modelled as a spherical cap (part of a hemisphere). Its volume is approximately 17.5% of the volume of a hemisphere of radius 192 metres and its surface area is approximately 33.9% of the curved surface area of the same hemisphere. The stadium is equipped with a state-of-the-art retractable roof which has two panels, each of area 10,000 m^2.

Source: www.static.dezeen.com



Calculate

(i) the estimated volume of the dome, [2]

(ii) the surface area of the dome when its retractable roof is **fully** open. [2]

(b) The organizers of the 2018 National Day Parade are deciding between two companies' drinking water for the expected 55,000 maximum-capacity spectators.

The table below summarizes some information for the two brands.

	Brand A	Brand B
Packaging type	Tetra pak rectangular prism with dimensions 5.7cm×5.3cm×10cm	Cylindrical with radius 2.75 cm and height 12.8 cm
Material	Recycled paperboard	Plastic
Fill volume	Nearest 10 ml of pack volume	Nearest 10 ml of bottle volume
Unit cost per package	60 cents	55 cents
Bulk refund for empty packages	$11000	$5200

Additional information:

1) Each spectator is projected to require an average of 600 ml of water during the Parade.
2) The chosen package is to be environmentally-friendly where possible.
3) Cost effectiveness to be taken into consideration.

Determine, with clear calculations, which brand of water should be awarded the contract. State an assumption you made to support your decision. [6]

~~ End of Paper 2 ~~ ☺

2018 Mathematics Prelim P2 Marking Scheme

1. (a) (i) $V = 2\pi(r^2 - x^2)$

$V = 2\pi[(4.5)^2 - (0.9)^2]$

$= 122$

(ii) $V = 2\pi r^2 - 2\pi x^2$

$2\pi x^2 = 2\pi r^2 - V$

$x^2 = \dfrac{2\pi r^2 - V}{2\pi}$

$\therefore x = \pm\sqrt{\dfrac{2\pi r^2 - V}{2\pi}}$

(b) $\dfrac{4x^3 - 25xy^2}{2x^2 + 7xy - 30y^2} = \dfrac{x(2x+5y)(2x-5y)}{(2x-5y)(x+6y)}$

$= \dfrac{x(2x+5y)}{x+6y}$

(c) (i) $\dfrac{1}{3-2x} = \dfrac{x}{9-12x+4x^2}$

$\dfrac{1}{3-2x} = \dfrac{x}{(3-2x)^2}$

$3 - 2x = x$

$\therefore x = 1$

Or $9 - 12x + 4x^2 = 3x - 2x^2$

$6x^2 - 15x + 9 = 0$

$2x^2 - 5x + 3 = 0$

$(2x-3)(x-1) = 0$

$\therefore x = 1$ or $x = \dfrac{3}{2}$ (N.A.)

(ii) If $x = \dfrac{3}{2}$, the denominator/s will be zero; fraction will be undefined.

(d) $\frac{x+7}{3} < \frac{4}{5}x+1 \le -9+x.$

$\frac{x+7}{3} < \frac{4}{5}x+1$ $\qquad$ $\frac{4}{5}x+1 \le -9+x.$

$5x+35 < 12x+15$ $\qquad$ $10 \le \frac{1}{5}x$

$20 < 7x$

$\therefore x > \frac{20}{7}$ $\qquad$ $\therefore x \ge 50$

$\therefore x \ge 50$

2. **(a)** $P = \begin{pmatrix} 25 & 6 & 7 \\ 23 & 8 & 7 \\ 21 & 12 & 5 \end{pmatrix}$

(b) **(i)** $Q = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

(ii) $PQ = \begin{pmatrix} 25 & 6 & 7 \\ 23 & 8 & 7 \\ 21 & 12 & 5 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 81 \\ 77 \\ 75 \end{pmatrix}$

(c) Disagree.

Liverpool had fewer losses than the other two teams but still ended up ranked behind them.

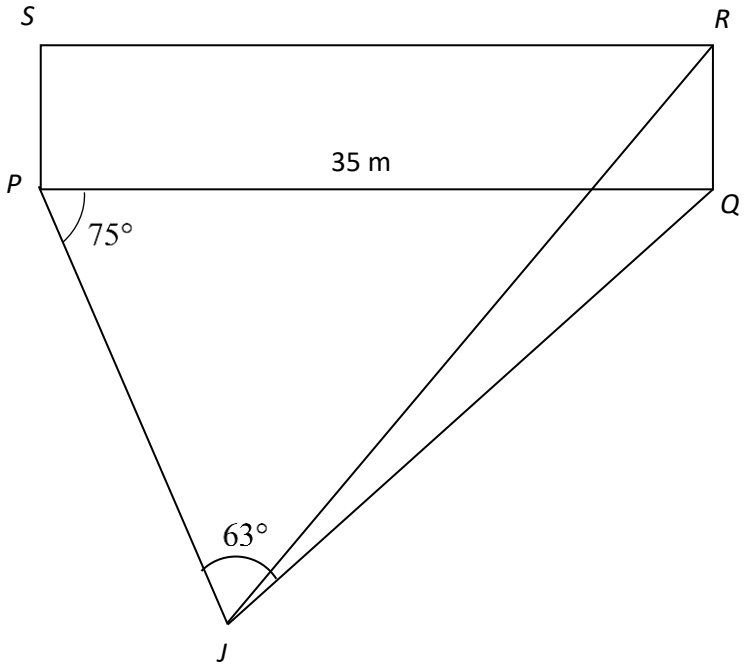
(d) **(i)** $G = (32 \quad 30 \quad 21)$

(ii) $G = (32 \quad 30 \quad 21)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$= (83)$

(ii) The element represents the total number of goals scored by the top three scorers.

3.



(a) $\angle PQJ = 180° - 75° - 63°$ (angles in a triangle)

$= 42°$

$\therefore$ bearing of J from $Q = 360° - 90° - 42°$ (angles at a point)

$= 228°$

(b) $\dfrac{JQ}{\sin 75°} = \dfrac{35}{\sin 63°}$

$\therefore JQ = \dfrac{35 \sin 75°}{\sin 63°}$

$= 37.9429$

$= 37.9$ m (3 s.f.)

(c) Area of triangle $JPQ = \dfrac{1}{2}(35)(37.9429)\sin 42°$

$= 444.303$

$= 444\,\text{m}^2$ (3 s.f.)

(d) Let shortest distance be d m .

$\sin 42° = \dfrac{d}{37.9429}$

$\therefore$ shortest distance $= 37.9429 \sin 42°$

$= 25.3888$

$= 25.4$ m (3 s.f.)

(e) **(i)** Let the height be h m.

$$\tan 18° = \frac{h}{37.9429}$$

$$\therefore \text{ height } = 12.3284$$

$$= 12.3 \text{ m (3 s.f.)}$$

(ii) Let the angle be θ .

$$\tan\theta = \frac{12.3284}{25.3888}$$

$\therefore$ greatest angle of elevation is $25.9°$ (1 d.p.)

---

4. **(a)** **(i)** Balance after deposit $= \frac{75}{100}(2595)$

$$= \$1946.25$$

$$\text{Total interest paid } = \frac{25}{100}(2595) + 60(12)(3) - 2595$$

$$= \$213.75$$

Let r be the rate of interest.

$$213.75 = 1946.25(\frac{r}{100})(3)$$

$$\therefore \text{ rate } = \frac{213.75(100)}{1946.25(3)}$$

$$= 3.66\% \quad (2 \text{ d.p.})$$

(ii) $\therefore$ percentage $= \frac{213.75}{2595} \times 100\%$

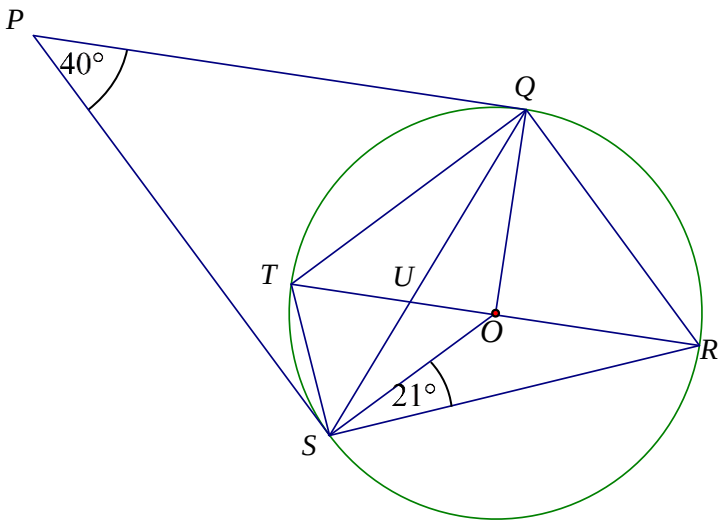
$$= 8.24\% \text{ (3 s.f.) or } 8\frac{41}{173}\%$$

(b) Let the sum of money be x.

$$5800.15 = x\left[1 + \frac{2.75}{4(100)}\right]^{3(4)}$$

$\therefore$ the sum of money $= \$5342.35$ (2 d.p.)

5.



(a) Angle PQO = angle PSO = $90°$ (tangent perpendicular to radius)

$\therefore$ obtuse angle QOS = $360° - 90° - 90° - 40°$ (angles in a quadrilateral)

$= 140°$

(b) Reflex angle QOS = $360° - 140°$ (angles at a point)

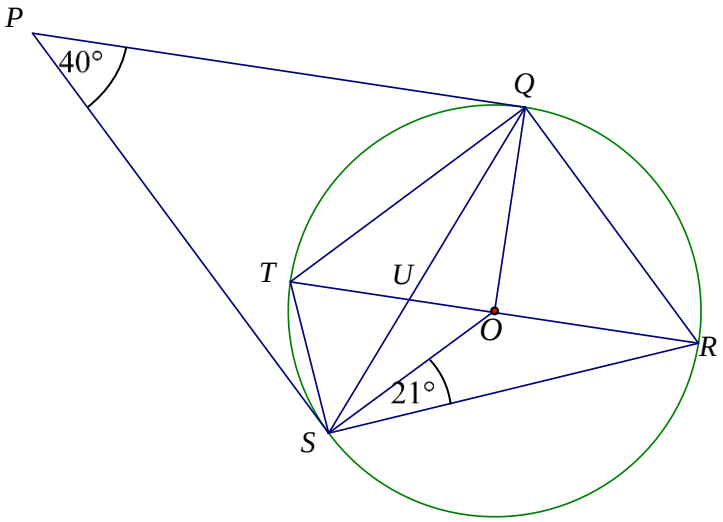
$= 220°$

$\therefore$ angle QTS = $\dfrac{220°}{2}$ (angle at centre = 2 angle at circumference)

$= 110°$

(c) Angle OSU = $\dfrac{180° - 140°}{2}$ (base angles of isosceles triangle)

$= 20°$



(d) Method 1

Angle $ORS = 21°$ (base angles of isosceles triangle)

$\therefore$ angle $TQU = 21°$ (angles in the same segment)

Angle $OQU = 20°$ (base angles of isosceles triangle)

$\therefore$ angle $PQT = 90° - 21° - 20°$ (tangent perpendicular to radius)

$= 49°$

Method 2

$PQ = PS$ (tangents from external point)

$\therefore$ triangle PQS is isosceles. Angle $PQS = \dfrac{180° - 40°}{2}$

$= 70°$

Angle $ORS = 21°$ (base angles of isosceles triangle)

$\therefore$ angle $TQU = 21°$ (angles in the same segment)

$\therefore$ angle $PQT = 70° - 21° = 49°$

Method 3

Angle $TSU = 90° - 20° - 21°$ (angle in semicircle)

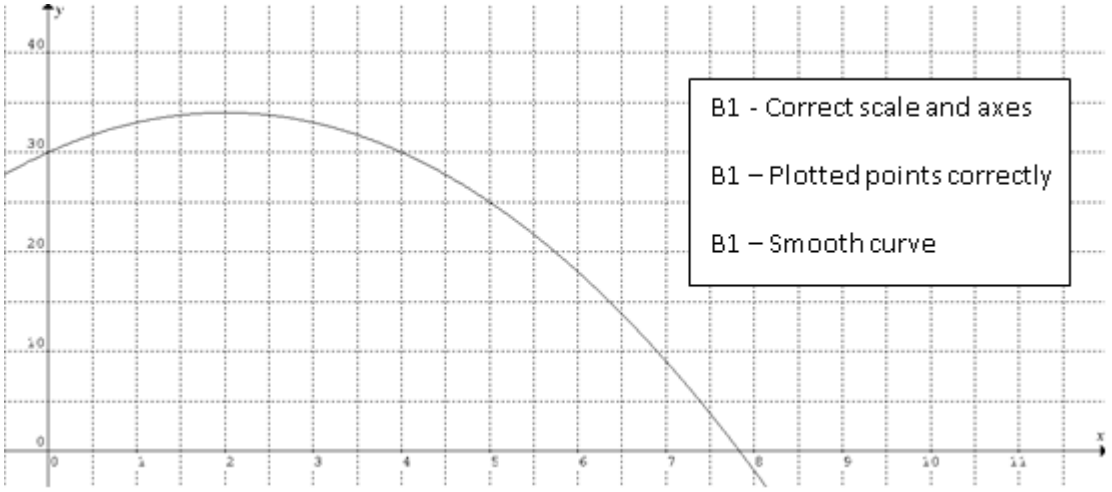
$= 49°$

$\therefore$ angle $PQT = 49°$ (angles in alternate segments/alternate segment theorem)

---

6. **(a)** $p = 18$

(b)



(c) The greatest height is 34 m.

(d) 3.41 m (3 s.f.) accept $3.35 \le x \le 3.45$

0.586 (3 s.f.) m accept $0.535 \le x \le 0.635$

(e) **(i)** tangent constructed

Gradient $= -6$ accept $-6.5 \le m \le -5.5$

(ii) The height of the ball above the ground was decreasing at a rate of 6 m for every 1 m it travels horizontally / falls.

7. (a) (i) $\overrightarrow{XY} = \begin{pmatrix} -6 \\ 10 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ 7 \end{pmatrix}$

(ii) $|\overrightarrow{XY}| = \sqrt{(-8)^2 + 7^2}$

= 10.6 u (3 s.f.)

(iii) $\overrightarrow{YZ} = \frac{1}{2}\overrightarrow{XY}$

$= \frac{1}{2}\begin{pmatrix} -8 \\ 7 \end{pmatrix}$

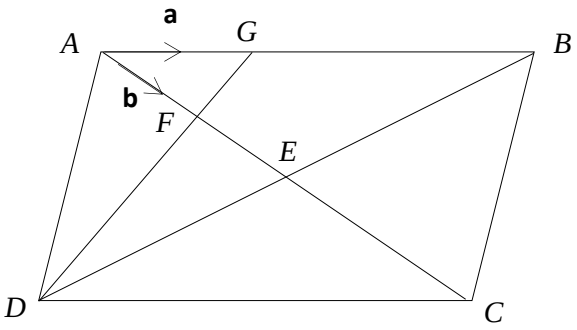
$= \begin{pmatrix} -4 \\ 3.5 \end{pmatrix}$

$\therefore \overrightarrow{OZ} = \begin{pmatrix} -6 \\ 10 \end{pmatrix} + \begin{pmatrix} -4 \\ 3.5 \end{pmatrix}$

$= \begin{pmatrix} -10 \\ 13.5 \end{pmatrix}$

$\therefore Z = (-10, 13.5)$

(b)



(i) **(a)** $\overrightarrow{FG} = \mathbf{a} - \mathbf{b}$

(b) $\overrightarrow{DF} = \mathbf{3a} - \mathbf{3b}$

(c) $\overrightarrow{BC} = \mathbf{4b} - \mathbf{3a}$

$\therefore \overrightarrow{GC} = \mathbf{2a} + \mathbf{4b} - \mathbf{3a}$

$= \mathbf{4b} - \mathbf{a}$

(ii) Since $\overrightarrow{DF} = 3\overrightarrow{FG}$ and F is a common point, D, F and G are collinear.

(iii) **(a)** $\dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta CFD} = \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$

(b) $\dfrac{\text{the area of } \Delta DEF}{\text{the area of } \Delta DEC} = \dfrac{1}{2} = \dfrac{3}{6}$

$\therefore \dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta DEC} = \dfrac{1}{6}$

8. **(a)** $23x + 20y = 345$

(b) $16x\left(\dfrac{345 - 23x}{20}\right) = 644$

$345x - 23x^2 = 805$

$23x^2 - 345x + 805 = 0$

$x^2 - 15x + 35 = 0$

(c) $x = \dfrac{15 \pm \sqrt{15^2 - 4(35)}}{2}$

$x = 2.89022$ or $x = 12.1097$

$= 2.89$ (3 s.f.) or $= 12.1$ (3 s.f.)

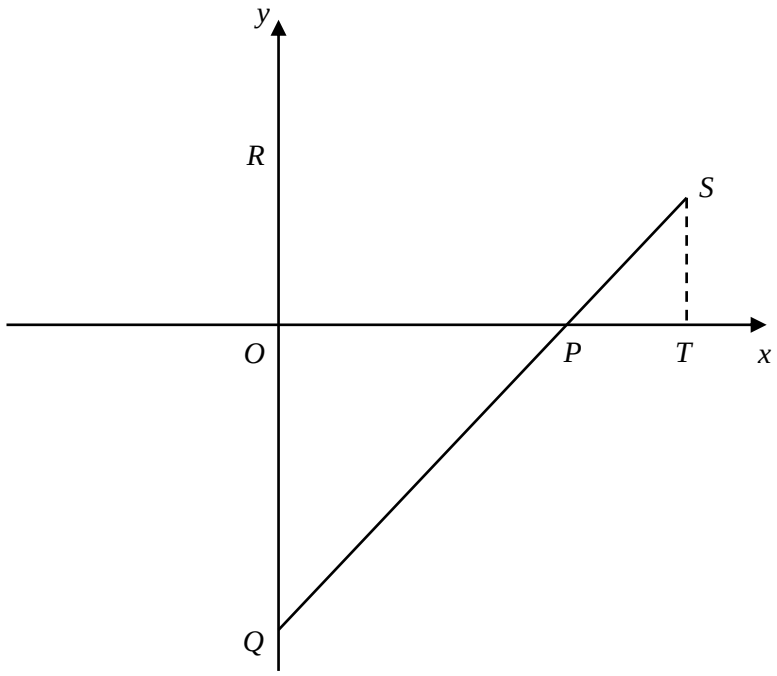
(d) Since $x > y$, $x = 12.1097$ and $y = 3.32376$

Area of path

$= [7(12.1097) + 2][4(3.32376) + 2] - 644 - 12(12.097)(3.32376)$

$= 200$ m^2 (3 s.f.)

9.



(a) Gradient of $PQ = \dfrac{0-(-6)}{5-0}$

$= \dfrac{6}{5}$

$\therefore PQ: y = \dfrac{6}{5}x - 6$ or $5y = 6x - 30$

(b) $\dfrac{1}{2}(10)(k) = 40$

$\therefore k = 8$ (shown)

(c) **(i)** Length of $PQ = \sqrt{(5-0)^2 + (0+6)^2}$

$= \sqrt{61}$

$= 7.81$ units (3 s.f.)

$y = \dfrac{6}{5}(8) - 6 = 3.6$

$\therefore S = (8, 3.6)$

Length of $PS = \sqrt{(5-8)^2 + (0-3.6)^2} = 4.69$ units (3 s.f.)

(ii) $\dfrac{PQ}{PS} = \dfrac{\sqrt{61}}{\sqrt{21.96}}$

$= \dfrac{5}{3}$ (exact) ~~ [1]

$\angle OPQ = \angle TPS$ (vertically opposite angles)

$\dfrac{PO}{PT} = \dfrac{5}{3}$ ~~ [2]

$\therefore \Delta PQO$ is similar to ΔPST (SAS similarity) test name optional

Or [1], [2] and

$\dfrac{OQ}{TS} = \dfrac{6}{3.6}$

$= \dfrac{5}{3}$

Since $\dfrac{PQ}{PS} = \dfrac{PO}{PT} = \dfrac{OQ}{TS} = \dfrac{5}{3}$,

$\therefore \Delta PQO$ is similar to ΔPST (SSS similarity)

10. **(a)** **(i)** $\text{Volume} = \frac{17.5}{100}\left(\frac{2}{3}\pi(192)^3\right)$

$= 2,590,000\,\text{m}^3$ (3 s.f.)

(ii) $\text{Surface area} = \frac{33.9}{100}[2\pi(192)^2] - 2(10000)$

$= 58,500\ \text{m}^2$ (3 s.f.)

(b) Fill volume of Brand A $= 5.7(5.3)(10)$

$= 302.1$

$= 300$ ml (nearest 10 ml)

Fill volume of Brand B $= \pi(2.75)^2(12.8)$

$= 304.106$

$= 300$ ml (nearest 10 ml)

Since both brands' packages are 300 ml, each spectator requires 2 bottles/packs – no difference.

Total cost for Brand A $= 0.6(110000)$

$= \$66000$

Less refund \$11000 Brand A: \$55 000

Total cost for Brand B $= 0.55(110000)$

$= \$60500$

Less refund \$5200 Brand B: \$55 300

Decision.

Possible reasoning:

Brand A should be chosen. Despite being slightly more expensive, Brand A has the advantage of using more environmentally-friendly material.

Assumption: Every pak/bottle is emptied and recycled.

Assumption: Total number of paks/bottles is 110000 (2/spectator); no extras.

Name: **Register Number:** **Class:**

For Marker's Use

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2018

SECONDARY FOUR EXPRESS

MATHEMATICS **4048/01**

Paper 1 **6 August 2018, Monday**

2 hours

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

Setter: Ms Ting Shi Yun

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

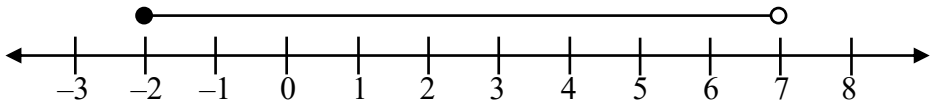
Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer all the questions.

1 A range of values of x is represented on the number line below.



Write down inequalities that represent this range of values of x.

Answer …… [1]

---

2 The stem-and-leaf diagram shows the masses, in grams, of some oranges.

19	1	2	2	*m*	5	6	6	8
20	0	0	3	4	6			
21	1	4	7					

Key: 19 | 2 represents 192 grams

(a) Find the median of these masses.

Answer median = …… g [1]

(b) Given that the interquartile range is 10, find the value of m.

Answer m = …… [2]

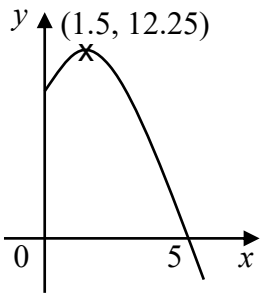
---

3 Given that $2^{x+1} + 2^x = 24$, find the value of x.

Answer x = …… [2]

---

4 The diagram shows part of the curve $y = ax^2 + bx + 10$. It cuts the x-axis at 5 and the coordinates of the maximum point is (1.5, 12.25). Find the value of a and of b.



Answer a = ……

b = …… [3]

---

5 The first four terms in a sequence of numbers T_1, T_2, T_3, T_4, …, are given below.

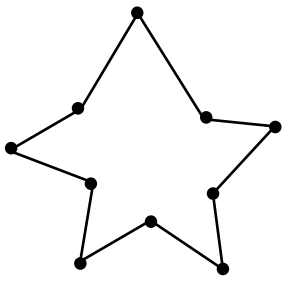
$$\frac{1}{3} \quad , \quad \frac{7}{15} \quad , \quad \frac{13}{35} \quad , \quad \frac{19}{63} \quad , \quad \ldots$$

Find an expression, in terms of n, for T_n.

Answer T_n = …… [2]

---

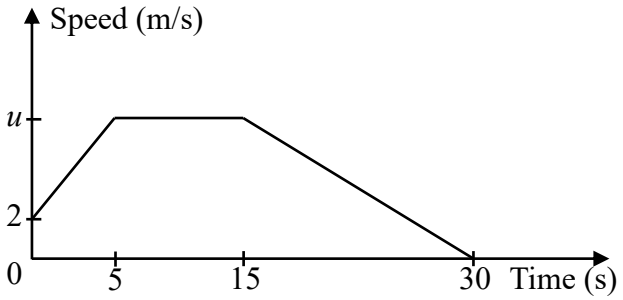
6 The diagram shows an irregular polygon.
Find the sum of all interior angles of this polygon.



Answer ……° [2]

---

7 The diagram shows the speed-time graph of a remote controlled toy car for the first 30 seconds of the journey.



(a) Given that the deceleration of the car is 0.5 m/s^2, find the greatest speed, u m/s.

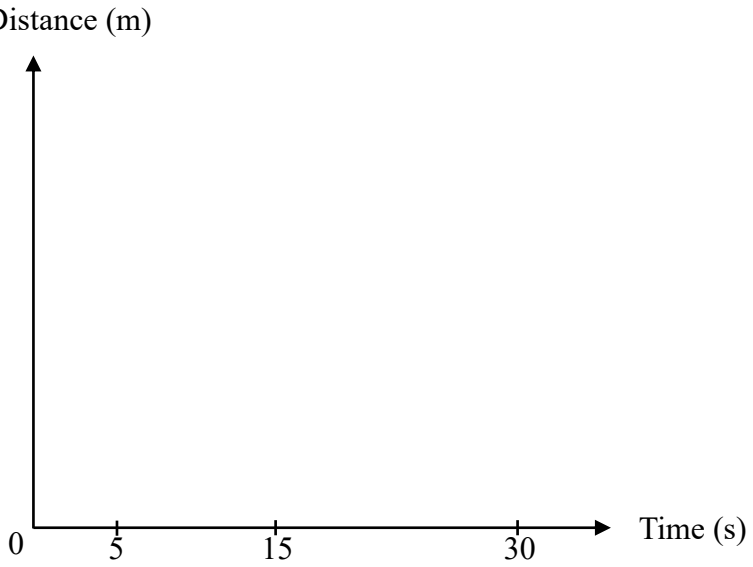
Answer u = ……..…… [1]

(b) Calculate the average speed of the car for the first 30 seconds of the journey.

Answer ……..…… m/s [2]

(c) Sketch the distance-time graph below for the car's journey.

Answer



[3]

8 $\varepsilon = \{\text{natural number less than } 10\}$
$A = \{\text{factors of } 6\}$
$B = \{\text{prime numbers}\}$
$C = \{\text{perfect squares}\}$

Use one of the symbols below to complete each statement.

$$\emptyset \quad \in \quad \subseteq \quad \subset \quad \notin$$

(a) $B \cap C =$ …… [1]

(b) $\{2, 3\}$ …… A [1]

(c) 8 …… $(A \cup B)' \cap C'$ [1]

---

9 **(a)** Factorise $9x^2 - 3x - 16y^2 + 4y$ completely.

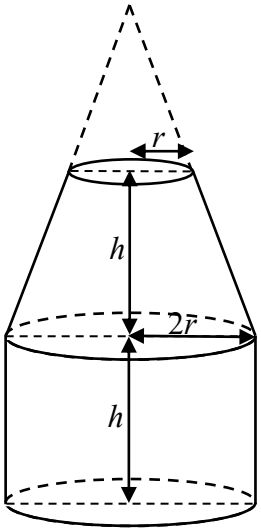
Answer …… [2]

(b) Given that $(2x - 1)^4 + (y + 2)^4 = 0$, find the value of $x - y$.

Answer $x - y =$ …… [3]

---

10 The diagram shows a milk container which is made up of a frustum and a cylinder. The height, *h* cm, of the cylinder is the same as the height of the frustum. The radius of the cylinder base is twice the radius of the top circular surface of the frustum, *r* cm. Given that the **curved** surface areas of the frustum and cylinder are equal, find an expression for *h*, in terms of *r*.



Answer $h =$ …… [3]

---

11 Solve $\frac{2}{1-x^2} - \frac{3}{x-1} = 5$.

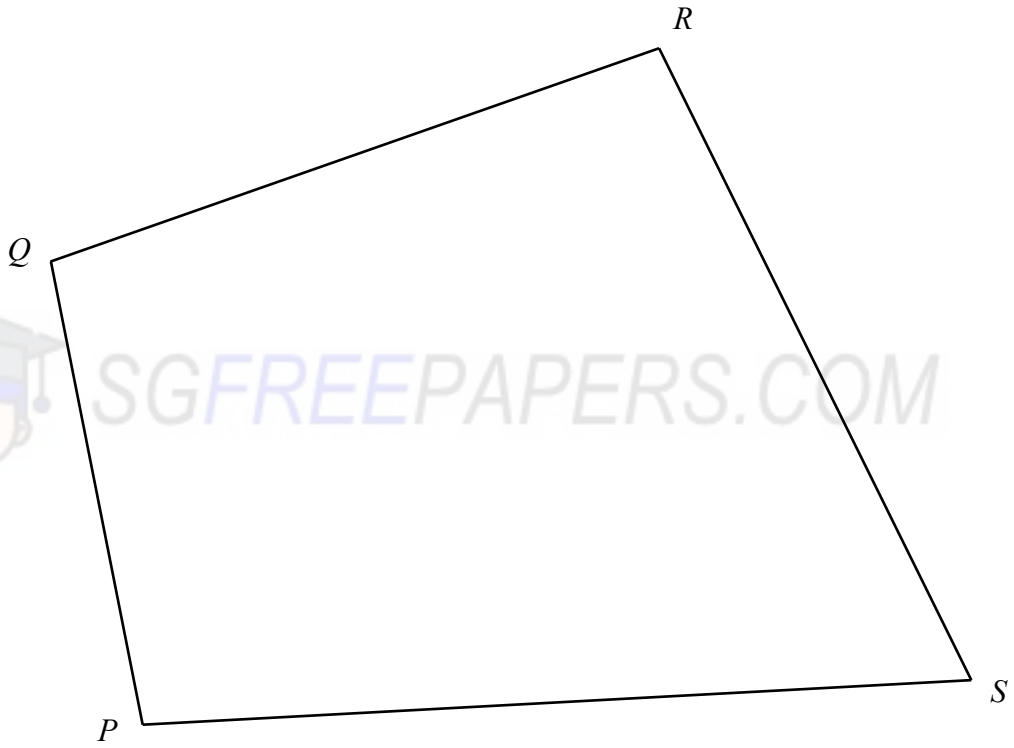
Answer $x =$ …… [3]

---

12 In the diagram *PQRS* represents a plot of land. A multi-storey carpark is to be built within *PQRS* with the following conditions:

- nearer to *PS* than *PQ*,
- nearer to *P* than *S* and
- nearer to *Q* than *P*.

Shade the region where the carpark is to be built. [3]



13 Given ABC is a triangle where $\overrightarrow{AB} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -1 \\ 8 \end{pmatrix}$.

(a) Find $\overrightarrow{BC}$.

Answer …… [2]

(b) Hence, or otherwise, show that $\angle BAC = 108.4°$.

Answer

[2]

(c) Hence, calculate the area of ΔABC.

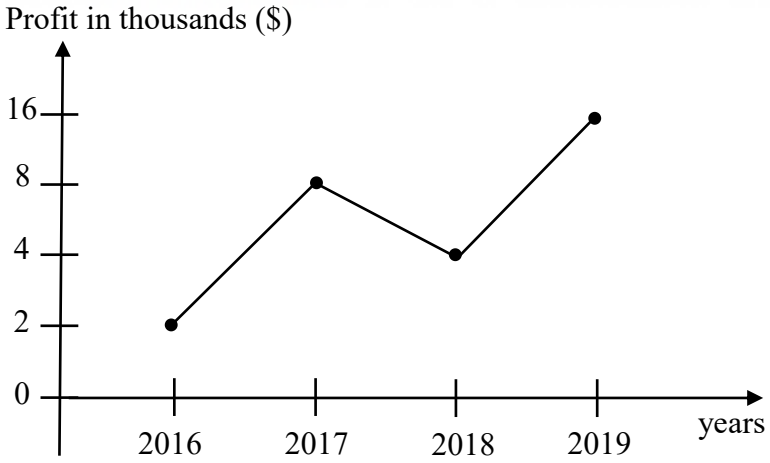
Answer …….…… units^2 [3]

14 Four numbers a, b, c, d are such that $a + b + c + d = 14$ and $a^2 + b^2 + c^2 + d^2 = 54$.
When the fifth number, e, is added, the standard deviation of the five numbers became 1.
Find the value of e.

Answer $e =$ …… [3]

---

15 The line graph below shows the profit made by Company *X* over the 4 years.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

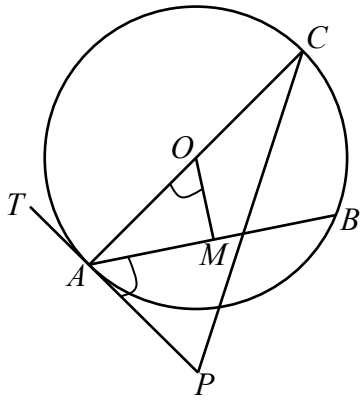
Answer ……..

……

……..... [2]

---

16 The diagram shows a circle *ABC*, with centre *O*. *AC* is the diameter of the circle. *M* is the midpoint of chord *AB* and *TAP* is tangent to the circle at point *A*.



Show, with reasons, that $\angle BAP = \angle AOM$.

Answer

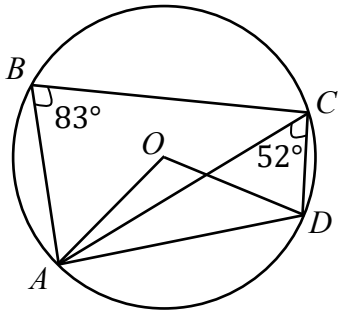
[3]

---

17 Mrs Teo wishes to open an account with a bank by investing \$5000 for 5 years.
Bank A offers 3% per annum, compounded half-yearly.
Bank B offers *r*% per annum simple interest.
Given that both banks offer the same amount of interest at the end of 5 years, find the value of *r*.

Answer $r =$ …… [3]

18



In the diagram, *A*, *B*, *C* and *D* are points on a circle, centre *O*.
Angle *ABC* = 83° and angle *ACD* = 52°.
Find angle *ODC*.

Answer $\angle ODC$ = …..……° [3]

---

19 **(a)** The volume of cube *A* is $1176x$ cm^3, where the length of the sides is an integer.
Find the smallest possible positive integer *x*.

Answer x = …… [2]

(b) What is the maximum number of cube *A* that a container of dimensions 5 m by 1m by 3 m can hold?

Answer ……. [3]

---

20 Singapore Chinese Dance Theatre put up a production in July.
The tickets pricing for senior citizens, students and adults were \$28, \$38 and \$48 respectively.

This information can be represented by the matrix $\mathbf{Q} = \begin{pmatrix} 28 \\ 38 \\ 48 \end{pmatrix}$

(a) 2 senior citizens, 15 students and 10 adults order tickets through NC School.
21 students and x adults order tickets through RV School.
Represent this information in a 2×3 matrix **P**.

Answer **P** = $\begin{pmatrix} & & \end{pmatrix} \begin{matrix} NC \\ RV \end{matrix}$ [1]

(b) Find the matrix **R**, in terms of x, such that **R** = **PQ**.

Answer **R** = $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

(c) Explain what each element in matrix **R** represents.

…….

……. [1]

(d) The total amount of money collected from NC School is less than RV School.
Work out the least value of x.

Answer x = …… [2]

(e) All tickets ordered through school will be entitled to a 25% discount for senior citizens, 20% discount for students and 15% discount for adults. Write down matrix **D** such that the elements in matrix multiplication of **PDQ** gives the total amount of money collected from each school after discount.

Answer **D** = $\begin{pmatrix} & & \\ & & \end{pmatrix}$ [1]

21 Anna and Betty have been given a task to complete 24 stamps in 15 days.
If Anna fall sick after 12 days, Betty will take additional 2 days to complete the task.
If Betty fall sick after 12 days, Anna will take additional n days to complete the task.

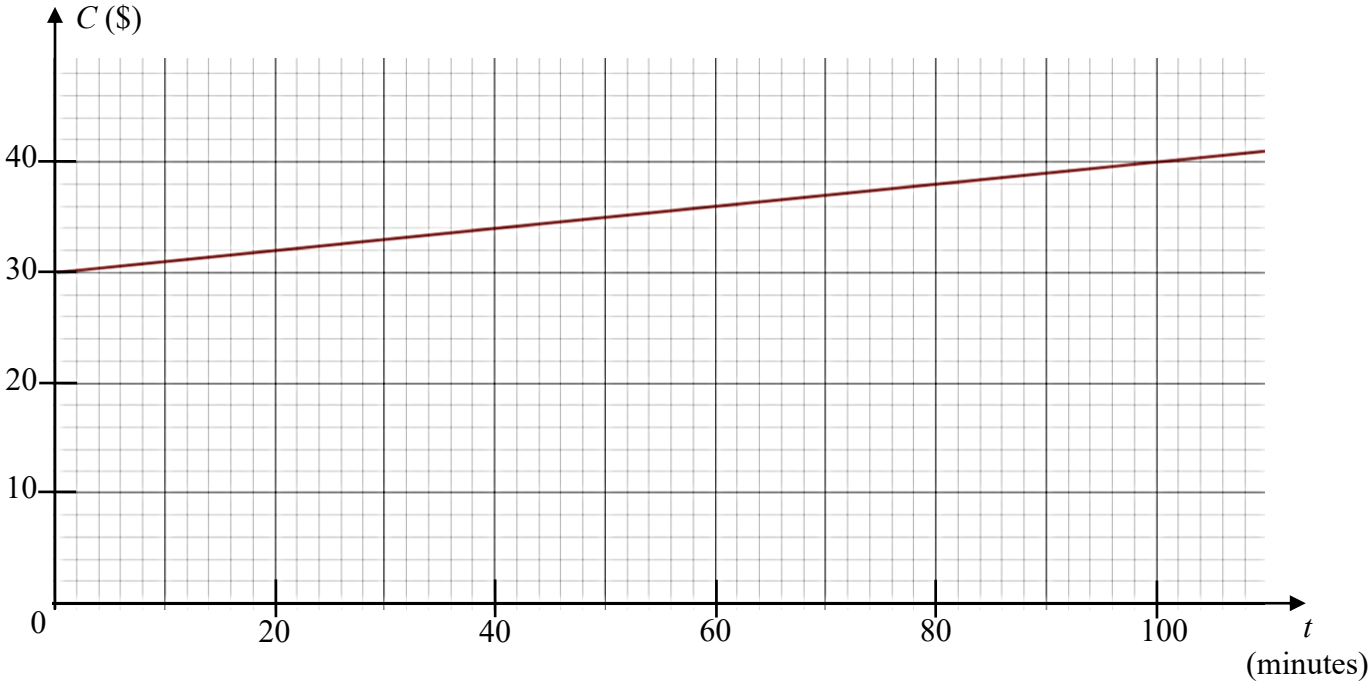
(a) Find the value of n.

Answer $n =$ …… [4]

(b) State an assumption you have made for part **(a)**.

Answer …….. [1]

22 The cost of a mobile phone plan, $\$C$, with respect to the amount of additional talktime, t minutes, by the user can be represented by the graph below.



(a) State the equation of this line.

Answer ……. [1]

(b) Explain what the vertical C-intercept value represents.

Answer ……..

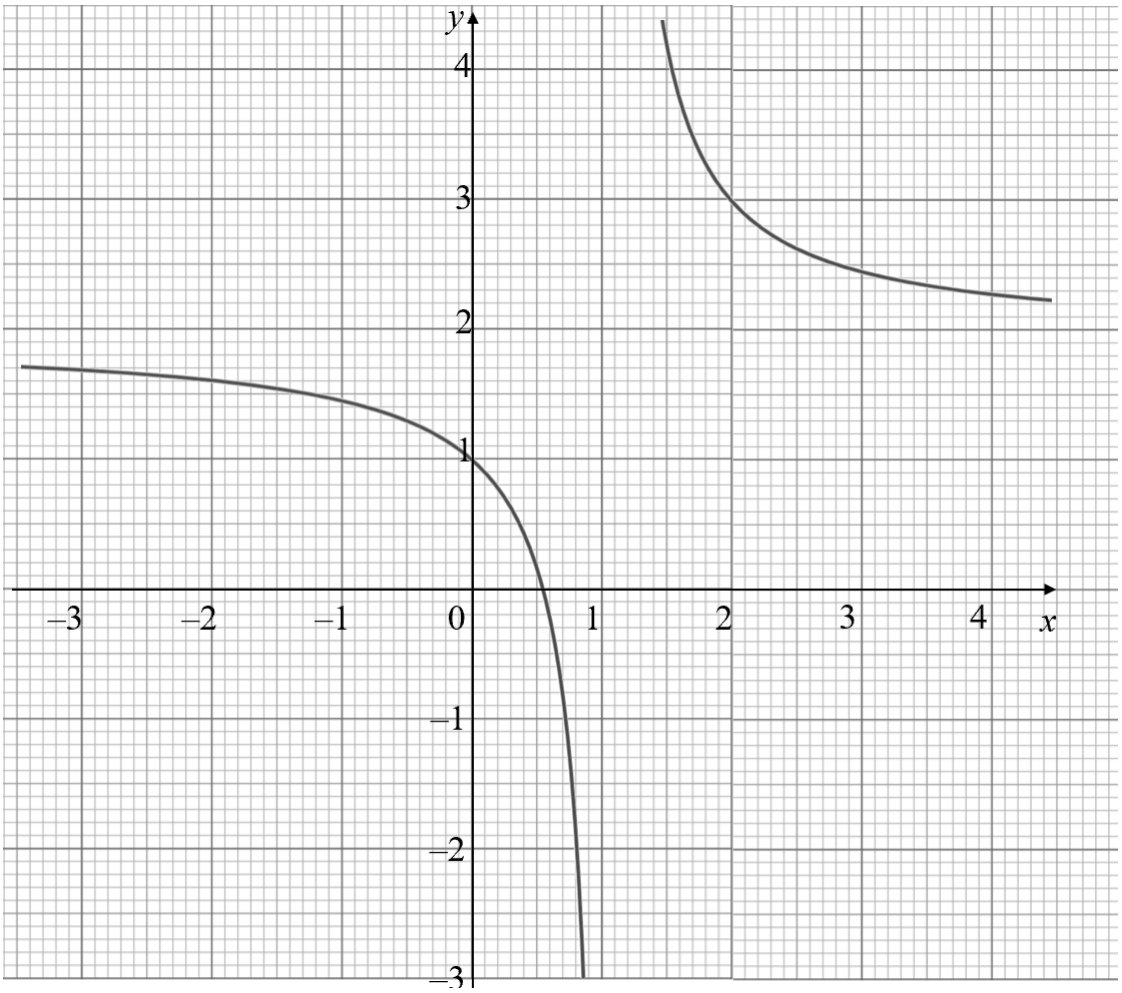
…… [1]

(c) Give a reason why the cost is **not** directly proportional to the additional talktime.

Answer ...……..……..……..

…… [1]

23 The graph of $y = \frac{1}{x-1} + 2$ is drawn on the grid.



(a) The equation $x^2 - x = 1$ can be solved by drawing a suitable straight line on the grid. Find the equation of this straight line.

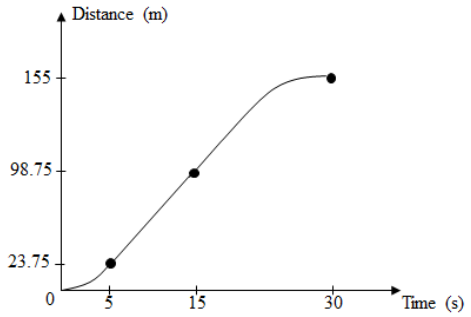
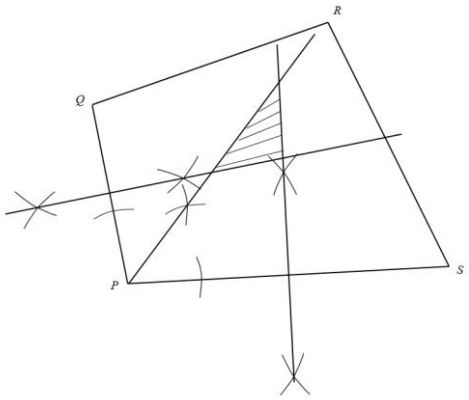
Answer …… [2]

(b) By drawing the straight line from part **(a)**, solve the equation $x^2 - x = 1$.

Answer x = …… or …… [2]

~ End of paper ~

Answers

1	$-2 \leq x < 7$
2a	199
2b	5
3	3
4	$a = -1 \ , \ \ b = 3$
5	$\dfrac{6n-5}{4n^2-1}$
6	1440
7a	7.5
7b	$5\dfrac{1}{6}$
7c	
8a	$\varnothing$
8b	$\subset$
8c	$\in$
9a	$(3x-4y)(3x+4y-1)$
9b	2.5
10	$1.13r$
11	0 or 0.6
12	
13a	$\begin{pmatrix}-6\\9\end{pmatrix}$
13b	Shown
13c	19.5

14	3.5
15	The vertical axis is inconsistent, hence making the increase from 2016 to 2017 looks the same as the increase from 2018 to 2019, but the actual is lesser. Or The data for 2019 is invalid. There can be a decrease instead of increase.
16	Shown
17	3.21
18	59
19a	63
19b	154
20a	$\mathbf{P} = \begin{pmatrix}2 & 15 & 10\\0 & 21 & x\end{pmatrix}$
20b	$\mathbf{R} = \begin{pmatrix}1106\\798+48x\end{pmatrix}$
20c	It represents the total amount collected/ to be paid by NC and RV respectively.
20d	$x = 7$
20e	$\mathbf{D} = \begin{pmatrix}0.75 & 0 & 0\\0 & 0.8 & 0\\0 & 0 & 0.85\end{pmatrix}$
21a	3.5
21b	Both Anna and Betty worked at a constant rate.
22a	$C = 30 + 0.1t$
22b	It represents the basic cost without any additional talktime.
22c	The graph does not start from origin. Or $\frac{C}{t} \neq$ constant.
23a	$y = x + 2$
23b	Accept -0.6 ± 0.05 and 1.6 ± 0.05

Name: **ANSWERS** **Register Number:** **Class:**

For Marker's Use

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2018

SECONDARY FOUR EXPRESS

MATHEMATICS **4048/01**

Paper 1 **6 August 2018, Monday**

2 hours

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

Setter: Ms Ting Shi Yun

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

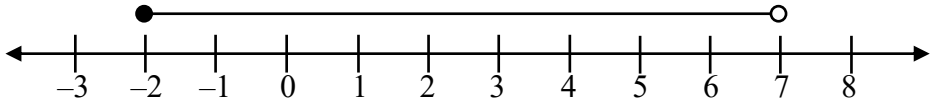
Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer all the questions.

1 A range of values of x is represented on the number line below.



Write down inequalities that represent this range of values of x.

Answer $-2 \leq x < 7$ **B1** [1]

---

2 The stem-and-leaf diagram shows the masses, in grams, of some oranges.

19	1	2	2	m	5	6	6	8
20	0	0	3	4	6			
21	1	4	7					

Key: 19 | 2 represents 192 grams

(a) Find the median of these masses.

Answer median = 199 **B1** g [1]

(b) Given that the interquartile range is 10, find the value of m.

$205 - Q_1 = 10$

$Q_1 = 195$ **M1**

$\therefore m = 5$ **A1**

Answer m = 5 [2]

---

3 Given that $2^{x+1} + 2^x = 24$, find the value of x.

$2^x(2 + 1) = 24$

$2^x = 8$ **M1**

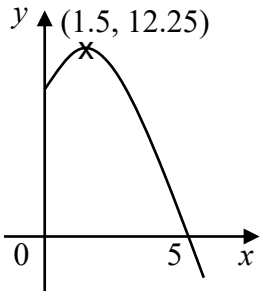
$2^x = 2^3$

$\therefore x = 3$ **A1**

Answer x = 3 [2]

---

4 The diagram shows part of the curve $y = ax^2 + bx + 10$. It cuts the x-axis at 5 and the coordinates of the maximum point is (1.5, 12.25).



Find the value of a and of b.

$y = a(x - 1.5)^2 + 12.25$

$y = a(x^2 - 3x + 2.25) + 12.25$

$\therefore a(2.25) + 12.25 = 10$ **M1**

$a = -1$ **A1**

$\therefore b = 3$ **B1**

Answer $a =$ -1

$b =$ 3 [3]

---

5 The first four terms in a sequence of numbers T_1, T_2, T_3, T_4, …, are given below.

$$\frac{1}{3} \ , \ \frac{7}{15} \ , \ \frac{13}{35} \ , \ \frac{19}{63} \ , \ \ldots$$

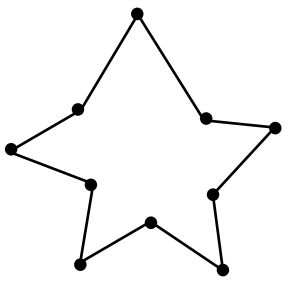
Find an expression, in terms of n, for T_n.

Note: If students only find numerator or denominator, award **B1**

Answer $T_n =$ $\dfrac{6n-5}{4n^2-1}$ **B2** [2]

---

6 The diagram shows an irregular polygon.
Find the sum of all interior angles of this polygon.



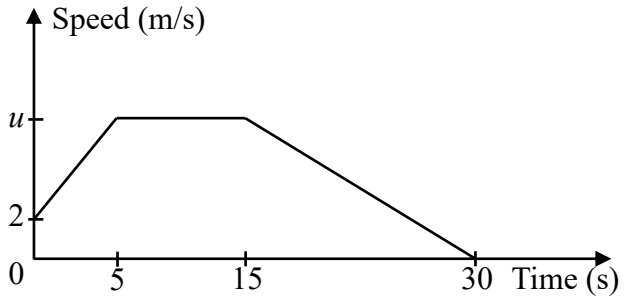
8×180 **M1**

$= 1440°$ **A1**

Answer 1440 ° [2]

---

7 The diagram shows the speed-time graph of a remote controlled toy car for the first 30 seconds of the journey.



(a) Given that the deceleration of the car is 0.5 m/s^2, find the greatest speed, u m/s.

Answer u = ……7.5…… **B1** [1]

(b) Calculate the average speed of the car for the first 30 seconds of the journey.

$$\text{Total distance} = \frac{1}{2}(2+7.5)(5)+(7.5)(10)+\frac{1}{2}(7.5)(15)$$
$$= 23.75+75+56.25$$
$$= 155 \quad \textbf{M1}$$

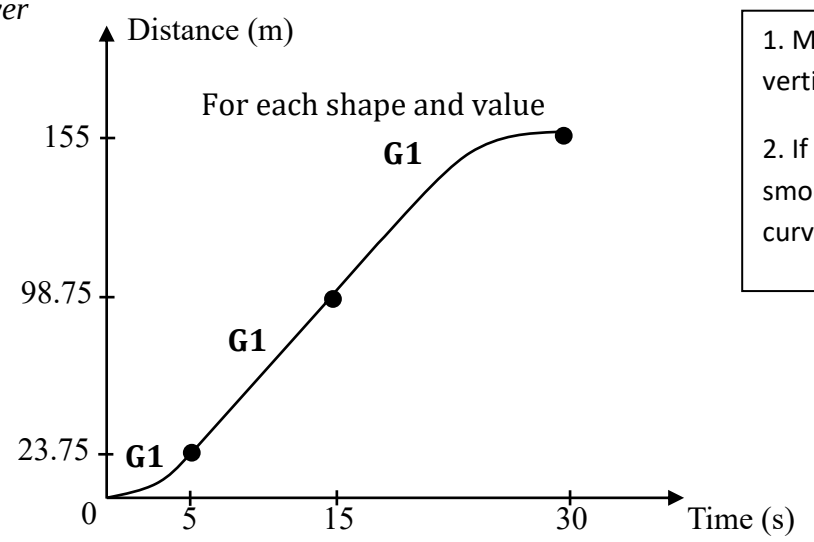
Only accept exact proper fraction

$$\textbf{Average speed} = \frac{155}{30} = 5\frac{1}{6} \quad \textbf{A1}$$

Answer ……$5\frac{1}{6}$…… m/s [2]

(c) Sketch the distance-time graph below for the car's journey.

Answer



1. Missing markings on vertical axis: –1 mark

2. If the sketch is not smooth (a.k.a. pimple curve), –1 mark

[3]

8 ε = {natural number less than 10}= {1, 2, 3, 4, 5, 6, 7, 8, 9}
A = {factors of 6} = {1, 2, 3, 6}
B = {prime numbers} = {2, 3, 5, 7}
C = {perfect squares} = {1, 4 , 9}

Use one of the symbols below to complete each statement.

$\emptyset$ $\in$ $\subseteq$ $\subset$ $\notin$

(a) $B \cap C =$ ..... $\emptyset$ **B1** [1]

(b) {2, 3} ..... $\subset$ ..... A **B1** [1]

(c) 8 ..... $\in$ ..... $(A \cup B)' \cap C'$ **B1** [1]

---

9 **(a)** Factorise $9x^2 - 3x - 16y^2 + 4y$ completely.

$9x^2 - 16y^2 - 3x + 4y$

$= (3x - 4y)(3x + 4y) - (3x - 4y)$ **M1**

$= (3x - 4y)(3x + 4y - 1)$ **A1**

Answer ..... $(3x - 4y)(3x + 4y - 1)$ [2]

(b) Given that $(2x - 1)^4 + (y + 2)^4 = 0$, find the value of $x - y$.

Since $(2x - 1)^4 \geq 0$ and $(y + 2)^4 \geq 0$, **B1** reason

$2x - 1 = 0$ and $y + 2 = 0$

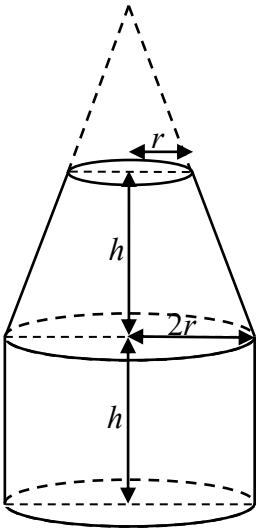
$x = \frac{1}{2}$ and $y = -2$ **M1**

$\therefore x - y = 2.5$ **A1**

Answer $x - y =$ ..... 2.5 [3]

---

10 The diagram shows a milk container which is made up of a frustum and a cylinder. The height, h cm, of the cylinder is the same as the height of the frustum. The radius of the cylinder base is twice the radius of the top circular surface of the frustum, r cm. Given that the **curved** surface areas of the frustum and cylinder are equal, find an expression for h, in terms of r.



$$\frac{\text{Area}_{\text{small cone}}}{\text{Area}_{\text{big cone}}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Surface area of frustum $= 3\pi r\sqrt{h^2 + r^2}$ **M1**

$$3\pi r\sqrt{h^2 + r^2} = 2\pi(2r)h \quad \textbf{M1}$$

$$3\sqrt{h^2 + r^2} = 4h$$

$$9(h^2 + r^2) = 16h^2$$

$$9r^2 = 7h^2$$

$$h^2 = \frac{9r^2}{7}$$

$$h = \frac{3}{\sqrt{7}}r \quad \text{or} \quad \frac{3\sqrt{7}}{7}r \quad \text{or} \quad 1.13r \quad \textbf{A1}$$

Answer $h =$ ..... 1.13r [3]

---

11 Solve $\frac{2}{1-x^2} - \frac{3}{x-1} = 5$.

$$\frac{2}{(1-x)(1+x)} + \frac{3}{1-x} = 5$$

$$\frac{2 + 3(1+x)}{(1-x)(1+x)} = 5 \quad \textbf{M1}$$

$$2 + 3(1+x) = 5(1-x)(1+x)$$

$$5x^2 + 3x = 0$$

$$x(5x+3) = 0 \quad \textbf{M1}$$

$$x = 0 \quad \text{or} \quad x = -\frac{3}{5} \quad \textbf{A1}$$

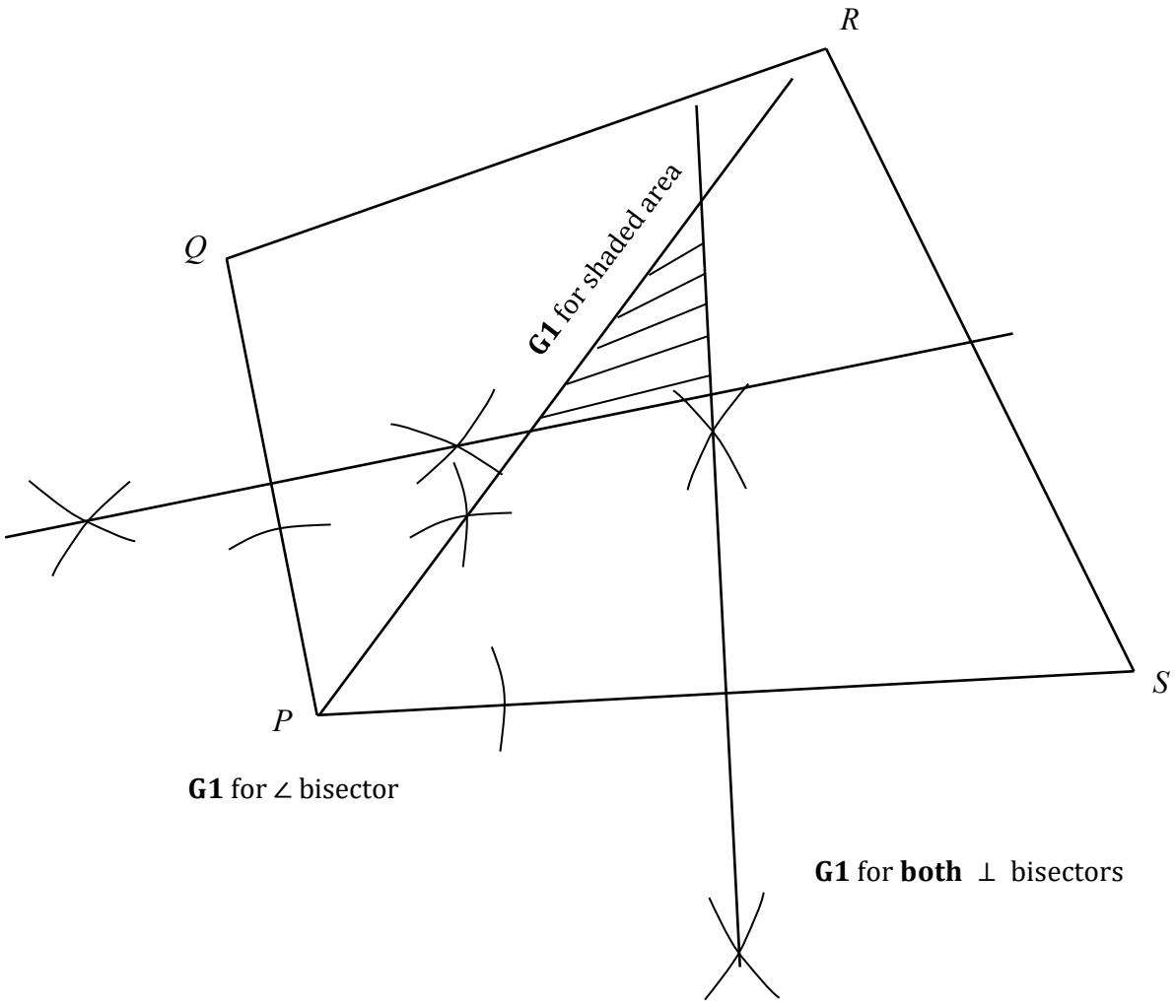
Answer $x =$ ..... 0 or $-\frac{3}{5}$ [3]

---

12 In the diagram *PQRS* represents a plot of land. A multi-storey carpark is to be built within *PQRS* with the following conditions:

- nearer to *PS* than *PQ*,
- nearer to *P* than *S* and
- nearer to *Q* than *P*.

Shade the region where the carpark is to be built. [3]



G1 for ∠ bisector

G1 for **both** ⊥ bisectors

13 Given ABC is a triangle where $\overrightarrow{AB} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -1 \\ 8 \end{pmatrix}$.

(a) Find $\overrightarrow{BC}$.

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$$
$$= -\begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 8 \end{pmatrix} \quad \textbf{M1}$$
$$= \begin{pmatrix} -6 \\ 9 \end{pmatrix} \quad \textbf{A1}$$

Answer $\begin{pmatrix} -6 \\ 9 \end{pmatrix}$ [2]

(b) Hence, or otherwise, show that $\angle BAC = 108.4°$.

Answer

[2]

"Hence" method

$|\overrightarrow{BC}| = \sqrt{117}$

$|\overrightarrow{AB}| = \sqrt{26}$

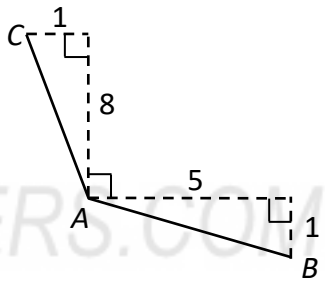
$|\overrightarrow{AC}| = \sqrt{65}$

$117 = 26 + 65 - (\sqrt{26})(\sqrt{65})cos\angle BAC$ **M1**

$cos\angle BAC = -0.3162277$

$\angle BAC = 108.4°$ **A1**

"Otherwise" method



$\angle BAC = 90 + tan^{-1}\frac{1}{5} + tan^{-1}\frac{1}{8}$ **M1**

$= 108.4°$ **A1**

(c) Hence, calculate the area of ΔABC.

Area = $\frac{1}{2}(\sqrt{26})(\sqrt{65})\sin 108.4$

= 19.5 units^2

M1 length of AB and AC
M1 formula
A1

If students calculate the length in part (b), award them the M1 too.

Answer 19.5 units^2

[3]

14 Four numbers a, b, c, d are such that $a + b + c + d = 14$ and $a^2 + b^2 + c^2 + d^2 = 54$.
When the fifth number, e, is added, the standard deviation of the five numbers became 1.
Find the value of e.

$$\sqrt{\frac{54 + e^2}{5} - \left(\frac{14 + e}{5}\right)^2} = 1 \qquad \textbf{M1}$$

$$\frac{5(54 + e^2) - (14 + e)^2}{25} = 1$$

$$270 + 5e^2 - 196 - 28e - e^2 = 25$$

$$4e^2 - 28e + 49 = 0 \qquad \textbf{M1}$$

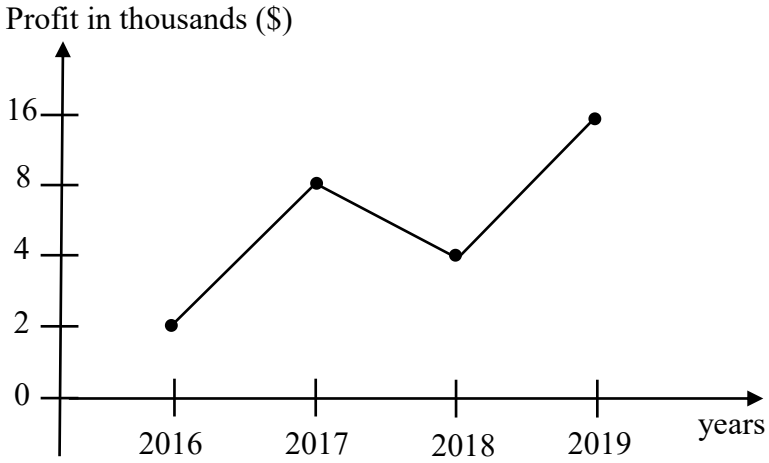
$$(2e - 7)^2 = 0$$

$$e = 3.5 \qquad \textbf{A1}$$

Answer $e =$ 3.5 [3]

---

15 The line graph below shows the profit made by Company *X* over the 4 years.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

B1

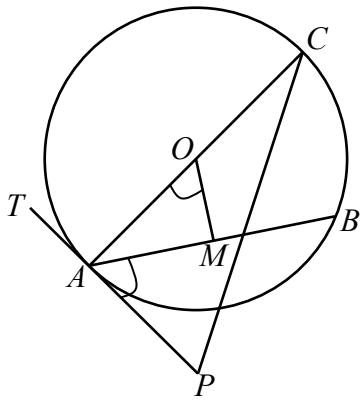
Answer The vertical axis is inconsistent, hence making the increase from 2016 to 2017 looks the same as the increase from 2018 to 2019, but the actual is lesser. **B1**

OR The data for 2019 is invalid. There can be a decrease instead of increase. [2]

B1 **B1**

---

16 The diagram shows a circle *ABC*, with centre *O*. *AC* is the diameter of the circle. *M* is the midpoint of chord *AB* and *TAP* is tangent to the circle at point *A*.



Show, with reasons, that $\angle BAP = \angle AOM$.

Answer

[3]

$\angle OAP = 90°$ (Tangent perpendicular to radius) **M1**

$\angle AM0 = 90°$ (Perpendicular bisector of chord) **M1**

$\angle AOM = 180 - 90 - \angle OAM$ (∠ sum of Δ)

$= 90 - (90 - \angle BAP)$ **A1**

$= \angle BAP$

Any missing reasons: **overall** –1 mark.

---

17 Mrs Teo wishes to open an account with a bank by investing $5000 for 5 years.
Bank A offers 3% per annum, compounded half-yearly.
Bank B offers *r*% per annum simple interest.
Given that both banks offer the same amount of interest at the end of 5 years, find the value of *r*.

$$\text{Interest} = 5000\left(1 + \frac{3}{200}\right)^{10} - 5000$$ **M1**

$$\frac{5000 \times r \times 5}{100} = 802.7041$$ **M1**

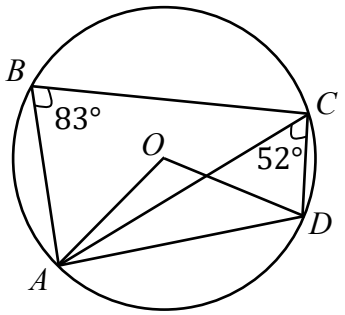
$$250r = 802.7041$$

$$r = 3.21$$ **A1**

If student write 3.21% on answer blank, –1 mark.

Answer $r =$ 3.21 [3]

18



In the diagram, *A*, *B*, *C* and *D* are points on a circle, centre *O*.
Angle *ABC* = 83° and angle *ACD* = 52°.
Find angle *ODC*.

Any missing reasons:
overall –1 mark.

$\angle AOD = 52 \times 2 = 104°$ (∠ at centre = 2∠ at circumference) **M1**

$\angle ODA = (180 - 104) \div 2 = 38°$ (base of isosceles Δ) **M1**

$\angle ODC = 180 - 83 - 38$ (∠ in opposite segment)
$= 59°$ **A1**

Answer ∠*ODC* = ……59……° [3]

---

19 **(a)** The volume of cube *A* is 1176*x* cm^3, where the length of the sides is an integer.
Find the smallest possible positive integer *x*.

$1176 = 2^3 \times 3 \times 7^2$ **M1**

$\therefore x = 3^2 \times 7 = 63$ **A1**

Answer *x* = ……63…… [2]

(b) What is the maximum number of cube *A* that a container of dimensions 5 m by 1m by 3 m can hold?

Length of cube = $2 \times 3 \times 7 = 42$ **M1**

$\frac{500}{42} = 11\frac{19}{21}$
$\frac{100}{42} = 2\frac{8}{21}$ **M1**
$\frac{300}{42} = 7\frac{1}{7}$

No. of cubes = $11 \times 2 \times 7 = 154$ **A1**

Answer ……154…… [3]

---

20 Singapore Chinese Dance Theatre put up a production in July.
The tickets pricing for senior citizens, students and adults were \$28, \$38 and \$48 respectively.

This information can be represented by the matrix $\mathbf{Q} = \begin{pmatrix} 28 \\ 38 \\ 48 \end{pmatrix}$

(a) 2 senior citizens, 15 students and 10 adults order tickets through NC School.
21 students and x adults order tickets through RV School.
Represent this information in a 2×3 matrix **P**.

B1

Answer $\mathbf{P} = \begin{pmatrix} 2 & 15 & 10 \\ 0 & 21 & x \end{pmatrix} \begin{matrix} NC \\ RV \end{matrix}$ [1]

(b) Find the matrix **R**, in terms of x, such that $\mathbf{R} = \mathbf{PQ}$.

Answer $\mathbf{R} = \begin{pmatrix} 1106 \\ 798 + 48x \end{pmatrix}$ **B1**
[1]

(c) Explain what each element in matrix **R** represents.

It represents the total amount collected/ to be paid by NC and RV respectively **B1**
.....
.....
[1]

(d) The total amount of money collected from NC School is less than RV School.
Work out the least value of x.

$798 + 48x > 1106$ **M1**

$x > 6\frac{5}{12}$

$\therefore x = 7$ **A1**

Answer $x =$ 7 ..... [2]

(e) All tickets ordered through school will be entitled to a 25% discount for senior citizens, 20% discount for students and 15% discount for adults. Write down matrix **D** such that the elements in matrix multiplication of **PDQ** gives the total amount of money collected from each school after discount.

Answer $\mathbf{D} = \begin{pmatrix} 0.75 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.85 \end{pmatrix}$ **B1**
[1]

21 Anna and Betty have been given a task to complete 24 stamps in 15 days.
If Anna fall sick after 12 days, Betty will take additional 2 days to complete the task.
If Betty fall sick after 12 days, Anna will take additional *n* days to complete the task.

(a) Find the value of *n*.

A+B: 15 days, 24 stamps
1 day, 1.6 stamps
3 days, 4.8 stamps left **M1** no. of stamps left

B: 5 days, 4.8 stamps
1 day, 0.96 stamp **M1** rate of B

A: (3+*n*) days, 4.8 stamps
1 day , $\frac{4.8}{3+n}$

$$\frac{4.8}{3+n} + 0.96 = 1.6 \quad \textbf{M1}$$

$$\frac{4.8}{3+n} = 0.64$$

$$3 + n = 7.5$$

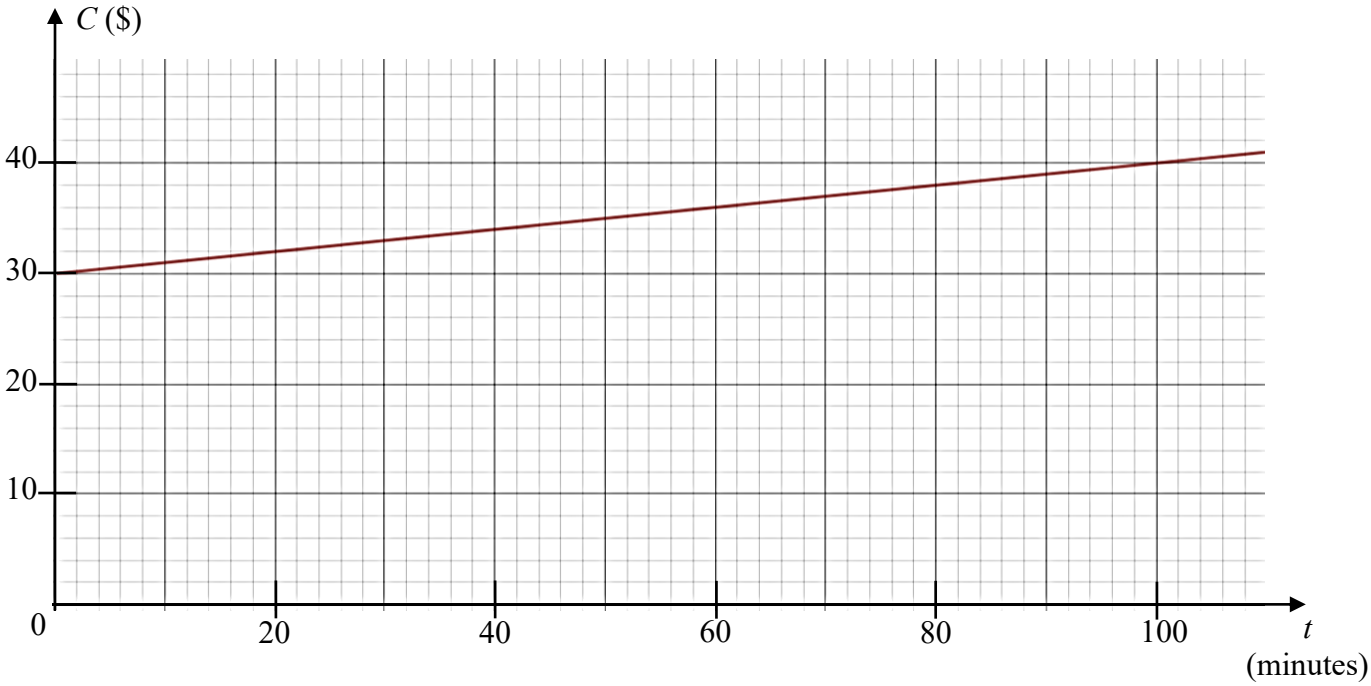
$$n = 3.5 \quad \textbf{A1}$$

Answer $n =$ 3.5 [4]

(b) State an assumption you have made for part **(a)**.

Answer Both Anna and Betty worked at a constant rate. **B1** [1]

22 The cost of a mobile phone plan, $\$C$, with respect to the amount of additional talktime, t minutes, by the user can be represented by the graph below.



(a) State the equation of this line.

B1

Answer $C = 30 + 0.1t$ [1]

(b) Explain what the vertical C-intercept value represents.

Answer It represents the basic cost without any additional talktime. **B1** [1]

(c) Give a reason why the cost is **not** directly proportional to the additional talktime.

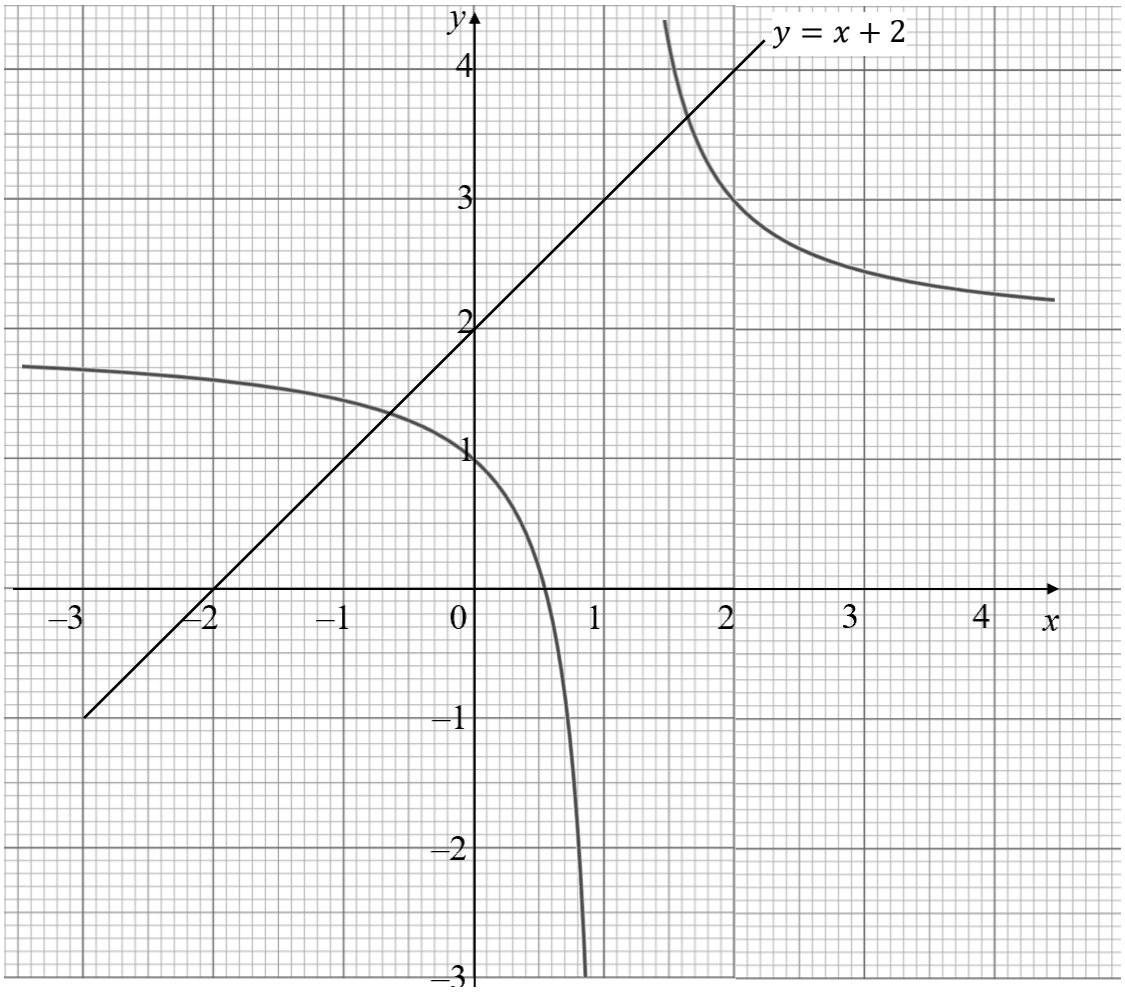
Answer The graph does not start from origin. **B1**

OR $\frac{C}{t} \neq constant$ **B1** [1]

Students cannot write $\frac{C}{t} \neq k$ and not defining what k is.

23 The graph of $y = \frac{1}{x-1} + 2$ is drawn on the grid.

G1



(a) The equation $x^2 - x = 1$ can be solved by drawing a suitable straight line on the grid. Find the equation of this straight line.

$x(x-1) = 1$ **M1**

$x = \frac{1}{x-1}$

$x + 2 = \frac{1}{x-1} + 2$

$y = x + 2$ **A1**

Answer $y = x + 2$ [2]

(b) By drawing the straight line from part **(a)**, solve the equation $x^2 - x = 1$.

B1 for both ans

Answer $x =$ -0.65 or 1.65 [2]

Accept **–0.6 ± 0.05** and **1.6 ± 0.05**

~ End of paper ~

Name: **Register Number:** **Class:**

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2018
SECONDARY FOUR EXPRESS

MATHEMATICS **4048/02**

PAPER 2 **10 September 2018, Monday**

2 hours 30 minutes

Additional materials: Writing Papers (8 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number at the top of the cover page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is **100**.

Setter: Mrs Sim Hwee Mung

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** questions.

1 **(a)** Simplify $\frac{(2xy)^2}{35xy^7} \div \left(\frac{x^{-1}y^{-2}}{4}\right)^{-2}$, leaving your answer in positive index form. [2]

(b) Solve the inequality $\frac{1}{4}x - \frac{3}{5}\left(x + \frac{1}{3}\right) \geq \frac{1}{2}(x - 9)$. [2]

(c) **(i)** Express $\frac{2}{1-2x} + \frac{3(x-1)}{2x^2-5x+2} + \frac{1}{x-2}$ as a single fraction in its simplest form. [4]

(ii) Hence solve the equation $\frac{2}{1-2x} + \frac{3(x-1)}{2x^2-5x+2} + \frac{1}{x-2} = \frac{3}{x-2}$. [2]

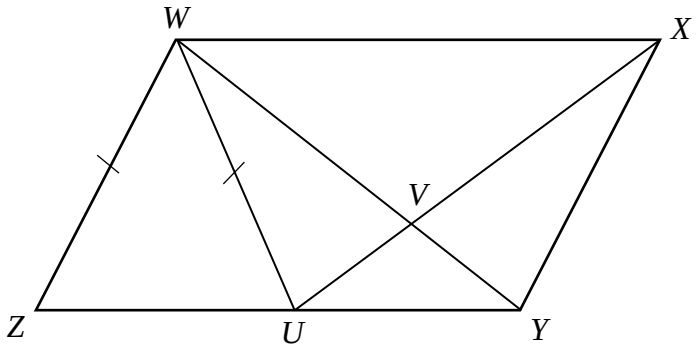
---

2 **(a)** *PQRS* is a parallelogram in which the coordinates of *P* and *Q* are $(p\,, 4)$ and $(7\,, 11)$ respectively. The line $9y + 27 - 7x = 0$ cuts the *y*-axis at *R* and is parallel to the line *PQ*.
Find

(i) the value of *p*, [3]

(ii) the coordinates of *S* by **vector method**. [3]

(b) In the diagram, *WXYZ* is a parallelogram and *U* is a point on *ZY* such that *WZ* = *WU*. The lines *WY* and *UX* intersect at *V*.



Prove that ΔWUY is congruent to ΔXYU. [3]

---

3 **(a)** The radius of a particular spherical cell is approximately 6.2×10^{-11} metre.

(i) 88 of these spherical cells are arranged to form a straight line such that each cell touches one another without overlapping. Calculate the length, in metre, of the straight line formed. Give your answer in standard form. [1]

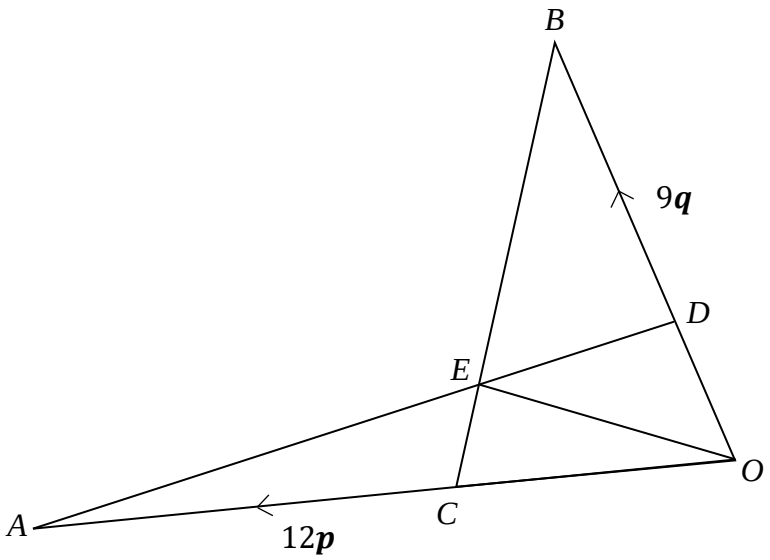
(ii) Calculate the volume of a spherical cell, in cubic centimetre. Give your answer in standard form correct to 2 decimal places. [1]

(b) The planet Mercury is approximately 77 billion metre from the Earth. Given that radio waves travel at a speed of 3×10^{8} m/s, find the time taken for radio waves to travel from the Earth to Mercury, giving your answer to the nearest minutes. [2]

(c) The word 'Googol' is defined as 1×10^{100}. If a man has ten 'Googol' cents and a Boeing 737 aircraft costs $72 million, how many such aircrafts can he buy? Give your answer in standard form correct to 5 significant figures. [3]

---

4 In the diagram, $\overrightarrow{OA} = 12\boldsymbol{p}$ and $\overrightarrow{OB} = 9\boldsymbol{q}$.
It is given that $3DB = 2OB$ and $OA = 3OC$.



(a) Express, as simply as possible, in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$,

(i) $\overrightarrow{BC}$, [2]

(ii) $\overrightarrow{DA}$. [2]

(b) Given that $\dfrac{\text{area of triangle } ODE}{\text{area of triangle } ODA} = \dfrac{1}{4}$, find $\overrightarrow{OE}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$. [2]

(c) Find the value of $\dfrac{\text{area of triangle } BDE}{\text{area of quadrailateral } EDOC}$. [2]

5 Answer the whole of this question on a sheet of graph paper.

A radioactive substance decays such that its mass, *m* grams, after *d* days is given by the equation $m = 43(3)^{-d}$.

The table below shows record of the mass, *m* grams of the substance, corrected to 1 decimal place, after *d* days.

d (days)	0	1	2	3	4	5	6
m (grams)	43	14.3	4.8	*p*	0.5	0.2	0.1

(a) Find the value of *p*. [1]

(b) Using a horizontal scale of 2 cm to represent 1 day and a vertical scale of 2 cm to represent 5 grams, draw the graph of $m = 43(3)^{-d}$ for $0 \le d \le 6$. [3]

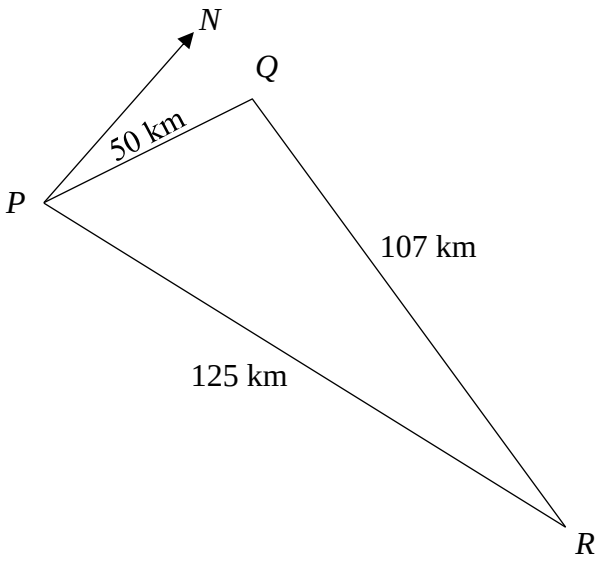
(c) Use your graph to estimate the value of *d* when the mass of substance is reduced to 65% of its original mass. [1]

(d) By drawing a tangent, find the gradient of the curve at the point when $d = 2.5$.

State briefly what this gradient represents. [3]

(e) Using your graph, find the range of *d* for $86(3)^{-d} + 12d < 60$. [2]

6 PQR represents a triangular plot of land on horizontal ground.
PQ = 50 km, QR = 107 km and PR = 125 km.
R is due east of P.



(a) Calculate

(i) the bearing of Q from P, [3]

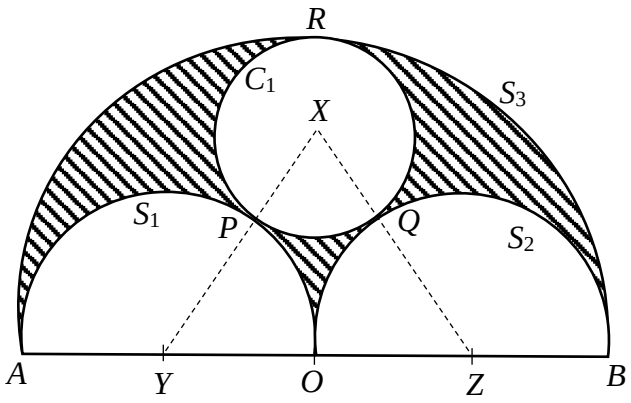
(ii) the bearing of P from Q, [2]

(iii) the obtuse angle PQR, [2]

(iv) the area of the triangular plot of land PQR. [2]

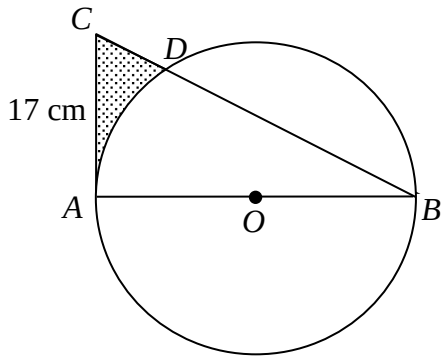
(b) A vertical pole of height 9 km is erected at Q. Calculate the greatest angle of elevation of the top of the pole from a point S along PR. [3]

7 In the diagram, the circle C_1 with centre X has a radius $(3r + 1)$ cm, where r is a constant. Two identical semicircles, S_1 and S_2 with centre at Y and Z respectively, have a radius $(13 - 6r)$ cm.
Another semicircle, S_3 with centre O has a diameter AB.
C_1 touches S_1 and S_2 at P and Q respectively while S_1 and S_2 touches one another at O.
S_3 touches C_1, S_1 and S_2 at R, A and B respectively.
$AYOZB$ is a straight line.



(a) Write down an expression, in its simplest form and in terms of r, for
(i) XZ, [1]
(ii) XO. [2]
(b) Hence, form an equation in terms of r and show that it reduces to $126r^2 - 411r + 299 = 0$. [3]
(c) Solve the equation $126r^2 - 411r + 299 = 0$. [2]
(d) Hence, find the area of the shaded region. [3]

8 **(a)** The diagram shows a circle with centre O and radius of 12 cm. AB is the diameter of the circle and AC is a tangent to the circle at A with AC = 17 cm. The circle intersects the line BC at D.



(i) Show that angle $DOA = 1.23$ radians. [2]

Calculate

(ii) the length of minor arc AD, [1]

(iii) the area of the shaded region. [3]

(b) Diagram I shows an open container which is made up of a cylinder and a cone. The cylinder has radius r cm and height 30 cm.
The cone has base radius r cm and slant height l cm.
The container is fully filled with water.

Diagram II shows a spherical object in which half of it is immersed into the container and some water is displaced. The radius of the sphere is the same as the radius of the cylinder. Assume the thickness of the container and the spherical object are negligible.

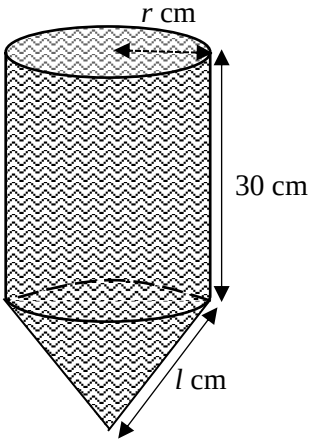


Diagram I

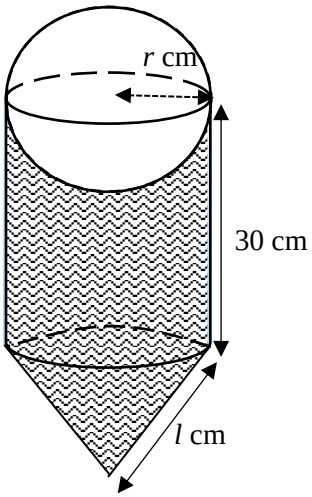


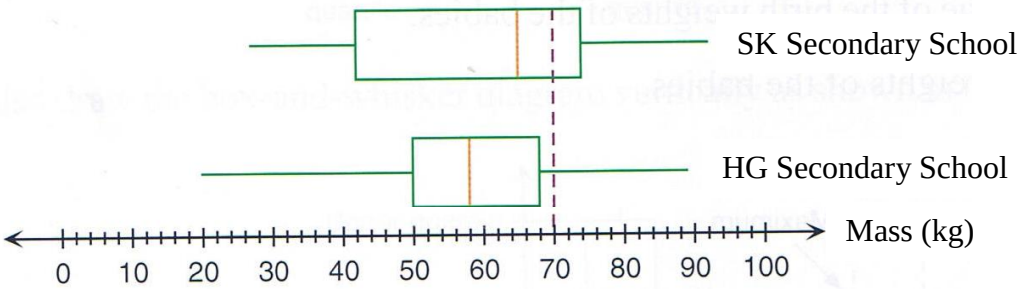
Diagram II

Given that the volume of the water displaced is 1152π cm^3 and the volume of the water left in the container is 3600π cm^3, find

(i) the value of r and of l, [4]

(ii) the total internal area of the container and the sphere that is in contact with water, leaving your answer in terms of π. [2]

9 **(a)** The following box-and-whisker diagrams show the distribution of the mass of 300 students from each school, SK Secondary School and HG Secondary School respectively.

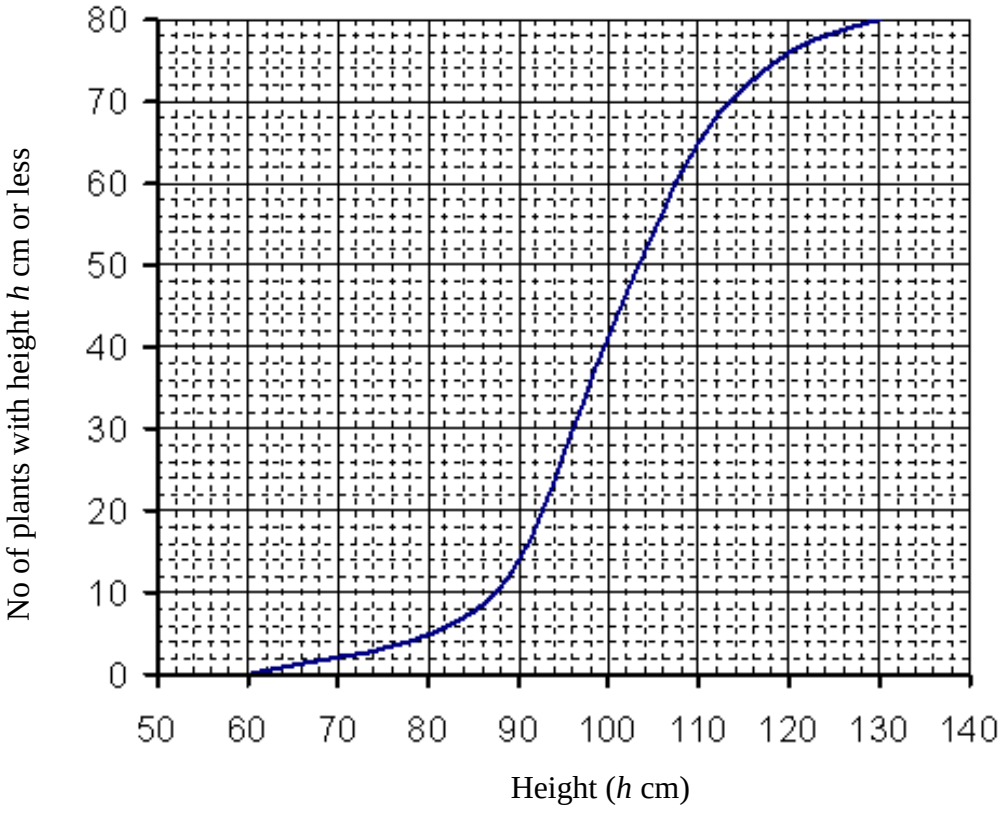


(i) What is the median mass for each school? [1]

(ii) Compare the mass of the students from SK Secondary School and HG Secondary School in two ways. [2]

(iii) Mary commented that there are more students in HG Secondary School than SK Secondary School who weigh more than or equal to 70 kg.
Do you agree with Mary? Support with a reason. [1]

(b) The cumulative frequency curve shows the height distribution of 80 plants.



(i) Use the graph to find

(a) the value of m, if 32.5% of the plants have heights more than m cm, [2]

(b) the probability that two plants chosen will each has a height of more than 118 cm. [2]

(ii) **(a)** The height distribution of the 80 plants was also recorded in the following frequency table. Find the value of a and of b.

Height (h cm)	**Number of plants**
$60 < h \leq 70$	2
$70 < h \leq 80$	a
$80 < h \leq 90$	9
$90 < h \leq 100$	27
$100 < h \leq 110$	23
$110 < h \leq 120$	b
$120 < h \leq 130$	4

[1]

(b) Hence find the mean and the standard deviation of the height of the 80 plants. [2]

10 Mr and Mrs Tan bought a 3-bedroom unit at a newly launched condominium project at Serangoon. The unit has a floor area of 1152 square foot (sqft). The selling price for the unit is at $1494 per sqft. The developer gives a 5% early bird discount to all buyers.

(a) Calculate the price that they paid for the unit. [1]

Mr and Mrs Tan are both Singapore citizens and they also owned a HDB 5-room flat and they do not have any intention of selling their flat. As such, they will have to pay Additional Buyer's Stamp Duty (ABSD) to the government on top of the Buyer's Stamp Duty (BSD). Table 1 shows the BSD rate and Table 2 shows the ABSD rate.

Table 1. BSD

Based on purchase price	**Rate**
First $180 000	1%
Next $180 000	2%
Next $640 000	3%
Remaining amount	4%

Table 2. ABSD

Based on purchase price	**Rate**
SC^1 buying first residential property	NA
SC^1 buying second residential property	12%
SC^1 buying third and subsequent residential property	15%

SC^1 denotes Singapore Citizens

(b) Calculate the **total** Buyer's Stamp Duty paid by Mr and Mrs Tan. [4]

Mr and Mrs Tan made a 20% down-payment based on the amount paid for the unit obtained in **(a)**. For the remaining amount, which exclude the total Buyer's Stamp Duty, they had decided to sign up either for a 20-years Home Loan plan from CBCO Bank or a 30-years Home Loan plan from BSOP Bank.

(c) Determine which bank they should sign up if they can only afford a monthly instalment of not more than $6000.
Support you answer with appropriate workings. [5]

Simple Interest Rate for CBCO Bank

Loan period: 20 years

1st year:	2nd Year:
2.18% p.a.	2.18% p.a.
Thereafter: 2.68% p.a.	

Simple Interest Rate for BSOP Bank

Loan period: 30 years

1st year:	2nd Year:
1.95% p.a.	1.95% p.a.
Thereafter: 2.15% p.a.	

***** End of Paper *****

Answer keys for 2018 NCHS Prelim Exam 2_E.Maths Paper 2

1a	$\frac{1}{140xy^9}$	7ai	14 – 3r
1b	$x \leq 5\frac{1}{17}$	7aii	25 – 15r
1ci	$\frac{3x}{(x-2)(2x-1)}$	7b	$(14-3r)^2 = (25-15r)^2 + (13-6r)^2$
1cii	$x = 1$	7c	$2\frac{1}{6}$ or $1\frac{2}{21}$
2ai	$p = -2$	7d	72.1 cm^2
2aii	S = (-9, -10)	8ai	In triangle CAB, tan $C\hat{B}A = \frac{17}{24}$ $C\hat{B}A = 0.61630\ rad$ $D\hat{O}A = 2 \times 0.61630$ (angle at centre = 2× angle at circumference) = 1.23 rad (shown)
2b	WZ = WU (given) = XY (opp sides of *ll*gram) UY (common length) $W\hat{U}Y = 180 - W\hat{U}Z$ (adj angles on a str. line) $X\hat{Y}U = 180 - W\hat{Z}U$ (int. angles) Since WZ = WU $\Rightarrow W\hat{Z}U = W\hat{U}Z$ Hence $W\hat{U}Y = X\hat{Y}U$ ΔWUY is congruent to ΔXYU (SAS)	8aii	14.8 cm
3ai	1.09×10^{-8} m	8aiii	47.3 cm^3
3aii	9.98×10^{-25} cm^3	8bi	h = 9 cm , $l = 15\ cm$
3b	4 min	8bii	1188π cm^2
3c	1.3889×10^{91}	9ai	For SK Sec, median = 65 kg For HG Sec, median = 58 kg
4ai	$-9\boldsymbol{q} + 4\boldsymbol{p}$	9aii	Generally, the students for SK Sec Sch are heavier compared to the students in HG Sec Sch because of the higher median. The mass of the students in HG Sec Sch is more consistent than that of SK Sec Sch.
4aii	$-3\boldsymbol{q} + 12\boldsymbol{p}$	9aiii	Disagree as there are more than 25% of the students from SK Sec Sch weigh more than 70 kg while there are less than 25% of the students fron HG Sec Sch weigh more than 70 kg. Accept: There are more students in SK Sec Sch weigh more than 70 kg because it has a higher upper quartile than HG Sec Sch.
4b	$3\boldsymbol{p} + \frac{9}{4}\boldsymbol{q}$	9bia	m = 105
4c	1	9bib	$\frac{3}{632}$
5a	1.6	9biia	a = 3, b = 12
5c	Accept 0.2 to 0.4	9biib	99.75 cm ; SD = 12.6 cm
5d	Accept -3.06 to -2.46 It represents the rate of change of mass with respect to day when d = 2.5.	10a	$1 635 033.60
5e	0.3 < d < 4.95	10b	$246 205.38
6ai	032.3°	10c	**CBCO Bank** Monthly instalment = $8316.87 > $6000 **BSOP Bank** Monthly instalment = $5962.42 < $6000Mr and Mrs Tan should sign up with BSOP Bank as the monthly instalment is less than $6000.
6aii	212.3°		
6aiii	99.0°		
6aiv	2640 km^2		
6b	12.0°		

Name: **Register Number:** **Class:**

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2018
SECONDARY FOUR EXPRESS

MATHEMATICS **4048/02**

PAPER 2 **10 September 2018, Monday**

2 hours 30 minutes

SUGGESTED SOLUTIONS

Setter: Mrs Sim Hwee Mung

1

(a) Simplify $\frac{(2xy)^2}{35xy^7} \div \left(\frac{x^{-1}y^{-2}}{4}\right)^{-2}$, leaving your answer in positive index form. [2]

(b) Solve the inequality $\frac{1}{4}x - \frac{3}{5}\left(x + \frac{1}{3}\right) \geq \frac{1}{2}(x - 9)$. [2]

(c) **(i)** Express $\frac{2}{1-2x} + \frac{3(x-1)}{2x^2-5x+2} + \frac{1}{x-2}$ as a single fraction in its simplest form. [4]

(ii) Hence solve the equation $\frac{2}{1-2x} + \frac{3(x-1)}{2x^2-5x+2} + \frac{1}{x-2} = \frac{3}{x-2}$. [2]

1	**(a)**	$\frac{(2xy)^2}{35xy^7} \div \left(\frac{x^{-1}y^{-2}}{4}\right)^{-2}$ $= \frac{4x^2y^2}{35xy^7} \div \left(\frac{4}{x^{-1}y^{-2}}\right)^2$ $= \frac{4x^2y^2}{35xy^7}$ x $\frac{x^{-2}y^{-4}}{16}$ [Able to remove both brackets][M1] $= \frac{1}{140xy^9}$ [A1]	
	(b)	$\frac{1}{4}x - \frac{3}{5}\left(x + \frac{1}{3}\right) \geq \frac{1}{2}(x - 9)$ $\frac{1}{4}x - \frac{3}{5}x - \frac{1}{5} \geq \frac{1}{2}x - \frac{9}{2}$ [Able to expand all terms correctly][M1] $-\frac{17}{20}x \geq -\frac{43}{10}$ $x \leq 5\frac{1}{17}$ [A1]	
	(c)(i)	$\frac{2}{1-2x} + \frac{3(x-1)}{2x^2-5x+2} + \frac{1}{x-2}$ $= \frac{-2}{2x-1} + \frac{3x-3}{(x-2)(2x-1)} + \frac{1}{x-2}$ [Able to factorise $2x^2 - 5x + 2$][M1] $= \frac{-2(x-2)+3(x-1)+2x-1}{(x-2)(2x-1)}$ [Able to combine into common denominator][M1] $= \frac{-2x+4+3x-3+2x-1}{(x-2)(2x-1)}$ [Able to expand correctly][M1] $= \frac{3x}{(x-2)(2x-1)}$ [A1]	
	(c)(ii)	$\frac{2}{1-2x} + \frac{3(x-1)}{2x^2-5x+2} + \frac{1}{x-2} = \frac{3}{x-2}$ $\frac{3x}{(x-2)(2x-1)} = \frac{3}{x-2}, x \neq 2$ [M1] $\frac{3x}{2x-1} = 3$ $6x - 3 = 3x$ $3x = 3$ $x = 1$ [A1]	**[10]**

2 (a) *PQRS* is a parallelogram in which the coordinates of *P* and *Q* are $(p\,,4)$ and $(7\,,11)$ respectively. The line $9y + 27 - 7x = 0$ cuts the *y*-axis at *R* and is parallel to the line *PQ*.

Find

(i) the value of *p*, [3]

(ii) the coordinates of *S* by **vector method**. [3]

(a)(i)

$9y = 7x - 27$

$y = \frac{7}{9}x - 3$

Gradient of the line = $\frac{7}{9}$

Gradient of *PQ* = $\frac{7}{9}$ (parallel lines) [M1]

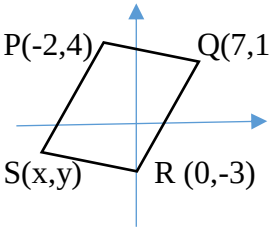
$m_{PQ} = \frac{11-4}{7-p}$

$\frac{11-4}{7-p} = \frac{7}{9}$ [M1]

$7 - p = 9$

$p = -2$ [A1] **[9]**

(a)(ii)



$\overrightarrow{OP} = \begin{pmatrix}-2\\4\end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix}7\\11\end{pmatrix},$

$\overrightarrow{OS} = \begin{pmatrix}x\\y\end{pmatrix}, \overrightarrow{OR} = \begin{pmatrix}0\\-3\end{pmatrix}$

Accept $\overrightarrow{SR} = \overrightarrow{PQ}$

By vector method, $\overrightarrow{PS} = \overrightarrow{QR}$ [Concept of equal vectors][M1]

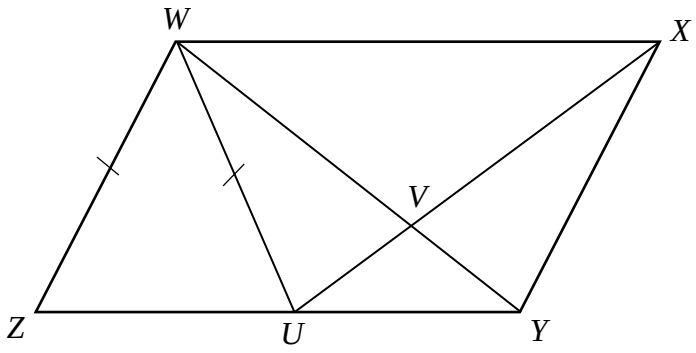
$-\begin{pmatrix}-2\\4\end{pmatrix} + \begin{pmatrix}x\\y\end{pmatrix} = -\begin{pmatrix}7\\11\end{pmatrix} + \begin{pmatrix}0\\-3\end{pmatrix}$ [M1]

$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-7\\-14\end{pmatrix} + \begin{pmatrix}-2\\4\end{pmatrix}$

$= \begin{pmatrix}-9\\-10\end{pmatrix}$

S = (-9, -10) [A1]

(b) In the diagram, *WXYZ* is a parallelogram and *U* is a point on *ZY* such that *WZ* = *WU*. The lines *WY* and *UX* intersect at *V*.



Prove that ΔWUY is congruent to ΔXYU. [3]

	(b)	WZ = WU (given) = XY (opp sides of *ll*gram) UY (common length) $W\hat{U}Y = 180 - W\hat{U}Z$ (adj angles on a str. line) $X\hat{Y}U = 180 - W\hat{Z}U$ (int. angles) Since WZ = WU $\Rightarrow W\hat{Z}U = W\hat{U}Z$ Hence $W\hat{U}Y = X\hat{Y}U$ ΔWUY is congruent to ΔXYU (SAS) [A1]	Prove 2 pairs of equal sides [M1] Prove included angle [M1]	

3 **(a)** The radius of a particular spherical cell is approximately 6.2×10^{-11} metre.

(i) 88 of these spherical cells are arranged to form a straight line such that each cell touches one another without overlapping. Calculate the length, in metre, of the straight line formed. Give your answer in standard form. [1]

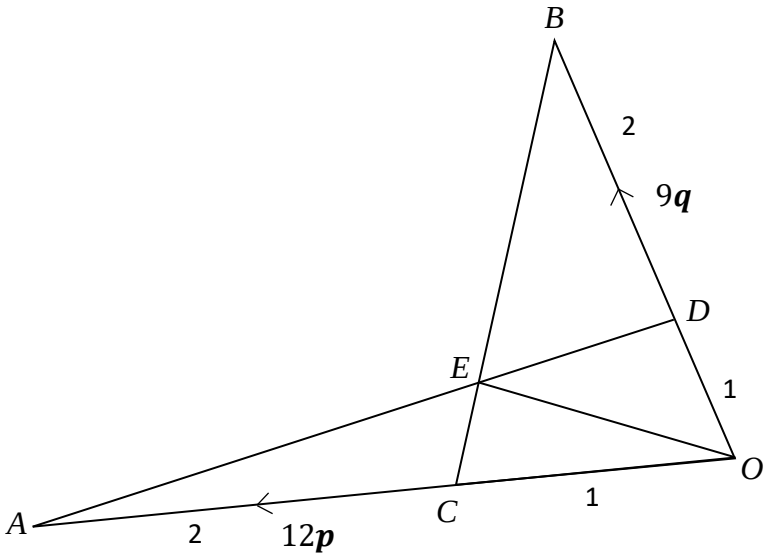
(ii) Calculate the volume of a spherical cell, in cubic centimetre. Give your answer in standard form correct to 2 decimal places. [1]

(b) The planet Mercury is approximately 77 billion metre from the Earth. Given that radio waves travel at a speed of 3×10^{8} m/s, find the time taken for radio waves to travel from the Earth to Mercury, giving your answer to the nearest minutes. [2]

(c) The word 'Googol' is defined as 1×10^{100}. If a man has ten 'Googol' cents and a Boeing 737 aircraft costs \$72 million, how many such aircrafts can he buy? Give your answer in standard form correct to 5 significant figures. [3]

	(a)(i)	Diameter = 2 x 6.2×10^{-11} Length = 88 x 2 x 6.2×10^{-11} = 1.09×10^{-8} m [B1]	
	(a)(ii)	Volume = $\frac{4}{3}\pi$ x $(6.2\times10^{-9})^3$ Note: r = 6.2×10^{-11} m = 6.2×10^{-9} cm = 9.98×10^{-25} cm^3 (2dp) [B1]	
	(b)	$t = \frac{77\times10^{9}}{3\times10^{8}}$ [M1] = $256\frac{2}{3}$ s = 4 min (nearest min) [A1]	
	(c)	ten 'Googol' cents = \$ $\frac{10 \text{ x } 1\times10^{100}}{100}$ = \$$10^{99}$ [Change to \$][M1] No of aircrafts = $\frac{10^{99}}{72\text{x}10^{6}}$ [M1] = $1.3889\,\text{x}\,10^{91}$ [A1]	**[7]**

4 In the diagram, $\overrightarrow{OA} = 12\boldsymbol{p}$ and $\overrightarrow{OB} = 9\boldsymbol{q}$.
It is given that $3DB = 2OB$ and $OA = 3OC$.



(a) Express, as simply as possible, in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$,

(i) $\overrightarrow{BC}$, [2]

(ii) $\overrightarrow{DA}$. [2]

(b) Given that $\dfrac{\text{area of triangle } ODE}{\text{area of triangle } ODA} = \dfrac{1}{4}$, find $\overrightarrow{OE}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$. [2]

(c) Find the value of $\dfrac{\text{area of triangle } BDE}{\text{area of quadrailateral } EDOC}$. [2]

	(a)(i)	$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$ $= -9\boldsymbol{q} + \frac{1}{3}\overrightarrow{OA}$ $= -9\boldsymbol{q} + 4\boldsymbol{p}$ [B1]	
	(a)(ii)	$\overrightarrow{DA} = \overrightarrow{DO} + \overrightarrow{OA}$ $= \frac{1}{3}\overrightarrow{BO} + 12\boldsymbol{p}$ [M1] $= -3\boldsymbol{q} + 12\boldsymbol{p}$ [A1]	
	(b)	$\dfrac{\text{area of triangle } ODE}{\text{area of triangle } ODA} = \dfrac{1}{4}$ $\dfrac{0.5 \text{ x ED x h}}{0.5 \text{ x AD x h}} = \dfrac{1}{4}$ [Shared same perpendicular height][M1] $\dfrac{\text{ED}}{\text{AD}} = \dfrac{1}{4}$ $\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$ $= 12\boldsymbol{p} + \frac{3}{4}\overrightarrow{AD}$ $= 12\boldsymbol{p} + \frac{3}{4}(3\boldsymbol{q} - 12\boldsymbol{p})$ [M1] $= 3\boldsymbol{p} + \frac{9}{4}\boldsymbol{q}$ [A1]	**[8]**

	(c)	$\frac{\text{area of triangle } ODE}{\text{area of triangle } DEB} = \frac{OD}{DB} = \frac{1}{2}$ $\frac{\text{area of triangle } AEC}{\text{area of triangle } OEC} = \frac{AC}{OC} = \frac{2}{1}$ M1 Triangle ODE: Triangle DEB: Triangle AEC : Triangle OEC = 1 : 2 : 2 : 1 $\frac{\text{area of triangle } BDE}{\text{area of quadrailateral } EDOC}$ $= \frac{2}{2}$ = 1 [A1]	

5 Answer the whole of this question on a sheet of graph paper.

A radioactive substance decays such that its mass, m grams, after d days is given by the equation $m = 43(3)^{-d}$.

The table below shows record of the mass, m grams of the substance, corrected to 1 decimal place, after d days.

d (days)	0	1	2	3	4	5	6
m (grams)	43	14.3	4.8	p	0.5	0.2	0.1

(a) Find the value of p. [1]

(b) Using a horizontal scale of 2 cm to represent 1 day and a vertical scale of 2 cm to represent 5 grams, draw the graph of $m = 43(3)^{-d}$ for $0 \le d \le 6$. [3]

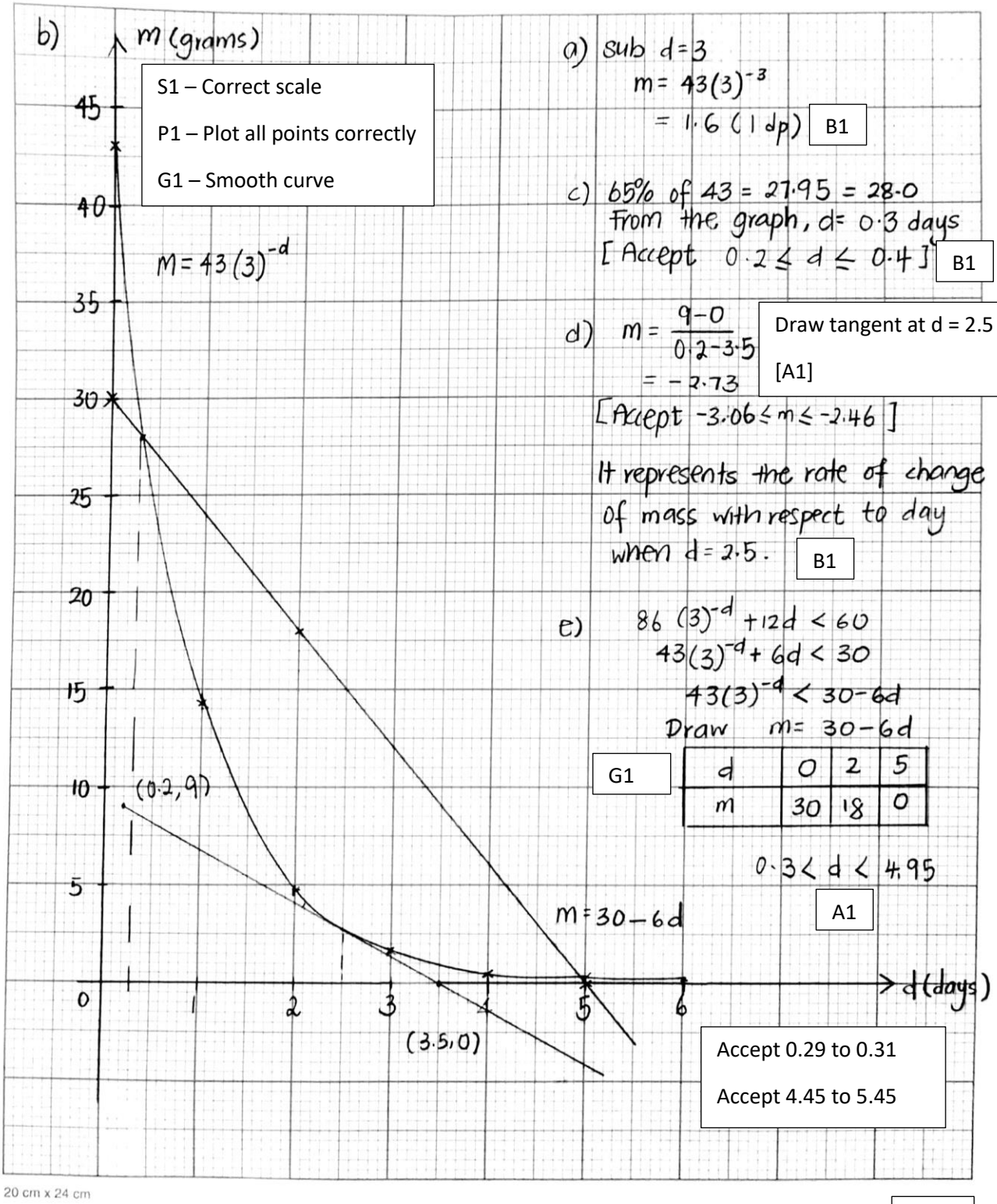
(c) Use your graph to estimate the value of d when the mass of substance is reduced to 65% of its original mass. [1]

(d) By drawing a tangent, find the gradient of the curve at the point when $d = 2.5$.

State briefly what this gradient represents. [3]

(e) Using your graph, find the range of d for $86(3)^{-d} + 12d < 60$. [2]

Subject Question 5 Class Date



S1 – Correct scale

P1 – Plot all points correctly

G1 – Smooth curve

a) sub $d = 3$

$m = 43(3)^{-3}$

$= 1.6$ (1 dp) B1

c) 65% of 43 = 27.95 = 28.0

From the graph, $d = 0.3$ days

[Accept $0.2 \leq d \leq 0.4$] B1

d) $m = \frac{9-0}{0.2-3.5}$

$= -2.73$

[Accept $-3.06 \leq m \leq -2.46$]

Draw tangent at d = 2.5 [G1]

[A1]

It represents the rate of change of mass with respect to day when $d = 2.5$. B1

e) $86(3)^{-d} + 12d < 60$

$43(3)^{-d} + 6d < 30$

$43(3)^{-d} < 30 - 6d$

Draw $m = 30 - 6d$

G1

d	0	2	5
m	30	18	0

$0.3 < d < 4.95$ A1

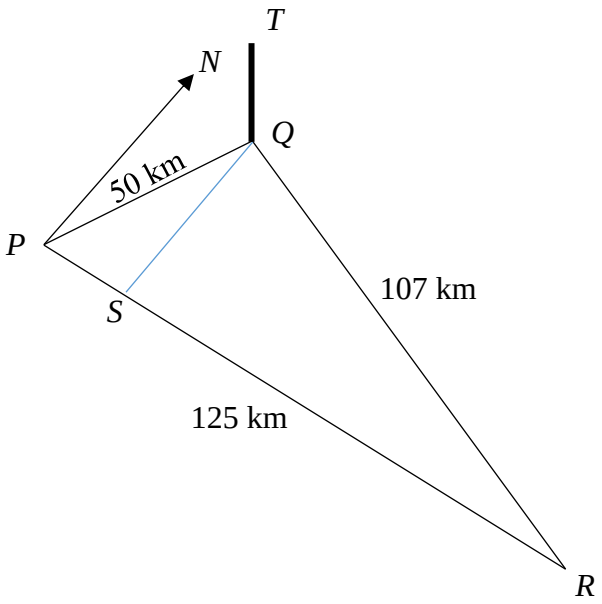
Accept 0.29 to 0.31

Accept 4.45 to 5.45

20 cm x 24 cm

[10]

6 *PQR* represents a triangular plot of land on horizontal ground.
PQ = 50 km, *QR* = 107 km and *PR* = 125 km.
R is due east of *P*.



(a) Calculate

(i) the bearing of *Q* from *P*, [3]

(ii) the bearing of *P* from *Q*, [2]

(iii) the obtuse angle *PQR*, [2]

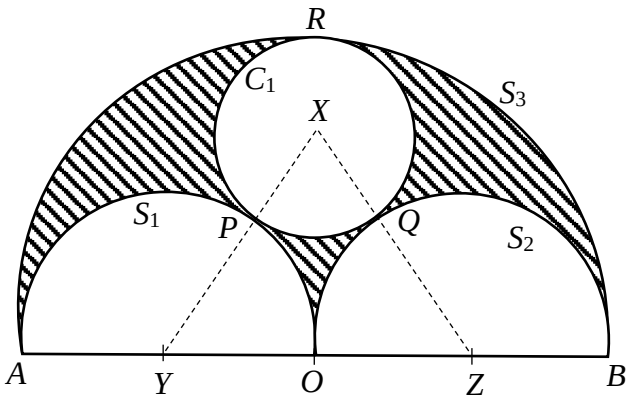
(iv) the area of the triangular plot of land *PQR*. [2]

(b) A vertical pole of height 9 km is erected at *Q*. Calculate the greatest angle of elevation of the top of the pole from a point *S* along *PR*. [3]

(a)(i)	$\cos \mathrm{Q}\hat{P}R = \frac{125^2+50^2-107^2}{(2)(125)(50)}$ [M1] $\mathrm{Q}\hat{P}R = 57.718$ Bearing of Q from P $= 90 - 57.718$ [M1] $=032.3°$ (1dp) [A1]	
(a)(ii)	Bearing of P from Q $=360 - (180 - 32.282)$ [M1] $=212.3°$(1dp) [A1]	
(a)(iii)	$\frac{\sin P\hat{Q}R}{125} = \frac{\sin 57.718}{107}$ [M1] $\mathrm{P}\hat{Q}R = 80.987°\ (rejected)\ or\ \mathrm{P}\hat{Q}R = 99.013°$ obtuse $\mathrm{P}\hat{Q}R = 99.0°$ [A1] or $\cos \mathrm{P}\hat{Q}R = \frac{107^2+50^2-125^2}{(2)(107)(50)}$ [M1] obtuse $\mathrm{P}\hat{Q}R = 99.0°$ [A1]	**[12]**

(a)(iv)	Area of the triangular plot of land *PQR* $=\frac{1}{2}(125)(50)\sin 57.718$ [M1] = 2640 km^2 [A1]	
(b)	In triangle PQS, $\sin 57.718 = \frac{d}{50}$ [M1] d = 42.271 In triangle PQT, $\tan e = \frac{9}{42.271}$ [M1] $e = 12.0°$ [A1]	

7 In the diagram, the circle C_1 with centre X has a radius $(3r + 1)$ cm, where r is a constant. Two identical semicircles, S_1 and S_2 with centre at Y and Z respectively, have a radius $(13 - 6r)$ cm.
Another semicircle, S_3 with centre O has a diameter AB.
C_1 touches S_1 and S_2 at P and Q respectively while S_1 and S_2 touches one another at O.
S_3 touches C_1, S_1 and S_2 at R, A and B respectively.
$AYOZB$ is a straight line.



(a) Write down an expression, in its simplest form and in terms of r, for
(i) XZ, [1]
(ii) XO. [2]

(b) Hence, form an equation in terms of r and show that it reduces to $126r^2 - 411r + 299 = 0$. [3]

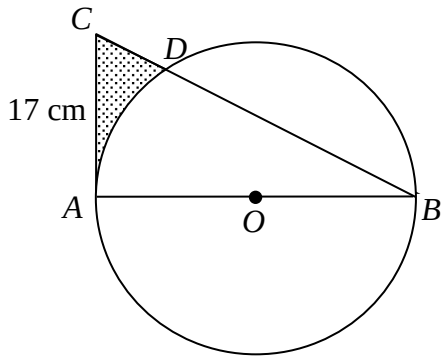
(c) Solve the equation $126r^2 - 411r + 299 = 0$. [2]

(d) Hence, find the area of the shaded region. [3]

(a)(i)	XZ = XQ + QZ = 3r + 1 + 13 – 6r = 14 – 3r [B1]	
(a)(ii)	XO = 2 (13 – 6r) – (3r + 1) [M1] = 25 – 15r [A1]	
(b)	$XZ^2 = XO^2 + OZ^2$ $(14 - 3r)^2 = (25 - 15r)^2 + (13 - 6r)^2$ [M1] $196 - 84r + 9r^2 = 625 - 750r + 225r^2 + 169 - 156r + 36r^2$ [M1] $252r^2 - 822r + 598 = 0$ [A1] $126r^2 - 411r + 299 = 0$ (shown)	
(c)	$r = \frac{411 \pm \sqrt{(-411)^2 - 4(126)(299)}}{2(126)}$ [M1] $= 2\frac{1}{6}$ or $1\frac{2}{21}$ [A1]	
(d)	If r = $2\frac{1}{6}$, then XO < 0. Hence r = $1\frac{2}{21}$	**[11]**

	Area of the shaded region $=\frac{1}{2}\pi(26-12\left(1\frac{2}{21}\right))^2-\pi\left(3\left(1\frac{2}{21}\right)+1\right)^2-\pi(13-6\left(1\frac{2}{21}\right))^2$ [M1- Area of S3][M1 – correct unshaded area] $=72.1\ cm^2$ [A1]	

8 **(a)** The diagram shows a circle with centre O and radius of 12 cm. AB is the diameter of the circle and AC is a tangent to the circle at A with AC = 17 cm. The circle intersects the line BC at D.



(i) Show that angle $DOA = 1.23$ radians. [2]

Calculate

(ii) the length of minor arc AD, [1]

(iii) the area of the shaded region. [3]

(b) Diagram I shows an open container which is made up of a cylinder and a cone. The cylinder has radius r cm and height 30 cm.
The cone has base radius r cm and slant height l cm.
The container is fully filled with water.

Diagram II shows a spherical object in which half of it is immersed into the container and some water is displaced. The radius of the sphere is the same as the radius of the cylinder. Assume the thickness of the container and the spherical object are negligible.

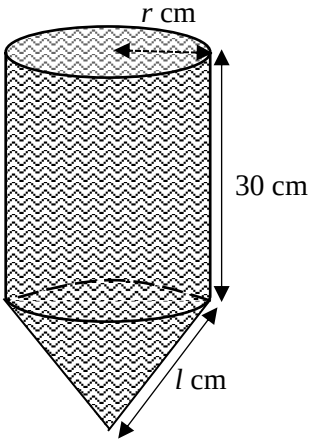


Diagram I

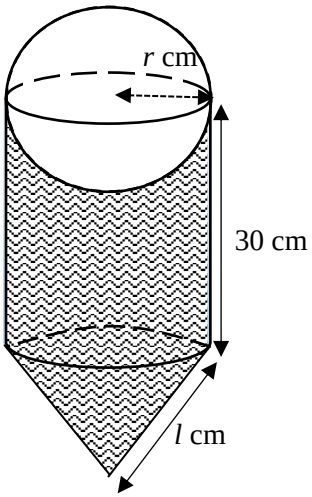
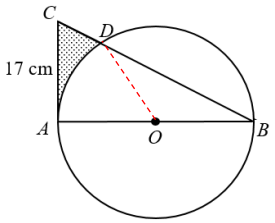
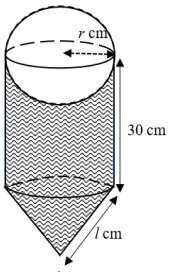


Diagram II

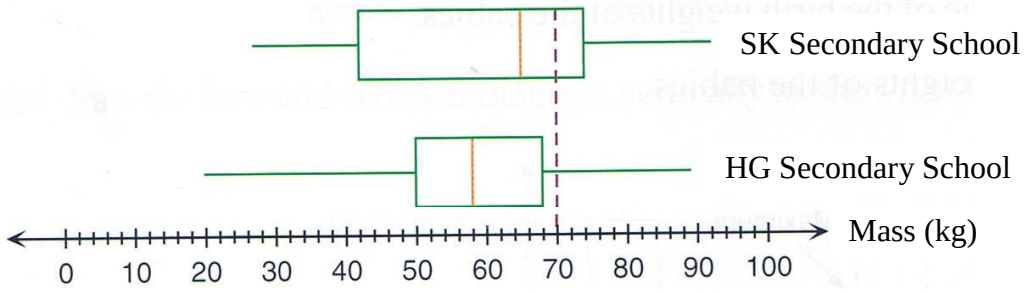
Given that the volume of the water displaced is 1152π cm^3 and the volume of the water left in the container is 3600π cm^3, find

(i) the value of r and of l, [4]

(ii) the total internal area of the container and the sphere that is in contact with water, leaving your answer in terms of π. [2]

	(a)(i)	 In triangle CAB, $\tan C\hat{B}A = \frac{17}{24}$ [M1] $C\hat{B}A = 0.61630\ rad$ $D\hat{O}A = 2 \times 0.61630$ (angle at centre = 2× angle at circumference) = 1.2326 = 1.23 rad (shown) [A1]	
	(a)(ii)	length of minor arc *AD* = 12 (1.2326) = 14.8 cm [B1]	
	(a)(iii)	Area of the shaded region (Join OD) $=\frac{1}{2}(17)(24) - \frac{1}{2}(12)^2(1.2326) - \frac{1}{2}(12)^2\sin(\pi - 1.2326)$ [M1-Sector area][M1-area of triangle] = 204 – 88.747 – 67.922 = 47.3 cm^3 [A1]	
	(b)(i)	Vol of water displaced = 1152π $\frac{1}{2} \times \frac{4}{3}\pi(r)^3 = 1152\pi$ [M1] $r^3 = 1728$ r = 12 cm [A1] Total volume $=1152\pi + 3600\pi$ $=4752\pi$ $\pi(12)^2(30) + \frac{1}{3}\pi(12)^2 h = 4752\pi$ [M1] 48 h = 432 h = 9 cm By Pythagoras' Theorem, $l^2 = 9^2 + 12^2$ $l = 15\ cm$ [A1]	
	(b)(ii)	Internal area $=\pi(12)(15) + 2\pi(12)(30) + 2\pi(12^2)$ [M1] = 1188π cm^2 [A1]	**[12]**

9 **(a)** The following box-and-whisker diagrams show the distribution of the mass of 300 students from each school, SK Secondary School and HG Secondary School respectively.



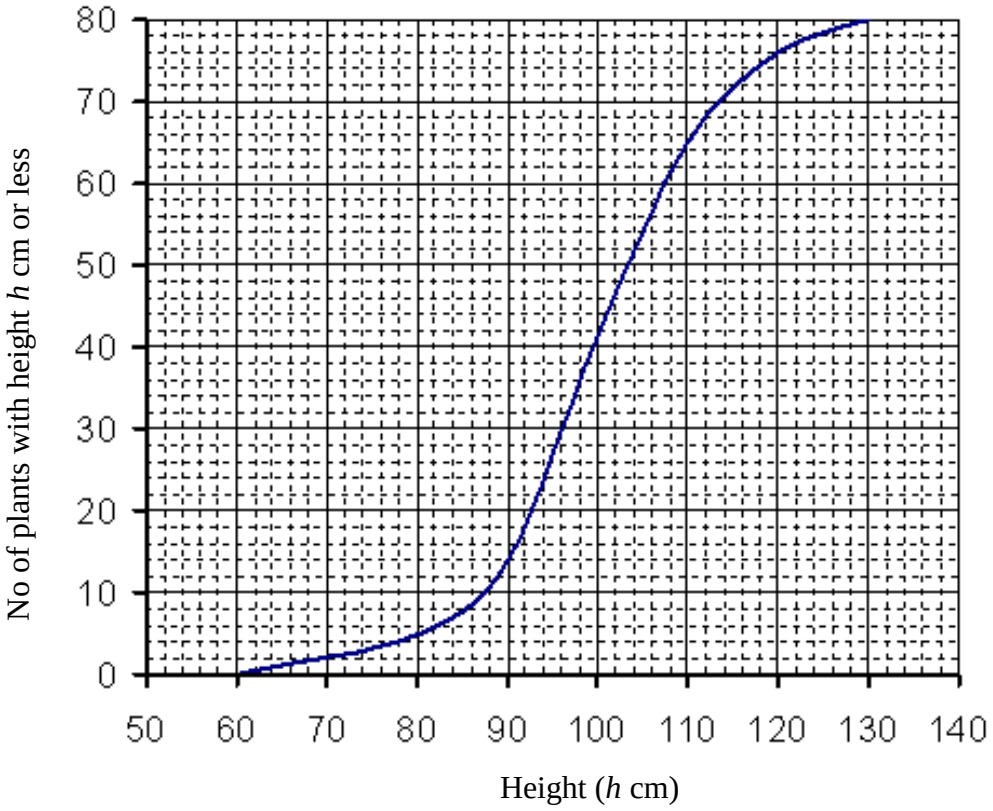
(i) What is the median mass for each school? [1]

(ii) Compare the mass of the students from SK Secondary School and HG Secondary School in two ways. [2]

(iii) Mary commented that there are more students in HG Secondary School than SK Secondary School who weigh more than or equal to 70 kg.
Do you agree with Mary? Support with a reason. [1]

	(i)	For SK Sec, median = 65 kg For HG Sec, median = 58 kg Both correct [B1]	
	(ii)	Generally, the students for SK Sec Sch are heavier compared to the students in HG Sec Sch because of the higher median. [B1] For SK, IQR= 32 kg; For HG, IQR = 18 kg The mass of the students in HG Sec Sch is more consistent than that of SK Sec Sch because of smaller interquartile range. [B1]	
	(iii)	Disagree as there are more than 25% of the students from SK Sec Sch weigh more than 70 kg while there are less than 25% of the students fron HG Sec Sch weigh more than 70 kg. [B1] Accept: There are more students in SK Sec Sch weigh more than 70 kg because it has a higher upper quartile than HG Sec Sch.	**[4]**

(b) The cumulative frequency curve shows the height distribution of 80 plants.



(i) Use the graph to find

(a) the value of m, if 32.5% of the plants have heights more than m cm, [2]

(b) the probability that two plants chosen will each has a height of more than 118 cm. [2]

(ii) **(a)** The height distribution of the 80 plants was also recorded in the following frequency table. Find the value of a and of b.

Height (h cm)	**Number of plants**
$60 < h \leq 70$	2
$70 < h \leq 80$	a
$80 < h \leq 90$	9
$90 < h \leq 100$	27
$100 < h \leq 110$	23
$110 < h \leq 120$	b
$120 < h \leq 130$	4

[1]

(b) Hence find the mean and the standard deviation of the height of the 80 plants. [2]

		(bi)	**(a)**	No of plants less than or equal to m cm = 67.5% of 80 = 54 plants [B1] m = 105 [A1]	
		(bi)	**(b)**	$\frac{6}{80} \times \frac{5}{79}$ [M1] $= \frac{3}{632}$ [A1]	
		(bii)	**(a)**	a = 3, b = 12 Both correct [B1]	
		(bii)	**(b)**	Mean = 99.75 cm [B1] Standard deviation = 12.6 cm [B1]	**[7]**

10 Mr and Mrs Tan bought a 3-bedroom unit at a newly launched condominium project at Serangoon. The unit has a floor area of 1152 square foot (sqft). The selling price for the unit is at $1494 per sqft. The developer gives a 5% early bird discount to all buyers.

(a) Calculate the price that they paid for the unit. [1]

Mr and Mrs Tan are both Singapore citizens and they also owned a HDB 5-room flat and they do not have any intention of selling their flat. As such, they will have to pay Additional Buyer's Stamp Duty (ABSD) to the government on top of the Buyer's Stamp Duty (BSD). Table 1 shows the BSD rate and Table 2 shows the ABSD rate.

Table 1. BSD

Based on purchase price	**Rate**
First $180 000	1%
Next $180 000	2%
Next $640 000	3%
Remaining amount	4%

Table 2. ABSD

Based on purchase price	**Rate**
SC^1 buying first residential property	NA
SC^1 buying second residential property	12%
SC^1 buying third and subsequent residential property	15%

SC^1 denotes Singapore Citizens

(b) Calculate the **total** Buyer's Stamp Duty paid by Mr and Mrs Tan. [4]

Mr and Mrs Tan made a 20% down-payment based on the amount paid for the unit obtained in **(a)**. For the remaining amount, which exclude the total Buyer's Stamp Duty, they had decided to sign up either for a 20-years Home Loan plan from CBCO Bank or a 30-years Home Loan plan from BSOP Bank.

(c) Determine which bank they should sign up if they can only afford a monthly instalment of not more than $6000.
Support you answer with appropriate workings. [5]

Simple Interest Rate for CBCO Bank

Loan period: 20 years

1^{st} year:	2^{nd} Year:
2.18% p.a.	2.18% p.a.
Thereafter: 2.68% p.a.	

Simple Interest Rate for BSOP Bank

Loan period: 30 years

1^{st} year:	2^{nd} Year:
1.95% p.a.	1.95% p.a.
Thereafter: 2.15% p.a.	

	(a)	Amount paid =1152 x 1494 x 0.95 = $1 635 033.60 [B1]	
	(b)	BSD paid = 0.01 x 180 000 + 0.02 x 180 000 + 0.03 x 640 000 + 0.04 x 635 033.60 [M1 – First 1 mil][M1 – remaining amount] = 50 001.344 ABSD paid = 0.12 x 1 635 033.60 [M1] = 196 204.032 Total stamp duty = 50 001.344 + 196 204.032 = $246 205.38 [A1]	
	(c)	Loan amount = 0.8 x 1 635 033.60 [M1] = $1 308 026.88 **<u>CBCO Bank</u>** Interest = 2.18% x 2 x 1308026.88 + 2.68% x 18 x 1308026.88 [M1] = 688 022.1389 Monthly instalment = (688 022.1389 + 1 308 026.88)÷(20 x 12) [M1 for either of the monthly instalments] = $8316.87 > $6000 **<u>BSOP Bank</u>** Interest = 1.95% x 2 x 1308026.88 + 2.15% x 28 x 1308026.88 [M1] = 838 445.2301 Monthly instalment = (838 445.2301+ 1 308 026.88)÷(30 x 12) = $5962.42 < $6000 Mr and Mrs Tan should sign up with BSOP Bank as the monthly instalment is less than $6000. [A1]	[10]

SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY FOUR
O-LEVEL PROGRAMME

CANDIDATE NAME							
CLASS	4			REGISTER NUMBER			
CENTRE NUMBER					INDEX NUMBER			

MATHEMATICS **4048/01**

Paper 1

Friday **20 July 2018** **2 hours**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Factorise $6f^2 - 11f + 3$.

Answer ……..…..…… [1]

(b) Hence solve $6f^2 + 3 = 11f$.

Answer f = ……..……...……..……. [1]

---

2 $10 000 was invested in an account which pays r % per annum compound interest.
At the end of 3 years, the interest earned was $1910.16.
Find the value of r.

Answer r = ……..…… [2]

---

3 Adam has written down seven numbers.
The mean of these numbers is 8, the median is 7 and the mode is 11.
The smallest number is an even prime number and the largest number is eight times the smallest number.
The second and third numbers are consecutive numbers.

Find the seven numbers.

Answer ……. , ……. , ……. , ……. , ……. , ……. , ……. [2]

---

4 China has an estimated land area of 9.39×10^6 square kilometres. In 2017, the country had an estimated population of 1.4 billion. Find, giving your answer to a reasonable degree of accuracy, the average number of people per square kilometre of the country in 2017.

Answer ……. [2]

---

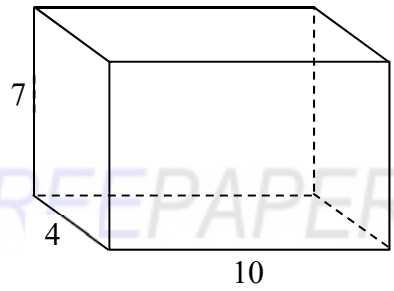
5 The area of a triangle XYZ is 166 cm^2. XY = 20.7 cm and YZ = 40.5 cm.

Find the two possible sizes of angle XYZ.

Answer ……..…… or …….. [2]

---

6 A solid cuboid has a length of 10 centimetres and a width of 4 centimetres.
Its height, when correct to the nearest centimetre, is 7 centimetres.



(a) Given that 1 cubic centimetre of the material used to make the cuboid has a mass of 0.65 grams, find the maximum possible mass of the cuboid.

Answer ……..…… grams [1]

(b) Cubes of side 3 centimetres are to be cut from the cuboid. Find the largest possible number of cubes which can be obtained.

Answer ……..……..…… [1]

---

7 **(a)** Simplify $a(5b-a)-(b-a)^2$.

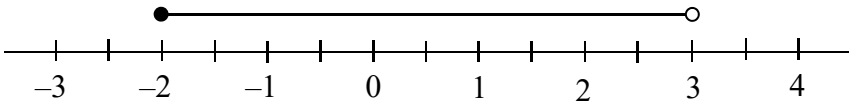
Answer ……. [1]

(b) Factorise $6d^2+30de-10e-2d$ completely.

Answer ……. [2]

---

8 The solution of the inequality $-6\frac{1}{4} < \frac{x-4}{4} - 2x \leq c$, where c is a constant, is represented on the number line below.



Find c.

Answer c = ……. [2]

---

9 On a particular day, the exchange rate between Singapore dollar ($) and Japanese yen (¥) was $100 = ¥8187. Anna bought a watch for ¥55 499 and sold it to her friend for $700. Express Anna's profit as a percentage of the price she paid for the watch.

Answer ……..……..…… % [2]

---

10 Express $\frac{3}{2-3x} - \frac{24x}{9x^2-4} + \frac{4}{3x+2}$ as a single fraction in its simplest form.

Answer ……..……..…… [3]

---

11 Expressed as the products of their prime factors,

$$196 = 2^2 \times 7^2,$$
$$252 = 2^2 \times 3^2 \times 7.$$

(a) Write down the highest common factor of 196 and 252.

Answer ……..…… [1]

(b) The lowest common multiple of 252 and an even number is 756.

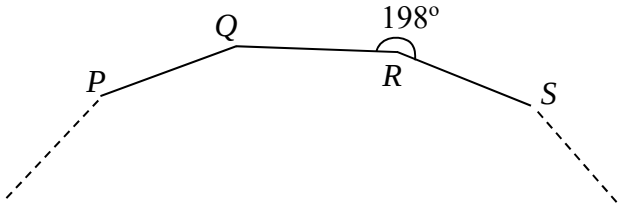
(i) Express 756 as the product of its prime factors.

Answer 756 = ……..…… [1]

(ii) Find the smallest possible value of the even number.

Answer ……..…… [1]

12 The diagram shows part of a regular polygon *PQRS* … .
Reflex angle *QRS* = 198°.



(a) Find angle *QPS*.

Answer ……..…..…… ° [1]

(b) Find the sum of interior angles of the polygon.

Answer ……..…..…… ° [2]

---

13 The map of a national park is drawn to a scale of 1 : *n*. A lake, which has an actual area of 7.5 km^2, is represented by an area of 4.8 cm^2 on the map.

(a) Find the value of *n*.

Answer n = ….…….…… [2]

(b) Calculate the actual perimeter of the lake, in km, if its perimeter on the map is 9 cm.

Answer ……..…..…… km [1]

---

14 **(a)** Simplify $\frac{a^7 b}{2} \times \left(a^3 b\right)^{-2}$.

Answer …… [1]

(b) Solve $\frac{2 \times 3^x}{\sqrt{3}} = 162$.

Answer x = …… [2]

---

15 **(a)** 12 men can complete a project in 27 days. Assuming that all the men work at the same rate, find the additional number of men needed to complete the project in 6 days.

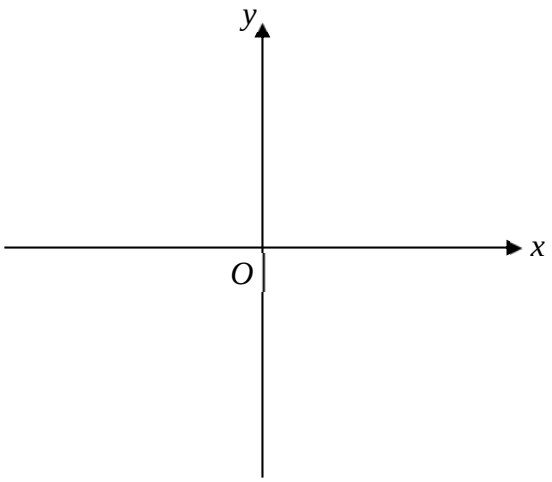
Answer …… [1]

(b) P is proportional to the square root of Q. Find the percentage increase in P if Q is increased by 300%.

Answer …… % [2]

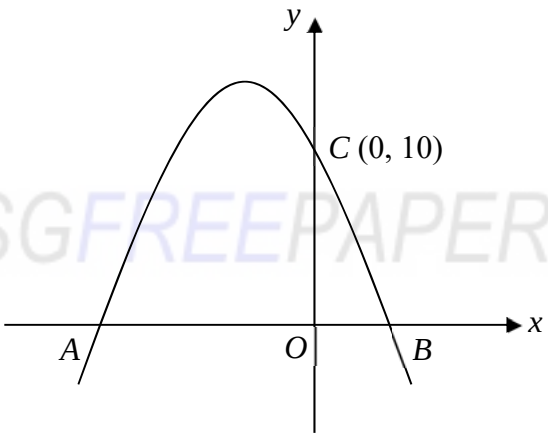
---

16 **(a)** Sketch the graph of $y = 3^x$ in the space below.



[1]

(b) The diagram shows the graph of $y = (x + p)(2 - x)$, where p is a constant. The graph cuts the x-axis at A and B and the y-axis at $C(0, 10)$.



(i) Write down the value of p.

Answer p = ... [1]

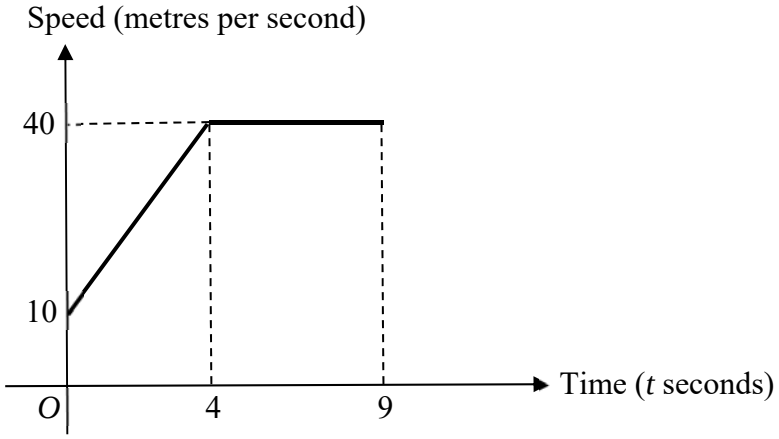
(ii) Write down the equation of the line of symmetry of the graph.

Answer ..... [1]

(iii) Find the maximum value of $y = (x + p)(2 - x)$.

Answer ..... [1]

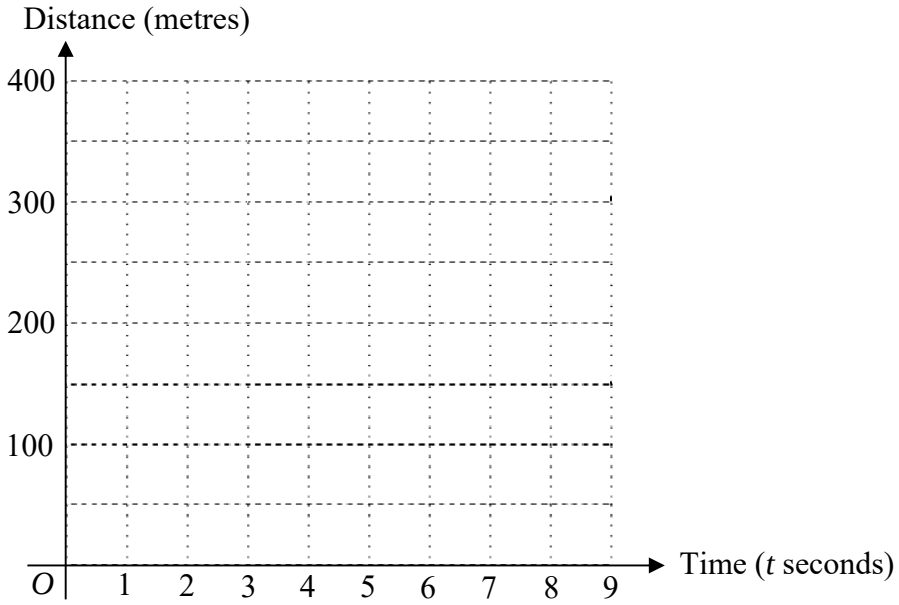
17 The diagram shows the speed-time graph for the first 9 seconds of an object's journey.



(a) Find the speed when $t = 2.5$.

Answer ..... m/s [1]

(b) Sketch the distance-time graph for the first 9 seconds of the object's journey in the space below.



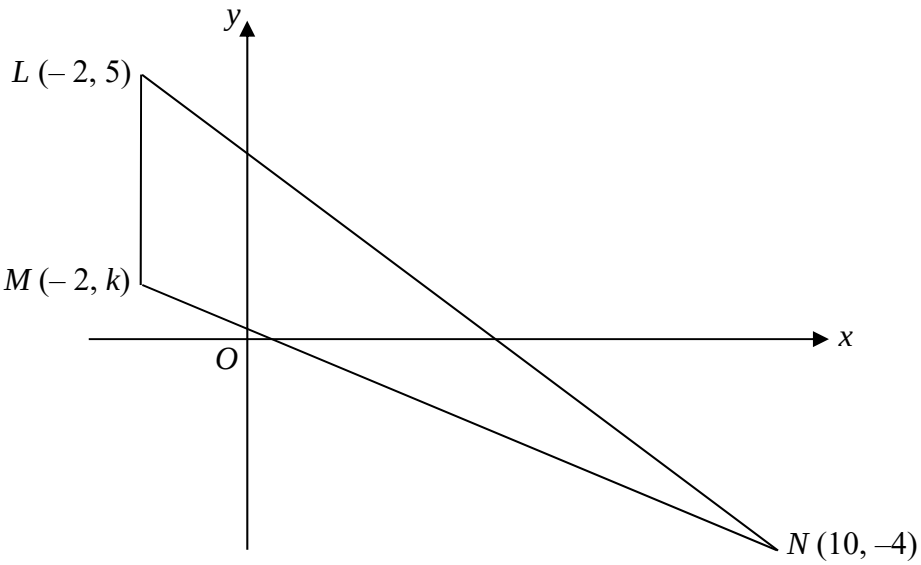
[2]

After the first 9 seconds, the object decelerated uniformly until it came to rest at $t = 17$.

(c) Calculate the deceleration.

Answer ..... m/s^2 [1]

18 L, M and N are the points $(-2, 5)$, $(-2, k)$ and $(10, -4)$ respectively.



The area of triangle LMN is 24 square units.

(a) Show that $k = 1$.

[1]

(b) Find, in its simplest form, the value of $\cos L\hat{M}N$.

Answer …… [1]

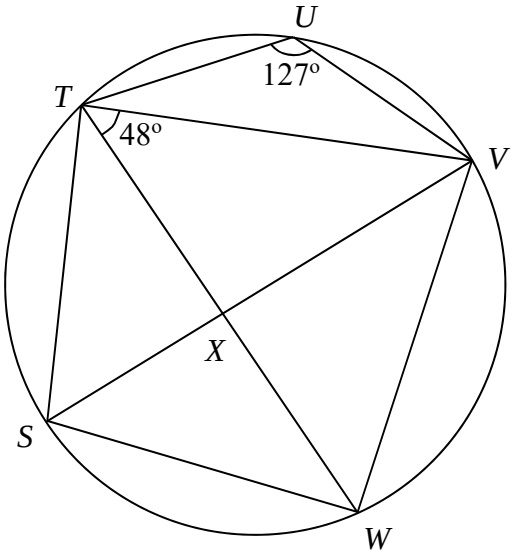
(c) Find the equation of the line passing through M and parallel to LN.

Answer …… [2]

(d) Write down the x-coordinate of the point P such that $LPMN$ is a parallelogram.

Answer x = …… [1]

19 S, T, U, V and W are points on a circle. SV is the diameter and it intersects TW at X. Angle $VTW = 48^o$ and angle $TUV = 127^o$.



(a) Find, stating your reasons clearly,

(i) angle VSW,

Answer ……° [1]

(ii) angle TVS,

Answer ……° [2]

(iii) angle WXV.

Answer ……° [1]

(b) Is X the centre of the circle? Explain your answer, stating your reasons clearly.

Answer

……

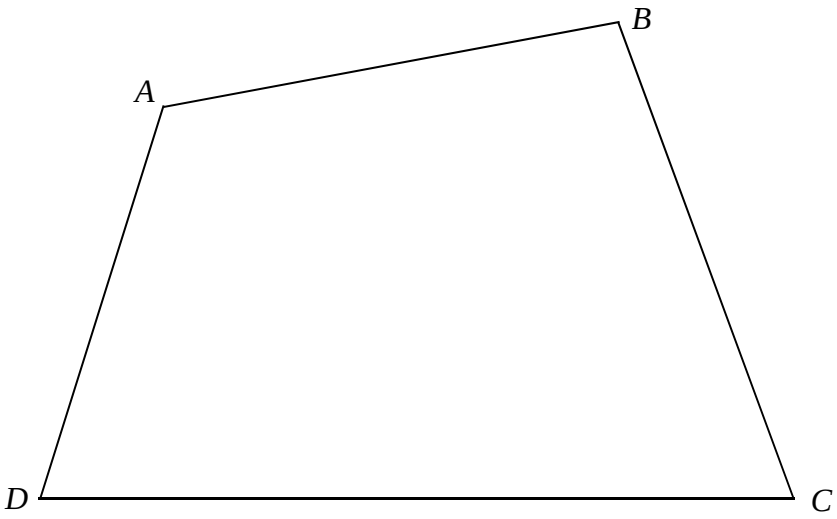
……

……

…… [1]

20 The diagram shows a scale drawing of a park *ABCD*.
D is due West of *C*.

Scale: 1 cm to 10 m



(a) Write down the bearing of *B* from *C*.

Answer …… ° [1]

(b) Construct the bisector of angle *DAB*. [1]

(c) Two benches are to be placed in the park such that they are equidistant from *AB* and *AD* and 50 metres from the point *D*. Write down the actual distance between the two benches.

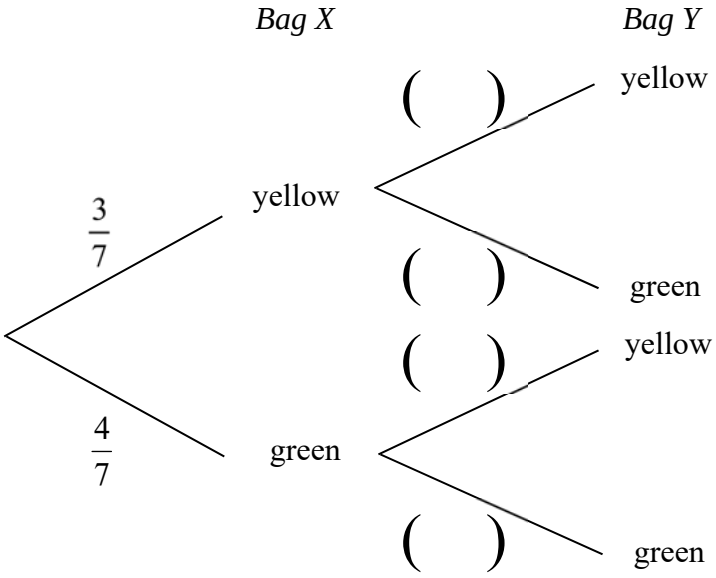
Answer …… m [1]

(d) Construct the perpendicular bisector of *CD*. [1]

(e) A lamp post, *P*, is to be erected in the park, nearer to *C* than to *D* and nearer to *AD* than to *AB*. Mark and label a possible position of *P*. [1]

21 Two bags contain coloured marbles.
Bag *X* contains 3 yellow marbles and 4 green marbles.
Bag *Y* contains 5 yellow marbles and 6 green marbles.
A marble is drawn at random from Bag *X* and put into Bag *Y*.
A marble is then drawn at random from Bag *Y*.

(a) Complete the tree diagram to show this information.



[2]

(b) Find, in its simplest form, the probability that the two marbles drawn are of different colours.

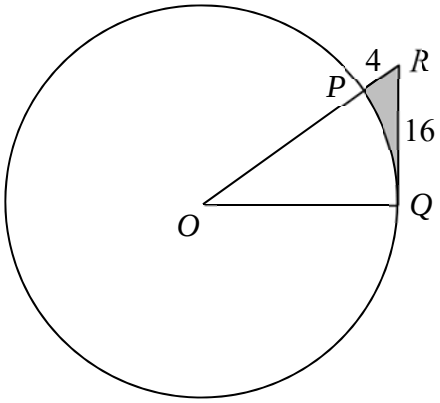
Answer ……. [2]

If the marble drawn from Bag *Y* has the same colour as the marble drawn from Bag *X*, it is set aside and another marble is drawn at random from Bag *Y*.

(c) Find, in its simplest form, the probability that all the three marbles drawn have the same colour.

Answer ……. [2]

22 P and Q are points on a circle, centre O. RQ is a tangent to the circle at Q.
$PR = 4$ cm and $RQ = 16$ cm.



(a) Show that the radius of the circle is 30 cm.

Answer

[2]

(b) Find angle ROQ in radians.

Answer ……..…… radians [2]

(c) Find the perimeter of the shaded region.

Answer ……..…… cm [2]

(d) Calculate the area of the shaded region.

Answer ……..…… cm^2 [2]

23 A unit fraction is a fraction with 1 as its numerator.

(a) The first four terms of a sequence of unit fractions are

$$\frac{1}{37}\ ,\ \frac{1}{31}\ ,\ \frac{1}{25}\ ,\ \frac{1}{19}\ ,\ \ldots\ .$$

(i) Write down the first negative term in the sequence.

Answer ..... [1]

(ii) Find the nth term of the sequence.

Answer ..... [1]

(b) A unit fraction can be expressed as the sum of two or more unit fractions.

For example, $\frac{1}{24} = \frac{1}{88} + \frac{1}{33}$.

The following method is used to find the two unit fractions that add up to $\frac{1}{24}$.

$$\begin{aligned}\frac{1}{24} &= \frac{1}{3\times 8}\\ &= \frac{11}{3\times 8\times 11}\\ &= \frac{3+8}{3\times 8\times 11}\\ &= \frac{3}{3\times 8\times 11} + \frac{8}{3\times 8\times 11}\\ &= \frac{1}{88} + \frac{1}{33}\end{aligned}$$

(i) In a similar manner, showing each step clearly, express $\frac{1}{24}$ as a sum of two **other** unit fractions.

Answer

[2]

(ii) Using a similar method, express $\frac{1}{24}$ as a sum of **three** different unit fractions.

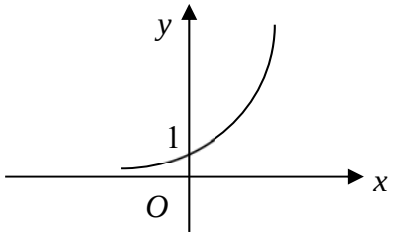
Answer

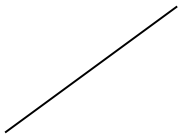
[2]

[BLANK PAGE]

Marking Scheme

Qn	Solution
1(a)	$(2f-3)(3f-1)$
(b)	$f = 1\frac{1}{2}$ or $\frac{1}{3}$
2	$10000 + 1910.16 = 10000\left(1+\frac{r}{100}\right)^3$ $r = 6$
3	2, 4, 5, 7, 11, 11, 16
4	Average number per km^2 $= \frac{1.4\times10^9}{9.39\times10^6}$ $= 150$ (2sf)
5	23.3° or 156.7° Alternatively, 0.407 rad or 2.73 rad
6(a)	Max mass = 195 grams
(b)	6 cubes
7(a)	$7ab - 2a^2 - b^2$
(b)	$6d^2 + 30de - 10e - 2d$ $= 2[3d(d+5e) - (d+5e)]$ $= 2(3d-1)(d+5e)$
8	$\frac{x-4}{4} - 2x \le c$ $x \ge \frac{-4c-4}{7}$ $\frac{-4c-4}{7} = -2$ $c = 2\frac{1}{2}$
9	Selling price in \$ $= \frac{55499}{8187}\times100$ $= 677.89$ Percentage $= \frac{700-677.89}{677.89}\times100$ $= 3.26\%$ Or, Cost price in ¥ $= 7\times8187$ $= 57309$ Percentage $= \frac{57309-55499}{55499}\times100$ $= 3.26\%$

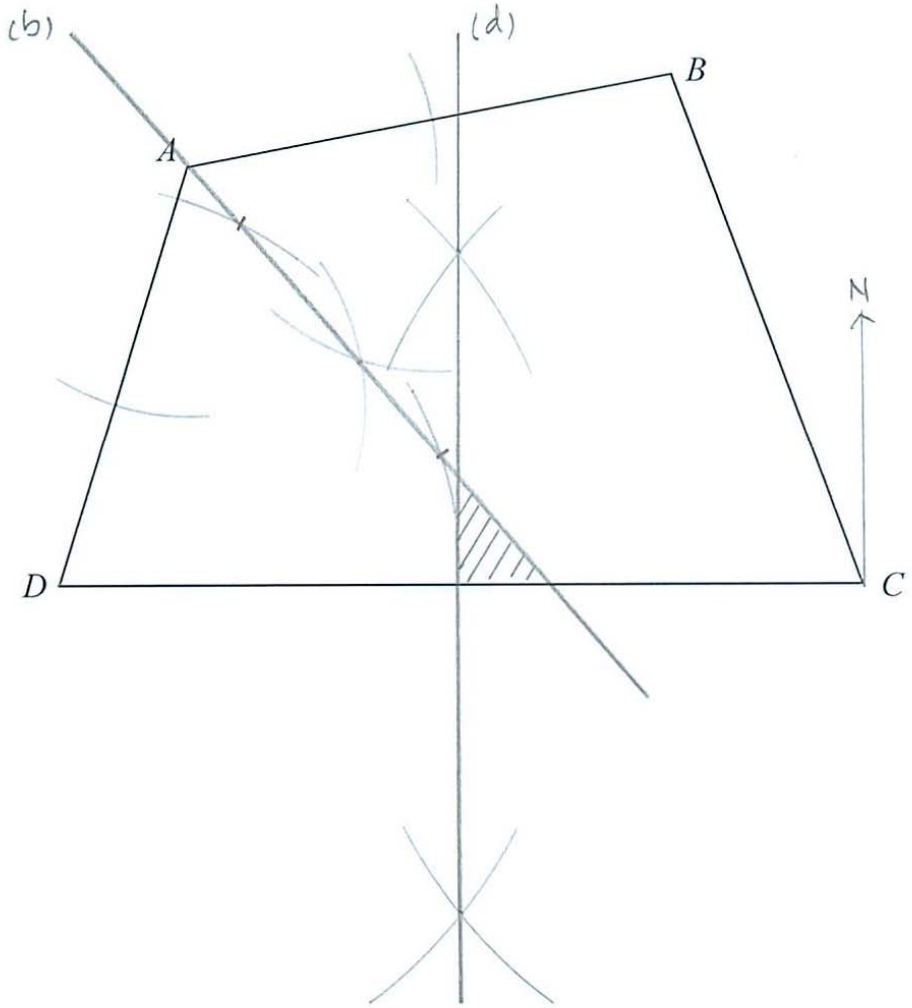
10	$\frac{3}{2-3x}-\frac{24x}{9x^2-4}+\frac{4}{3x+2}$ $=\frac{-3(3x+2)-24x+4(3x-2)}{(3x+2)(3x-2)}$ $=\frac{-21x-14}{(3x+2)(3x-2)}$ $=\frac{7}{2-3x}$ or $-\frac{7}{3x-2}$
11(a)	28
(b)(i)	$2^2\times3^3\times7$
(ii)	54
12(a)	18^o
(b)	No. of sides = 20 Sum of interior angles = 3240^o
13(a)	1 cm ----- $\sqrt{\frac{25}{16}}$ km n = 125 000
(b)	11.25 km
14(a)	$\frac{a}{2b}$
(b)	$\frac{2\times3^x}{\sqrt{3}}=162$ $3^{x-\frac{1}{2}}=3^4$ $x-\frac{1}{2}=4$ $x=4\frac{1}{2}$
15(a)	42 men
(b)	$P_{new}=\frac{P}{\sqrt{Q}}\left(\sqrt{4Q}\right)$ Percentage increase = 100%
16(a)	
(b)(i)	p = 5
(ii)	x = –1.5

(iii)	Maximum value = 12.25
17(a)	28.75 m/s
(b)	 , 100  , 300
(c)	5 m/s^2
18(a)	$\frac{1}{2}(5-k)(12)=24$ $k=1$
(b)	cos $\angle LMN = -\frac{5}{13}$
(c)	Gradient = $-\frac{3}{4}$ At $M(-2, 1)$, $1=-\frac{3}{4}(-2)+c$ $y=-\frac{3}{4}x-\frac{1}{2}$
(d)	$x=-14$
19(a)(i)	Angle $VSW = 48^o$ (angles in same segment)
(ii)	Angle $STV = 90^o$ (right angle in semicircle) Angle $TSV = 53^o$ (angles in opposite segments) Angle $TVS = 37^o$
(iii)	Angle $WXV = 85^o$ (vertically opposite angles)
(b)	Since angle $VXW \neq$ twice of angle VTW, X is not the centre (*angle at centre = 2 angle at circumference*) Or, Angle $TSW = 53^o + 48^o \neq 90^o$ TW is not the diameter (*right angle in semicircle*) Therefore, X is not the centre.
20(a)	340^o
(b)	Bisector of angle DAB
(c)	38 m
(d)	Perpendicular bisector of CD
(e)	Correct possible position of P

21(a)	$\frac{1}{2}$ and $\frac{1}{2}$ $\frac{5}{12}$ and $\frac{7}{12}$
(b)	P(different) = $\frac{3}{7}\left(\frac{1}{2}\right)+\frac{4}{7}\left(\frac{5}{12}\right)$ = $\frac{19}{42}$
(c)	P(same) = $\frac{3}{7}\left(\frac{1}{2}\right)\left(\frac{5}{11}\right)+\frac{4}{7}\left(\frac{7}{12}\right)\left(\frac{6}{11}\right)$ = $\frac{43}{154}$
22(a)	Let r cm be the radius. $(r+4)^2 = r^2+16^2$ $r^2+8r+16 = r^2+256$ $r = 30$
(b)	$\tan\angle ROQ = \frac{16}{30}$ $\angle ROQ = 0.48995$ = 0.490 rad (3sf)
(c)	Arc length $PQ = 30(0.48995)$ Perimeter = $30(0.48995)+16+4$ = 34.7 cm (3sf)
(d)	Area of sector $OPQ = \frac{1}{2}(30^2)(0.48995)$ Shaded area = $\frac{1}{2}(30)(16)-\frac{1}{2}(30^2)(0.48995)$ = 19.5 cm^2 (3sf)
23(a)(i)	$-\frac{1}{5}$
(ii)	$\frac{1}{43-6n}$
(b)(i)	$\frac{1}{24} = \frac{1}{2\times12}$ $= \frac{14}{2\times12\times14}$ $= \frac{2+12}{2\times12\times14}$ $= \frac{2}{2\times12\times14}+\frac{12}{2\times12\times14}$ $= \frac{1}{168}+\frac{1}{28}$ Or,

	$\frac{1}{24} = \frac{1}{4\times 6}$ $= \frac{10}{4\times 6\times 10}$ $= \frac{4+6}{4\times 6\times 10}$ $= \frac{4}{4\times 6\times 10} + \frac{6}{4\times 6\times 10}$ $= \frac{1}{60} + \frac{1}{40}$ Or, $\frac{1}{24} = \frac{1}{1\times 24}$ $= \frac{25}{1\times 24\times 25}$ $= \frac{1+24}{1\times 24\times 25}$ $= \frac{1}{1\times 24\times 25} + \frac{24}{1\times 24\times 25}$ $= \frac{1}{600} + \frac{1}{40}$
(ii)	$\frac{1}{24} = \frac{1}{2\times 3\times 4}$ $= \frac{2+3+4}{2\times 3\times 4\times 9}$ $= \frac{2}{2\times 3\times 4\times 9} + \frac{3}{2\times 3\times 4\times 9} + \frac{4}{2\times 3\times 4\times 9}$ $= \frac{1}{108} + \frac{1}{72} + \frac{1}{54}$


N


(b)
(d)
B
A
N
D
C

SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY FOUR
O-LEVEL PROGRAMME

CANDIDATE NAME								
CLASS	4				REGISTER NUMBER			
CENTRE NUMBER					INDEX NUMBER				

MATHEMATICS **4048/02**
PAPER 2

Monday **30 July 2018** **2 hours 30 minutes**

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This question paper consists of **12** printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Given that $\sqrt{\frac{r}{7+q^2}} - p = r$,

(i) evaluate p when $q = -3$ and $r = 36$, [1]

(ii) express q in terms of p and r. [3]

(b) Mr Ang makes x bowls and y jugs.

(i) He has 22.9 kilograms of clay.
He uses 300 grams of clay for a bowl and 800 grams of clay for a jug.
Write down an equation in terms of x and y, and show that it simplifies to $3x + 8y = 229$. [1]

(ii) He has $6\frac{3}{5}$ hours to make the bowls and the jugs.
It takes him 8 minutes to make a bowl and 12 minutes to make a jug.
Write down an equation in terms of x and y, to represent this information. [1]

(iii) Solve these two equations to find the value of x and the value of y. [3]

---

2 Mr Lee went on a journey of 190 km.
For the first 100 km, he drove at an average speed of v km/h.
For the remaining journey, he drove at an average speed which was 25 km/h slower than the speed for the first 100 km.

(a) Write down an expression, in terms of v, for the time taken in hours for the first 100 km. [1]

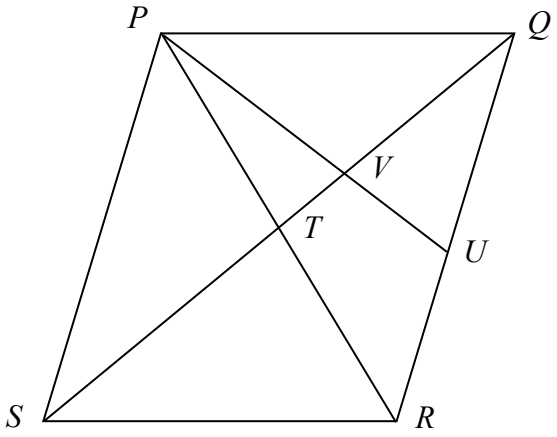
(b) Given that the journey took a total of 2 hours 45 minutes, form an equation in v and show that it simplifies to $11v^2 - 1035v + 10000 = 0$. [4]

(c) Solve the equation $11v^2 - 1035v + 10000 = 0$, giving each answer correct to two decimal places. [3]

(d) Which solution in **part (c)** represents the speed for the first 100 km of Mr Lee's journey?
Give a reason for rejecting the other solution. [1]

(e) Find the difference between the times taken for the first and second parts of the journey.
Give your answer correct to the nearest minute. [2]

3



The diagram shows a parallelogram $PQRS$ with diagonals PR and QS intersecting at T. The point U is the mid-point of QR and the line PU cuts QS at V.

(a) Name a triangle that is congruent to triangle PTQ. [1]

(b) Prove that

(i) triangles PVS and UVQ are similar, [2]

(ii) $QV = 2VT$. [2]

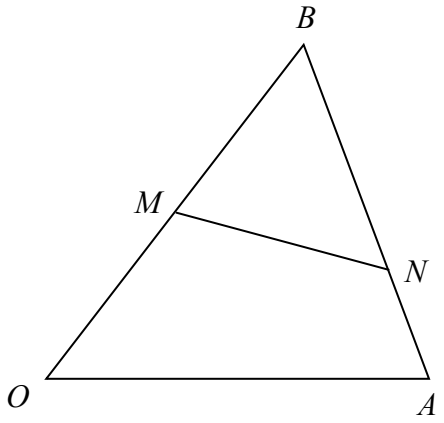
(c) Find the ratio of the area of triangle QVU to the area of trapezium $PURS$. [2]

4 **(a)** The point P is $(5, -8)$ and the point Q is $(-4, 7)$.

The point L is such that $\overrightarrow{QP} = \frac{1}{2}\overrightarrow{PL}$ and O is the origin.

Find $\left|\overrightarrow{OL}\right|$. [3]

(b) In the diagram, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
M is a point on OB where $OB = 2MB$ and the point N lies on AB such that $3BN = 2BA$.



(i) Express in terms of **a** and **b**, simplifying your answers where possible,

(a) $\overrightarrow{BN}$, [1]

(b) $\overrightarrow{MN}$. [2]

The point P lies on OA produced such that $OA : OP = 1 : 2$.

(ii) Determine whether the points M, N and P are collinear.
Justify your answer. [2]

The point Q is such that $k\overrightarrow{BQ} = \mathbf{a}$ and ONQ is a straight line.

(iii) Write down the value of k. [1]

5

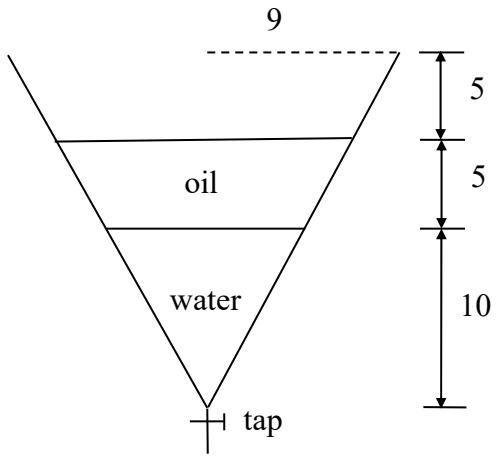


Diagram 1

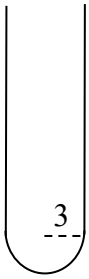


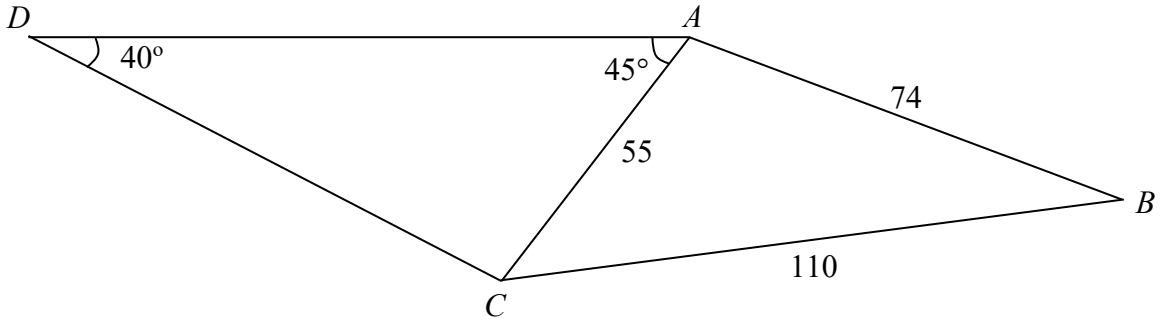
Diagram 2

Diagram 1 shows the vertical cross-section of a separating funnel with a small tap at its vertex.
The funnel is in the shape of an inverted right circular cone of base radius 9 cm and height 20 cm.
It contains water and oil, which do not mix, of depths 10 cm and 5 cm respectively, with the water at the bottom.

(a) Find the ratio of volume of water : volume of oil : capacity of the funnel. [3]

(b) All the water in the funnel is drained through the tap into a glass test-tube.
The test-tube consists of a hollow cylindrical upper part of internal radius 3 cm and a hollow hemispherical lower part of the same radius, as shown in **Diagram 2**.
Find the total surface area of the test-tube in contact with the water. [5]

6 The diagram shows four points A, B, C and D on a horizontal land where A is due east of D.
$AB = 74$ m, $BC = 110$ m and $AC = 55$ m.
Angle $ADC = 40^o$ and angle $CAD = 45^o$.



(a) Calculate

(i) AD, [2]

(ii) the area of triangle ACD, [2]

(iii) angle BAC, [3]

(iv) the bearing of A from B. [2]

(b) A vertical tower stands at B.
A man walks along CA produced such that the largest angle of elevation of the top of the vertical tower is 10^o.
Find the height of the tower. [3]

---

7 **(a)** $\mathscr{E}$ = { students in Class 4B }
H = { students who study History }
G = { students who study Geography }

There are 32 students in Class 4B.
13 students study History, 23 students study Geography and 11 students study both subjects.

(i) Draw a Venn diagram to illustrate this information. [1]

(ii) Find the total number of students who study only one of these two subjects. [1]

(b) D = { students who play the drums }
P = { students who play the piano }

(i) Express, in set notation, 'All students who play the drums also play the piano'. [1]

(ii) Write the set notation $D \cap P \neq \phi$ in words. [1]

(c) A factory supplies boxes of cereals to 3 supermarkets X, Y and Z.
The number of boxes of cereals supplied per delivery to each supermarket, the sizes and sale prices of the boxes, together with the number of deliveries made to each supermarket over a 4-month period are shown in the table below.

		Number of boxes per delivery			Number of deliveries over 4 months
Size of box		small	medium	large	
Supermarket	X	300	350	0	8
	Y	250	0	200	11
	Z	100	200	150	13
Sale price per box		\$2.50	\$4.25	\$7.80	

It is given that $\mathbf{A} = \begin{pmatrix} 300 & 350 & 0 \\ 250 & 0 & 200 \\ 100 & 200 & 150 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 8 & 11 & 13 \end{pmatrix}$.

(i) Calculate **BA**. [1]

(ii) Explain what the elements of **BA** represent. [1]

(iii) Given that $\mathbf{C} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, calculate **AC**. [1]

(iv) Describe what is represented by the elements of **AC**. [1]

(v) Using matrix multiplication, calculate the total amount of money that will be collected from the sale of all the boxes of cereals to Supermarket Z over 4 months. [2]

8 Answer the whole of this question on a sheet of graph paper.

A particle moves in a straight line so that at time t seconds, its distance y metres from a fixed point, O, is given by $y = t + \frac{32}{t+2} - 8$.

The following table gives some corresponding values of t and y.

t (seconds)	0	1	2	3	4	5	6	8	10	12	14
y (metres)	8	3.67	2	1.4	1.33	1.57	2	3.2	4.67	k	8

(a) Calculate the value of k. [1]

(b) Using a scale of 1 cm to represent 1 second, draw a horizontal t–axis for $0 \leq t \leq 14$.
Using a scale of 2 cm to represent 1 metre, draw a vertical y–axis for $0 \leq y \leq 8$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Explain the significance of the y–intercept. [1]

(d) Find the time when the particle is nearest to the fixed point, O. [1]

(e) Mark and label P, the point on your graph when the particle is 4 metres from the fixed point, O and moving away from O. [1]

(f) Find the length of time for which the particle is less than or equal to 2.5 metres from the fixed point, O. [1]

(g) By drawing a tangent, find the gradient of the curve at $t = 6$. [2]

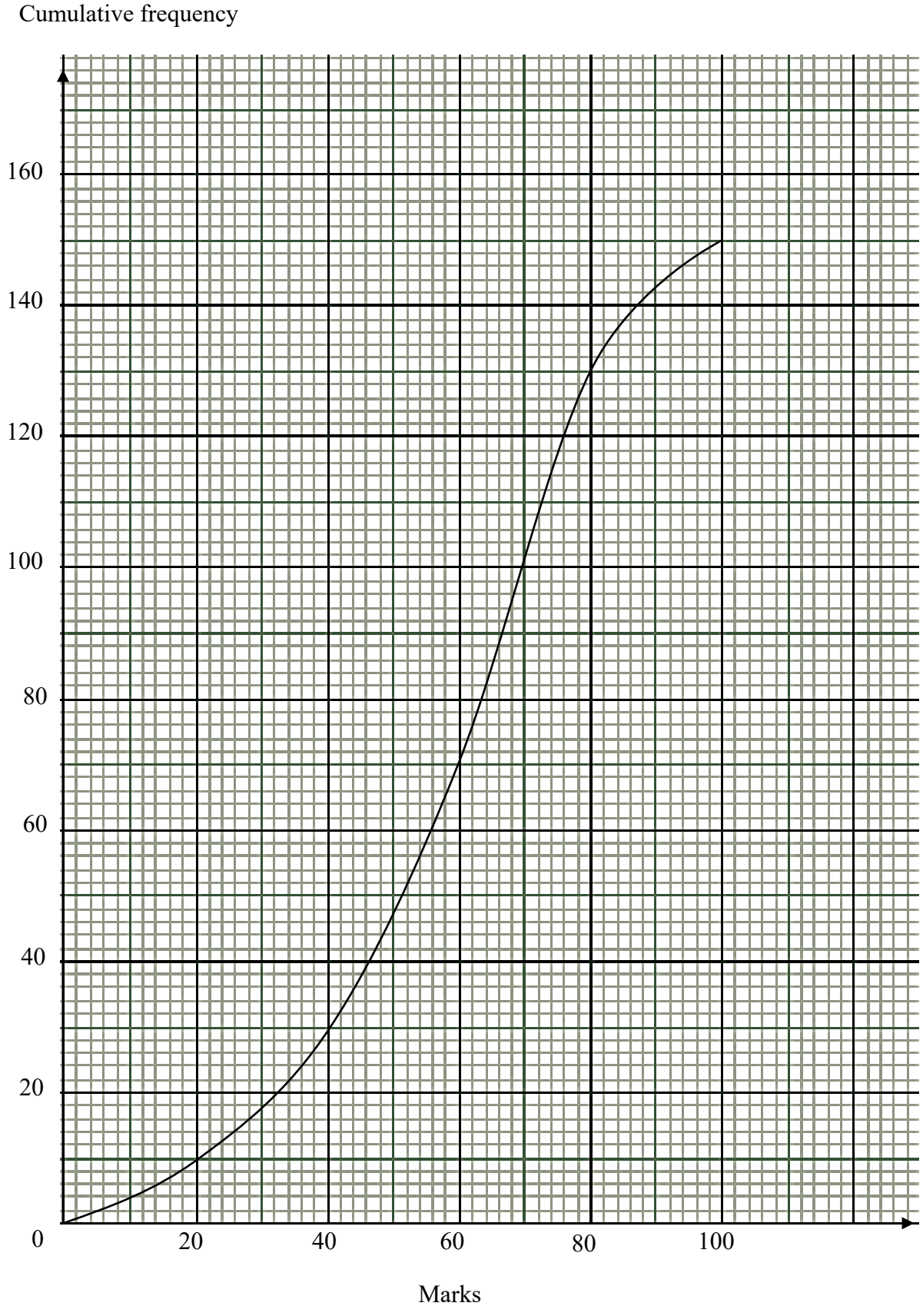
(h) The equation $t + \frac{32}{t+2} = 13 - \frac{1}{4}t$ can be solved by drawing a straight line on the same axes.

(i) Draw this line for $0 \leq t \leq 14$. [1]

(ii) Write down the t–coordinates of the points where the line intersects the curve. [2]

---

9 The cumulative frequency graph shows the distribution of marks of 150 students in a Mathematics examination.



9 **(a)** Use the graph to estimate

(i) the number of students who score more than 36 marks, [1]

(ii) the interquartile range. [2]

(b) Two students are selected at random.
Find the probability that

(i) both students score more than 36 marks, [1]

(ii) one student scores at most 64 marks while the other student scores more than 80 marks. [2]

(c) **(i)** Copy and complete the grouped frequency table of the marks of the 150 students. [1]

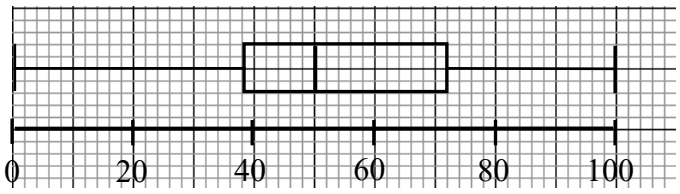
x (marks)	$0 < x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$
Number of students					

(ii) Using your grouped frequency table, calculate an estimate of

(a) the mean mark, [1]

(b) the standard deviation. [1]

(d) The same group of students took a Science examination.
The box and whisker plot shows the distribution of their marks.

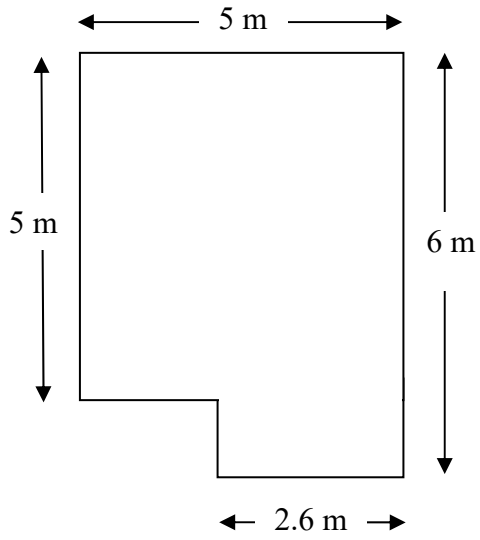


(i) Which examination was more difficult? Justify your answer. [1]

(ii) Compare and comment on the consistency of the performances of the students in the two examinations. [1]

---

10 **(a)** Mrs Wong had a budget of $2000 to spend on buying new kitchen flooring.
A sketch of the kitchen floor plan is shown below.
All angles shown in the diagram are right angles.



Mrs Wong planned to use either tiles or single coloured vinyl floor covering to cover the whole of the kitchen floor.

Each square tile measures 20 cm by 20 cm.
A box of 25 floor tiles costs $66.25.
Floor tiles are only sold in complete boxes.

The vinyl floor covering is cut to the required length from a roll.
A roll of vinyl floor covering is 1.8 metres wide and is sold at a price of $118.70 per metre length.
It is sold in lengths measured in a whole number of metres only.

Are both types of flooring materials within Mrs Wong's budget?
Show all your working and give reasons for your answers. [5]

(b) Mrs Wong saw an advertisement for a refrigerator and bought it using a payment plan.
The total price of the payment plan is 12% more than the advertised price.
The payments are calculated as shown.

- deposit of one-third of total price
- 8 equal instalments of $92.60 per month
- final payment of $200

Find the advertised price. [3]

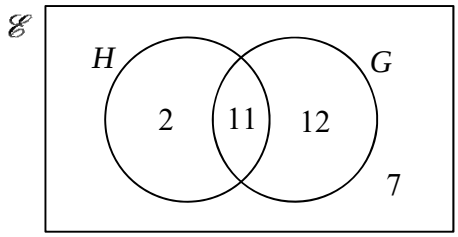
(c) Mrs Wong also bought a washer which cost $569.24, inclusive of 7% Goods and Services Tax (GST).
Find the amount of GST paid by Mrs Wong. [2]

Preliminary Examination 2018
Secondary Four O Level Mathematics
Paper 2 Solutions

1	**(a)(i)**	–34.5
	(a)(ii)	$\sqrt{\frac{r}{7+q^2}} - p = r$ $\frac{r}{7+q^2} = (r+p)^2$ $q^2 = \frac{r}{(r+p)^2} - 7$ $q = \pm\sqrt{\frac{r}{(r+p)^2} - 7}$
	(b)(i)	$300x + 800y = 22900$
	(b)(ii)	$8x + 12y = 396$ or $2x + 3y = 99$
	(b)(iii)	substitution or elimination $x = 15$ $y = 23$
2	**(a)**	$\frac{100}{v}$
	(b)	$\frac{100}{v} + \frac{90}{v-25} = \frac{11}{4}$ $\frac{100(v-25)+90v}{v(v-25)} = \frac{11}{4}$ $760v - 10000 = 11v^2 - 275v$ $11v^2 - 1035v + 10000 = 0$
	(c)	$\frac{-(-1035) \pm \sqrt{(-1035)^2 - 4(11)(10000)}}{2(11)}$ $= 83.16\,(2\text{dp})$ or $10.93\,(2\text{dp})$
	(d)	83.16 The other solution is rejected because it will give a negative speed for the second part of the journey.
	(e)	$\frac{90}{83.1589-25} - \frac{100}{83.1589}$ $= 21\,\text{min (nearest min)}$

3	**(a)**	Triangle *RTS*
	(b)(i)	$P\hat{V}S = U\hat{V}Q$ (vertically opposite angles) $V\hat{P}S = V\hat{U}Q$ (alternate angles, PS parallel to QR) $V\hat{S}P = V\hat{Q}U$ (alternate angles, PS parallel to QR) Thus, triangles *PVS* and *UVQ* are similar.
	(b)(ii)	$\frac{QV}{QS} = \frac{1}{3}$ $QS = 2QT$ $\frac{QV}{2QT} = \frac{1}{3}$ $\frac{QV}{QT} = \frac{2}{3}$ Thus, $QV = 2VT$
	(c)	$\frac{\text{area of } \Delta QVU}{\text{area of trapezium} PURS}$ $= \frac{\text{area of } \Delta QVU}{\text{area of } \Delta QPU} \times \frac{\text{area of } \Delta QPU}{\text{area of trapezium} PURS}$ $= \frac{1}{3} \times \frac{3}{9}$ $= \frac{1}{9}$ Ratio is 1 : 9
4	**(a)**	$\overrightarrow{QP} = \frac{1}{2}\overrightarrow{PL}$ $2\left(\overrightarrow{OP} - \overrightarrow{OQ}\right) = \overrightarrow{OL} - \overrightarrow{OP}$ $\overrightarrow{OL}$ $= 2\left[\begin{pmatrix} 5 \\ -8 \end{pmatrix} - \begin{pmatrix} -4 \\ 7 \end{pmatrix}\right] + \begin{pmatrix} 5 \\ -8 \end{pmatrix}$ $= \begin{pmatrix} 23 \\ -38 \end{pmatrix}$ $\left\lvert\overrightarrow{OL}\right\rvert = \sqrt{23^2 + (-38)^2}$ $= 44.4$ units (3sf)
	(b)(i) (a)	$\frac{2}{3}\mathbf{a} - \frac{2}{3}\mathbf{b}$
	(b)(i)(b)	$\frac{1}{2}\mathbf{b} + \frac{2}{3}\mathbf{a} - \frac{2}{3}\mathbf{b}$ $= \frac{2}{3}\mathbf{a} - \frac{1}{6}\mathbf{b}$

4	**(b)(ii)**	$\overrightarrow{MP}$ $= 2\mathbf{a} - \frac{1}{2}\mathbf{b}$ $= 3\left(\frac{2}{3}\mathbf{a} - \frac{1}{6}\mathbf{b}\right)$ $= 3\overrightarrow{MN}$ Since $\overrightarrow{MP} = 3\overrightarrow{MN}$, points *M*, *N* and *P* are collinear.
	(b)(iii)	$\frac{1}{2}$
5	**(a)**	vol of water : vol of oil : vol of funnel = 10^3 : $15^3 - 10^3$: 20^3 = 8 : 19 : 64
	(b)	Volume of water $= \frac{8}{64} \times \frac{1}{3}\pi(9^2)20$ $= 67.5\pi \text{ cm}^3$ Vol of water in cyindrical part of test-tube $= 67.5\pi - \frac{2}{3}\pi(3)^3$ $= 49.5\pi$ Height of water in cylindrical part of test-tube $= \frac{49.5\pi}{\pi(3)^2}$ $= 5.5$ Surface area in contact with water $= 2\pi(3)^2 + 2\pi(3)(5.5)$ $= 160 \text{ cm}^2$ (3sf)
6	**(a)(i)**	$\frac{55\sin 95^o}{\sin 40^o}$ $= 85.2 \text{ m}$ (3sf)
	(a)(ii)	$\frac{1}{2} \times 55 \times 85.2392 \times \sin 45^o$ $= 1660 \text{ m}^2$ (3sf)
	(a)(iii)	$\angle BAC = \cos^{-1}\left(\frac{110^2 - 55^2 - 74^2}{-2(55)(74)}\right)$ $= 116.2^o$ (1dp)
	(a)(iv)	$360^o - (116.2403^o - 45^o)$ $= 288.8^o$ (1dp)

6	**(b)**	perpendicular distance from B to AC produced $= 74\sin 63.7597^o$ $= 66.3741$ height of tower $= 66.3741\tan 10^o$ $= 11.7$ m (3sf)
7	**(a)(i)**	
	(a)(ii)	14 students
	(b)(ii)	$D \subset P$
	(b)(ii)	There are some students who play the drums <u>and</u> the piano.
	(c)(i)	$\begin{pmatrix}6450 & 5400 & 4150\end{pmatrix}$
	(c)(ii)	Total number of boxes of cereals of <u>each size</u> delivered to all supermarkets over <u>4 months</u>.
	(c)(iii)	$\begin{pmatrix}650\\450\\450\end{pmatrix}$
	(c)(iv)	Total number of boxes of cereals delivered <u>to each of the supermarkets per delivery</u>.
	(c)(v)	$13\begin{pmatrix}100 & 200 & 150\end{pmatrix}\begin{pmatrix}2.5\\4.25\\7.8\end{pmatrix}$ $= \begin{pmatrix}29510\end{pmatrix}$ Total amount is \$29510
8	**(a)**	6.29
	(b)	correct plotted points (at least 8 points) smooth curve
	(c)	Object is <u>8 metres from O</u> at <u>$t = 0$</u>
	(d)	3.6 ± 0.1 and 3.8 seconds
8	**(e)**	P labelled on graph at $(9.1 \pm 0.1, 4)$
	(f)	5.3 ± 0.2

	(g)	Tangent drawn and calculation of gradient 0.5 ± 0.05
	(h)(i)	Draw the line $y = 5 - \frac{1}{4}t$ for $0 \le t \le 14$.
	(h)(ii)	0.6 ± 0.1 7.8 ± 0.1
9	**(a)(i)**	126 students
	(a)(ii)	$73 - 45$ $= 28$ marks
	(b)(i)	$\frac{105}{149}$
	(b)(ii)	$\frac{82}{150} \times \frac{20}{149} \times 2$ $= \frac{328}{2235}$

	(c)(i)	x (marks)	$0 < x \le 20$	$20 < x \le 40$	$40 < x \le 60$	$60 < x \le 80$	$80 < x \le 100$
		No of students	10	20	40 or 41	60 or 59	20

	(c)(ii)(a)	58 marks
	(c)(ii)(b)	21.7 marks
	(d)(i)	Science examination is more difficult as it has a lower median mark.
	(d)(ii)	The interquartile range for the Science examination is larger. Hence the performance for Science examination is less consistent.
10	**(a)**	No of boxes of tiles required $= \frac{5 \times 5 + 2.6}{0.2 \times 0.2} \div 25$ $= 27.6$ $= 28 \text{ (whole number)}$ Total cost $= 28 \times 66.25$ $= \$1855$ Length required

		$= \frac{27.6}{1.8}$ $= 15.333$ $= 16\,(\text{whole number})$ Cost of vinyl flooring $= 16 \times 118.7$ $= \$1899.20$ Both types of floorings are within Mrs Wong's budget.
	(b)	Let advertised price be A $\frac{1}{3} \times \frac{112A}{100} + 8 \times 92.6 + 200 = \frac{112A}{100}$ $940.8 = \frac{56A}{75}$ $A = \$1260$
	(c)	$\frac{569.24}{107} \times 7$ $= \$37.24$

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION

MATHEMATICS
Paper 1

4048/01

28 AUGUST 2018 (Tuesday)

2 hours

CANDIDATE NAME

CLASS

INDEX NUMBER

READ THESE INSTRUCTIONS FIRST

Do not turn over the page until you are told to do so.

Write your name, class and index number in the spaces above.

Write in dark blue or black pen in the space provided for each question.

You may use a pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

INFORMATION FOR CANDIDATES

Answer **all** the questions.

Write your answers in the space provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use		
Q1	1	
Q2	1	
Q3	2	
Q4	2	
Q5	2	
Q6	2	
Q7	2	
Q8	4	
Q9	2	
Q10	3	
Q11	3	
Q12	3	
Q13	3	
Q14	3	
Q15	3	
Q16	2	
Q17	4	
Q18	5	
Q19	6	
Q20	5	
Q21	3	
Q22	7	
Q23	9	
Q24	3	
Total	/ 80	

The total number of marks for this paper is **80**.

This document consists of 24 printed pages including the Cover Sheet.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle ABC $= \frac{1}{2}ab\sin C$

Arc length $= r\theta$, *where* θ *is in radians*

Sector area $= \frac{1}{2}r^2\theta$, *where* θ *is in radians*

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Factorise $5(3x-y)^2-(3x-y)$ completely.

Answer ..... [1]

---

2 The enrolment for a school in 2017 was 3450. This was 15% more than the enrolment in 2016. Calculate the enrolment in 2016.

Answer ..... [1]

---

3 Ms Chew invested $\$P$ in a bank that pays compound interest at the rate of 4 % per annum compounded half yearly. If she received \$6341.21 from the bank after 6 years, find the value of P, giving your answer to the nearest whole number.

Answer $P =$ \$..... [2]

---

4 **(a)** Express 315 as a product of its prime factors.

Answer ..... [1]

(b) Find the **smallest whole number** by which 315 must be multiplied to obtain a perfect square.

Answer ..... [1]

---

5 Given that $4^{\left(\frac{1}{2n}\right)} \div 64^{-2} = 2^5$, find the value of n.

Answer $n =$ ..... [2]

---

6 Given that $OABC$ is a parallelogram such that $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and A (5, 1). Find $\left|\overrightarrow{OB}\right|$.

Answer .....units [2]

---

7 A map is drawn to a scale of 1 : 50000.

(a) Two towns are 24 km apart. Calculate, in centimetres, their distance apart on the map.

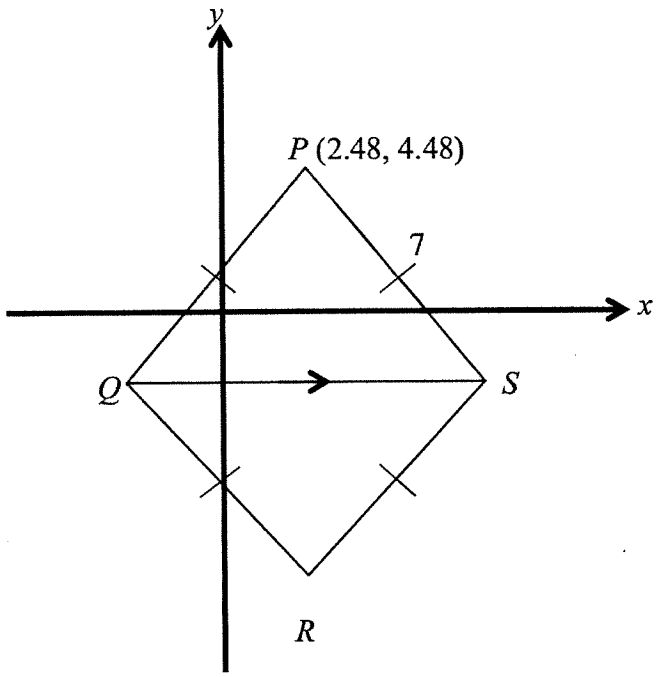
Answer .....cm [1]

(b) On the map, a farm has an area of 20 cm^2. Calculate, in square kilometres, the actual area of the farm.

Answer .....km^2 [1]

---

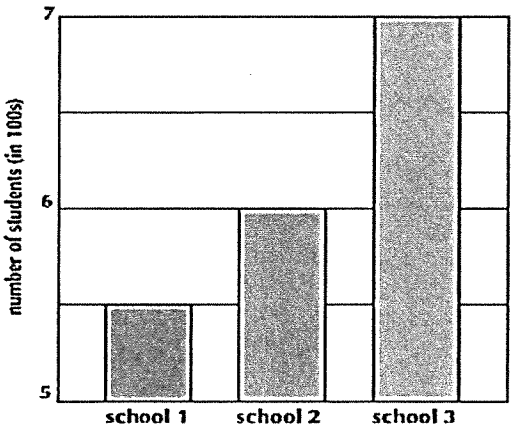
8 The diagram below shows 2 congruent equilateral triangles PQS and SQR with sides 7 cm. Point P has coordinates (2.48, 4.48).
The base SQ of the equilateral triangle PSQ is parallel to the x-axis. Find the coordinates of R and S, giving your answers correct to two decimal places.



Answer $R = (\ldots\ldots, \ldots\ldots)$

$S = (\ldots\ldots, \ldots\ldots)$ [4]

9 Jenny drew a bar chart to compare the enrolment (number of students) in school 1, 2 and 3.



State one aspect of the bar chart that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

.....

.....

..... [2]

---

10 Given that $3xy + x = \sqrt{3yz + x^2}$, express x in terms of y and z.

Answer $x =$..... [3]

---

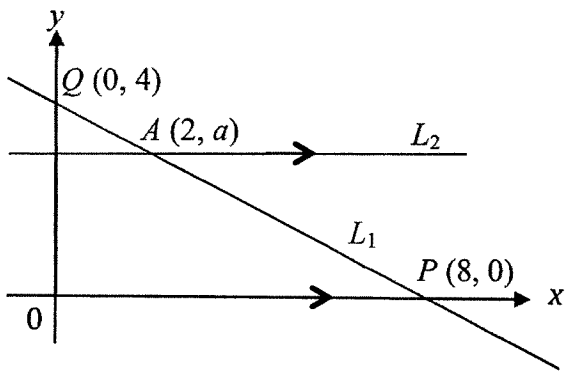
11 (a) Solve the inequalities $4 \leq 7 - \frac{x+3}{2} < \frac{13}{2}$.

Answer (a)..... [2]

(b) Write down all the integers that satisfy $4 \leq 7 - \frac{x+3}{2} < \frac{13}{2}$.

Answer (b)..... [1]

12 In the figure below, line L_1 cuts the x-axis at P (8, 0) and the y-axis at Q (0, 4). On the same axes line L_2 meets line L_1 at A $(2, a)$. Line L_2 is parallel to the x-axis.



(a) Write down the equation of line L_1.

Answer ..... [1]

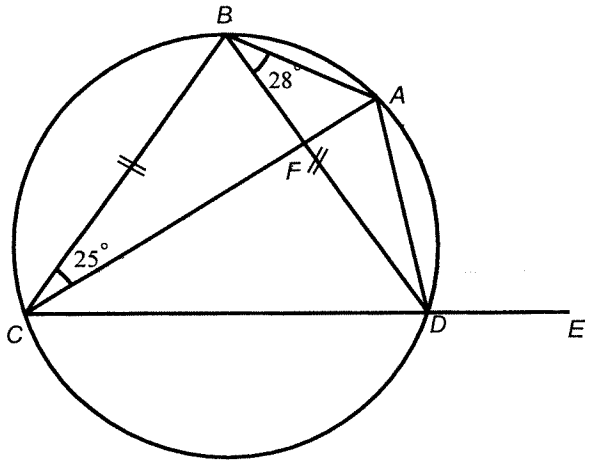
(b) Calculate the value of a.

Answer $a =$ ..... [1]

(c) Write down the value of $\frac{QA}{AP}$.

Answer ..... [1]

13 The diagram shows a circle $ABCD$ with $BC = BD$. CDE is a straight line. Given that angle $ABD = 28°$ and angle $ACB = 25°$,



(a) explain why is angle $ACD = 28°$.

Answer

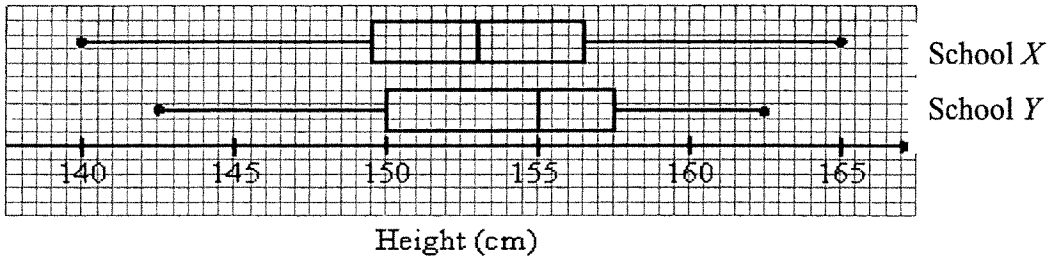
.....

..... [1]

(b) Hence, find angle BAD, giving reasons for your answer.

Answer..... ° [2]

14 The box-and-whisker plots show the distribution of heights of girls in 2 schools.



(a) Find the median height for School *X*.

Answer .....cm [1]

(b) Find the interquartile range for School *Y*.

Answer .....cm [1]

(c) Janet said the girls in School *X* are generally taller than the girls in School *Y*. Do you agree? Give a reason for your answer.

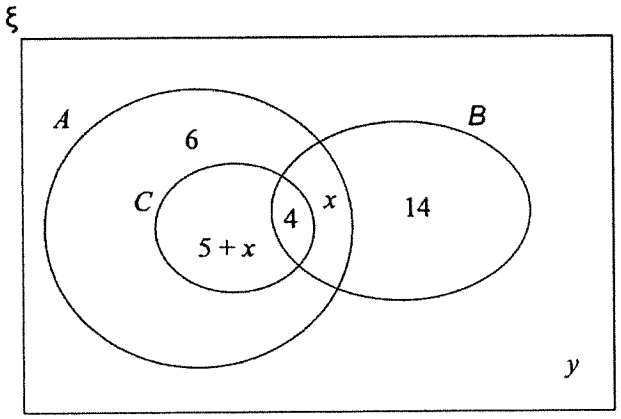
Answer

.....

..... [1]

15 The Venn diagram shows the number of elements of sets A, B and C. Given that $n(A) = 27$ and $n(A \cup B)' = 4$

(a) find the value of x and y,



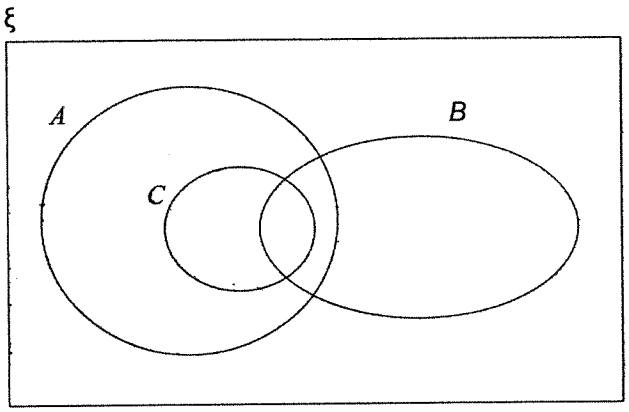
Answer $x =$ ..... [1]

$y =$ ..... [1]

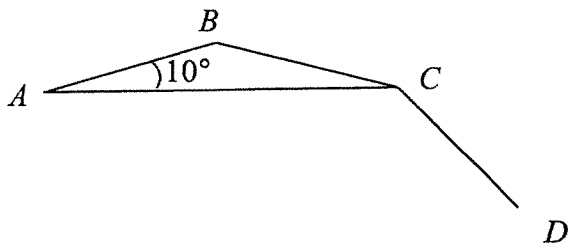
(b) shade the region $A \cap B'$.

Answer

[1]



16 AB, BC and CD are adjacent sides of a regular polygon. Given that $\angle CAB = 10°$,



calculate

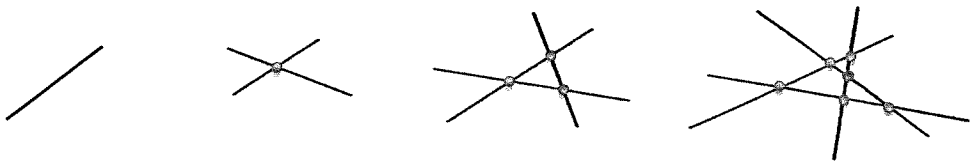
(a) the exterior angle of the polygon,

Answer .....° [1]

(b) the number of sides of the polygon.

Answer ..... [1]

17



The above diagrams show the maximum number of intersections obtained from 1, 2, 3 and 4 lines respectively.

Number of lines, n	Maximum number of line segments, E	Maximum number of intersections, P	Maximum number of regions, R
1	1	0	$2 = 1+1$
2	$2^2 = 4$	$1 = \frac{2(1)}{2}$	$4 = 1+3$
3	$3^2 = 9$	$3 = \frac{3(2)}{2}$	$7 = 1+6$
4		$6 = \frac{4(3)}{2}$	$11 = 1+10$
5		$10 = \frac{5(4)}{2}$	
6	$6^2 = 36$	$15 = \frac{6(5)}{2}$	$22 = 1+21$

(a) Complete the above table. [1]

(b) What is the maximum number of intersections P obtained from n straight lines in terms of n?

Answer $P =$..... [1]

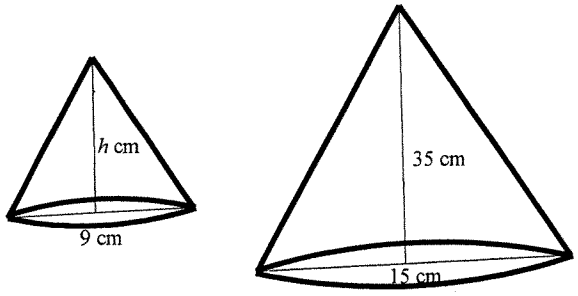
(c) What is the maximum number of regions R obtained by using n straight lines in terms of n?

Answer $R =$..... [1]

(d) Hence, write down an expression connecting R, E and P.

Answer ..... [1]

18 Two solid cones are geometrically similar. The diameters of the base of the smaller cone and the base of the larger cone are 9 cm and 15 cm respectively. The heights of the smaller cone and the larger cone are h cm and ~~15~~ 35 cm respectively.



(a) Find the value of h.

Answer $h =$ .....cm [1]

(b) If it costs \$9 to paint the smaller cone with 1 coat of paint, how much does it cost to paint the larger cone with 1 coat of the same kind of paint?

Answer \$..... [2]

(c) Given that the mass of the larger cone is 25 g, find the mass of the smaller cone, assuming that both cones are made of the same kind of material.

Answer .....g [2]

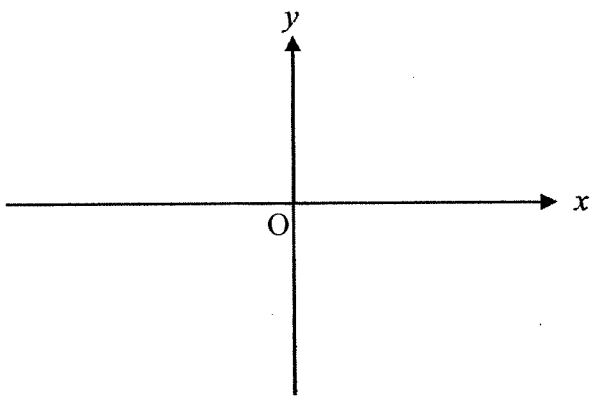
19 **(a)** Express $-x^2-6x-7$ in the form $-(x+a)^2+b$, [1]

Answer

(b) hence, solve $-x^2-6x-7=0$, showing your working clearly.
Give your answers correct to two decimal places.

Answer $x =$ ..... *or* ..... [2]

(c) Sketch the graph of $y = -x^2-6x-7$,



[2]

(d) With reference to graph drawn above, explain why there is no solution for the equation $-x^2-6x-7=3$.

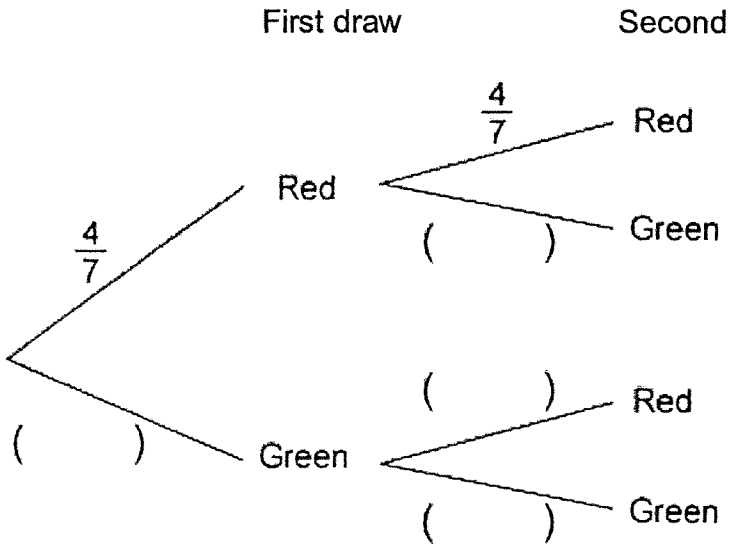
.....

.....

..... [1]

20 A box contains 4 red balls and 3 green balls. One ball is drawn at random. If a green ball is drawn, it will **not** be replaced, a second ball is then drawn. If a red ball is drawn, it will be replaced, a second ball is then drawn from the box.
Complete the probability tree diagram to show the probabilities of possible outcomes.

Answer [1]



Find the probability that

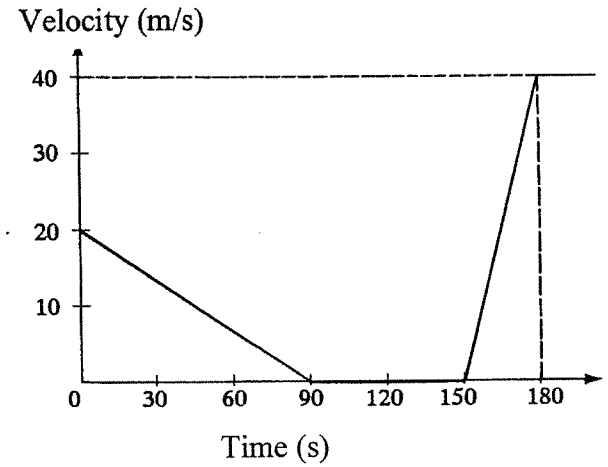
(i) the two balls are of different colours,

Answer ..... [2]

(ii) at least 2 green balls are left in the box after the second draw.

Answer ..... [2]

21 A train slows down to a stop on entering a station P as shown in the velocity-time graph. After a brief stop of 60 s, it starts to move off with an acceleration of $1\frac{1}{3}$ m/s^2 for 30 s before it gets out of station P. It then continues its journey with this velocity until it reaches another station Q.



(a) Find the deceleration of the train when it enters the station P.

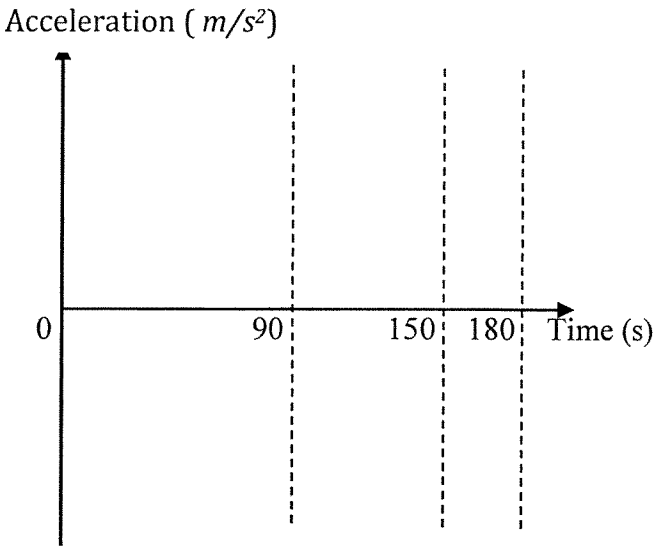
Answer .....m/s^2 [1]

(b) Calculate the total distance travelled by the train in its first 3 minutes journey.

Answer .....m [1]

(c) On the axes below, sketch an acceleration-time graph of the train for the whole 3 minutes of its motion.

[1]



22

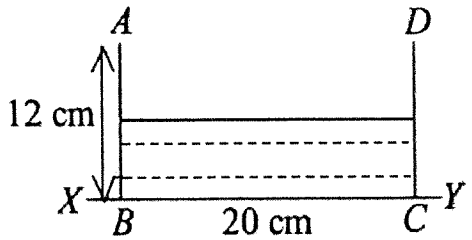


Figure 1

Figure 1 shows a vertical cross-section of a rectangular tank that stands on a horizontal table represented by *XY*. The tank is 12 cm high and has a square base of side 20 cm and contains 3000 cm^3 of water. Calculate

(a) **(i)** the volume of the tank,

Answer .....cm^3 [1]

(ii) the depth of the water.

Answer .....cm [2]

The tank is now tilted about a base edge through C, so that some of the water spills out until the position shown in Figure 2 below. Calculate

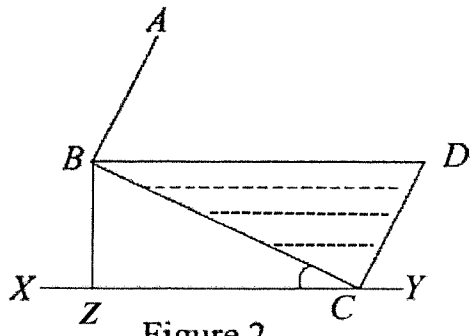


Figure 2

(b) Calculate

(i) the volume of water remaining in the tank,

Answer .....cm^3 [1]

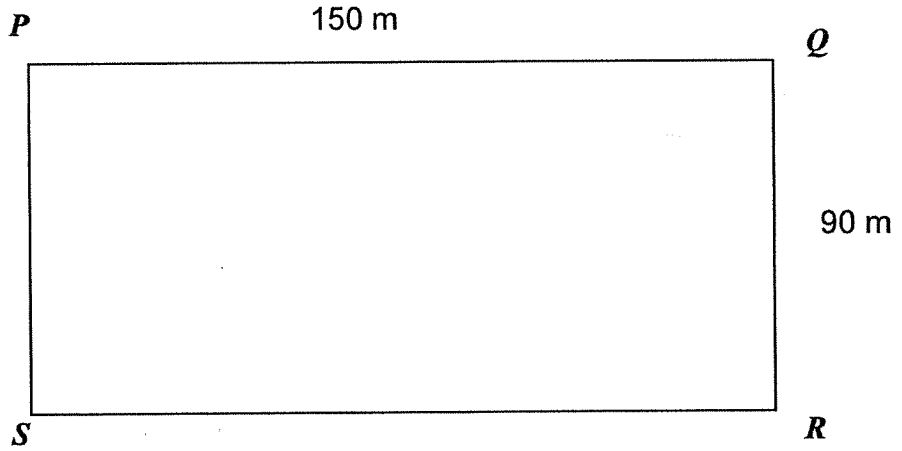
(ii) angle BCZ,

Answer ..... ° [2]

(iii) BZ, the vertical height of B above the table where Z is the foot of the perpendicular of B to YZ.

Answer .....cm [1]

23



The diagram shows a rectangular field where $PQ = 150$ m and $QR = 90$ m. Jason starts from P and walks towards Q at a constant speed of 1.5 m/s. At the same time, John starts from Q and walks towards R at a constant speed of 0.5 m/s.

(a) Write down in terms of t (where $t < 100$s)

(i) the distance of Jason from Q after t seconds,

Answer .....m [1]

(ii) the distance of John from Q after t seconds.

Answer .....m [1]

(b) Given that after t seconds, the two men are h m apart, show that

$$h^2 = 2.5t^2 - 450t + 22500.$$

Answer [2]

(c) Find the distance between the two men one minute after the start.

Answer .....m [2]

(d) Find the value of t when the two men are 100 m apart.

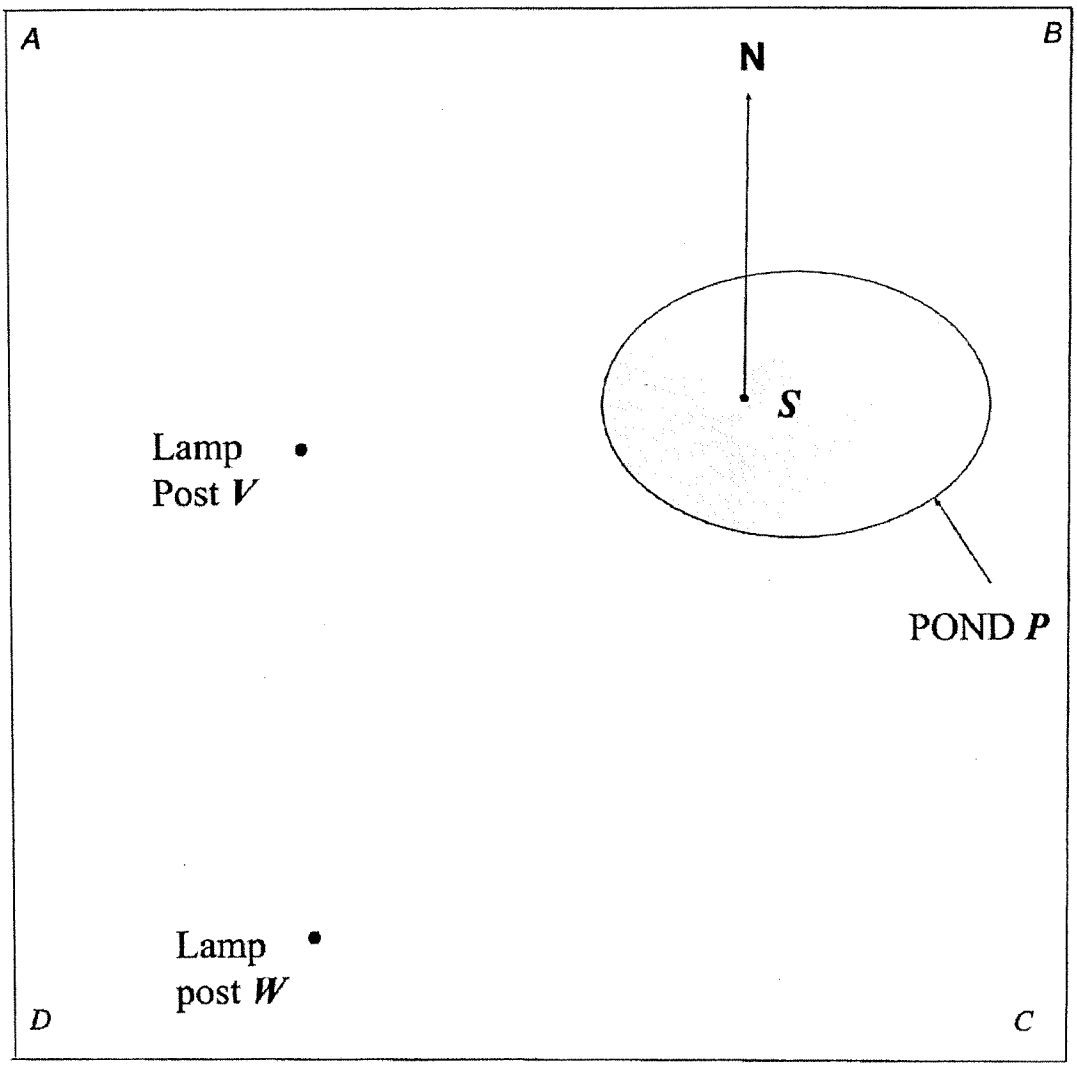
Answer .....s [2]

(e) Hence, find the distance that Jason is from Q when the two men are 100 m apart.

Answer .....m [1]

---

24 **(a)** The diagram shows a plot of land *ABCD* with a pond *P* at a corner and two lamp-posts *V* and *W*. *S* is the fixed position of a stick placed in the pond.
A gardener wants to plant a tree *T* equidistant from the two lamp posts *V* and *W* and also equidistant from the lines *VS* and *WS*. By appropriate constructions, mark the point *T* on the diagram below. [2]



(b) Measure the bearing of *W* from *S*.

Answer .....° [1]

End of Paper

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION

MATHEMATICS
Paper 1

4048/01

28 AUGUST 2018 (Tuesday)

2 hours

CANDIDATE NAME: **Solutions**

CLASS

INDEX NUMBER

READ THESE INSTRUCTIONS FIRST

Do not turn over the page until you are told to do so.

Write your name, class and index number in the spaces above.

Write in dark blue or black pen in the space provided for each question.

You may use a pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

INFORMATION FOR CANDIDATES

Answer **all** the questions.

Write your answers in the space provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use		
Q1	1	
Q2	1	
Q3	2	
Q4	2	
Q5	2	
Q6	2	
Q7	2	
Q8	4	
Q9	2	
Q10	3	
Q11	3	
Q12	3	
Q13	3	
Q14	3	
Q15	3	
Q16	2	
Q17	4	
Q18	5	
Q19	6	
Q20	5	
Q21	3	
Q22	7	
Q23	9	
Q24	3	
Total	/ 80	

The total number of marks for this paper is **80**.

This document consists of 24 printed pages including the Cover Sheet.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle ABC $= \frac{1}{2}ab\sin C$

Arc length $= r\theta$, *where* θ *is in radians*

Sector area $= \frac{1}{2}r^2\theta$, *where* θ *is in radians*

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Factorise $5(3x-y)^2-(3x-y)$ completely.

$(3x-y)[5(3x-y)-1]$

$=(3x-y)[15x-5y-1]$

Answer $(3x-y)(15x-5y-1)$ [1]

---

2 The enrolment for a school in 2017 was 3450. This was 15% more than the enrolment in 2016. Calculate the enrolment in 2016.

115% represents 3450

$\therefore$ 100% represents $\frac{100}{115} \times 3450$

$= 3000$

Answer 3000 [1]

---

3 Ms Chew invested \$$P$ in a bank that pays compound interest at the rate of 4 % per annum compounded half yearly. If she received \$6341.21 from the bank after 6 years, find the value of P, giving your answer to the nearest whole number.

$$6341.21 = P\left[1+\frac{\frac{4}{2}}{100}\right]^{6\times 2}$$

$$6341.21 = P(1.02)^{12}$$

$$\Rightarrow P = \frac{6341.21}{1.02^{12}}$$

$= \$5000$

Answer $P =$ \$5000 [2]

---

4 (a) Express 315 as a product of its prime factors.

$3 \mid 315$
$3 \mid 105$
$5 \mid 35$
$7 \mid 7$
1

$315 = 3^2 \times 5 \times 7$

Answer $3^2 \times 5 \times 7$ [1]

(b) Find the **smallest whole number** by which 315 must be multiplied to obtain a perfect square.

$3^2 \times 5 \times 7$

Answer 35 [1]

---

5 Given that $4^{\frac{1}{2n}} \div 64^{-2} = 2^5$, find the value of n.

$(2^2)^{\frac{1}{2n}} \div (2^6)^{-2} = 2^5$

$2^{\frac{1}{n}} \div 2^{-12} = 2^5$

$2^{\frac{1}{n} - (-12)} = 2^5$

$\frac{1}{n} + 12 = 5$

$\frac{1}{n} = -7$

$\Rightarrow n = -\frac{1}{7}$

Answer $n =$ $-\frac{1}{7}$ [2]

---

6 Given that $OABC$ is a parallelogram such that $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and A (5, 1). Find $\left|\overrightarrow{OB}\right|$.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ 6 \end{pmatrix}$

$|\overrightarrow{OB}| = \sqrt{8^2 + 6^2}$

$= 10$

Answer 10 units [2]

---

7 A map is drawn to a scale of 1 : 50000.

(a) Two towns are 24 km apart. Calculate, in centimetres, their distance apart on the map.

1 cm represent 0.5 km

0.5 km is represented by 1 cm

∴ 24 km is represented by $\frac{24}{0.5} = 48$ cm

Answer 48 cm [1]

(b) On the map, a farm has an area of 20 cm^2. Calculate, in square kilometres, the actual area of the farm.

1 cm^2 represents $(0.5\text{ km})^2$

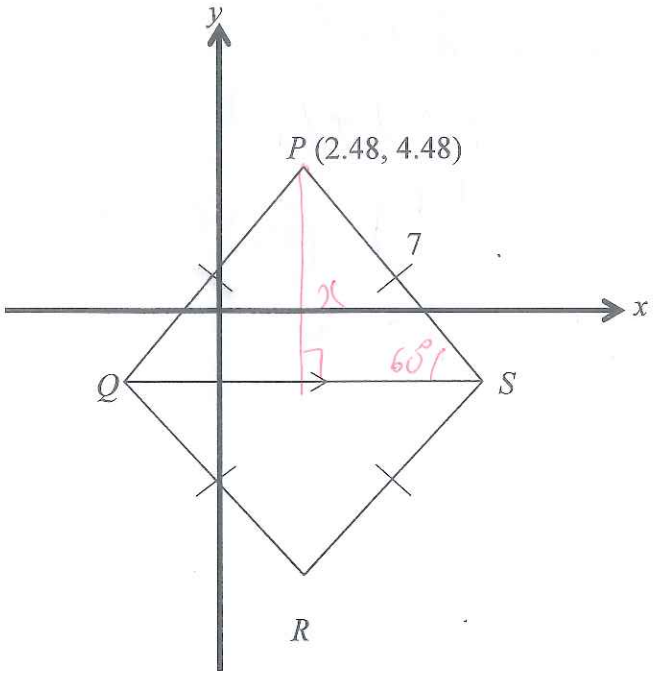
1 cm^2 represents 0.25 km^2

∴ 20 cm^2 represents $0.25 \times 20 = 5$ km^2

Answer 5 km^2 [1]

---

8. The diagram below shows 2 congruent equilateral triangles PQR and SQR with sides 7 cm. Point P has coordinates (2.48, 4.48).
The base SQ of the equilateral triangle PSQ is parallel to the x-axis. Find the coordinates of R and S, giving your answers correct to two decimal places.



Let perpendicular line from P to QS be x

$\sin 60° = \frac{x}{7} \Rightarrow x = 7\sin 60° = 6.0622$

$PR = 6.0622 \times 2 = 12.1244$

$\therefore$ y-coord. of $R = 4.48 - 12.1244$
$= -7.64$

$\therefore R(2.48, -7.64)$

$\frac{y}{7} = \cos 60° \Rightarrow y = 3.5$

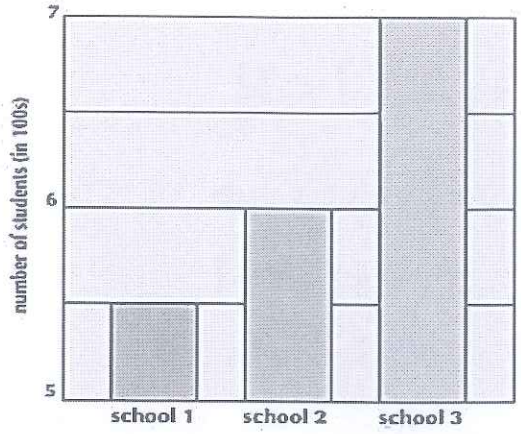
x-coord of $S = 5.98$

y-coord of $S = 4.48 - 6.0622$
$= -1.58$

Answer $R = (2.48, -7.64)$

$S = (5.98, -1.58)$ [3]

9 Jenny drew a bar chart to compare the enrolment in school 1, 2 and 3.



State one aspect of the bar chart that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

The vertical axis did not start from 0

This would exaggerate the difference in the enrolment of the schools

[2]

---

10 Given that $3xy + x = \sqrt{3yz + x^2}$, express x in terms of y and z.

$(3xy+x)^2 = 3yz + x^2$

$9x^2y^2 + 6x^2y + x^2 = 3yz + x^2$

$9x^2y + 6x^2y = 3yz$

$3x^2y(3y+2) = 3yz$

$x^2 = \frac{3yz}{3y(3y+z)}$

$x = \pm\sqrt{\frac{z}{3y+2}}$

Answer $x = \pm\sqrt{\frac{z}{3y+2}}$ [3]

---

11 (a) Solve the inequalities $4 \leq 7 - \frac{x+3}{2} < \frac{13}{2}$.

$4 \leq 7 - \left(\frac{x+3}{2}\right)$

$\frac{x+3}{2} \leq 7 - 4$

$\frac{x+3}{2} \leq 3$

$x + 3 \leq 6$

$x \leq 3$

$7 - \left(\frac{x+3}{2}\right) < \frac{13}{2}$

$14 - (x+3) < 13$

$-(x+3) < -1$

$x + 3 > 1$

$x > -2$

$-2 < x \leq 3$

$-2 \leq x \leq 3$

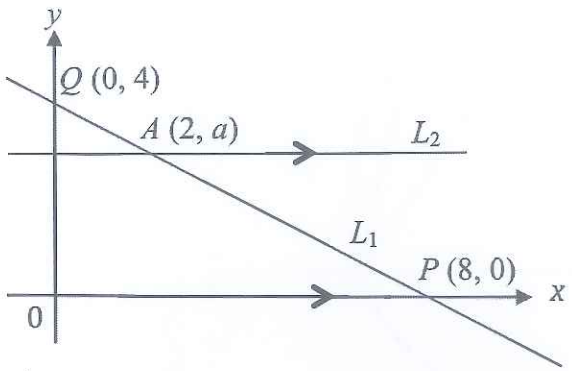
Answer (a)..... [2]

(b) Write down all the integers that satisfy $4 \leq 7 - \frac{x+3}{2} < \frac{13}{2}$.

$-1, 0, 1, 2, 3$

Answer (b) $-1, 0, 1, 2, 3$ [1]

12 In the figure below, line L_1 cuts the x-axis at P (8, 0) and the y-axis at Q (0, 4). On the same axes line L_2 meets line L_1 at A (2, a). Given that lines L_1 and L_2 are parallel.



(a) Write down the equation of line L_1.

Gradient of L_1 =

Equation of L_1 is y

Answer $y = -\frac{1}{2}x + 4$ [1]

(b) Calculate the value of a.

Sub. $x = 2$, $y = a$ into $y = -\frac{1}{2}x + 4$

$a = -\frac{1}{2}(2) + 4 \Rightarrow a = 3$

Answer $a =$ 3 [1]

(c) Write down the value of $\frac{QA}{AP}$.

$\frac{QA}{AP} = \frac{2}{6}$

$= \frac{1}{3}$

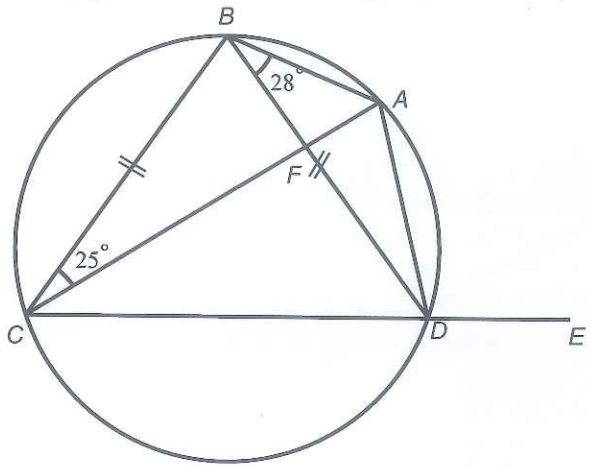
Alternative Method:

$\frac{QA}{AP} = \frac{\sqrt{2^2+1^2}}{\sqrt{6^2+(-3)^2}} = \sqrt{\frac{1}{9}}$

$= \frac{1}{3}$

Answer $\frac{1}{3}$ [1]

13 The diagram shows a circle $ABCD$ with $BC = BD$. CDE is a straight line. Given that angle $ABD = 28°$ and angle $ACB = 25°$,



(a) explain why is angle $ACD = 28°$.

Answer ∠ACD = ∠DBA (∠ in the same segment)

[1]

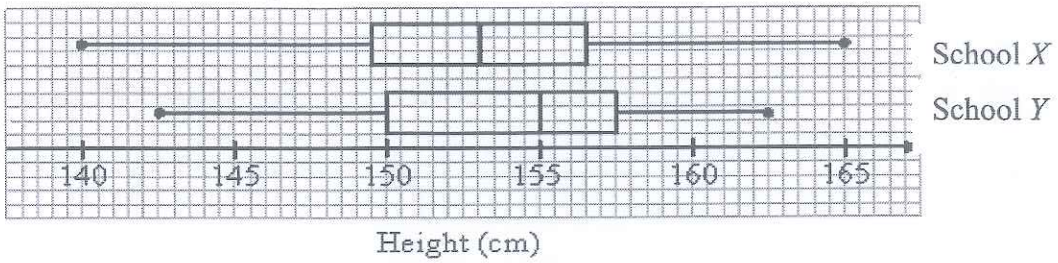
(b) Hence, find angle BAD, giving reasons for your answer.

∠BAD = 180° − 25° − 28° (Angles in opposite segment OR opposite ∠s in a cyclic quadrilateral)

= 127°

Answer 127 ° [2]

14 The box-and-whisker plots show the distribution of heights of girls in 2 schools.



(a) Find the median height for School *X*.

Median height for school X = 153cm

Answer 153 cm [1]

(b) Find the interquartile range for School *Y*.

Interquartile Range for School Y
= 157.5 − 150
= 7.5 cm

Answer 7.5 cm [1]

(c) Janet said the girls in School *X* are generally taller than the girls in School *Y*. Do you agree? Give a re

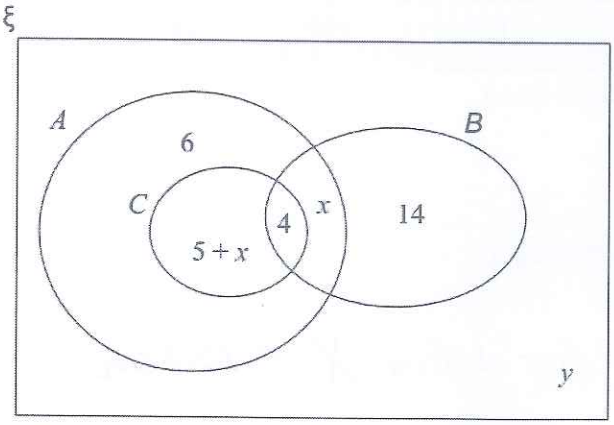
Answer

No I disagree median of School Y
= 153cm is larger than the median of School X [1]
(153 cm)

15 The Venn diagram shows the number of elements of sets A, B and C. Given that $n(A) = 27$ and $n(A \cup B)' = 4$

Find

(a) the value of x and y,



$6+5+x+4+x = 27$

$2x = 27-15$

$= 12$

$x = 6$

$y = 4$

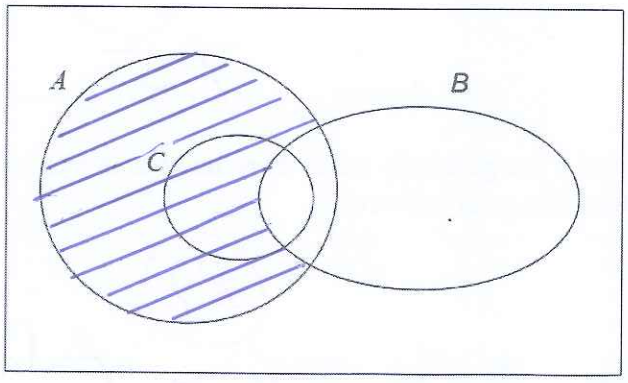
Answer $x =$ ..... 6 ..... [1]

$y =$ ..... 4 ..... [1]

(b) Shade the region $A \cap B'$.

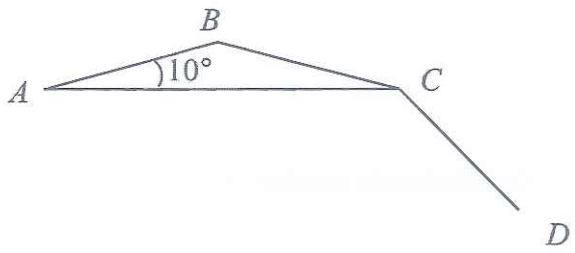
Answer

ξ



[1]

16 AB, BC and CD are adjacent sides of a regular polygon. Given that $\angle CAB = 10°$,



calculate

(a) the exterior angle of the polygon,

$\angle BCA = 10°$ ($AB = BC$)

Exterior $\angle = 10° + 10°$

$= 20°$

OR

$\angle ABC = 180° - 10° - 10°$

$= 160°$

Exterior $\angle = 180° - 160°$

$= 20°$

Answer 20 ° [1]

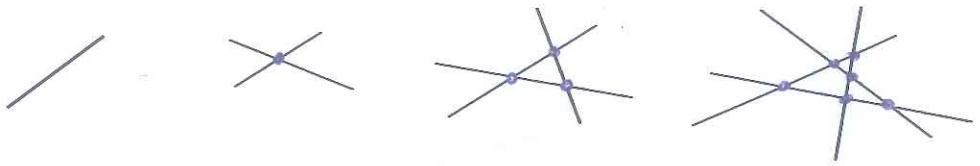
(b) the number of sides of the polygon.

Number of sides $= \frac{360}{20}$

$= 18$

Answer 18 [1]

17



The above diagrams show the maximum number of intersections obtained from 1, 2, 3 and 4 lines respectively.

Number of lines, n	Maximum number of line segments, E	Maximum number of intersections, P	Maximum number of regions, R
1	1	0	$2 = 1+1$
2	$2^2 = 4$	$1 = \frac{2(1)}{2}$	$4 = 1+3$
3	$3^2 = 9$	$3 = \frac{3(2)}{2}$	$7 = 1+6$
4	$4^2 = 16$	$6 = \frac{4(3)}{2}$	$11 = 1+10$
5	$5^2 = 25$	$10 = \frac{5(4)}{2}$	$16 = 1+15$
6	$6^2 = 36$	$15 = \frac{6(5)}{2}$	$22 = 1+21$

(a) Complete the above table. [1]

(b) What is the maximum number of intersections P obtained from n straight lines in terms of n?

$P = \frac{n(n-1)}{2}$ or $\frac{n^2-n}{2}$ or $\frac{1}{2}(n^2-n)$

Answer $P =$ $\frac{n^2-n}{2}$ [1]

(c) What is the maximum number of regions R obtained by using n straight lines in terms of n?

$R = 1 + \frac{n(n+1)}{2}$ or $\frac{2+n+n^2}{2}$

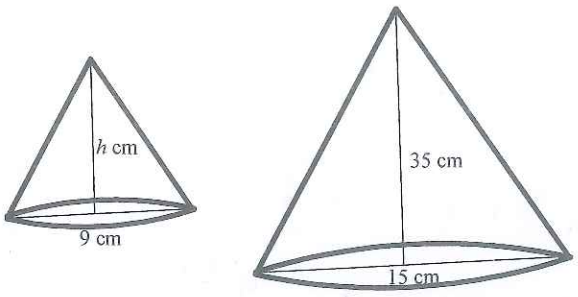
Answer $R =$ $1 + \frac{n(n+1)}{2}$ [1]

(d) Hence, write down an expression connecting R, E and P.

$R+P-E=1$

Answer $R+P-E=1$ [1]

18 Two solid cones are geometrically similar. The diameters of the base of the smaller cone and the base of the larger cone are 9 cm and 15 cm respectively. The heights of the smaller cone and the larger cone are h cm and ~~15~~ 35 cm respectively.



(a) Find the value of h.

$\frac{h}{35} = \frac{9}{15} \Rightarrow h = \frac{9}{15} \times 35 = 21$

Answer $h =$ 21 cm [1]

(b) If it costs \$9 to paint the smaller cone with 1 coat of paint, how much does it cost to paint the larger cone with 1 coat of the same kind of paint?

$\frac{A_1}{A_2} = \left(\frac{L_1}{L_2}\right)^2$

$\frac{9}{A} = \left(\frac{3}{5}\right)^2$

$\frac{9}{A} = \frac{9}{25}$

$\Rightarrow A = 25$

Answer \$ 25 [2]

(c) Given that the mass of the larger cone is 25 g, find the mass of the smaller cone, assuming that both cones are made of the same kind of material.

$\frac{M_1}{M_2} = \left(\frac{L_1}{L_2}\right)^3$

$\frac{M_1}{25} = \left(\frac{3}{5}\right)^3$

$M_1 = \frac{27}{125} \times 25 = 5.4$

Answer 5.4 g [2]

19 **(a)** Express $-x^2-6x-7$ in the form $-(x+a)^2+b$, [1]

Answer:

$$-x^2-6x-7 = -[x^2+6x+7]$$
$$= -[(x+3)^2-9+7]$$
$$= -[(x+3)^2-2]$$
$$= -(x+3)^2+2$$

(b) hence, solve $-x^2-6x-7=0$, showing your working clearly.
Give your answers correct to two decimal places.

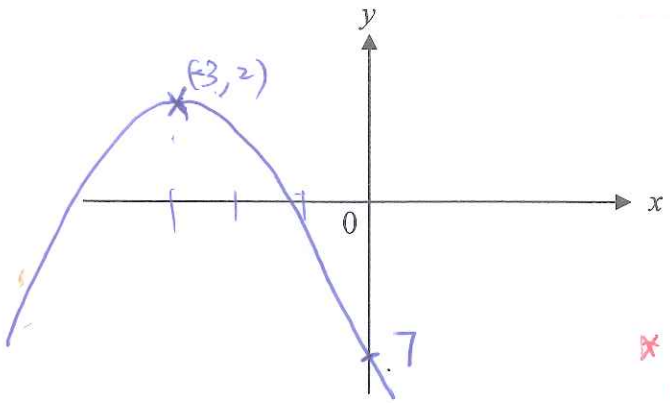
$-(x+3)^2+2=0$ $\quad x=-1.59$ or -4.41

$(x+3)^2=2$

$x+3=\pm\sqrt{2}$

Answer $x=$ -1.59 *or* -4.41 [2]

(c) Sketch the graph of $y=-x^2-6x-7$,



[2]

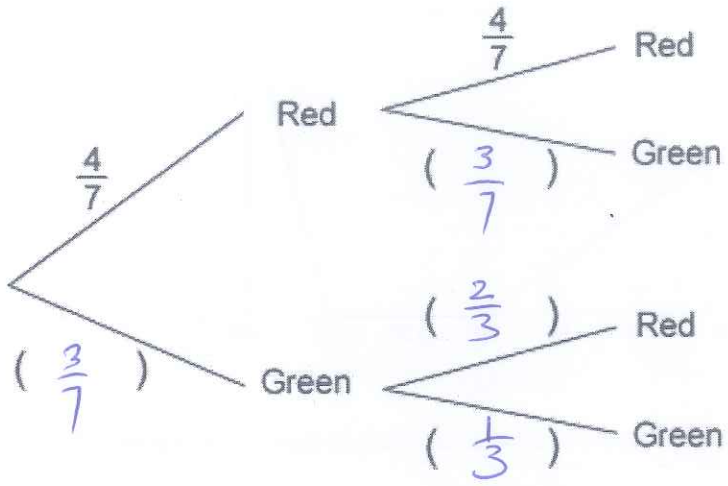
* Need to show both roots are negative
* Need to show y-intercept of -7.

(d) With reference to graph drawn above, explain why there is no solution for the equation $-x^2-6x-7=3$.

Inserting the graph $y=3$ will not intercept as max $y=2$. the graph $y=-x^2-6x-7$. Therefore, there is no solution. [1]

20 A box contains 4 red balls and 3 green balls. One ball is drawn at random. If a green ball is drawn, it will **not** be replaced, a second ball is then drawn. If a red ball is drawn, it will be replaced, a second ball is then drawn from the box.
Complete the probability tree diagram to show the probabilities of possible outcomes.

Answer [1]



Find the probability that

(i) the two balls are of different colours,

$P(R,G) + P(G,R)$

$= \frac{4}{7} \times \frac{3}{7} + \frac{3}{7} \times \frac{2}{3}$

$= \frac{26}{49}$

Answer $\frac{26}{49}$ [2]

(ii) at least 2 green balls are left in the box after the second draw.

$1 - P(G,G) = 1 - \frac{3}{7} \times \frac{1}{3}$

$= \frac{6}{7}$

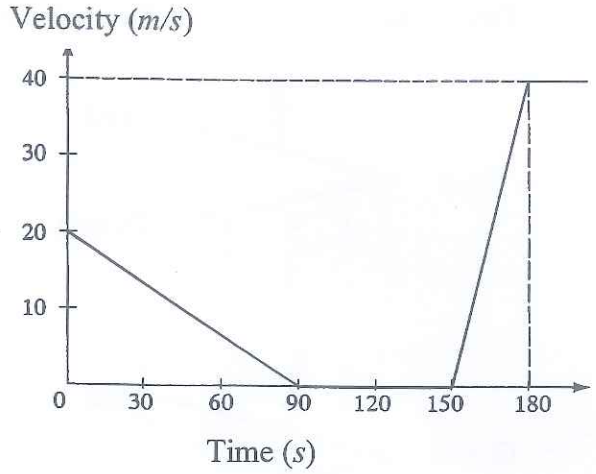
OR $P(R,R) + P(R,G) + P(G,R)$

$= \frac{4}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{3}{7} + \frac{3}{7} \times \frac{1}{3}$

$= \frac{6}{7}$

Answer $\frac{6}{7}$ [2]

21 A train slows down to a stop on entering a station P as shown in the velocity-time graph. After a brief stop of 60 s, it starts to move off with an acceleration of $1\frac{1}{3}$ m/s^2 for 30 s before it gets out of station P. It then continues its journey with this velocity until it reaches another station Q.



(a) Find the deceleration of the train when it enters the station P.

Deceleration $= \frac{20}{90}$

$= \frac{2}{9}$ m/s

Answer $\frac{2}{9}$ m/s^2 [1]

(b) Calculate the total distance travelled by the train in its first 3 minutes journey.

Total Distance travelled

$= \frac{1}{2} \times 90 \times 20 + \frac{1}{2} \times 30 \times 40$

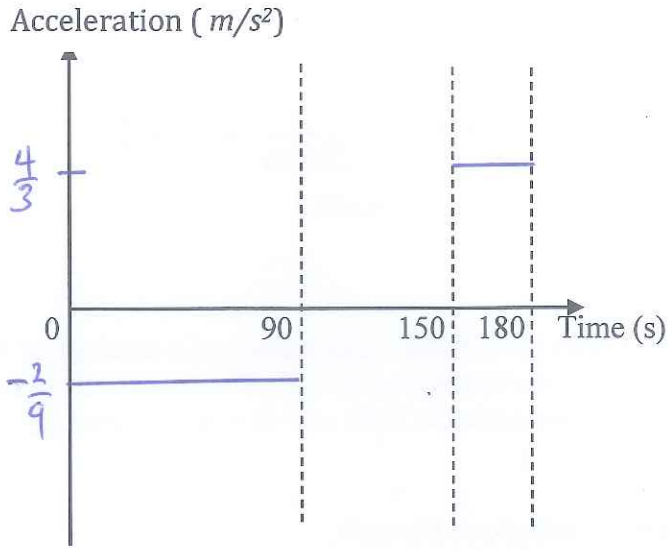
$= 900 + 600$

$= 1500$ m

Answer 1500 m [1]

(c) On the axes below, sketch an acceleration-time graph of the train for the whole 3 minutes of its motion.

[1]



22

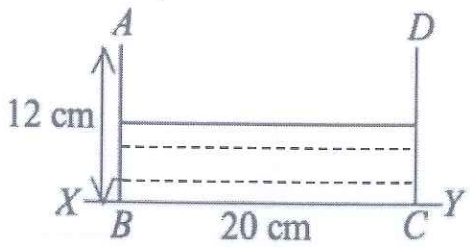


Figure 1

Figure 1 shows a vertical cross-section of a rectangular tank that stands on a horizontal table represented by XY. The tank is 12 cm high and has a square base of side 20 cm and contains 3000 cm^3 of water. Calculate

(a) **(i)** the volume of the tank,

$20 \times 20 \times 12 = 4800$

Answer 4800 cm^3 [1]

(ii) the depth of the water.

$20 \times 20 \times d = 3000$

$d = \frac{3000}{20 \times 20}$

$= 7.5$

Answer 7.5 cm

The tank is now tilted about a base edge through C, so that some of the water spills out until the position shown in Figure 2 below. Calculate

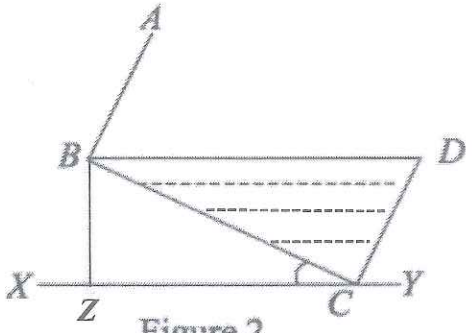


Figure 2

(b) **(i)** the volume of water remaining in the tank,

$\frac{1}{2} \times 12 \times 20 \times 20 = 2400\ cm^3$

Answer 2400 cm^3 [1]

(ii) angle BCZ,

$\tan \angle BCZ = \frac{12}{20}$

$\angle BCZ = 30.9638 \approx 31.0$

Answer 31.0 ° [2]

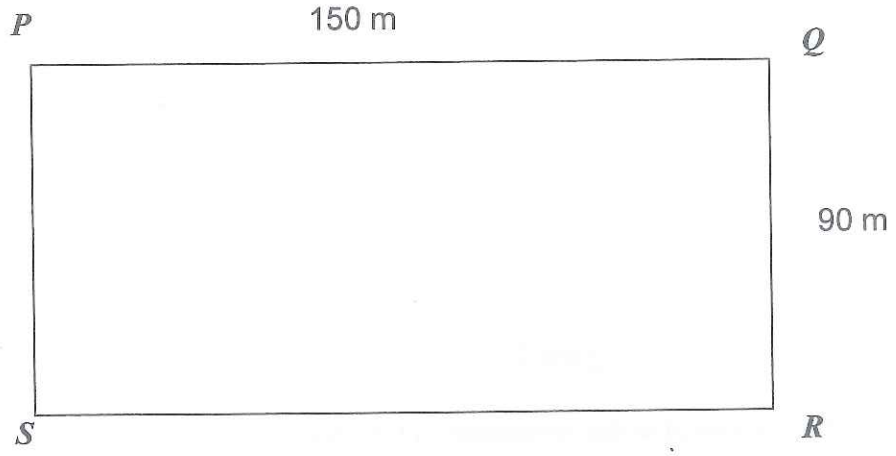
(iii) BZ, the vertical height of B above the table where Z is the foot of the perpendicular of B to YZ.

$\sin 30.9638 = \frac{BZ}{20}$

$= 10.3\ cm$

Answer 10.3 cm [1]

23



The diagram shows a rectangular field where $PQ = 150$ m and $QR = 90$ m.
Jason starts from P and walks towards Q at a constant speed of 1.5 m/s.
At the same time, another man, John starts from Q and walks towards R at a constant speed of 0.5 m/s.

(a) Write down in terms of t (where $t < 100$s)

(i) the distance of Jason from Q after t seconds,

$150 - 1.5t$

Answer $150 - 1.5t$ .....m [1]

(ii) the distance of John from Q after t seconds.

$0.5t$

Answer $0.5t$ .....m [1]

(b) Given that after t seconds, the two men are h m apart, show that

$$h^2 = 2.5t^2 - 450t + 22500$$

Answer [2]

$h^2 = (150 - 1.5t)^2 + (0.5t)^2$
$= 22500 - 450t + 2.25t^2 + 0.25t^2$
$= 22500 - 450t + 2.5t^2$
$= 2.5t^2 - 450t + 22500$ (shown)

(c) Find the distance between the two men one minute after the start.

$h^2 = 2.5(60)^2 - 450(60) + 22500$

$= 4500$

$\therefore h = 67.1$

Answer ..... 67.1 .....m [2]

(d) Find the value of t when the two men are 100 m apart

$100^2 = 2.5t^2 - 450t + 22500$

$2.5t^2 - 450t + 12500 = 0$

$t = \dfrac{450 \pm \sqrt{450^2 - 4(2.5)(12500)}}{2(2.5)}$

$= 145.68,\ 34.3224$

(Rej)

Answer ..... 34.3 .....s [2]

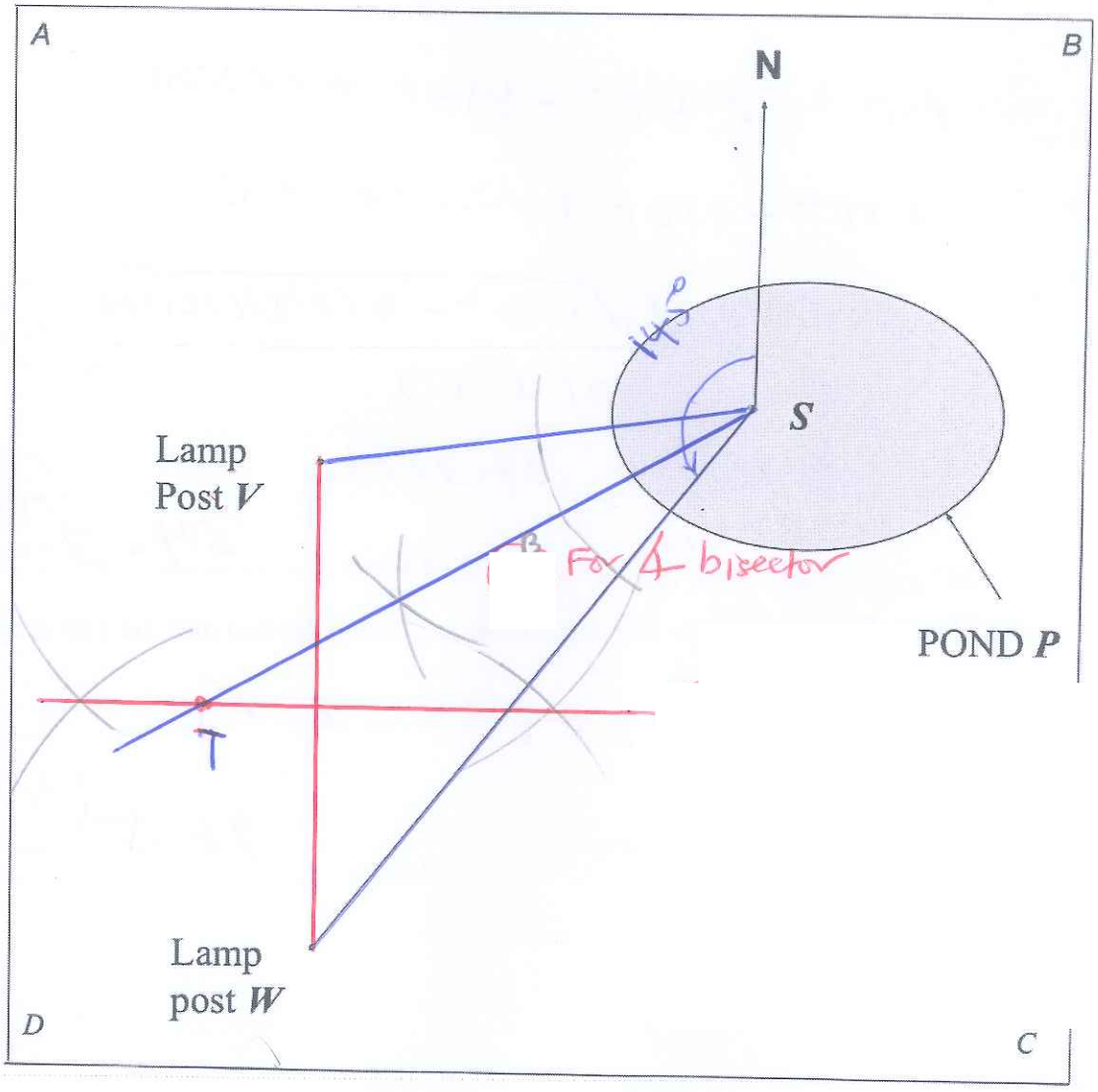
(e) Hence, find the distance that Jason is from Q when the two men are 100 m apart.

$150 - 1.5(34.3224)$

$= 98.5\ m$

Answer ..... 98.5 .....m [1]

24 **(a)** The diagram shows a plot of land $ABCD$ with a pond P at a corner and two lamp-posts V and W. S is the fixed position of a stick placed in the pond.
A gardener wants to plant a tree T equidistant from the two lamp posts V and W and also equidistant from the lines VS and WS. By appropriate constructions, mark the point T on the diagram below.



(b) Measure the bearing of W from S.

Bearing of W from S = 360° − 145°
= 218°

Answer 217° ± 1 ° [3]

Accept 217° ± 1°

End of Paper

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION

MATHEMATICS
Paper 2 4048/02

11 SEPTEMBER 2018 (Tuesday) **2 hours 30 minutes**

CANDIDATE NAME

CLASS

INDEX NUMBER

READ THESE INSTRUCTIONS FIRST

Do not turn over the page until you are told to do so.

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

INFORMATION FOR CANDIDATES

Answer **all** the questions.

Write your answers on the separate writing paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The use of a scientific calculator is expected, where appropriate.

If working is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

For Examiner's Use		
Q1	9	
Q2	8	
Q3	6	
Q4	7	
Q5	10	
Q6	11	
Q7	9	
Q8	11	
Q9	13	
Q10	16	
Total	/ 100	

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab \sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1. **(a)** The following table shows some information on the population of a country in 2016 and 2017. The total population of the country consists of residents and non-residents.

Year	Total population	Residents	Non-residents
2016	55.9 million	x	y
2017	56.1 million	54.45 million	1.65 million

[1 million people = 1×10^6 people]

From 2016 to 2017, the number of non-residents decreased by 1.6%.

(i) Find the value of y, expressing your answer as k million, where k is a constant correct to 3 decimal places. **[1]**

(ii) Hence, find the value of x, giving your answer in standard form correct to 2 decimal places. **[2]**

(b) P is proportional to Q^n, where n is an integer.

State the value of n when

(i) P units is the force between two particles which is inversely proportional to the square of the distance Q mm between them, **[1]**

(ii) P m^3 is the volume of a sphere with radius Q m, **[1]**

(iii) P cm^3 is the volume of a cone with radius Q cm and a fixed height. **[1]**

(c) Express as a single fraction in its simplest form

$$\frac{2}{2c-b}-\frac{3c-11b}{5b^2-20c^2}.$$ **[3]**

2. Jenny bought some jars of cookies for \$900. She paid \$$n$ for each jar of cookies.

(a) Write down an expression, in terms of n, for the number of jars of cookies she bought. **[1]**

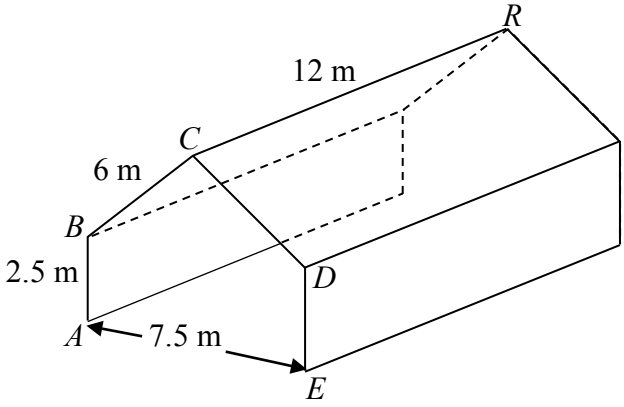
(b) Jenny found that 2 jars of cookies were spoilt and could not be sold. Jenny sold each remaining jar of cookies for \$3 more than she paid for it. Write down an expression, in terms of n, for the total sum of money she received from the sale of the jars of cookies. **[1]**

(c) Given that she made a profit of \$92 from the sale of the jars of cookies, form an equation in n and show that it reduces to $n^2+49n-1350=0$. **[2]**

(d) Solve $n^2+49n-1350=0$, giving your solutions correct to 3 decimal places. **[2]**

(e) Hence, find the selling price of each jar of cookies sold by Jenny, giving your solution correct to the nearest cent. **[2]**

3. The diagram shows a tent. The cross-section of the tent forms a pentagon *ABCDE* with two vertical sides of height 2.5 m and two slant sides of equal length 6 m. It is also given that the length of the tent *CR* is 12 m and the width of the tent *AE* is 7.5 m.



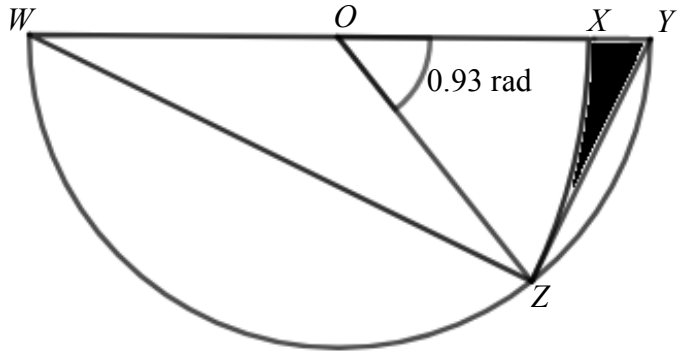
Find

(a) the area of the cross section *ABCDE*, **[2]**

(b) the angle of elevation of *R* from *A*. **[4]**

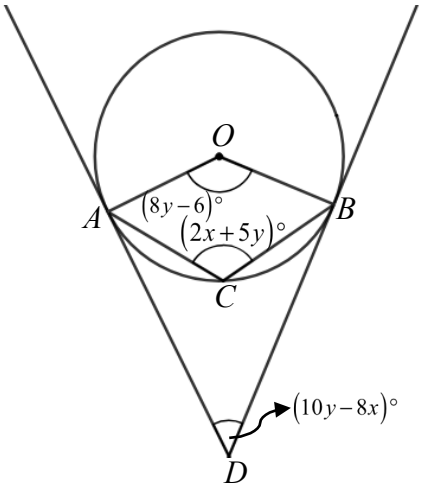
4. **(a)** Explain, with mathematical calculations, why it is not possible to fold a sector of area 115 cm^2 into a cone of base radius 8 cm. **[3]**

(b) In the diagram below, *WZY* is a semicircle with centre *O*, radius 7 cm and angle *ZOX* = 0.93 radians. *WZX* is a sector of another circle with centre *W* and radius 12.5 cm.



Find the perimeter of the shaded area. **[4]**

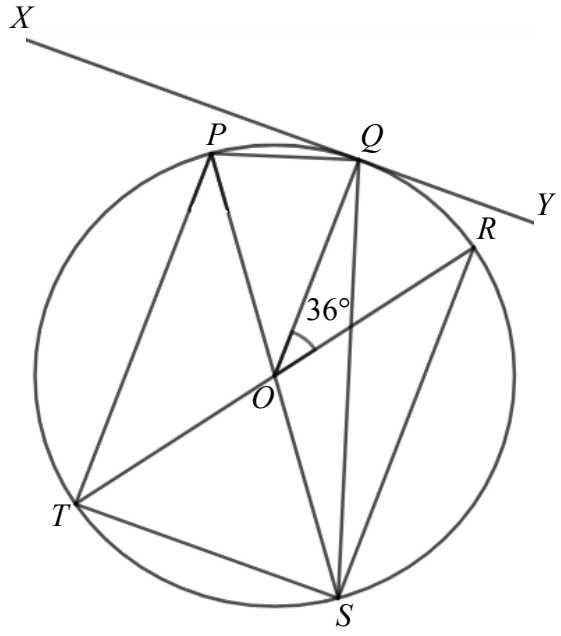
5. **(a)** In the diagram, A, B and C lie on a circle with centre O.
The tangents at A and B meet at D.
It is given that angle $AOB = (8y-6)°$, angle $ACB = (2x+5y)°$ and angle $ADB = (10y-8x)°$.



(i) Stating your reasons clearly, show that
$(8y-6)° + (10y-8x)° = 180°$ **[3]**

(ii) Hence, by solving a pair of simultaneous equations, find the value of x and of y. **[3]**

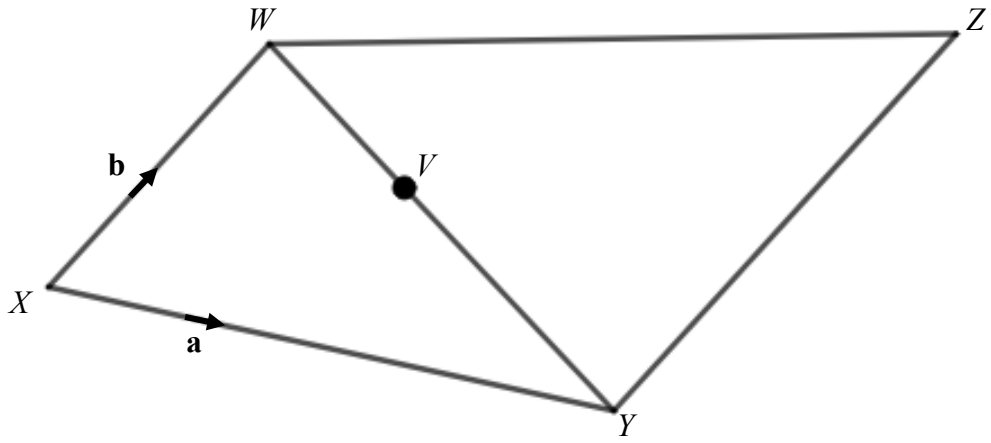
(b) The diagram shows a circle $PQRST$, with centre O. XY is a tangent to the circle at Q. It is given that angle $QOR = 36°$. TOR and POS are straight lines.



(i) Find, giving reasons for your answer, angle RSQ. **[1]**

(ii) Prove that triangle PTS and triangle RST are congruent. **[3]**

6. In the diagram, $WXYZ$ is a quadrilateral such that $\overrightarrow{XY} = \mathbf{a}$, $\overrightarrow{XW} = \mathbf{b}$ and $\overrightarrow{XW} = \frac{2}{3}\overrightarrow{YZ}$. V is a point on WY such that $5\overrightarrow{VY} = 3\overrightarrow{WY}$.



(a) What is the special name given to the quadrilateral $WXYZ$? **[1]**

(b) Express, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{WY}$, **[1]**

(ii) $\overrightarrow{XZ}$, **[1]**

(iii) $\overrightarrow{XV}$. **[2]**

(c) Explain why X, V and Z lie on a straight line. **[2]**

(d) Prove that triangle XWV and triangle ZYV are similar. **[2]**

(e) Find

(i) $\dfrac{\text{Area of triangle } XWV}{\text{Area of triangle } ZYV}$, **[1]**

(ii) $\dfrac{\text{Area of triangle } ZWV}{\text{Area of triangle } ZYV}$. **[1]**

7. **(a)** The cash price of a waffle maker is \$149.

Rose wants to start a waffle shop business and buys 5 waffle makers on hire purchase. She pays a deposit of 15% of the cash price followed by 24 equal monthly instalments with interest charged at a flat rate of 1.5% per annum.

Calculate the amount of the monthly instalment, correct to the nearest cent. **[2]**

(b) Rose offers three types of waffle fillings at her shop: chocolate, cheese and blueberry.

The price of each type of waffle is shown in the table below.

Chocolate	Cheese	Blueberry
\$1.80	\$2.50	\$1.50

The table below shows the sale of waffles at Rose's shop for the months of June and July.

Month/ Fillings	Chocolate	Cheese	Blueberry
June	52	8	27
July	48	13	21

(i) Represent the prices of each type of waffle in a column matrix **P**. **[1]**

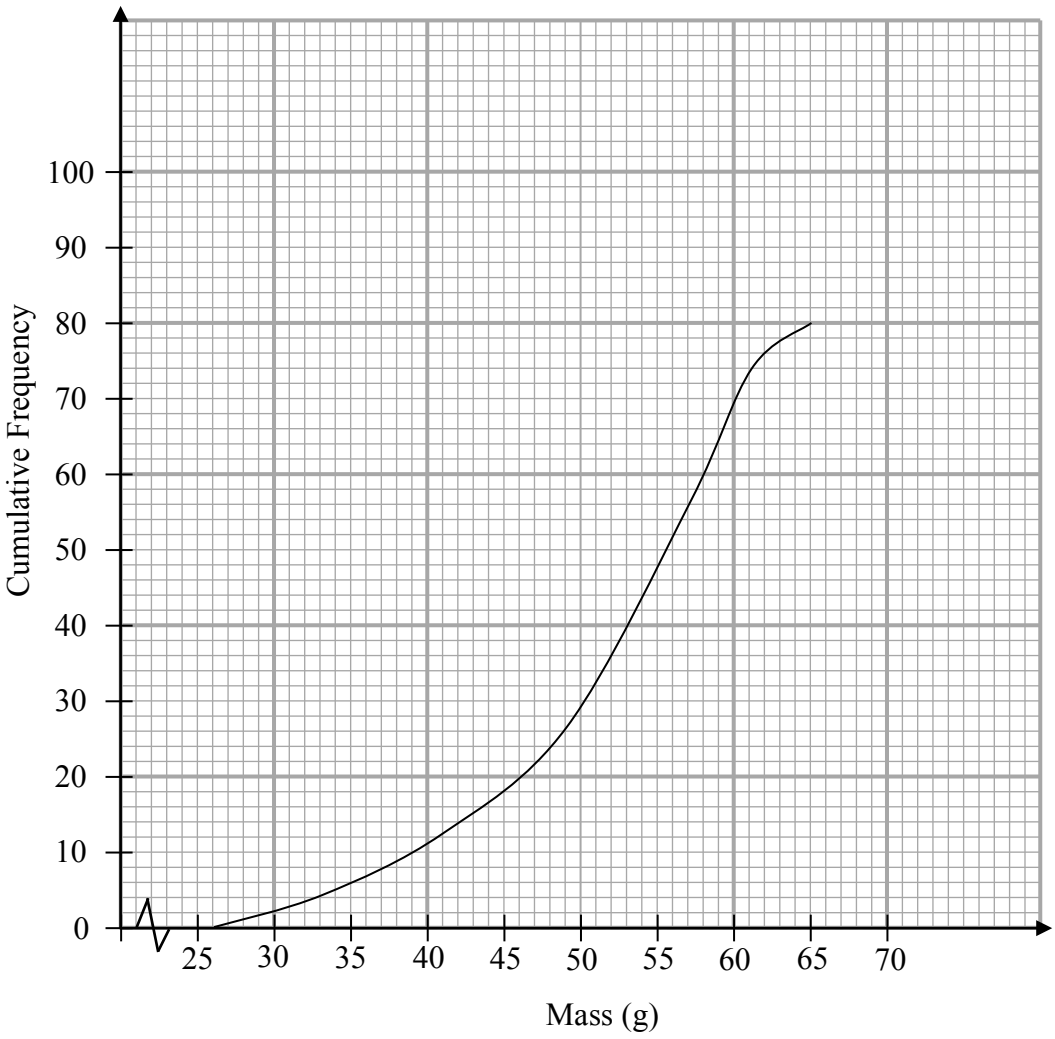
(ii) Represent the sale of waffles at Rose's shop for the months of June and July in a 2×3 matrix **W**. **[1]**

(iii) Evaluate the matrix **R** = **WP**. **[2]**

(iv) State what the elements of **R** represent. **[1]**

(v) By multiplying matrix **W** with a row matrix, find the matrix that represents the total number of each type of waffles Rose sold in June and July. **[2]**

8. The masses of 80 eggs collected at Farm A are recorded.
The cumulative frequency curve below shows the distribution of their masses.



(a) Use the curve to estimate

(i) the median mass, **[1]**

(ii) the 30^{th} percentile, **[1]**

(iii) the interquartile range. **[2]**

(b) The distribution of the masses of the eggs can be represented by the grouped frequency table below.

Mass (x g)	$25 < x \leq 35$	$35 < x \leq 45$	$45 < x \leq 55$	$55 < x \leq 65$
Frequency	6	m	30	32

(i) Show that the value of m is 12. **[1]**

(ii) A worker in Farm A select two eggs at random, one after another, without replacement.
Find, as a fraction in its simplest form, the probability that both eggs are more than 55g. **[2]**

8 **(c)** The masses of eggs at Farm B are also measured and recorded.
Information relating to the masses of eggs at Farm B are given below.

Mean = 53g
Standard Deviation = 9g

(i) A worker at Farm B says to a worker at Farm A:

"The masses of the eggs at my farm are more consistent than the masses of the eggs at your farm."

Do you agree with the worker at Farm B? Explain with mathematical calculations. **[2]**

(ii) The worker at Farm B realises that the weighing machine is spoilt. Hence the mass of each egg should be 1g more than the measured mass.
State the correct mean and standard deviation of the masses of eggs at Farm B. **[2]**

9. Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = \frac{x}{4}\left(8 - 6x + x^2\right).$$

Some corresponding values of x and y, correct to 2 decimal places, are given in the table below.

x	–1	0	0.5	1	1.5	2	2.5	3	5
y	–3.75	0	k	0.75	0.47	0	–0.47	–0.75	3.75

(a) Calculate the value of k. **[1]**

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-1 \le x \le 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $-4 \le y \le 4$.
On your axes, plot the points given in the table and join them with a smooth curve. **[3]**

(c) The equation $x^3 - 6x^2 + 8x = -12$ has only one solution.
Explain how this can be seen from your graph. **[2]**

(d) By drawing a tangent, find the gradient of the curve at the point (1, 0.75). **[2]**

(e) **(i)** Line L has gradient – 0.5 and passes through the point (3, 1).
Draw line L on the same axes for $-1 \le x \le 5$. **[2]**

(ii) Write down the x-coordinate of the point where the two graphs intersect. **[1]**

(iii) This value of x is a solution of a cubic equation. Write down the cubic equation in the form $x^3 + Px^2 + Qx + R = 0$, where P, Q and R are integers. **[2]**

10. A soda can may be modelled as a cylinder with a closed top and a hollow hemisphere hollowed in at the base of the can as shown in the diagram below.

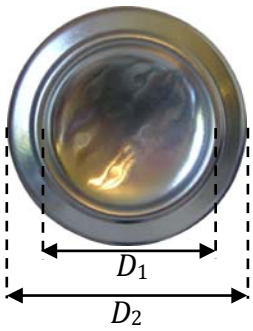
Information about the model of the soda can is given below.

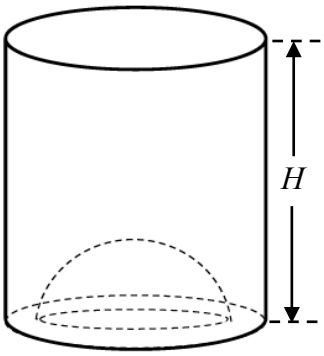
Height (H) = 12.4 cm

Inner Diameter (D_1) of base = 6.7 cm

Outer diameter (D_2) of base = 7.9 cm

Mass of empty can = 15g





(a) Using the model of the soda can in the diagram above, calculate

(i) the total surface area, in square centimetres, of the soda can. **[5]**

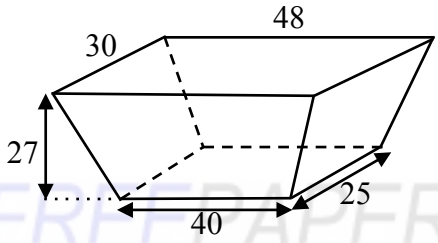
(ii) the volume, in cubic centimetres, of the soda can. **[3]**

10 (b) Harry uses a shopping basket to transport the soda cans filled with carbonated drink.

The soda cans will be placed with the base of the can lying on the base of the basket then stacked up vertically within the basket.

For safety reasons, all the soda cans must be contained inside the shopping basket. The maximum load that the shopping basket can carry is 55 pounds.

The shopping basket can be modelled by a frustrum of a inverted pyramid as shown in the diagram below.



The frustrum above is obtained by removing the top portion of an inverted right rectangular pyramid. The flat rectangular base of the frustum has length 40 cm and width 25 cm. The remaining vertical height is 27 cm. The flat rectangular top of the frustum is 48 cm by 30 cm.

Other Useful Information

- Density of carbonated drink = 1.3 g/cm^3
- 1 pound is equivalent to 0.45 kg
- Mass = Volume × Density
- Safety information: Soda can is filled with carbonated drink up to a maximum of 90% of its total volume.

Assuming that each soda can is filled with carbonated drink to the maximum safe volume, find the maximum number of soda cans Harry can transport with the shopping basket at any one time.
Justify your answer with mathematical calculations. **[8]**

END OF PAPER 2

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION

MATHEMATICS
Paper 2 4048/02

11 SEPTEMBER 2018 (Tuesday) **2 hours 30 minutes**

CANDIDATE NAME **Solutions (for students)**

CLASS

INDEX NUMBER

READ THESE INSTRUCTIONS FIRST

Do not turn over the page until you are told to do so.

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

INFORMATION FOR CANDIDATES

Answer **all** the questions.

Write your answers on the separate writing paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The use of a scientific calculator is expected, where appropriate.

If working is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

For Examiner's Use		
Q1	9	
Q2	8	
Q3	6	
Q4	7	
Q5	10	
Q6	11	
Q7	9	
Q8	11	
Q9	13	
Q10	16	
Total	/ 100	

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab \sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1. **[9]**

(a) The following table shows some information on the population of a country in 2016 and 2017. The total population of the country consists of residents and non-residents.

Year	Total population	Residents	Non-residents
2016	55.9 million	x	y
2017	56.1 million	54.45 million	1.65 million

[1 million people = 1×10^6 people]

From 2016 to 2017, the number of non-residents decreased by 1.6%.

(i) Find the value of y, expressing your answer as k million, where k is a constant correct to 3 decimal places. **[1]**

(ii) Hence, find the value of x, giving your answer in standard form correct to 2 decimal places. **[2]**

(b) P is proportional to Q^n, where n is an integer.
State the value of n when

(i) P units is the force between two particles which is inversely proportional to the square of the distance Q mm between them, **[1]**

(ii) P m^3 is the volume of a sphere with radius Q m, **[1]**

(iii) P cm^3 is the volume of a cone with radius Q cm and a fixed height. **[1]**

(c) Express as a single fraction in its simplest form

$$\frac{2}{2c-b}-\frac{3c-11b}{5b^2-20c^2}.$$

[3]

Solution:

(a)(i)

$$y=\frac{1.65\text{million}}{100-1.6}\times100$$

$$y=1.677\text{million (3dp)}$$

(a)(ii)

$$x=55.9\text{million}-1.677\text{million}$$

$$x=54.22317073\text{million}$$

$$x=54.22317073\times10^6$$

$$x=5.422317073\times10^7$$

$$x=5.42\times10^7\text{(2dp)}$$

(b)(i)

P is proportional to Q^n,

P units is the force between two particles which is inversely proportional to the square of the distance Q mm between them

$n=-2$

(b)(ii) P is proportional to Q^n, P m^3 is the volume of a sphere with radius Q m $n = 3$	
(b)(iii) P is proportional to Q^n, P cm^3 is the volume of a cone with radius Q cm and a fixed height. $n = 2$	
(c) $\frac{2}{2c-b} - \frac{3c-11b}{5b^2-20c^2}$ $= \frac{2}{2c-b} - \frac{3c-11b}{5\left(b^2-4c^2\right)}$ $= -\frac{2}{b-2c} - \frac{3c-11b}{5\left(b+2c\right)\left(b-2c\right)}$ $= \frac{-2(5)\left(b+2c\right)-\left(3c-11b\right)}{5\left(b+2c\right)\left(b-2c\right)}$ $= \frac{-10b-20c-3c+11b}{5\left(b+2c\right)\left(b-2c\right)}$ $= \frac{b-23c}{5\left(b+2c\right)\left(b-2c\right)}$	
OR	

$\frac{2}{2c-b}-\frac{3c-11b}{5b^2-20c^2}$ $=\frac{2}{2c-b}-\frac{3c-11b}{5\left(b^2-4c^2\right)}$ $=\frac{2}{2c-b}-\frac{3c-11b}{5\left(b+2c\right)\left(b-2c\right)}$ $=\frac{2(5)\left(b+2c\right)\left(b-2c\right)-\left(3c-11b\right)\left(2c-b\right)}{5\left(b+2c\right)\left(b-2c\right)\left(2c-b\right)}$ $=\frac{-b^2+25bc-46c^2}{5\left(b+2c\right)\left(b-2c\right)\left(2c-b\right)}$ $=\frac{\left(2c-b\right)\left(b-23c\right)}{5\left(b+2c\right)\left(b-2c\right)\left(2c-b\right)}$ $=\frac{b-23c}{5\left(b+2c\right)\left(b-2c\right)}$	
OR	
$\frac{2}{2c-b}-\frac{3c-11b}{5b^2-20c^2}$ $=\frac{2\left(5b^2-20c^2\right)-\left(2c-b\right)\left(3c-11b\right)}{\left(2c-b\right)\left(5b^2-20c^2\right)}$ $=\frac{-b^2+25bc-46c^2}{\left(2c-b\right)\left(5b^2-20c^2\right)}$ $=\frac{\left(2c-b\right)\left(b-23c\right)}{\left(2c-b\right)(5)\left(b+2c\right)\left(b-2c\right)}$ $=\frac{b-23c}{5\left(b+2c\right)\left(b-2c\right)}$	

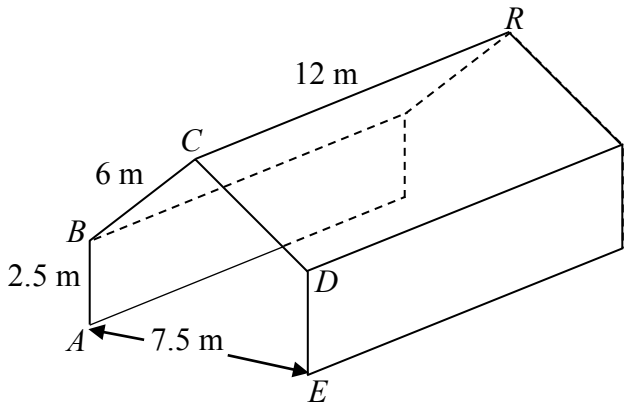
2. [8]	Jenny bought some jars of cookies for \$900. She paid \$$n$ for each jar of cookies.		
	(a)	Write down an expression, in terms of n, for the number of jars of cookies she bought.	**[1]**
	(b)	Jenny found that 2 jars of cookies were spoilt and could not be sold. Jenny sold each remaining jar of cookies for \$3 more than she paid for it. Write down an expression, in terms of n, for the total sum of money she received from the sale of the jars of cookies.	**[1]**
	(c)	Given that she made a profit of \$92 from the sale of the jars of cookies, form an equation in n and show that it reduces to $n^2 + 49n - 1350 = 0$.	**[2]**
	(d)	Solve $n^2 + 49n - 1350 = 0$, giving your solutions correct to 3 decimal places.	**[2]**
	(e)	Hence, find the selling price of each jar of cookies sold by Jenny, giving your solution correct to the nearest cent.	**[2]**

Remark: Deduct U for missing/ incorrect units in any part for this question where applicable

Solution: **(a)** $\left(\frac{900}{n}\right)$ jars of cookies	
(b) $\$\left(\frac{900}{n} - 2\right)(n+3)$ **OR** $\$\left(900 + \frac{2700}{n} - 2n - 6\right)$	**Remark:** Remember to write the correct units
(c) $\left(\frac{900}{n} - 2\right)(n+3) - 900 = 92$ $900 + \frac{2700}{n} - 2n - 6 - 900 = 92$ $\frac{2700}{n} - 2n - 6 = 92$ $\frac{2700}{n} - 2n - 98 = 0$ $2700 - 2n^2 - 98n = 0$ $n^2 + 49n - 1350 = 0$ (Shown)	

(d) $n^2 + 49n - 1350 = 0$ $n = \dfrac{-49 \pm \sqrt{(49)^2 - 4(1)(-1350)}}{2(1)}$ $n = 19.662$ (3dp) or $n = -68.662$ (3dp)	**Remark:** Must show working
(e) (Reject $n = -68.662$ as n>0) Selling price $= 19.662 + 3$ =\$22.66 (nearest cent)	

3. [6] The diagram shows a tent. The cross-section of the tent forms a pentagon *ABCDE* with two vertical sides of height 2.5 m and two slant sides of equal length 6 m. It is also given that the length of the tent *CR* is 12 m and the width of the tent *AE* is 7.5 m.



Find

(a) the area of the cross section *ABCDE*, **[2]**

(b) the angle of elevation of *R* from *A*. **[4]**

Solution:

(a)

$$\text{height of triangle BCD} = \sqrt{6^2 - \left(\frac{7.5}{2}\right)^2} = \sqrt{\frac{351}{16}}$$

Area of the cross section *ABCDE*

$$= \left(\frac{1}{2} \times 7.5 \times \sqrt{\frac{351}{16}}\right) + (7.5 \times 2.5)$$

$$= 17.56405687 + 18.75$$

$$= 36.31405687$$

$$= 36.3 \text{m}^2 \text{(3sf)}$$

Remark: Remember to write the correct units

OR

$$7.5^2 = 6^2 + 6^2 - 2(6)(6)\cos\angle BCD$$

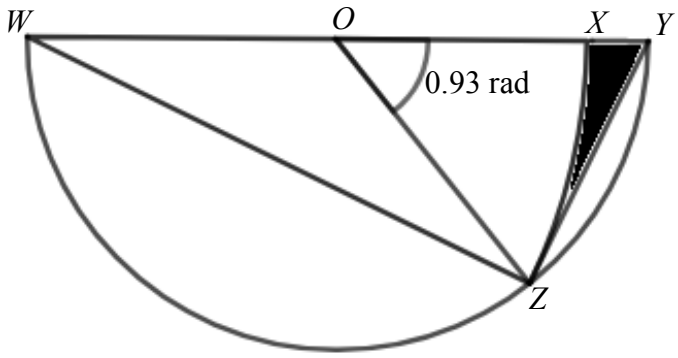
$$\cos\angle BCD = \frac{6^2 + 6 - 7.5^2}{2(6)(6)}$$

$$\angle BCD = \cos^{-1}\frac{6^2 + 6 - 7.5^2}{2(6)(6)}$$

$$\angle BCD = 77.36437491°$$

OR

Let M is midpoint of BD. $\sin\angle BCM = \frac{(7.5 \div 2)}{6}$ $\angle BCM = \sin^{-1}\frac{(7.5 \div 2)}{6}$ $\angle BCD = 2\sin^{-1}\frac{(7.5 \div 2)}{6}$ $\angle BCD = 77.36437491°$ Area of the cross section $ABCDE$ $= \left(\frac{1}{2} \times 6 \times 6 \times \sin 77.36437491°\right) + (7.5 \times 2.5)$ $= 17.56405687 + 18.75$ $= 36.31405687$ $= 36.3\text{m}^2\text{(3sf)}$	**Remark:** Remember to write the correct units
(b) Let N be the point vertically below R on the ground $RN = \sqrt{\frac{351}{16}} + 2.5$ $= 7.183748499m$ $AN = \sqrt{12^2 + \left(\frac{7.5}{2}\right)^2}$ $= 12.57229096m$ $\tan\angle RAN = \frac{RN}{AN}$ $\tan\angle RAN = \frac{7.183748499}{12.57229096}$ $\angle RAN = \tan^{-1}\frac{7.183748499}{12.57229096}$ $\angle RAN = 29.7°\text{(1dp)}$ Angle of elevation = 29.7°(1dp)	**Remark:** -Remember to write statements to define clearly what you are finding -Remember to write the correct units

4. [7]	**(a)**	Explain, with mathematical calculations, why it is not possible to fold a sector of area 115 cm^2 into a cone of base radius 8 cm.	**[3]**
	(b)	In the diagram below, *WZY* is a semicircle with centre *O*, radius 7 cm and angle *ZOX* = 0.93 radians. *WZX* is a sector of another circle with centre *W* and radius 12.5 cm.  Find the perimeter of the shaded area.	**[4]**

Solution:

(a)

$\pi(8)(\text{slant height}) = 115$

$\text{slant height} = \dfrac{115}{8\pi}$

$\text{height} = \sqrt{\left(\dfrac{115}{8\pi}\right)^2 - 8^2}$

$\left(\dfrac{115}{8\pi}\right)^2 - 8^2 = -43.06 < 0$

As height^2 is negative, height is undefined.

Hence it is not possible to fold a sector of area 115 cm^2 into a cone of base radius 8 cm.

OR

Curved surface area of cone = 115

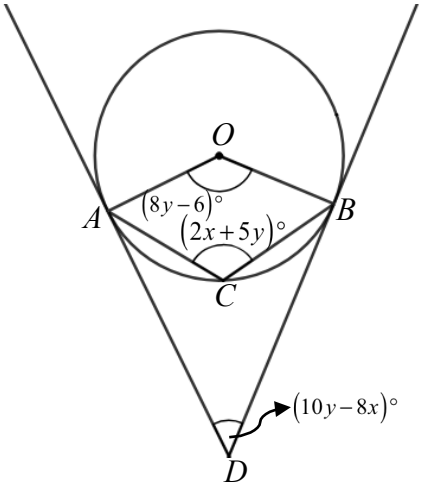
Base area of cone = $\pi(8)^2 = 201.06(2dp) > 115$

As base area of cone is greater than the curved surface area of cone, it is not possible to fold a sector of area 115 cm^2 into a cone of base radius 8 cm.

(b) $XY = 2(7) - 12.5 = 1.5$ Arc XZ = $12.5\left(\frac{0.93}{2}\right)$ $\angle WZY = 90°$(angle in a semicircle is a right angle) $\sin\left(\frac{0.93}{2}\right) = \frac{YZ}{14}$ $YZ = 14\sin\left(\frac{0.93}{2}\right)$ Perimeter $= 14\sin\left(\frac{0.93}{2}\right) + 12.5\left(\frac{0.93}{2}\right) + 1.5$ $= 13.6\text{cm(3sf)}$	**Remark:** Remember to write the correct units

5. [10]

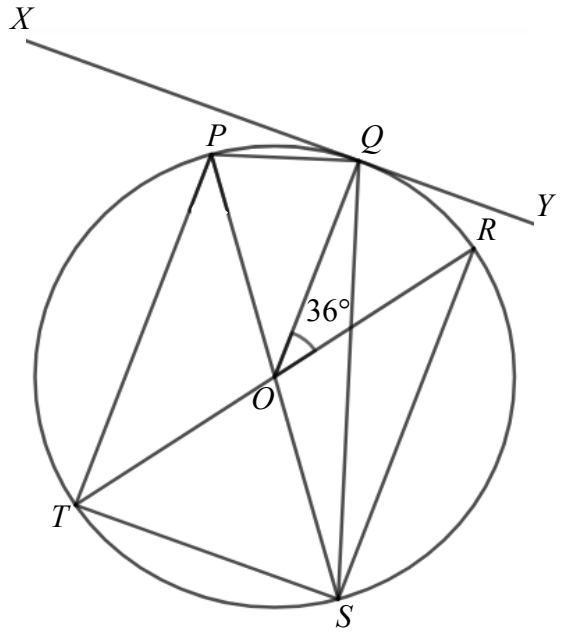
(a) In the diagram, A, B and C lie on a circle with centre O.
The tangents at A and B meet at D.
It is given that angle $AOB = (8y-6)°$, angle $ACB = (2x+5y)°$ and angle $ADB = (10y-8x)°$.



(i) Stating your reasons clearly, show that
$(8y-6)° + (10y-8x)° = 180°$ **[3]**

(ii) Hence, by solving a pair of simultaneous equations, find the value of x and of y. **[3]**

(b) The diagram shows a circle $PQRST$, with centre O. XY is a tangent to the circle at Q. It is given that angle $QOR = 36°$. TOR and POS are straight lines.



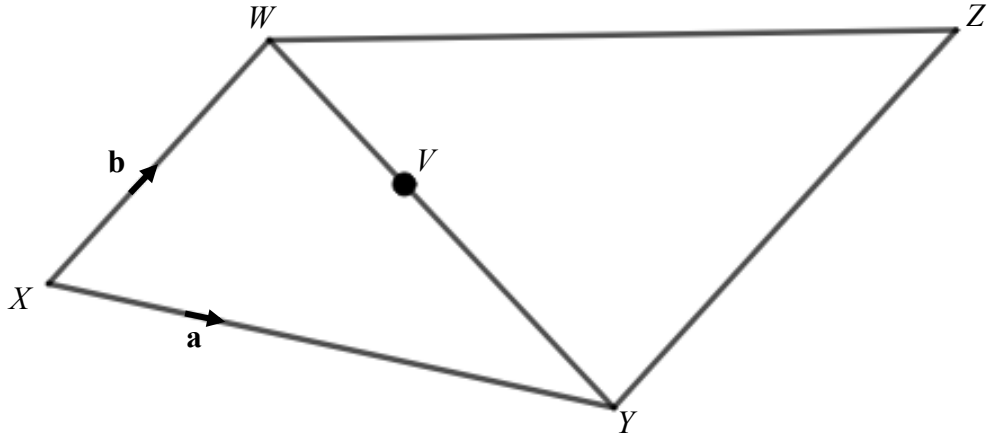
(i) Find, giving reasons for your answer, angle RSQ. **[1]**

(ii) Prove that triangle PTS and triangle RST are congruent. **[3]**

Solution: **(a)(i)** $\angle OAD = \angle OBD = 90°$ ($\angle$ between tangent and radius of a circle is a right angle) $90° + 90° + (8y - 6)° + (10y - 8x)° = 360°$ ($\angle$ sum of quadrilateral) $(8y - 6)° + (10y - 8x)° = 180°$ (Shown)	**Remark:** Remember to write the correct reasons in full, no short form
(a)(ii) $(8y - 6) + (10y - 8x) = 180$ $-8x + 18y = 186$ $-4x + 9y = 93$ -----(1) $360 - (8y - 6) = 2(2x + 5y)$ $360 - 8y + 6 = 4x + 10y$ $4x + 18y = 366$ $2x + 9y = 183$ -----(2) Equations (1) + 2(2): $-4x + 9y + 2(2x + 9y) = 93 + 2(183)$ $27y = 459$ $y = 17$ Sub y=17 into (1): $-4x + 9(17) = 93$ $-4x = -60$ $x = 15$ $\therefore x = 15, y = 17$	
(b) Remark: Remember to write the correct units	
(b)(i) $\angle RSQ = \frac{36°}{2} = 18°$ ($\angle$ at the centre is twice the $\angle$ at the circumference)	**Remark:** Remember to write the correct reasons in full, no short form

(b)(ii) $\angle PTS = \angle RST = 90°$ ($\angle$ in a semicircle is a right angle) $TS = ST$ (common length) $PS = RT$ (diameter of circle) Triangle PTS and Triangle RST are congruent (RHS)	**Remark:** -Remember to write the correct reasons in full, no short form -Remember to write the vertices in their corresponding order for all statements and conclusion -Remember to state the congruency test used
OR	
$\angle PTS = \angle RST = 90°$ ($\angle$ in a semicircle is a right angle) $\angle TPS = \angle SRT$ ($\angle$s in the same segment are equal) $\angle OTS = \angle OST$ (base angles of isosceles triangle) so $\angle PST = \angle RTS$ $TS = ST$ (common length) **OR** $PS = RT$ (diameter of circle) Triangle PTS and Triangle RST are congruent (AAS/ ASA)	**Remark:** -Remember to write the correct reasons in full, no short form -Remember to write the vertices in their corresponding order for all statements and conclusion -Remember to state the congruency test used -2 angles and 1 side sufficient for AAS or ASA congruency test
OR	
$\angle OTS = \angle OST$ (base angles of isosceles triangle) so $\angle PST = \angle RTS$ $TS = ST$ (common length) $PS = RT$ (diameter of circle) Triangle PTS and Triangle RST are congruent (SAS)	**Remark:** -Remember to write the correct reasons in full, no short form -Remember to write the vertices in their corresponding order for all statements and conclusion -Remember to state the congruency test used -For SAS congruency test, the angle must be the included angle between the 2 sides

6. **[11]** In the diagram, $WXYZ$ is a quadrilateral such that $\overrightarrow{XY} = \mathbf{a}$, $\overrightarrow{XW} = \mathbf{b}$ and $\overrightarrow{XW} = \frac{2}{3}\overrightarrow{YZ}$. V is a point on WY such that $5\overrightarrow{VY} = 3\overrightarrow{WY}$.



	(a)	What is the special name given to the quadrilateral $WXYZ$?		**[1]**
	(b)	Express, as simply as possible, in terms of **a** and **b**,		
		(i)	$\overrightarrow{WY}$,	**[1]**
		(ii)	$\overrightarrow{XZ}$,	**[1]**
		(iii)	$\overrightarrow{XV}$.	**[2]**
	(c)	Explain why X, V and Z lie on a straight line.		**[2]**
	(d)	Prove that triangle XWV and triangle ZYV are similar.		**[2]**
	(e)	Find		
		(i)	$\frac{\text{Area of triangle } XWV}{\text{Area of triangle } ZYV}$,	**[1]**
		(ii)	$\frac{\text{Area of triangle } ZWV}{\text{Area of triangle } ZYV}$.	**[1]**
	Remark: Remember to write correct vector notation for vectors			

Solution:
(a) Trapezium

(b)(i)
$\overrightarrow{WY} = \overrightarrow{WX} + \overrightarrow{XY}$
$\overrightarrow{WY} = -\mathbf{b} + \mathbf{a}$
OR
$\overrightarrow{WY} = \overrightarrow{XY} - \overrightarrow{XW}$
$\overrightarrow{WY} = \mathbf{a} - \mathbf{b}$

(b)(ii)
$\overrightarrow{XZ} = \overrightarrow{XY} + \overrightarrow{YZ}$
$\overrightarrow{XZ} = \mathbf{a} + \frac{3}{2}\mathbf{b}$

(b)(iii) $5\overrightarrow{VY} = 3\overrightarrow{WY}$ $\overrightarrow{VY} = \frac{3}{5}\overrightarrow{WY}$ $\overrightarrow{WV} = \frac{2}{5}\overrightarrow{WY}$ $\overrightarrow{XV} = \overrightarrow{XW} + \overrightarrow{WV}$ $\overrightarrow{XV} = \overrightarrow{XW} + \frac{2}{5}\overrightarrow{WY}$ $\overrightarrow{XV} = \mathbf{b} + \frac{2}{5}(\mathbf{a} - \mathbf{b})$ $\overrightarrow{XV} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ **OR** $\frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$ **OR** $\frac{2}{5}(\mathbf{a} + \frac{3}{2}\mathbf{b})$	
OR	
$\overrightarrow{XV} = \overrightarrow{XY} + \overrightarrow{YV}$ $\overrightarrow{XV} = \overrightarrow{XY} + \frac{3}{5}(-\overrightarrow{WY})$ $\overrightarrow{XV} = \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$ $\overrightarrow{XV} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ **OR** $\frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$ **OR** $\frac{2}{5}(\mathbf{a} + \frac{3}{2}\mathbf{b})$	
(c) $\overrightarrow{XZ} = \mathbf{a} + \frac{3}{2}\mathbf{b} = \frac{1}{2}(2\mathbf{a} + 3\mathbf{b})$ $\overrightarrow{XV} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$ $\frac{XV}{XZ} = \frac{\left(1/5\right)}{\left(1/2\right)} = \frac{2}{5}$ $\overrightarrow{XV} = \frac{2}{5}\overrightarrow{XZ}$ $\overrightarrow{XV}$ and $\overrightarrow{XZ}$ are parallel. X is a common point. Hence X, V and Z lie on a straight line.	**Remarks:** -Can also use vectors XV and VZ or vectors VZ and XZ. -Division of vectors is undefined

<table>
<tr><td>(d)
$\angle XVW = \angle ZVY$ (vertically opposite $\angle$s)
$\angle XWV = \angle ZYV$
(alternate $\angle$s, XW parallel to YZ)
$\angle WXV = \angle YZV$
(alternate $\angle$s, XW parallel to YZ)
Triangle XWV and Triangle ZYV are similar
(AA similarity test)</td><td rowspan="2">Remark:
-Remember to write the correct reasons in full, no short form
-Remember to write the vertices in their corresponding order for all statements and conclusion</td></tr>
<tr><td>OR
$\frac{XW}{ZY} = \frac{XV}{ZV} = \frac{WV}{YV} = \frac{2}{3}$
Triangle XWV and Triangle ZYV are similar
(SSS similarity test)</td></tr>
<tr><td colspan="2">OR
$\frac{XV}{ZV} = \frac{WV}{YV} = \frac{2}{3}$
$\angle XVW = \angle ZVY$ (vertically opposite $\angle$s)
Triangle XWV and Triangle ZYV are similar (SAS similarity test)
OR
$\frac{XW}{ZY} = \frac{WV}{YV} = \frac{2}{3}$
$\angle XWV = \angle ZYV$ (alternate $\angle$s, XW parallel to YZ)
OR
$\frac{XW}{ZY} = \frac{XV}{ZV} = \frac{2}{3}$
$\angle WXV = \angle YZV$ (alternate $\angle$s, XW parallel to YZ)
Triangle XWV and Triangle ZYV are similar (SAS similarity test)</td></tr>
<tr><td>(e)(i)
$\frac{\text{Area of triangle } XWV}{\text{Area of triangle } ZYV} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$</td><td>Remarks: Division of vectors is undefined</td></tr>
<tr><td></td><td></td></tr>
<tr><td>(e)(ii)
$\frac{\text{Area of triangle } ZWV}{\text{Area of triangle } ZYV} = \frac{\left(\frac{1}{2} \times WV \times h\right)}{\left(\frac{1}{2} \times YV \times h\right)} = \frac{2}{3}$</td><td>Remarks: Division of vectors is undefined</td></tr>
</table>

7. [9]

(a) The cash price of a waffle maker is \$149. Rose wants to start a waffle shop business and buys 5 waffle makers on hire purchase. She pays a deposit of 15% of the cash price followed by 24 equal monthly instalments with interest charged at a flat rate of 1.5% per annum. Calculate the amount of the monthly instalment, correct to the nearest cent. **[2]**

(b) Rose offers three types of waffle fillings at her shop: chocolate, cheese and blueberry.

The price of each type of waffle is shown in the table below.

Chocolate	Cheese	Blueberry
\$1.80	\$2.50	\$1.50

The table below shows the sale of waffles at Rose's shop for the months of June and July.

Month/ Fillings	Chocolate	Cheese	Blueberry
June	52	8	27
July	48	13	21

(i) Represent the prices of each type of waffle in a column matrix **P**. **[1]**

(ii) Represent the sale of waffles at Rose's shop for the months of June and July in a 2×3 matrix **W**. **[1]**

(iii) Evaluate the matrix **R** = **WP**. **[2]**

(iv) State what the elements of **R** represent. **[1]**

(v) By multiplying matrix **W** with a row matrix, find the matrix that represents the total number of each type of waffles Rose sold in June and July. **[2]**

Solution:

(a)

Balance

$$= \frac{100-15}{100} \times (149 \times 5) = \$633.25$$

Interest

$$= 633.25 \times \frac{1.5}{100} \times \frac{24}{12}$$
$$= \$18.9975$$

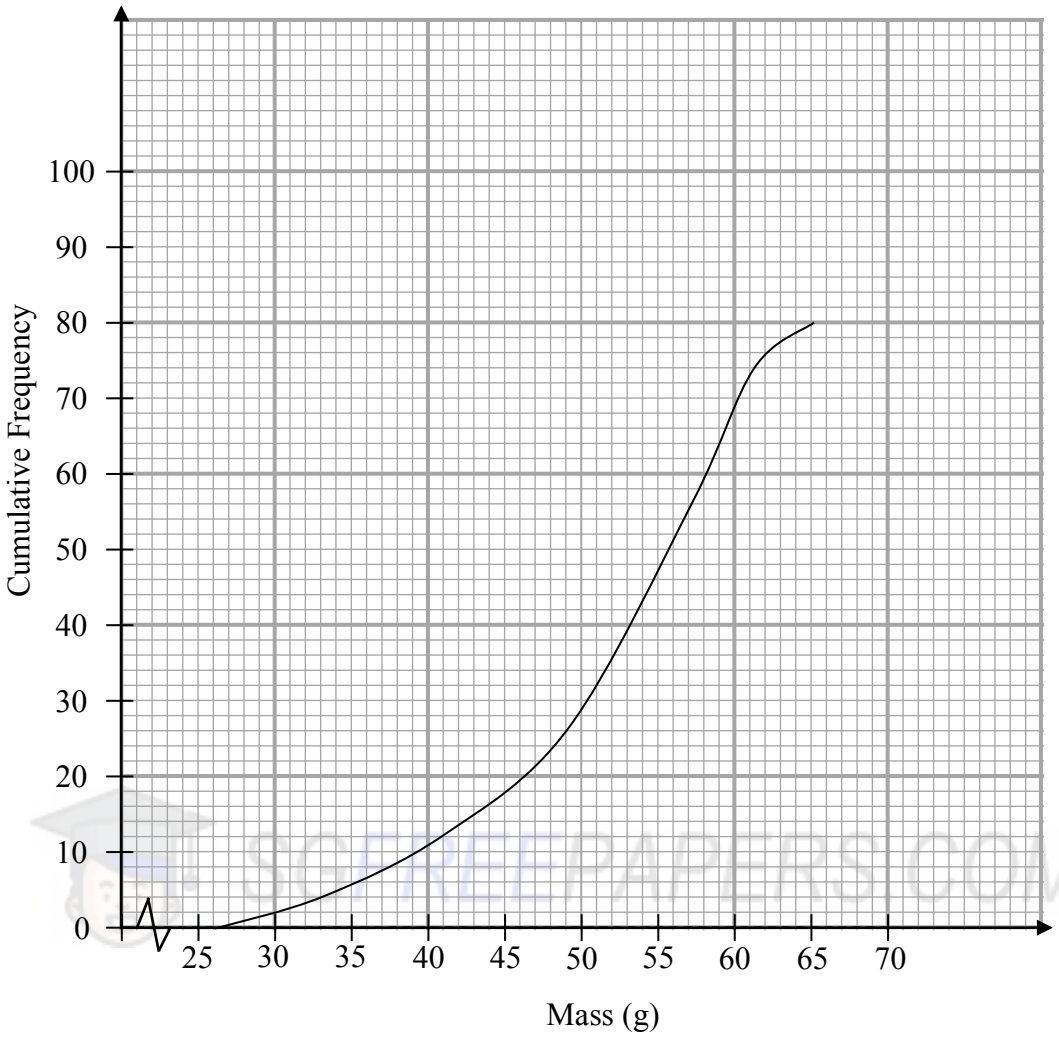
Amount of monthly instalment

$$= \frac{633.25 + 18.9975}{24}$$
$$= \$27.17697917 \text{ (8dp)}$$
$$= \$27.18 \text{ (2dp)}$$

Remark: Remember to write the correct units

(b)(i) $\mathbf{P} = \begin{pmatrix} 1.8 \\ 2.5 \\ 1.5 \end{pmatrix}$	**OR** $\begin{pmatrix} 1.80 \\ 2.50 \\ 1.50 \end{pmatrix}$
(b)(ii) $\mathbf{W} = \begin{pmatrix} 52 & 8 & 27 \\ 48 & 13 & 21 \end{pmatrix}$	
(b)(iii) $\mathbf{R} = \mathbf{WP}$ $= \begin{pmatrix} 52 & 8 & 27 \\ 48 & 13 & 21 \end{pmatrix} \begin{pmatrix} 1.8 \\ 2.5 \\ 1.5 \end{pmatrix}$ $= \begin{pmatrix} 154.1 \\ 150.4 \end{pmatrix}$	
(b)(iv) The elements of matrix **R** represent the amount collected from the sales of waffles for the months of June and July **respectively**. **OR** The elements of matrix **R** represent the amount collected from the sales of waffles for **each** month. **OR** 154.1 represents the amount collected from the sales of waffles for the month of June. 150.4 represents the amount collected from the sales of waffles for the month of July.	**Remark:** Profit means selling price – cost price, different from amount of money collected
(b)(v) $\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 52 & 8 & 27 \\ 48 & 13 & 21 \end{pmatrix}$ $= \begin{pmatrix} 100 & 21 & 48 \end{pmatrix}$	

8. [11] The masses of 80 eggs collected at Farm A are recorded.
The cumulative frequency curve below shows the distribution of their masses.



(a) Use the curve to estimate

(i) the median mass, **[1]**

(ii) the 30th percentile, **[1]**

(iii) the interquartile range. **[2]**

(b) The distribution of the masses of the eggs can be represented by the grouped frequency table below.

Mass (x g)	$25 < x \leq 35$	$35 < x \leq 45$	$45 < x \leq 55$	$55 < x \leq 65$
Frequency	6	m	30	32

(i) Show that the value of m is 12. **[1]**

(ii) A worker in Farm A select two eggs at random, one after another, without replacement.
Find, as a fraction in its lowest form, the probability that both eggs are more than 55g. **[2]**

8	**(c)**	The masses of eggs at Farm B are also measured and recorded. Information relating to the masses of eggs at Farm B are given below. Mean = 53g Standard Deviation = 9g		
		(i)	A worker at Farm B says to a worker at Farm A: "The masses of the eggs at my farm are more consistent than the masses of the eggs at your farm." Do you agree with the worker at Farm B? Explain with mathematical calculations.	**[2]**
		(ii)	The worker at Farm B realises that the weighing machine is spoilt. Hence the mass of each egg should be 1g more than the measured mass. State the correct mean and standard deviation of the masses of eggs at Farm B.	**[2]**

Remark: Deduct U for missing/ incorrect units in any part for this question where applicable

Solution: **(a)(i)** $\frac{50}{100} \times 80 = 40\text{eggs}$ Median = 53g	**Remark:** Remember to write the correct units
(a)(ii) $\frac{30}{100} \times 80 = 24\text{eggs}$ 30^{th} percentile = 48g	**Remark:** Remember to write the correct units
(a)(iii) $\frac{25}{100} \times 80 = 20\text{eggs}$ Lower quartile = 46g $\frac{75}{100} \times 80 = 60\text{eggs}$ Upper quartile = 58g Interquartile range = 58 – 46 = 12g	**Remark:** -Remember to show working -Remember to write the correct units
(b)(i) Frequency for mass, $x \le 45$ = 18 Frequency for mass, $x \le 35$ = 6 $m = 18 - 6 = 12$ (Shown) **OR** $6 + m + 30 + 32 = 80$ $m = 80 - 6 - 30 - 32 = 12$ (Shown)	

(b)(ii)
P(both eggs more than 55g)
$= \frac{32}{80} \times \frac{31}{79}$
$= \frac{62}{395}$

(c)(i)

Mid value (x)	30	40	50	60
Mass (x g)	$25 < x \leq 35$	$35 < x \leq 45$	$45 < x \leq 55$	$55 < x \leq 65$
Frequency	6	12	30	32

Mean of masses of eggs at Farm A = 51g
Standard deviation of masses of eggs at Farm A
= 9.16515139=9.17g (3sf)

The standard deviation of the masses of eggs at Farm B, 9g, is smaller than that of Farm A. Hence the masses of eggs at Farm B is more consistent than that of Farm A. So **I agree** with the worker at Farm B.

(c)(ii)
Correct Mean = 54g
Correct Standard Deviation = 9g

9. [13] **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{x}{4}\left(8 - 6x + x^2\right).$$

Some corresponding values of x and y, correct to 2 decimal places, are given in the table below.

x	–1	0	0.5	1	1.5	2	2.5	3	5
y	–3.75	0	k	0.75	0.47	0	–0.47	–0.75	3.75

(a) Calculate the value of k. **[1]**

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-1 \le x \le 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $-4 \le y \le 4$.
On your axes, plot the points given in the table and join them with a smooth curve. **[3]**

(c) The equation $x^3 - 6x^2 + 8x = -12$ has only one solution.
Explain how this can be seen from your graph. **[2]**

(d) By drawing a tangent, find the gradient of the curve at the point (1, 0.75). **[2]**

(e) **(i)** Line L has gradient – 0.5 and passes through the point (3, 1).
Draw line L on the same axes for $-1 \le x \le 5$. **[2]**

(ii) Write down the x-coordinate of the point where the two graphs intersect. **[1]**

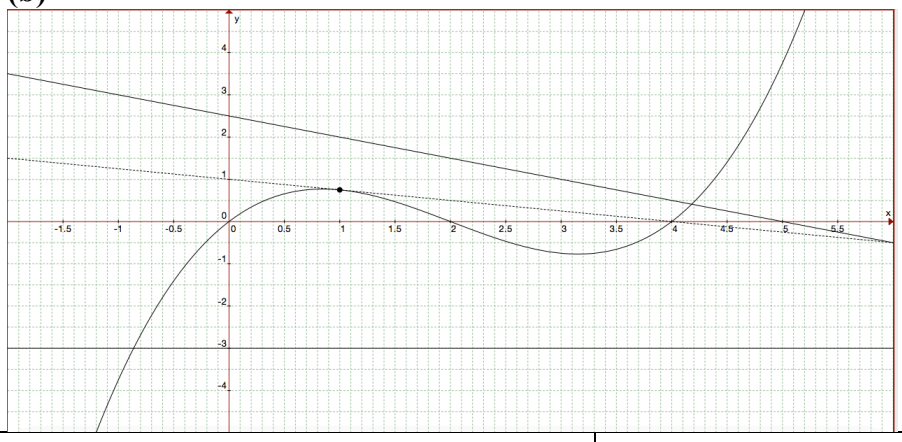
(iii) This value of x is a solution of a cubic equation. Write down the cubic equation in the form $x^3 + Px^2 + Qx + R = 0$, where P, Q and R are integers. **[2]**

Solution: [Graph attached]

(a) k = 0.66 (2dp)

Remark: Remember to write k=, not y=

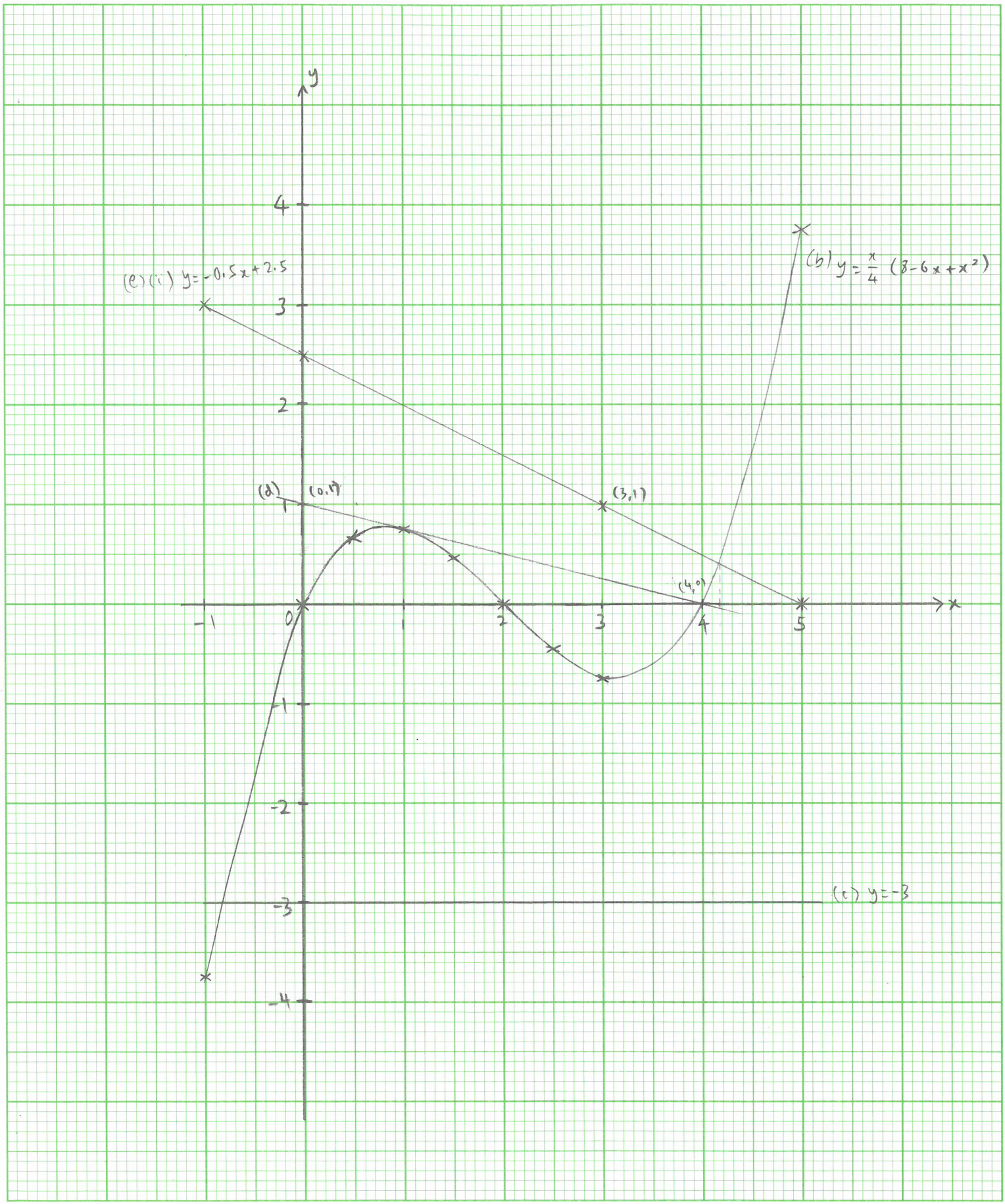
(b)



(c) $x^3 - 6x^2 + 8x = -12$ $x\left(x^2 - 6x + 8\right) = -12$ $\frac{x}{4}\left(x^2 - 6x + 8\right) = -3$ $\frac{x}{4}\left(8 - 6x + x^2\right) = -3$ $y = -3$ (Draw line $y = -3$. See graph in (b)) The line $y = -3$ cuts the curve $y = \frac{x}{4}\left(8 - 6x + x^2\right)$ at only one point. Hence $x^3 - 6x^2 + 8x = -12$ has only one solution.	
(d) Gradient = –0.25	(Accept $-0.4 \le$ gradient ≤ -0.2 (1dp))
(e)(i) $y = -\frac{1}{2}x + c$ $1 = -\frac{1}{2}(3) + c$ $c = \frac{5}{2}$ $y = -\frac{1}{2}x + \frac{5}{2}$ See graph in (b)	**Remark:** line $y = -\frac{1}{2}x + \frac{5}{2}$ must be drawn for $-1 \le x \le 5$ as defined in question: Passes through (–1, 3), (3, 1) and (5, 0)
(e)(ii) $x = 4.18$	(Accept $4.1 \le x \le 4.\mathbf{3}$)
(e)(iii) $\frac{x}{4}\left(8 - 6x + x^2\right) = -\frac{1}{2}x + \frac{5}{2}$ $x\left(8 - 6x + x^2\right) = -2x + 10$ $8x - 6x^2 + x^3 = -2x + 10$ $x^3 - 6x^2 + 10x - 10 = 0$	

Name _____ Index No. _____

Subject SST 2018 Sec 4 Prelim EMP2 Class _____ Date _____



20 cm x 24 cm

(a) $k = \frac{0.5}{4}(8 - 6(0.5) + (0.5)^2)$

$= 0.66$ (2dp)

(c) $x^3 - 6x^2 + 8x = -12$

$x(x^2 - 6x + 8) = -12$

$\frac{x}{4}(8 - 6x + x^2) = -3$

Draw line $y = -3$

$y = -3$ intersects graph $y = \frac{x}{4}(8 - 6x + x^2)$ only at 1 point.

Hence $x^3 - 6x^2 + 8x = -12$ only has one solution.

(d) Gradient at $(1, 0.75) = \frac{0-1}{4-0}$

$= -0.25$

(Accept $-0.4 \leq$ gradient ≤ -0.2)

(e)(i) $y = -0.5x + b$

$1 = -0.5(3) + b$

$b = 2.5$

$y = -0.5x + 2.5$

x	-1	0	5
y	3	2.5	0

(ii) $x = 4.2$ (Accept $4.1 \leq x \leq$ **4.3**)

(iii) $\frac{x}{4}(8 - 6x + x^2) = -0.5x + 2.5$

$8x - 6x^2 + x^3 = -2x + 10$

$x^3 - 6x^2 + 10x - 10 = 0$

10. [16]

A soda can may be modelled as a cylinder with a closed top and a hollow hemisphere hollowed in at the base of the can as shown in the diagram below.

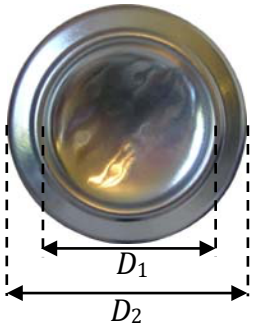
Information about the model of the soda can is given below.

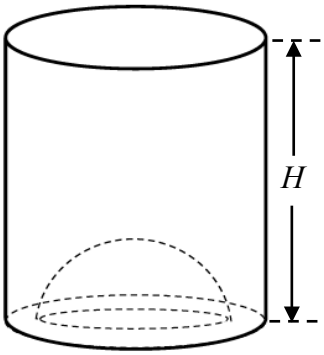
Height (H) = 12.4 cm

Inner Diameter (D_1) of base = 6.7 cm

Outer diameter (D_2) of base = 7.9 cm

Mass of empty can = 15g





(a) Using the model of the soda can in the diagram above, calculate

(i) the total surface area, in square centimetres, of the soda can. **[5]**

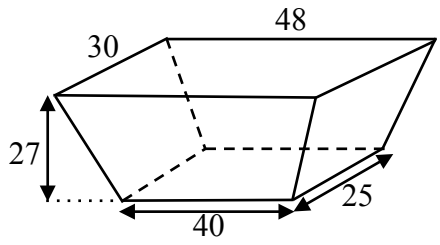
(ii) the volume, in cubic centimetres, of the soda can. **[3]**

(b) Harry uses a shopping basket to transport the soda cans filled with carbonated drink.

The soda cans will be placed with the base of the can lying on the base of the basket then stacked up vertically within the basket.

For safety reasons, all the soda cans must be contained inside the shopping basket. The maximum load that the shopping basket can carry is 55 pounds.

The shopping basket can be modelled by a frustrum of a inverted pyramid as shown in the diagram below.



The frustrum above is obtained by removing the top portion of an inverted right rectangular pyramid. The flat rectangular base of the frustum has length 40 cm and width 25 cm. The remaining vertical height is 27 cm. The flat rectangular top of the frustum is 48 cm by 30 cm.

Other Useful Information

- Density of carbonated drink = 1.3 g/cm^3
- 1 pound is equivalent to 0.45 kg
- Mass = Volume $\times$ Density
- Safety information: Soda can is filled with carbonated drink up to a maximum of 90% of its total volume.

Assuming that each soda can is filled with carbonated drink to the maximum safe volume, find the maximum number of soda cans Harry can transport with the shopping basket at any one time.
Justify your answer with mathematical calculations. **[8]**

Solution:

(a)(i)

Total surface area of soda can

$$= \pi\left(\frac{7.9}{2}\right)^2 + 2\pi\left(\frac{7.9}{2}\right)(12.4)$$

$$+\left(\pi\left(\frac{7.9}{2}\right)^2 - \pi\left(\frac{6.7}{2}\right)^2\right) + 2\pi\left(\frac{6.7}{2}\right)^2$$

$$= 441.0403387$$

$$= 441\text{cm}^2 \text{ (3sf)}$$

Remark: Remember to write the correct units

(a)(ii)

Volume of soda can

$$= \pi\left(\frac{7.9}{2}\right)^2 (12.4) - \frac{2}{3}\pi\left(\frac{6.7}{2}\right)^3$$

$$= 529.067503$$

$$= 529\text{cm}^3 \text{ (3sf)}$$

Remark: Remember to write the correct units

(b) Volume of carbonated drink in each soda can $= \frac{90}{100} \times 529.067503$ $= 476.1607527 \text{cm}^3$ Mass of each soda can and carbonated drink $= \left(476.1607527 \times 1.3\right) + 15$ $= 634.0089785 \text{g}$ Maximum number of cans based on mass $= \frac{55 \times 0.45 \times 1000}{634.0089785}$ $= 39.03730205$ $= 39 \text{cans}$ (nearest whole number rounded down) Number of layers of cans based on height $= \frac{27}{12.4}$ $= 2.177419355$ $= 2$ layers of cans (nearest whole number rounded down) Number of cans based on length of basket $= \frac{40}{7.9}$ $= 5.063291139$ $= 5$ cans (nearest whole number rounded down) Number of cans based on width of basket $= \frac{25}{7.9}$ $= 3.164556962$ $= 3$ cans (nearest whole number rounded down) Maximum number of cans based on dimensions $= 5 \times 3 \times 2$ $= 30$ cans Since the maximum number of cans by dimensions, 30 cans, is less than the maximum number of cans by mass, 39 cans, the maximum number of cans Harry can transport by basket is 30 cans.	**Remarks:** Remember to write clear statements and the correct units at each solution step

CHIJ St Nicholas Girls' School 2018 Preliminary Examination Mathematics Paper 1

1 **(a)** Given $x^9 = 9^0$, find the value of *x*.

Answer …….. [1]

(b) Simplify $\frac{x^2}{3y} \div \frac{x}{9y^2}$.

Answer …….. [1]

---

2 Factorise $2p - 2q - p^2 + pq$.

Answer …….. [2]

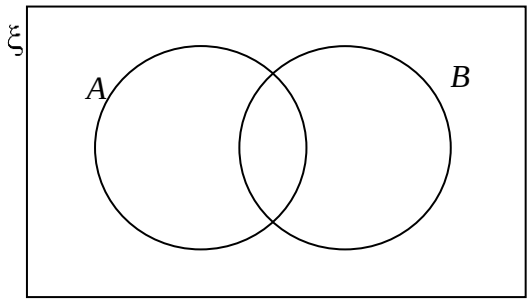
---

3 Write as a single fraction in its simplest form $\frac{5x}{(3-x)^2} - \frac{1}{x-3}$.

Answer …….. [2]

---

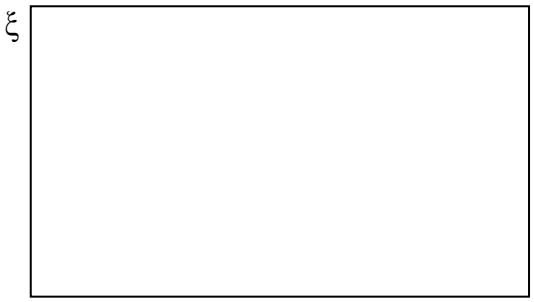
4 **(a)** On the Venn diagram, shade the region which represents $A' \cap B$.



[1]

(b) Given that P is a subset of Q, simplify $(P \cup Q)'$.

You may use the space below to help in your investigation.



Answer $(P \cup Q)'$ = …….. [1]

---

5 A shuttle bus is due to arrive at the ABC station at a certain time every morning.

The numbers of minutes by which the bus was late on ten successive days are shown below.

3 0 −2 −3 8 4 10 5 −4 9

(a) Explain the meaning of the number '–2' in the list of numbers of minutes.

Answer .....

.....

..... [1]

(b) Find the mean number of minutes by which the bus was late.

Answer …….. [1]

---

6 Given that p is a positive integer,

(a) write down expressions for the next two even numbers after $2(p - 1)$.

Answer …… and …….. [1]

(b) **(i)** find, in its simplest form, an expression for the sum of the squares of these three even numbers,

Answer …….. [2]

(ii) explain why this sum is a multiple of 4.

Answer .....

.....

..... [1]

---

7 **(a)** Express 40 and 138 as the product of their prime factors.

Answer 40 = …….....……, 138 = ...…….. [2]

(b) Hence, find the smallest positive integer k such that $138k$ is divisible by 40.

Answer smallest positive integer k = ……....…….. [1]

---

8 A wooden cube with side 8 cm is cut into two-centimetre cubes.
All of the two-centimetre cubes are then arranged to form a cuboid with height greater than 8 cm.
The perimeter of the top of the cuboid is 36 cm.

Find the height of the cuboid.

Answer …… cm [2]

---

9 A map is drawn to a scale of 1 : 40 000.

(a) This scale can be expressed as 1 cm represents *n* km.

Find *n*.

Answer n = ……..…….. [1]

(b) The distance between a seaport and an airport on the map is 60 cm.

Find the actual distance, in kilometres, between the seaport and the airport.

Answer ……..…….. km [1]

(c) A bus depot has an actual area of 8 km^2.

Find the area, in square centimetres, of the bus depot on the map.

Answer ……..…….. cm^2 [2]

---

10 **(a)** Fynn deposited $\$m$ into an account that paid a compound interest of 1.85% per annum. He made no other deposits or withdrawals for three years.
At the end of three years, he had \$2509.26 in his account.

Find the value of m, giving your answer correct to the nearest dollar.

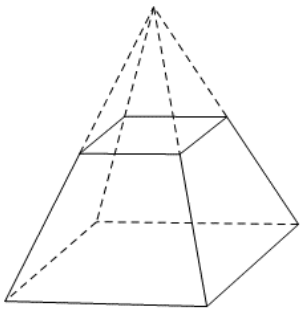
Answer m = ……..……. [2]

(b) Fynn withdrew all his money from the bank and used 30% of it to buy a watch. Subsequently he sold the watch for a profit of 60%.

Find the selling price of the watch.

Answer \$……..…….. [2]

11 The diagram shows a frustum obtained by removing a small pyramid with height half of that of the original pyramid.
[A frustum is a portion of a pyramid that is left after a smaller pyramid is removed from the top.]



Find the ratio of the volume of the frustum to the volume of the original pyramid.

Answer ……..… : ……..…….. [2]

12 **(a)** Express $3x^2 - 12x$ in the form $3[(x + a)^2 + b]$.

Answer …….. [2]

(b) Write down the smallest value of $3x^2 - 12x$.

Answer …….. [1]

---

13 Cooking oil is sold in two sizes:

\$4.80 for each 2 kg-bottle

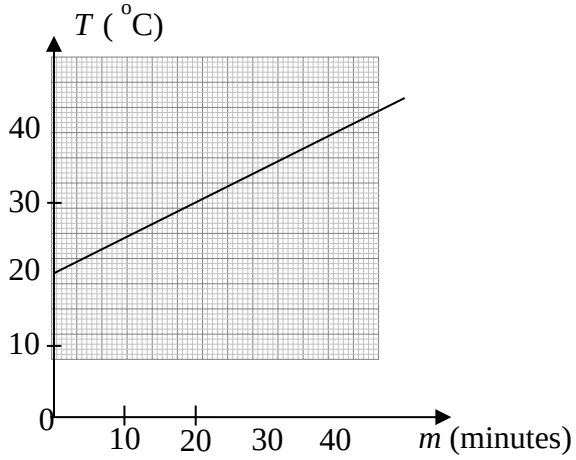
\$6.95 for each 3 kg-bottle

Which bottle gives the better value?

You must show your calculations.

Answer The …… kg-bottle gives the better value. [2]

14 The graph shows the temperature, $T\,^{o}$C, of the water in a hot water tank after the heater is switched on for m minutes.



Use the graph to find

(a) the increase in temperature of the heater when it is switched on for 20 minutes,

Answer …… °C [1]

(b) an equation for T in terms of m.

Answer …….…….. [1]

---

15 The distance between the points $M(k, 7)$ and $N(9, k)$ is $\sqrt{20}$.

Given that $k > 10$, find the value of k.

Answer k = ……..……. [3]

---

16 The table shows some corresponding values of x and y of the equation of a line.

x	–1	0	b	3
y	2	a	0	–18

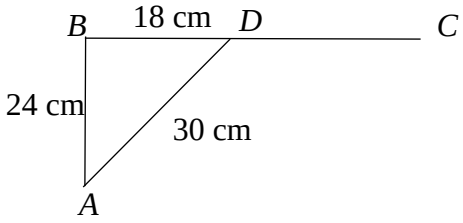
(a) Find the equation of the line.

Answer …….. [2]

(b) Hence find the value of a and of b.

Answer a = ……, b = …….. [2]

17



In triangle ABD, AB = 24 cm, BD = 18 cm, AD = 30 cm. BD is produced to C.

(a) Explain why angle ABD is a right angle.

Answer ……..

……..

……..

……..

……..

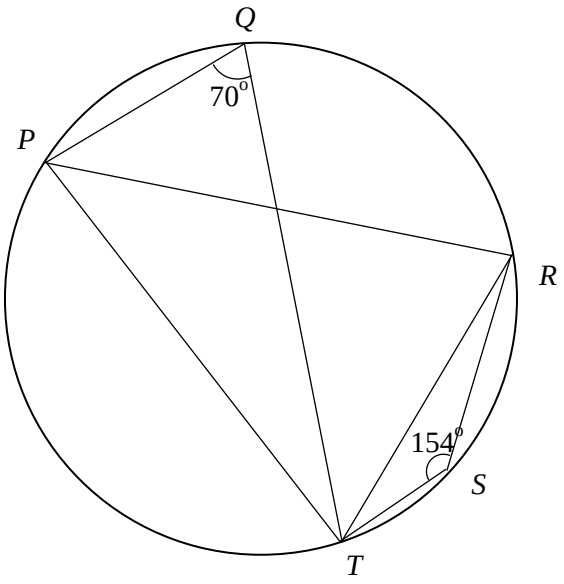
……..

…….. [1]

(b) Write down the value of $\cos \angle ADC$.

Answer …….. [1]

18



In the diagram, *P*, *Q*, *R*, *S* and *T* are points on the circumference of a circle.
Angle *TQP* = 70 ^{o} and angle *TSR* = 154 ^{o}.

(a) Find angle *PTR*.
Give a reason for each step of your working.

Answer angle *PTR* =…….. [3]

(b) There is a point *A* on the same side of *PT* as point *R*. Angle *TAP* = 90^{o}.

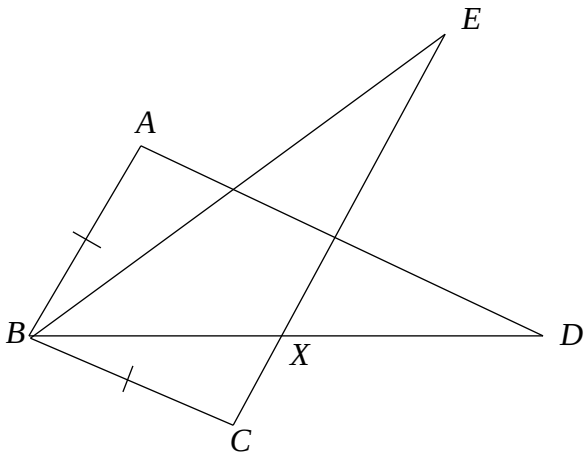
Determine if point *A* lies on the circumference of the circle, inside or outside the circle.
Justify your answer.

Answer Point *A* lies …… the circle because .....

.....

..... [1]

19



(a) In the diagram, *BA* = *BC*, angle *ABE* = angle *CBD* and angle *BEC* = angle *BDA*.

Explain why triangles *ABD* and *CBE* are congruent.

Answer ……

……

……

……

……

……

…… [2]

(b) Given further that angle *ABE* = angle *BEC*, what type of quadrilateral is *ABCE* ? Justify your answer.

Answer Quadrilateral *ABCE* is a …… .

……

……

……

…… [2]

20 The table shows the number of people in groups of 1, 2, 3 and 4 people who attended a travel fair exhibition during a two-hour period.

No. of people in each group	1	2	3	4
No. of groups	20	94	85	26

Find

(a) the total number of people who attended the exhibition during the two-hour period,

Answer ……..…….. [1]

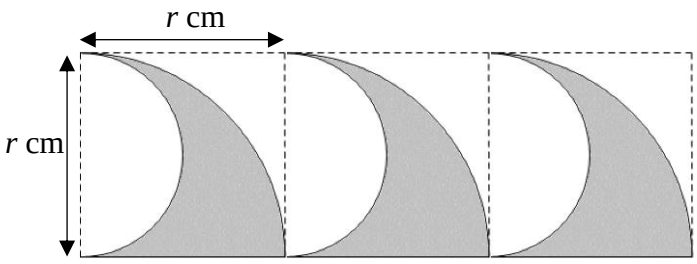
(b) the median number of people per group,

Answer ……..…….. [1]

(c) the percentage of groups with at least 2 people.

Answer ……..……..% [1]

21



A surfing brand's logo consists of 3 waves.
Each wave is made up of a quadrant with a semicircle removed.

(a) Find the perimeter of the logo in terms of r.

Answer ……...…… cm [3]

The logo is drawn and then cut from a piece of fabric measuring $3r$ cm by r cm.

(b) Given that the area of the remaining fabric is 16.4 cm^2, find the value of r.

Answer r = ……...…… [3]

22 A is the point (–4, 11). The position vector of B is $\begin{pmatrix} 10 \\ 4 \end{pmatrix}$.

(a) Express $\overrightarrow{BA}$ as a column vector.

Answer ……..……..[1]

(b) Calculate $|\overrightarrow{AB}|$.

Answer ……..…… units [1]

$\overrightarrow{BC} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$ and D is the point $(0, d)$.

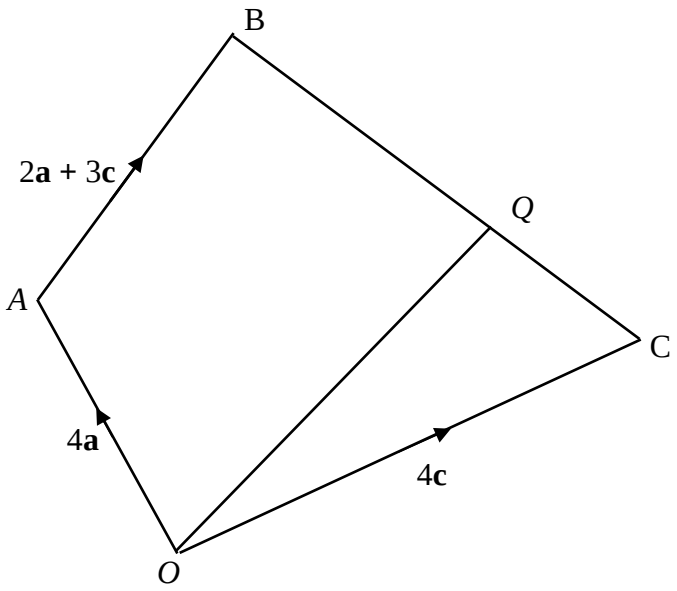
(c) (i) Find the column vector $\overrightarrow{OC}$.

Answer ……..……..[1]

(ii) If $\overrightarrow{BA}$ is parallel to $\overrightarrow{CD}$, find the value of d.

Answer d = ……..…..……..[3]

23



OABC is a quadrilateral.
$\overrightarrow{OA} = 4\mathbf{a}$, $\overrightarrow{OC} = 4\mathbf{c}$, *and* $\overrightarrow{AB} = 2\mathbf{a} + 3\mathbf{c}$.
$CQ : QB = 2 : 3$.

(a) Write each of the following in terms of **a** and **c**.
Give your answers in their simplest form.

(i) $\overrightarrow{BC}$,

Answer ……..……..[1]

(ii) $\overrightarrow{OQ}$.

Answer ……..……..[1]

(b) Use your answer to part **(a) (ii)** to explain why *AB* is parallel to *OQ*.

Answer .....

.....

..... [1]

(c) Find

(i) $OQ : AB$,

Answer ……..…….[1]

(ii) $\dfrac{\text{Area of triangle } OAB}{\text{Area of triangle } OQB}$,

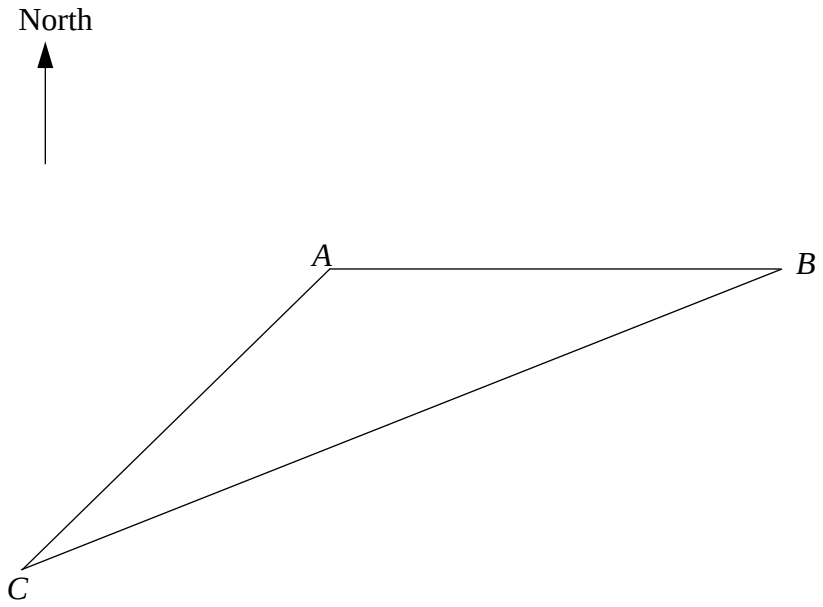
Answer ……..…….[1]

(iii) $\dfrac{\text{Area of triangle } OQC}{\text{Area of triangle } OBC}$.

Answer ……..…….[1]

24 The diagram below shows a scale drawing of triangle *ABC*.

Answer (a), (b), (c) **Scale: 1 cm to 10 m**



(a) Measure the bearing of *B* from *C*.

Answer ……..…….[1]

Points *A*, *B* and *C* are on the ground and a WiFi router is placed at *B*.

The WiFi router's signal can reach a distance of up to 42 m.

(b) Construct the range of the WiFi signal from the WiFi router at *B*. [1]

(c) Stacia is currently at *C* and starts walking along a path that is equidistant to *AC* and *BC*.
She stops at a point that is equidistant from *A* and *B*.

(i) Locate this point by construction and label it *S*. [3]

(ii) Hence state if Stacia is able to receive the Wifi signal at *S*, giving a reason for your answer.

Answer Stacia is ..... to receive the Wifi signal at *S*, ...

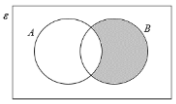
.....

.....

..... [1]

CHIJ St Nicholas Girls' School 2018 Preliminary Examination Mathematics Paper 1

Answers

1.(a) $x = 1$, (b) $3xy$	15. $k = 11$
2. $(p - q)(2 - p)$	16. (a) $y = -5x - 3$
3. $\frac{4x+3}{(x-3)^2}$ or $\frac{4x+3}{(3-x)^2}$	(b) a = – 3, b = $-\frac{3}{5}$
4. (a) 	17.(a) $(AB)^2 + (BD)^2 = 24^2 + 18^2$ $= 900$ $= (30)^2$ $= (AD)^2$ angle $ABD = 90°$ by Pythagoras Theorem
(b) Q'	
5. (a) The number '–2' means the bus was early by 2 minutes	(b) $-\frac{3}{5}$
(c) 3 minutes	18. (a) angle $PTR = 84°$
6. (a) $2p$ and $2p + 2$	(b) Point A lies inside the circle because if A lies on the circumference, $\angle TAP = 70°$, and since $\angle TAP = 90° > 70°$, A lies inside the circle.
(b) $12p^2 + 8$	
(b) $12p^2 + 8 = 4(3p^2 + 2)$, hence the sum has a factor of 4, this means the sum is a multiple of 4.	
7. (a) $40 = 2^3 \times 5$, $138 = 2 \times 3 \times 23$	19(a) $\angle ABE = \angle CBD$ (given) $\angle ABE + \angle EBD = \angle CBD + \angle EBD$ $\therefore \angle ABD = \angle CBE$ $\angle BDA = \angle BEC$ (given) $BA = BC$ (given) $\Delta ABD \equiv \Delta CBE$ (AAS)
(b) smallest positive integer $k = 20$	
8. height = 16 cm	
9. (a) n = 0.4	
(b) 24 km	
(c) 50 cm^2	
10. (a) $m = 2375$	(b) $ABCE$ is a trapezium. AB is parallel to CE because alternate angles, $\angle ABE$ and $\angle BEC$, are equal.
(b) \$1204.44	
11. ratio = 7 : 8	
12. (a) $3\left((x-2)^2 - 4\right)$	20.(a) 567
(b) – 12	(b) 2 people per group
13. The 3 kg-bottle	(c) $91\frac{1}{9}$% or 91.1 % (to 3 s.f.)
14. (a) 10°C	21. (a) $(3\pi r + 3r)$ cm or $12.4r$ cm
(b) $T = 20 + \frac{1}{2}m$	(b) r = 3.00 (to 3 s.f.)

CHIJ St Nicholas Girls' School 2018 Preliminary Examination Mathematics Paper 1

Answers

22.(a) $\overrightarrow{BA} = \begin{pmatrix} -14 \\ 7 \end{pmatrix}$

(b) 15.7 units (to 3 s.f.)

(c) (i) $\overrightarrow{CA} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$

(ii) $d = 15$

23. (a) (i) $\overrightarrow{BC} = -6\mathbf{a} + \mathbf{c}$

(ii) $\overrightarrow{OQ} = \frac{12}{5}\mathbf{a} + \frac{18}{5}\mathbf{c}$

or $\frac{6}{5}(2\mathbf{a} + 3\mathbf{c})$

23(b) $\overrightarrow{OQ} = \frac{6}{5}(2\mathbf{a} + 3\mathbf{c})$

$\therefore \overrightarrow{OQ} = \frac{6}{5}\overrightarrow{AB}$

Hence AB is parallel to OQ.

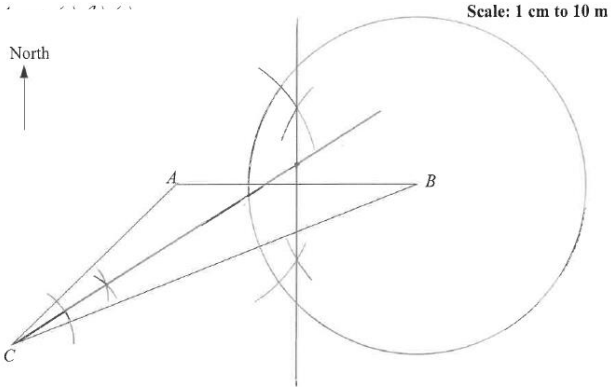
(d) (i) 6 : 5

(ii) $\frac{5}{6}$

(iii) $\frac{2}{5}$

24. (a) 068°

(b),(c) (i)



(c) (ii) Stacia is <u>able</u> to receive the Wifi signal at *S*, <u>because she is within the 42 m radius circular region, centre at the router position at *B*.</u>

1 **(a)** Given $x^9 = 9^0$, find the value of x.

$x^9 = 9^0$

$x^9 = 1$

$x = 1$ A1

Answer …….. [1]

(b) Simplify $\frac{x^2}{3y} \div \frac{x}{9y^2}$.

(b) $\frac{x^2}{3y} \div \frac{x}{9y^2}$

$= \frac{x^2}{3y} \times \frac{9y^2}{x}$

$= 3xy$ A1

Answer …….. [1]

---

2 Factorise $2p - 2q - p^2 + pq$.

$2p - 2q - p^2 + pq$

$= 2(p-q) - p(p-q)$ M1

$= (p-q)(2-p)$ A1

Answer …….. [2]

---

3 Write as a single fraction in its simplest form $\frac{5x}{(3-x)^2} - \frac{1}{x-3}$.

$\frac{5x}{(3-x)^2} - \frac{1}{x-3}$

$= \frac{5x}{(x-3)^2} - \frac{1}{x-3}$ B1 for $3-x = -(x-3)$

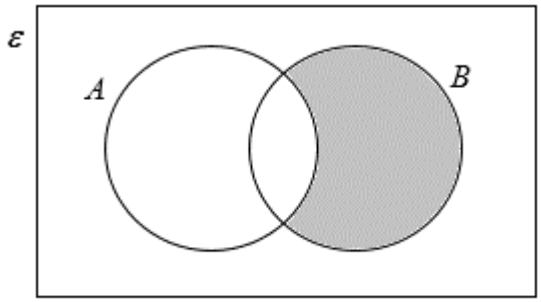
$= \frac{5x-x+3}{(x-3)^2}$

$= \frac{4x+3}{(x-3)^2}$ or $\frac{4x+3}{(3-x)^2}$ A1

Answer …….. [2]

---

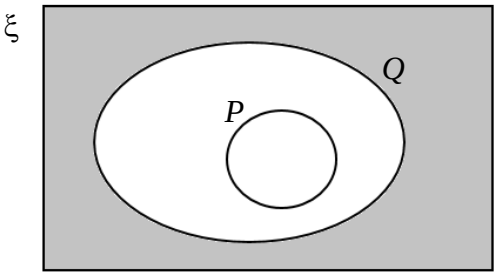
4 **(a)** On the Venn diagram, shade the region which represents $A' \cap B$.



[1]

(b) Given that P is a subset of Q, simplify $(P \cup Q)'$.

You may use the space below to help in your investigation.



Answer $(P \cup Q)' =$ ……Q'…….. [1]

---

5 A shuttle bus is due to arrive at the ABC station at 09 30 daily.

The numbers of minutes by which the bus was late on ten successive days are shown below.

3 0 −2 −3 8 4 10 5 −4 9

(a) Explain the meaning of the number '−2' in the list of numbers of minutes.

Answer ....

The number '−2' means the bus was early by 2 minutes

.....

..... [1]

(b) Find the mean number of minutes by which the bus was late.

(a) $\bar{x} = \frac{30}{10} = 3$ min A1

Answer …….. [1]

---

6 Given that p is a positive integer,

(a) write down expressions for the next two even numbers after 2(p – 1).

$2p$ and $2p + 2$ or $2(p + 1)$ A1

Answer …… and …….. [1]

(b) **(i)** find, in its simplest form, an expression for the sum of the squares of these three even numbers,

$(2p-2)^2+(2p)^2+(2p+2)^2$

$=4p^2-8p+4+4p^2+4p^2+8p+4$ M1 follow thru

$=12p^2+8$ A1

Answer …….. [2]

(ii) explain why this sum is a multiple of 4.

Answer $12p^2 + 8 = 4(3p^2 + 2)$, hence the sum has a factor of 4, this means the sum is a multiple of 4.

OR $4(3p^2 + 2)$ is a multiple of 4. [1]

---

7 **(a)** Express 40 and 138 as a product of their prime factors.

$40 = 2^3 \times 5$ A1

$138 = 2 \times 3 \times 23$ A1

Answer 40 = ……, 138 = ……. [2]

(b) Hence, find the smallest positive integer k such that $138k$ is divisible by 40.

$\frac{138k}{40} = \frac{2 \times 3 \times 23 \times k}{2^3 \times 5}$

$k = 2^2 \times 5$

$= 20$ A1

Answer smallest positive integer k = …….. [1]

---

8 A wooden cube with side 8 cm is cut into two-centimetre cubes.
All of the two-centimetre cubes are then arranged to form a cuboid with height greater than 8 cm.
The perimeter of the top of the cuboid is 36 cm.

Find the height of the cuboid.

Total volume = 8^3 = 512 cm^3

Total number of cubes = 4^3 = 64 cubes

Breadth of cuboid = 2 cm

Length of cuboid = 16 cm

Height of cuboid = $512 \div 2 \div 16$

= 16 cm

B1 for breadth or length

B1 for height

Answer …… cm [2]

---

9 A map is drawn to a scale of 1 : 40 000.

(a) This scale can be expressed as 1cm represents *n* km.

Find *n*.

1 : 40 000
1 cm : 0.4 km
n = 0.4 A1

Answer n = ……..……. [1]

(b) The distance between a seaport and an airport on the map is 60 cm.

Find the actual distance, in kilometres, between the seaport and airport.

$0.4 \times 60 = 24$ km A1

Answer ……..……. km [1]

(c) A bus depot has an actual area of 8 km^2.

Find the area, in square centimetres, of the bus depot on the map.

1 cm^2 : 0.16 km^2 B1

$\frac{8}{0.16} = 50\,cm^2$ A1

Answer ……..……. cm^2 [2]

---

10 **(a)** Fynn deposited \$*m* into an account that paid a compound interest of 1.85% per annum. He made no other deposits or withdrawals for three years.
At the end of three years, he had \$2509.26 in his account.

Find the value of *m*, giving your answer correct to the nearest dollar.

$2509.26 = m\left(1+\frac{1.85}{100}\right)^3$ B1

m = \$2374.994..

m = \$2375 (nearest dollar) A1

Answer m = ……..……. [2]

(b) Fynn withdrew all his money from the bank and used 30% of it to buy a watch. Subsequently he sold the watch for a profit of 60%.

Find the selling price of the watch.

2509.26×0.3 M1

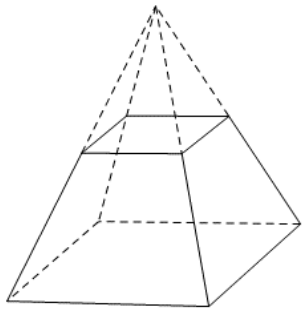
= \$752.778

$\frac{160}{100} \times \$752.788$

= \$1204.44 (2 d.p.) A1

Answer \$……..……. [2]

11 The diagram shows a frustum obtained by removing a small pyramid with height half of that of the original pyramid.
[A frustum is a portion of a pyramid that is left after a smaller pyramid is removed from the top.]



Find the ratio of the volume of the frustum to the volume of the original pyramid.

$V_{top} : V_{original} = \left(\frac{1}{2}\right)^3 : 1^3$ B1 for cube

$= \frac{1}{8} : 1 = 1 : 8$

$V_{frustum} : V_{original} = 1 - \frac{1}{8} : 1$

$= \frac{7}{8} : 1$

= 7 : 8 B1

Answer ……..… : ……..…….. [2]

12 **(a)** Express $3x^2 - 12x$ in the form $3\left[(x+a)^2 + b\right]$.

(a) $3x^2 - 12x$
$= 3(x^2 - 4x)$ B1 for factor 3
$= 3\left((x-2)^2 - 4\right)$ A1

Answer …….. [2]

(b) Write down the smallest value of $3x^2 - 12x$.

(b) – 12 A1

Answer …….. [1]

---

13 Cooking oil is sold in two sizes:
$4.80 for each 2 kg bottle

$6.95 for each 3 kg bottle

Which bottle gives the better value?

You must show your calculations.

$\frac{4.80}{2} = \$2.40$ per kg
$\frac{6.95}{3} = \$2.32$ per kg
The 3kg bottle gives the better value

B1 for either
B1 for answer

OR

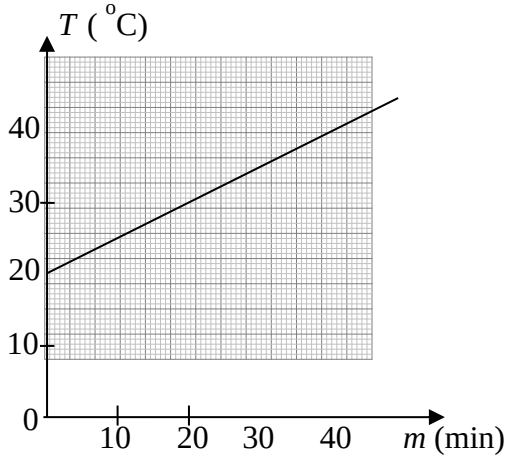
$\frac{2}{4.80} = 0.416666...$ kg per $
$\frac{3}{6.95} = 0.431654...$ kg per $
The 3kg bottle gives the better value

B1 for either
B1 for answer

Answer The …… kg-bottle gives the better value. [2]

---

14 The graph shows the temperature, $T\,^{o}$C, of the water in a hot water tank after the heater is switched on for m minutes.



Use the graph to find

(a) the increase in temperature of the heater when it is switched on for 20 minutes,

(a) 10 °C A1

Answer …… °C [1]

(b) an equation for T in terms of m.

(b) $T = 20 + \frac{1}{2}m$ A1

Answer …….…….. [1]

---

15 The distance between the points $M\,(k,7)$ and $N\,(9,k)$ is $\sqrt{20}$.

Given that $k > 10$, find the value of k.

$\sqrt{(k-9)^2+(7-k)^2} = \sqrt{20}$	B1	
$k^2-18k+81+49-14k+k^2 = 20$ $2k^2-32k+110 = 0$ $k^2-16k+55 = 0$		
$(k-5)(k-11) = 0$ $k = 5 \quad or \quad k = 11$ (NA)	M1 DA1	For factorising

Answer k = ……....…….. [3]

---

16 The table shows some corresponding values of x and y of the equation of a line.

x	–1	0	b	3
y	2	a	0	–18

(a) Find the equation of the line.

$m = \dfrac{-18-2}{3-(-1)} = -5$ M1

$y = -5x + c$

$2 = -5(-1) + c$

$\therefore c = -3$

$y = -5x - 3$

A1

Answer …….. [2]

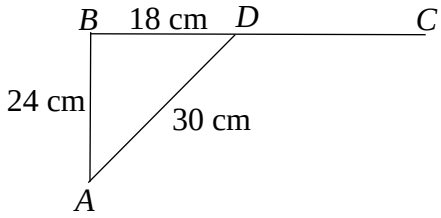
(b) Hence find the values of a and b.

$a = -3$ A1

$b = -\dfrac{3}{5}$ A1

Answer a = ……, b = …….. [2]

17



In triangle ABD, AB = 24 cm, BD = 18 cm, AD = 30 cm. BD is produced to C.

(a) Explain why angle ABD is a right angle.

$(AB)^2 + (BD)^2 = 24^2 + 18^2$
$= 900$
$= (30)^2$
$= (AD)^2$

angle $ABD = 90°$ by Pythagoras Theorem A1

Answer …….. [1]

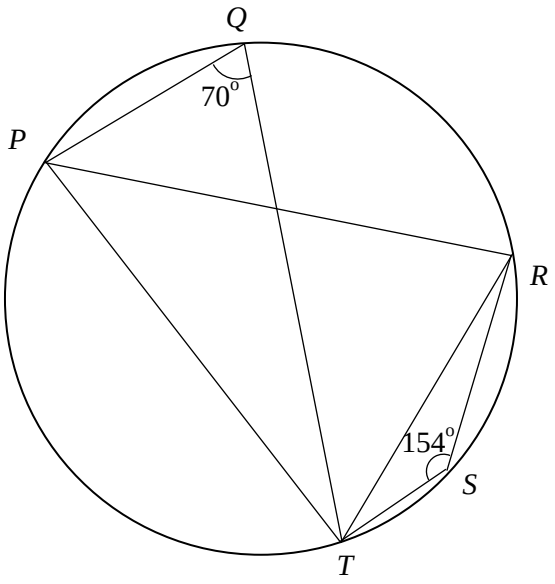
(b) Write down $\cos \angle ADC$.

$\cos \angle ADC = -\cos \angle ADB$

$= -\dfrac{18}{30} = -\dfrac{3}{5}$ A1

Answer …….. [1]

18



In the diagram, *P*, *Q*, *R*, *S* and *T* are points on the circumference of a circle.
Angle *TQP* = 70° and angle *TSR* = 154°.

(a) Find angle *PTR*.
Give a reason for each step of your working.

$\angle TPR = 180^o - \angle TSR$ (∠s in opp segment are supp)
$= 180^o - 154^o$ M1
$= 26^o$

$\angle PRT = \angle TQP = 70°$ (∠s in same segment) M1

$\angle PTR = 180° - 70° - 26°$ (∠ sum of Δ) (can don't see this reason)
$= 84°$ A1

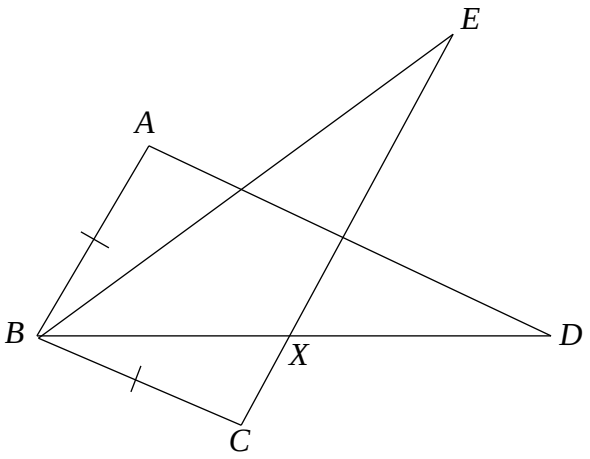
Answer angle *PTR* = …….. [3]

(b) There is a point *A* on the same side of *PT* as point *R*. Angle *TAP* = 90°.

Determine if point *A* lies on the circumference of the circle, inside or outside the circle. Justify your answer.

Answer Point *A* lies inside the circle because .....
if *A* lies on the circumference, $\angle TAP = 70°$ (∠s in the same segment), and since
$\angle TAP = 90° > 70°$, *A* lies inside the circle. A1
..... [1]

19



(a) In the diagram, *BA* = *BC*, angle *ABE* = angle *CBD* and angle *BEC* = angle *BDA*.

Explain why triangles *ABD* and *CBE* are congruent.

Answer …….....……..…….……

…….

…….

…….

…….

…….

…….

……....…… [2]

(b) Given further that angle *ABE* = angle *BEC*, what type of quadrilateral is *ABCE* ? Justify your answer.

19(b) *ABCE* is a trapezium. A1

AB is parallel to *CE* because alternate angles, as angle *ABE* and angle *BEC*, are equal. A1

Answer ABCE is a …….…….. [2]

19(a) $\angle ABE = \angle CBD$ (given)

$\angle ABE + \angle EBD = \angle CBD + \angle EBD$

$\therefore \angle ABD = \angle CBE$

$\angle BDA = \angle BEC$ (given)

BA = *BC* (given) M1 for any correct two conditions

$\Delta ABD \equiv \Delta CBE$ (AAS) A1

20 The table shows the number of people in groups of 1, 2, 3 and 4 people who attended a travel fair exhibition during a two -hour period.

No. of people in each group	1	2	3	4
No. of groups	20	94	85	26

Find

(a) the total number of people who attended the exhibition during the two-hour period,

(a)	$20 + 2\times94 + 3\times85 + 4\times26$ $= 567$	A1	

Answer ……..……. [1]

(b) the median number of people per group,

(b)	225 groups in total, 113^{th} group is the middle Median is 2 people per group	A1	

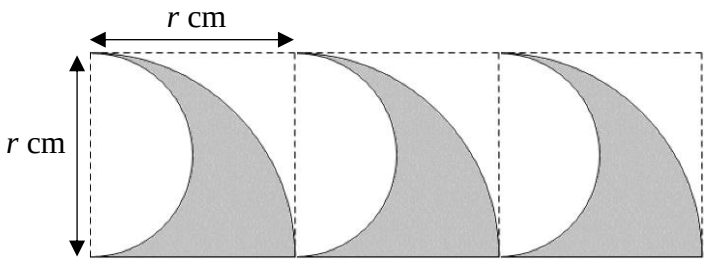
Answer ……..……. [1]

(c) the percentage of groups with at least 2 people,

(c)	Total groups = 20 + 94 + 85 + 26 = 225 205 groups with at least 2 people. $\frac{205}{225}\times100\%$ $= 91\frac{1}{9}\%$ or 91.1 % (to 3 s.f.)	A1	

Answer ……..……..% [1]

21



A surfing brand's logo consists of 3 waves.
Each wave is made up of a quadrant with a semicircle removed.

(a) Find the perimeter of the logo in terms of *r*.

Arc length of quadrant = $\frac{1}{4}(2\times\pi\times r)$ B1

$= \frac{\pi r}{2}$ cm

Arc length of semi-circle

$= \frac{1}{2}\left(2\pi r\times\frac{1}{2}r\right)$ B1 *no / wrong unit, –1 per paper*

$= \frac{\pi r}{2}$ cm

Perimeter = $3\left(\frac{\pi r}{2}+\frac{\pi r}{2}+r\right)$

$= 3(\pi r + r)$ cm B1

or $(3\pi r + 3r)$ cm

Answer ……..…… cm [3]

The logo is drawn and then cut from a piece of fabric measuring 3*r* cm by *r* cm.

(b) Given that the area of the remaining fabric is 16.4 cm^2, find the value of *r*.

$r^2\left(1-\frac{\pi}{8}\right)=\frac{16.4}{3}$ M1√ for atempt to factorise

$r = \sqrt{\frac{16.4}{3\left(1-\frac{\pi}{8}\right)}}$, r is positive

r = 3.00 (to 3 s.f.) A1 cannot '3'

$r^2\left(1-\frac{\pi}{8}\right)=\frac{16.4}{3}$ M1√ for atempt to factorise

$r = \sqrt{\frac{16.4}{3\left(1-\frac{\pi}{8}\right)}}$, r is positive

r = 3.00 (to 3 s.f.) A1 cannot '3'

Area of logo = $3\left[\frac{1}{4}\pi r^2-\frac{1}{2}\pi\left(\frac{r}{2}\right)^2\right]$ = = $\frac{3}{8}\pi r^2$ cm^2 B1

$3r^2 - \frac{3}{8}\pi r^2 = 16.4$ M1√ for attempting to factorise

$r = \sqrt{\frac{16.4}{\left(3-\frac{3\pi}{8}\right)}}$, where r >0 , *r* = 3.00 (to 3 s.f.) A1

Answer r = ……..…… [3]

22 A is the point (–4, 11). The position vector of B is $\begin{pmatrix}10\\4\end{pmatrix}$.

(a) Express $\overrightarrow{BA}$ as a column vector.

(a)	$\overrightarrow{BA} = \begin{pmatrix}-4\\11\end{pmatrix} - \begin{pmatrix}10\\4\end{pmatrix} = \begin{pmatrix}-14\\7\end{pmatrix}$	A1	

Answer ……..……..[1]

(b) Calculate $|\overrightarrow{AB}|$.

(b)	$\left\|\overrightarrow{AB}\right\| = \left\|\overrightarrow{BA}\right\| = \sqrt{(-14)^2 + 7^2}$ $= \sqrt{245}$ = 15.7 units (3 s.f.)	A1	

Answer ……..…… units [1]

$\overrightarrow{BC} = \begin{pmatrix}0\\6\end{pmatrix}$ and D is the point (0, d).

(c) (i) Find the column vector $\overrightarrow{OC}$.

(c)	$\therefore\ \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix}0\\6\end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix}0\\6\end{pmatrix} + \begin{pmatrix}10\\4\end{pmatrix}$ $= \begin{pmatrix}10\\10\end{pmatrix}$	A1	

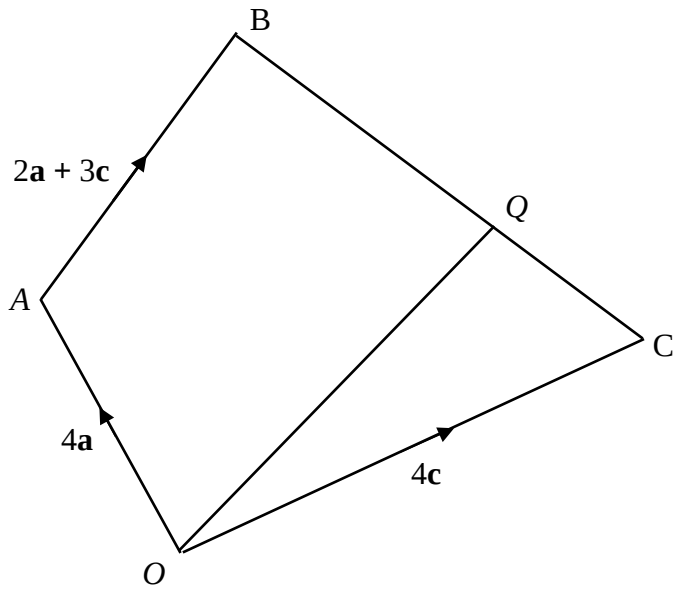
Answer ……..……..[1]

(ii) If $\overrightarrow{BA}$ is parallel to $\overrightarrow{CD}$, find the value of d.

(iii)	$\overrightarrow{BA} = h\ \overrightarrow{CD}$ $\begin{pmatrix}-14\\7\end{pmatrix} = h\left[\begin{pmatrix}0\\d\end{pmatrix} - \begin{pmatrix}10\\10\end{pmatrix}\right]$ $= \begin{pmatrix}-10h\\h(d-10)\end{pmatrix}$ $-10h = -14$ $h = \frac{7}{5}$ $\frac{7}{5}(d-10) = 7$ $d - 10 = 5,\quad d = 15$	M1 M1 A1	can be $h = \frac{5}{7}$ if $h\overrightarrow{BA} = \overrightarrow{CD}$

Answer d = ……..…….……..[3]

23



OABC is a quadrilateral.
$\overrightarrow{OA} = 4\mathbf{a}$, $\overrightarrow{OC} = 4\mathbf{c}$, *and* $\overrightarrow{AB} = 2\mathbf{a} + 3\mathbf{c}$.
$CQ : QB = 2 : 3$.

(a) Write each of the following in terms of **a** and **c**.
Give your answers in their simplest form.

(i) $\overrightarrow{BC}$,

$$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{BA} + \overrightarrow{AO} + \overrightarrow{OC} \\ &= -\overrightarrow{AB} + (-\overrightarrow{OA}) + \overrightarrow{OC} \\ &= -2\mathbf{a} - 3\mathbf{c} - 4\mathbf{a} + 4\mathbf{c} \\ &= -6\mathbf{a} + \mathbf{c}\end{aligned}$$

Answer ……..……..[1]

(ii) $\overrightarrow{OQ}$.

(ii) $\overrightarrow{OQ} = \overrightarrow{OC} + \overrightarrow{CQ}$

$= 4\mathbf{c} + \frac{2}{5}\overrightarrow{CB}$

$= 4\mathbf{c} + \frac{2}{5}(6\mathbf{a} - \mathbf{c})$

$= \frac{12}{5}\mathbf{a} + \frac{18}{5}\mathbf{c}$ or $\frac{6}{5}(2\mathbf{a} + 3\mathbf{c})$ A1

Answer ……..……..[1]

(b) Use your answer to part **(a) (ii)** to explain why *AB* is parallel to *OQ*.

Answer .....

.....

..... [1]

(b)	$\overrightarrow{OQ} = \frac{6}{5}(2\mathbf{a} + 3\mathbf{c})$ $\therefore \overrightarrow{OQ} = \frac{6}{5}\overrightarrow{AB}$ Hence *AB* is parallel to *OQ*.	A1	

(c) Find

(i) *OQ* : *AB*, 6 : 5

Answer ……..……..[1]

(ii) $\frac{\text{Area of triangle } OAB}{\text{Area of triangle } OQB}$, $\frac{5}{6}$

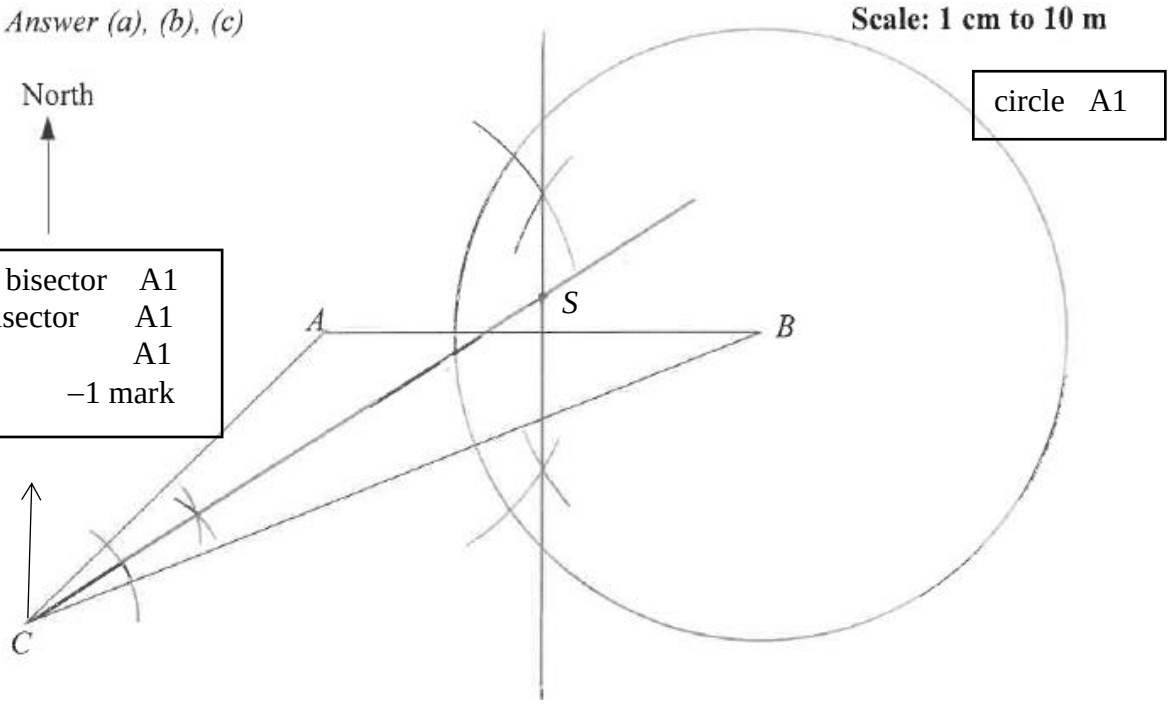
Answer ……..……..[1]

(iii) $\frac{\text{Area of triangle } OQC}{\text{Area of triangle } OBC}$. $\frac{2}{5}$

Answer ……..……..[1]

24 The diagram below shows a scale drawing of triangle *ABC*.

Answer (a), (b), (c)



circle A1

(c) (i) angle bisector A1
Perp. bisector A1
label *S* A1
Imperfect −1 mark

Measure the bearing of *B* from *C*.

Answer ……068° A1……[1]

Points *A*, *B* and *C* are on the ground and a WiFi router is placed at *B*.

The WiFi router's signal can reach a distance of up to 42 m.

(a) Construct the range of the WiFi signal from the WiFi router at *B*. [1]

(b) Stacia is currently at *C* and starts walking along a path that is equidistant to *AC* and *BC*.
She stops at a point that is equidistant from *A* and *B*.

(i) Locate this point by construction and label it *S*. [3]

(ii) Hence state if Stacia is able to receive the Wifi signal at *S*, giving a reason for your answer.

Answer Stacia is ..... able ..... to recieve the Wifi signal at S, ... because .....

..... she is within the 42 m radius circular region, centre at the router positon at *B*. .....

..... [1]

CHIJ St. Nicholas Girls' School
2018 Preliminary Examination 2018 Mathematics Paper 2

1 **(a)** It is given that $h = \dfrac{k+h}{3h-k}$.

(i) Find the positive value of h when $k = 2h$. [1]

(ii) Express k in terms of h. [3]

(b) Solve the equation $\dfrac{3x}{4} + \dfrac{1}{x} = 2$. [3]

(c) Solve these simultaneous equations

$$x + 4y + 3 = 0,$$
$$5x - 2y - 29 = 0.$$ [2]

(d) Simplify $\dfrac{2-5x-7x^2}{1-x^2}$. [3]

---

2 **(a)** The interior angles of a hexagon are

$$(2x + 17)^\circ, \quad (3x - 4)^\circ, \quad (2x + 49)^\circ, \quad (x + 40)^\circ, \quad (x - 17)^\circ, \quad (3x - 25)^\circ.$$

Find the smallest exterior angle. [3]

(b) The areas of the two similar octagons are 25 cm^2 and 576 cm^2.
The length of the sides of the octagons are x cm and 7 cm.

Find the two possible values of x. [3]

---

3 A group of volunteers pack goodie bags for the residents of a nursing home.
The table shows the contents of one of each type of goodie bag.

	Bag Type		
	P	*Q*	*R*
Number of buns	5	3	4
Number of toothbrushes	2	1	2
Number of packets of Milo	2	3	2
Number of packets of coffee	1	2	3

This information can be represented by the matrix $\mathbf{A} = \begin{pmatrix} 5 & 3 & 4 \\ 2 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$.

There are 20 bags of type *P*, 30 bags of type *Q* and 10 bags of type *R*.

(a) **(i)** Represent the numbers of the three types of goodie bags in a 3×1 column matrix **B**. [1]

(ii) Evaluate the matrix **C** = **AB**. [2]

(iii) State what the elements of **C** represent. [1]

(b) A bun costs \$1.
A toothbrush costs \$1.50.
A packet of Milo costs \$6.40.
A packet of coffee costs \$5.60.

The elements of the matrix **E**, where **E** = **DA**, represent the costs, in dollars, of each bag of *P*, of *Q* and of *R* respectively.

(i) Write down the matrix **D**. [1]

(ii) Evaluate the matrix **E**. [1]

(c) Evaluate the matrix **F** = **EB.** [1]

(d) State what the element(s) of **F** represent. [1]

4 The first four terms in a sequence of numbers are given below.

$T_1 = 1^2 + 8 \quad = 9$

$T_2 = 2^2 + 12 = 16$

$T_3 = 3^2 + 16 = 25$

$T_4 = 4^2 + 20 = 36$

(a) Find T_5. [1]

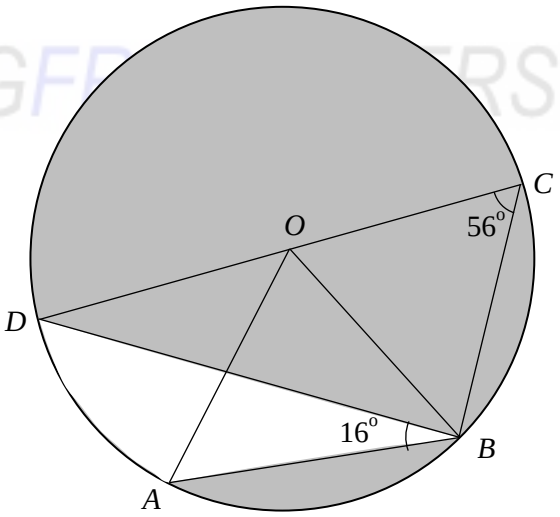
(b) Find an expression, in terms of n, for the nth term, T_n , of the sequence. [2]

(c) T_p and T_{p+1} are consecutive terms in the sequence.

Find and simplify an expression, in terms of p, for $T_{p+1} - T_p$. [2]

(d) Explain why two consecutive terms of the sequence cannot have a difference of less than 7. [1]

5



The diagram shows a circle $ABCD$, centre O and radius 4 cm.
COD is a diameter of the circle.

Angle ABD = 16° and angle BCD = 56°.

(a) Find the reflex angle DOB. [2]

(b) Find angle AOB. [2]

(c) Find the shaded area. [4]

---

6 Answer the whole of this question on a sheet of graph paper.

A ball was thrown from the top of a building.
The height, h metres, of the ball above ground level t seconds after it was thrown was measured every second.

Some corresponding values of t and h are given in the table below.

t	0	1	2	3	4	5	6	7
h	210	250	250	237	206	155	84	0

(a) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \leq t \leq 7$.
Using a scale of 4 cm to represent 100 metres, draw a vertical h-axis for $0 \leq h \leq 300$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Explain what the h-intercept of the curve represents. [1]

(c) Use your graph to estimate

(i) the maximum height of the ball, [1]

(ii) the time taken for the ball to reach its maximum height. [1]

(d) **(i)** By drawing a tangent, find the gradient of the curve at (4, 206). [2]

(ii) Use your answer in **(d)(i)** to explain what was happening to the ball at $t = 4$. [1]

7 (a) A shopkeeper mixed 30 kg of Brand *A* tea, which he bought at \$32 per kg, with 20 kg of Brand *B* tea, which he bought at \$35 per kg.
He sold all the mixture at \$40 per kg.

Determine whether the shopkeeper made a gain or loss from this transaction. Show your working clearly. [2]

(b) Mrs Tan bought some packets of coffee for \$800. Each packet of coffee costs \$$x$.

(i) Write down an expression, in terms of x, for the number of packets of coffee bought. [1]

It was found that 2 packets were damaged and had to be thrown away. Mrs Tan then sold each of the remaining packets of coffee for \$2 more than what she had paid for.

(ii) Write down an expression, in terms of x, for the total sum received from the sale of the packets of coffee. [1]

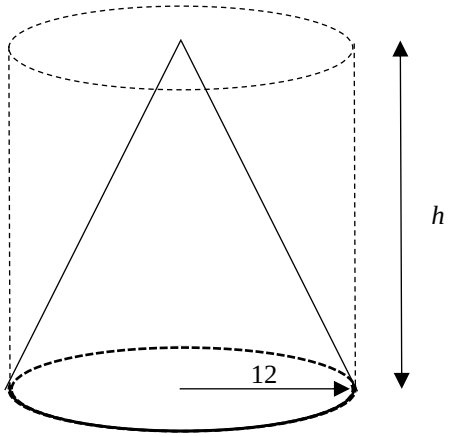
(iii) Given that Mrs Tan made a profit of \$99 from the sale of the packets of coffee, form an equation in x and show that it reduces to

$$2x^2 + 103x - 1600 = 0.$$ [3]

(iv) Solve the equation $2x^2 + 103x - 1600 = 0$. [3]

(v) Find the number of packets of coffee sold. [1]

8



The diagram shows a solid cone of radius 12 cm and height h cm cut from a solid cylindrical steel block of the same radius and height.

(a) The cylinder has a volume of 4320π cm^3. Find the value of h. [2]

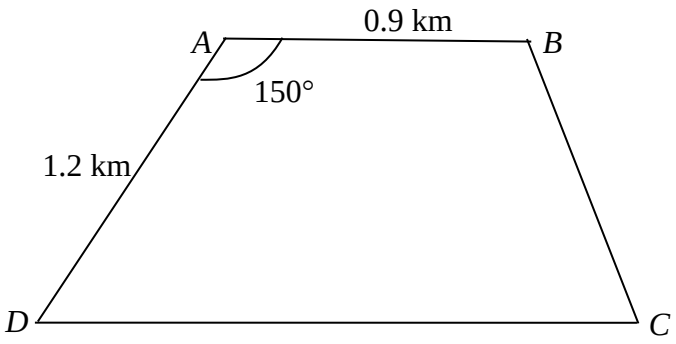
(b) Find the total surface area of the cone. [3]

(c) After the cone is cut from the steel block, the remaining steel is melted down and made into a solid sphere.

(i) Find the radius of the sphere. [3]

(ii) Find the surface area of the sphere. [1]

9



The diagram shows four towns A, B, C and D on a piece of horizontal land.
$ABCD$ is a trapezium.

AB = 0.9 km, AD = 1.2 km and angle BAD = 150°.

(a) Calculate the distance between Town B and Town D. [2]

(b) Calculate the value of angle BDC. [2]

(c) A tower is standing at Town B.
The greatest angle of elevation of the top of the tower, T, from the path CD is 18°.

Find the height of the tower in metres. [3]

---

10 (a) A chicken farmer fed 15 new-born chicks with a new variety of grain.

The stem-and-leaf diagram shows the weight gains of the chicks after three weeks.

37	8			
38	1	9		
39	0	5	6	
40	2	3	7	9
41	8	9		
42	5	7		
43	9			

Key 37 | 8 means 378 grams

(i) Find the median weight gain. [1]

(ii) Find the interquartile range. [2]

(iii) Calculate

(a) the mean of the weight gain, [1]

(b) the standard deviation. [2]

Chicks fed on the standard variety of grain had weight gains after three weeks.

The mean of these weight gains was 392 grams while the standard deviation was 12 grams.

(c) State briefly how the new variety of grain compares to the standard variety. [2]

(b) Box *A* contains 6 red cards, 4 blue cards and 2 green cards.
Box *B* contains 3 red cards and 5 blue cards.

A card is drawn at random from Box *A* and put into Box *B*.
Next, a card is drawn at random from Box *B*.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

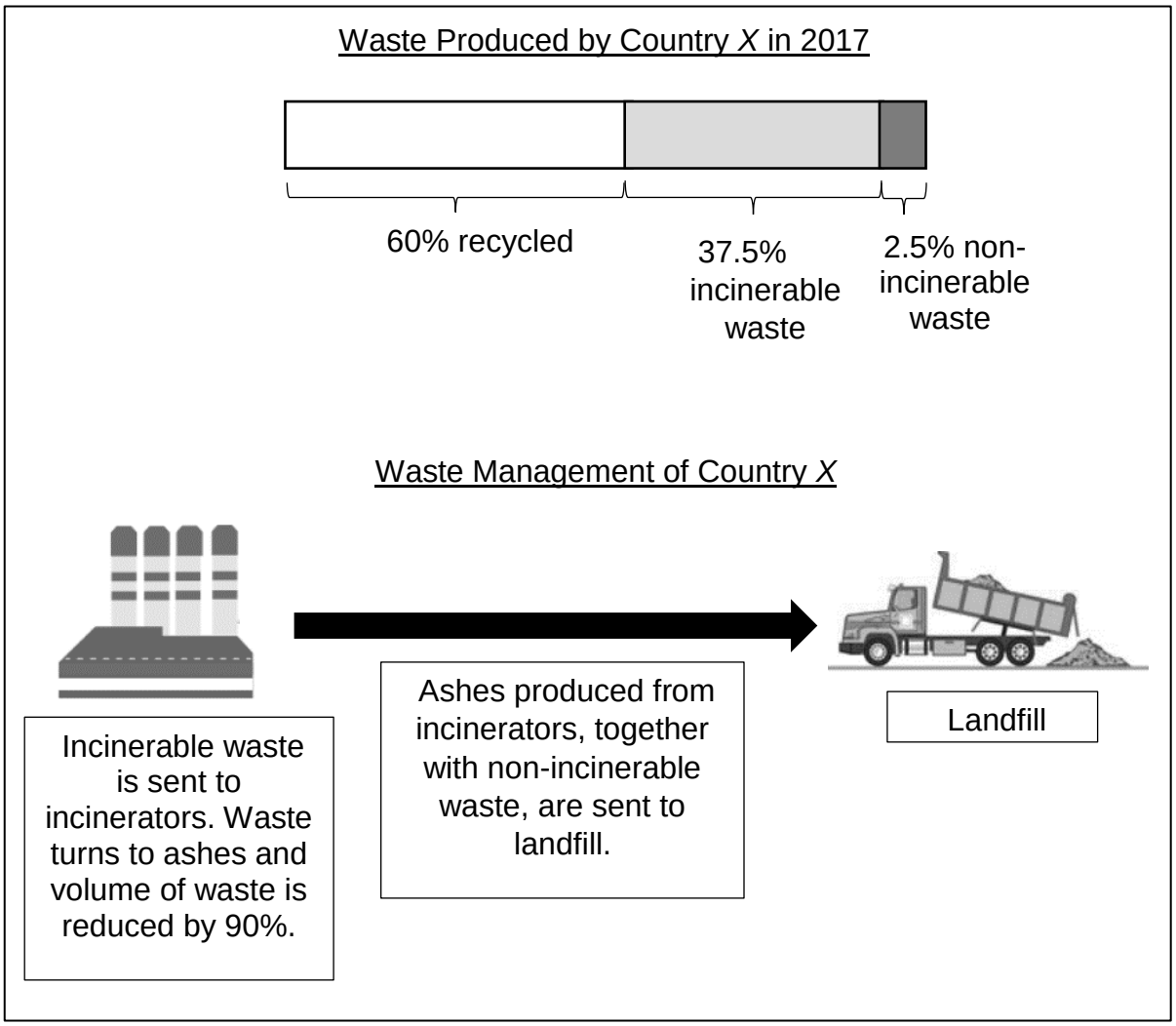
(ii) Find, as a fraction in its simplest form, the probability that

(a) two green cards are drawn, [1]

(b) neither of the cards is green, [1]

(c) the two cards are of different colours. [2]

11 Country *X* produced 3 million tonnes of waste in 2017. The infographic below shows more information on the waste produced and the waste management of Country *X*.



Images from: https://www.dreamstime.com/illustration/dumptruck.html, https://www.mewr.gov.sg/topic/landfill

(a) Given that the density of waste is 125 kg/m^3 and 1 tonne = 1000 kg, calculate the volume of waste, in m^3, that was incinerated in 2017.
Give your answer in standard form. [3]

The landfill used by Country *X* has a total capacity of 42 000 000 m^3.
By the end of 2017, 30% of the landfill has already been used.
A news article claims it will take another 32 years before the landfill is completely used.

(b) Is the news article correct?
Justify your decision with calculations. [6]

(c) State one assumption you made in your calculations in **(b)**. [1]

CHIJ St. Nicholas Girls' School 2018 Preliminary Examination Mathematics Paper 2
Answers

1 (a) (i) 3 (ii) $k = \dfrac{h(3h-1)}{(1+h)}$ (b) $x = 2,\ x = \frac{2}{3}$ (c) $x = 5, y = -2$ (d) $\dfrac{2-7x}{1-x}$

2 (a) 19° (b) 1.46 , 33.6

3 (a) (i) $\begin{pmatrix} 20 \\ 30 \\ 10 \end{pmatrix}$ (ii) $\begin{pmatrix} 230 \\ 90 \\ 150 \\ 110 \end{pmatrix}$

(iii) The elements of **C** represent the **total numbers** of buns, of toothbrushes, of packets of Milo and of packets of coffee respectively, needed to pack **all** the bags.

(b) (i) $\begin{pmatrix} 1 & 1.5 & 6.4 & 5.6 \end{pmatrix}$ (ii) $\begin{pmatrix} 26.4 & 34.9 & 36.6 \end{pmatrix}$

(c) $\begin{pmatrix} 1941 \end{pmatrix}$

(d) The element in F represents the **total cost** in dollars for packing the goodie bags.

4 (a) 49 (b) $n^2 + 4n + 4$ (c) $2p + 5$

(d) As the difference between two consecutive terms is $(2p + 5)$, and p is a positive integer, the smallest difference is $2(1) + 5$, which is 7. Hence the difference cannot be less than 7.

5 (a) 248^o (b) 80^o (c) 45.3 $cm^{2.}$

6 (b) h-intercept represents the height of the building is 210 m.

(c) (i) 255 m (ii) 1.5 s

(d) (i) –36.8 (ii)The ball is falling at a speed of 36.8 m /s.

7 (a) Cost per kg \$33.20 < \$40 , Gain

(b)(i) $\dfrac{800}{x}$ (ii) $\$\left(\dfrac{800}{x} - 2\right)(x + 2)$ (iv) 12.5, -64 (v) 62

8 (a) 30 (b) 1670 cm^2 (c) (i) 12.9 cm (ii) 2100 cm^2

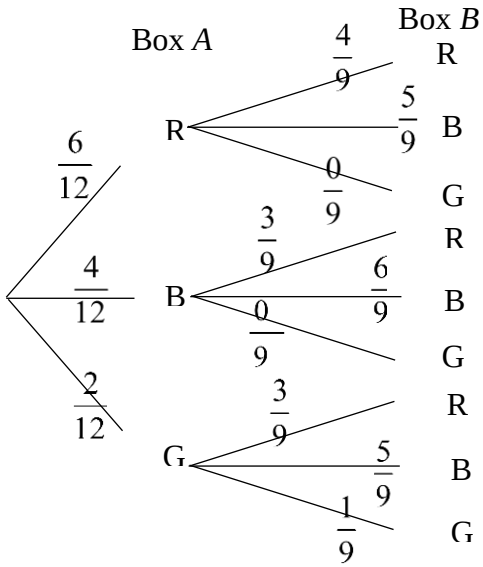
9 (a) 2.03 km (b) 17.2° (c) 195 m

10 (a) (i) 403 grams (ii) 29 grams (iii)(a) 405.2 grams (b) 17.1 grams

(a) (iii) (c) As 392 < 405.3, chicks had more weight gain when fed with the new variety of grain.

As 12 < 17.1, the weight gain from the new variety of grain shows more spread in the results.

10 (b) (i)



(ii) (a) $\frac{1}{54}$ (b) $\frac{5}{6}$ (c) $\frac{29}{54}$

11 (a) 9×10^6 m^3

(b) No, the news article is incorrect.
(with working to show it takes less than 32 yrs)
(1 possible solution is the waste produced per year will take only 19.6 years before the landfill is completely used)

(c) Possible answers:

- Amount of incinerated and non-incinerable waste remains the same every year
- The percentage breakdown of waste remains the same every year.

1 **(a)** It is given that $h = \dfrac{k+h}{3h-k}$.

(i) Find the positive value of h when $k = 2h$. [1]

(ii) Express k in terms of h. [3]

(b) Solve the equation $\dfrac{3x}{4} + \dfrac{1}{x} = 2$. [3]

(c) Solve these simultaneous equations

$$x + 4y + 3 = 0,$$
$$5x - 2y - 29 = 0.$$ [2]

(d) Simplify $\dfrac{2-5x-7x^2}{1-x^2}$. [3]

1(a)	**(i)** $h = \dfrac{2h+h}{3h-(2h)}$ $h = \dfrac{3h}{h}$ $h = 3$	A1	
	(ii) $h = \dfrac{k+h}{3h-k}$ $k+h = h(3h-k)$ $k+h = 3h^2 - hk$ $k+hk = 3h^2 - h$ $k(1+h) = 3h^2 - h$ $k = \dfrac{h(3h-1)}{(1+h)}$ or $\dfrac{3h^2-h}{(1+h)}$	M1 M1 A1	No fraction group like terms
(b)	$\dfrac{3x}{4} + \dfrac{1}{x} = 2$ $\dfrac{3x^2+4}{4x} = 2$ $3x^2 - 8x + 4 = 0$ $3x^2 - 8x + 4 = 0$ $(x-2)(3x-2) = 0$ $x = 2,\ x = \frac{2}{3}$	M1 B1 A1	single fraction
(c)	$x + 4y + 3 = 0$ ..... (1) $5x - 2y - 29 = 0$ .... (2) (1)×5, $5x + 20y + 15 = 0$ ..... (3) (3) – (2), $22y + 44 = 0$ $\therefore y = -2,\ \ x = 5$	 M1 A1	
(d)	$\dfrac{2-5x-7x^2}{1-x^2} = \dfrac{(2-7x)(1+x)}{(1-x)(1+x)}$ $= \dfrac{2-7x}{1-x}$	B1, B1 A1	for each factorisation

2 **(a)** The interior angles of a hexagon are

$(2x + 17)°$, $(3x – 4)°$, $(2x + 49)°$, $(x + 40)°$, $(x – 17)°$, $(3x – 25)°$.

Find the smallest exterior angle. [3]

(b) The areas of the two similar octagons are 25 cm^2 and 576 cm^2.
The length of the sides of the octagons are x cm and 7 cm.

Find the two possible values of x. [3]

(a)	sum of interior angles = $12x + 60$ $12x + 60 = (6 – 2) \times 180$ $12x = 720 – 60$ $x = \frac{660}{12}$ $= 55$ smallest exterior angle = 180° – largest interior angle $= 180° – (3\times55–4)°$ $= 180° – 161°$ $= 19°$	M1✓ M1✓ A1	
(b)	$\frac{x}{7} = \sqrt{\frac{25}{576}}$ $\frac{x}{7} = \frac{5}{24}$ $\therefore x = 1.46$ or $1\frac{11}{24}$ $\frac{x}{7} = \sqrt{\frac{576}{25}}$ $\frac{x}{7} = \frac{24}{5}$ $\therefore x = 33.6$ or $33\frac{3}{5}$	M1 A1 A1	Either sq rt

3 A group of volunteers pack goodie bags for the residents of a nursing home.
The table shows the contents of one of each type of goodie bag.

	Bag Type		
	P	*Q*	*R*
Number of buns	5	3	4
Number of toothbrushes	2	1	2
Number of packets of Milo	2	3	2
Number of packets of coffee	1	2	3

This information can be represented by the matrix $\mathbf{A} = \begin{pmatrix} 5 & 3 & 4 \\ 2 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$.

There are 20 bags of type *P*, 30 bags of type *Q* and 10 bags of type *R*.

(a) **(i)** Represent the numbers of the three types of goodie bags in a 3×1 column matrix **B**. [1]

(ii) Evaluate the matrix **C** = **AB**. [2]

(iii) State what the elements of **C** represent. [1]

(b) A bun costs $1.
A toothbrush costs $1.50.
A packet of Milo costs $6.40.
A packet of coffee costs $5.60.

The elements of the matrix **E**, where **E** = **DA**, represent the costs, in dollars, of each bag of *P*, of *Q* and of *R* respectively.

(i) Write down the matrix **D**. [1]

(ii) Evaluate the matrix **E**. [1]

(c) Evaluate the matrix **F** = **EB.** [1]

(d) State what the element(s) of **F** represent. [1]

3(a)	(i) $\mathbf{B} = \begin{pmatrix} 20 \\ 30 \\ 10 \end{pmatrix}$	B1	
(a)	(ii) $\mathbf{C} = \begin{matrix} P & Q & R \end{matrix}$ $\begin{pmatrix} 5 & 3 & 4 \\ 2 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 20 \\ 30 \\ 10 \end{pmatrix} \begin{matrix} P \\ Q \\ R \end{matrix}$ $= \begin{pmatrix} 230 \\ 90 \\ 150 \\ 110 \end{pmatrix} \begin{matrix} \text{buns} \\ \text{toothbrush} \\ \text{Milo} \\ \text{coffee} \end{matrix}$	M1√ A1	
(a)	(iii)The elements of **C** represent the **<u>total numbers</u>** of buns, of toothbrushes, of packets of Milo and of packets of coffee respectively, needed to pack **<u>all</u>** the bags.	A1	accept 'no packet'
(b)	(i) $\mathbf{D} = \begin{pmatrix} 1 & 1.5 & 6.4 & 5.6 \end{pmatrix}$	A1	accept 6.40, 5.60
(b)	(ii) $\mathbf{E} = \begin{matrix} \text{B} & \text{TB} & \text{Milo} & \text{coffee} \end{matrix}$ $\begin{pmatrix} 1 & 1.50 & 6.40 & 5.60 \end{pmatrix}$ $\begin{matrix} \text{P} & \text{Q} & \text{R} \end{matrix}$ $\begin{pmatrix} 5 & 3 & 4 \\ 2 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix} \begin{matrix} \text{B} \\ \text{TB} \\ \text{Milo} \\ \text{coffee} \end{matrix}$ $= \begin{matrix} \text{P} & \text{Q} & \text{R} \end{matrix}$ $\begin{pmatrix} 26.4 & 34.9 & 36.6 \end{pmatrix}$	A1	accept 26.40, 34.90, 36.60
(c)	$\mathbf{F} = \begin{matrix} \text{P} & \text{Q} & \text{R} \end{matrix}$ $\begin{pmatrix} 26.4 & 34.9 & 36.6 \end{pmatrix} \begin{pmatrix} 20 \\ 30 \\ 10 \end{pmatrix} \begin{matrix} \text{P} \\ \text{Q} \\ \text{R} \end{matrix}$ $= \begin{pmatrix} 1941 \end{pmatrix}$	A1	cannot 1940, cannot ($1941)
(d)	The element of **F** represents the **<u>total cost</u>** in dollars of ***all the*** **<u>items</u>** needed to pack **<u>all the goodie bags</u>** altogether. OR The element in F represents the **<u>total cost</u>** in dollars for packing the goodie bags.	A1	

4 The first four terms in a sequence of numbers are given below.

$T_1 = 1^2 + 8 \quad = 9$

$T_2 = 2^2 + 12 = 16$

$T_3 = 3^2 + 16 = 25$

$T_4 = 4^2 + 20 = 36$

(a) Find T_5. [1]

(b) Find an expression, in terms of n, for the nth term, T_n , of the sequence. [2]

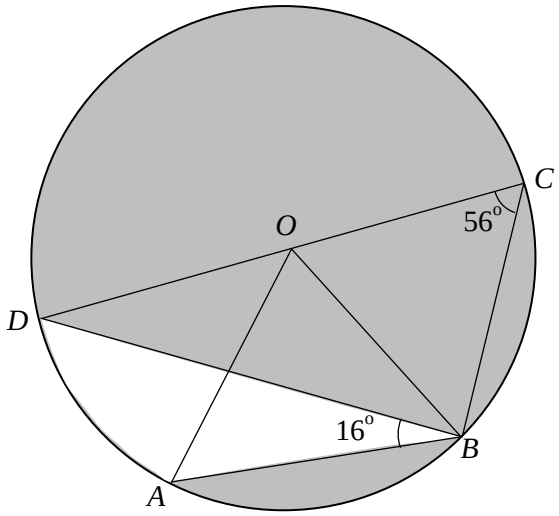
(c) T_p and T_{p+1} are consecutive terms in the sequence.

Find and simplify an expression, in terms of p, for $T_{p+1} - T_p$. [2]

(d) Explain why two consecutive terms of the sequence cannot have a difference of less than 7. [1]

4(a)	$T_5 = 5^2 + 24 = 7^2 = 49$	A1	accept just 49
(b)	$T_n = (n + 2)^2$ or $T_n = n^2 + 4(n + 1)$ $= n^2 + 4n + 4$	A1 + A1	1 mark for $(n+2)$, 1 mark for perfect or 1 mark for n^2 or $4(n + 1)$ 1 mark for perfect
(c)	$T_{p+1} - T_p = (p + 3)^2 - (p + 2)^2$ $= (p^2 + 6p + 9) - (p^2 + 4p + 4)$ $= 2p + 5$ or $T_{p+1} - T_p = (p + 1)^2 + 4(p+2) - p^2 - 4p - 4$ $= 2p + 5$	M1 A1 or M1 A1	1 mark for $(p + 3)^2$ or $(p + 2)^2$. 1 mark for answer 1 mark for $(p+1)^2+4(p+2) -p^2-4p-4$
(d)	As the difference between two consecutive terms is $(2p + 5)$, and p is a positive integer, the smallest difference is $2(1) + 5$, which is 7. Hence the difference cannot be less than 7.	A1	

5



The diagram shows a circle *ABCD*, centre *O* and radius 4 cm.
COD is a diameter of the circle.

Angle *ABD* = 16° and angle *BCD* = 56°.

(a) Find the reflex angle *DOB*. [2]

(b) Find angle *AOB*. [2]

(c) Find the shaded area. [4]

5(a)	$\angle DOB = 2\times 56^o = 112^o$ ($\angle$at the centre=2$\angle$ at circumf) reflex angle $\angle DOB = 360^o - 112^o$ $= 248^o$	M1 A1	
(b)	$\angle DOA = 2\times 16^o$ $= 32^o$ ($\angle$ at centre = 2$\times$ $\angle$ at circumf) $\angle AOB = 112^o - 32^o$ $= 80^o$	M1 A1	
(c)	Area of major sector *DOB* $= \frac{248^o}{360^o}\times\pi(4)^2$ $= 34.627$ cm^2 Area of minor sector *AOB* $= \frac{80^o}{360^o}\times\pi(4)^2$ $= 11.170$ cm^2 Area of ΔDOB $= \frac{1}{2}(4)^2\sin 112^o$ $= 7.417$ cm^2 Area of ΔAOB $= \frac{1}{2}(4)^2\sin 80^o$ $= 7.878$ cm^2 Total area $= 34.627 + 7.417 + 11.170 - 7.878$ $= 45.3$ cm^2	M1 M1 M1 A1	1 mark for either sector, 1 mark for either area of triangle 1 mark for either segment 1 mark for total area

6 Answer the whole of this question on a sheet of graph paper.

A ball was thrown from the top of a building.
The height, h metres, of the ball above ground level t seconds after it was thrown was measured every second.

Some corresponding values of t and h are given in the table below.

t	0	1	2	3	4	5	6	7
h	210	250	250	237	206	155	84	0

(a) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \leq t \leq 7$.
Using a scale of 4 cm to represent 100 metres, draw a vertical h-axis for $0 \leq h \leq 300$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Explain what the h-intercept of the curve represents. [1]

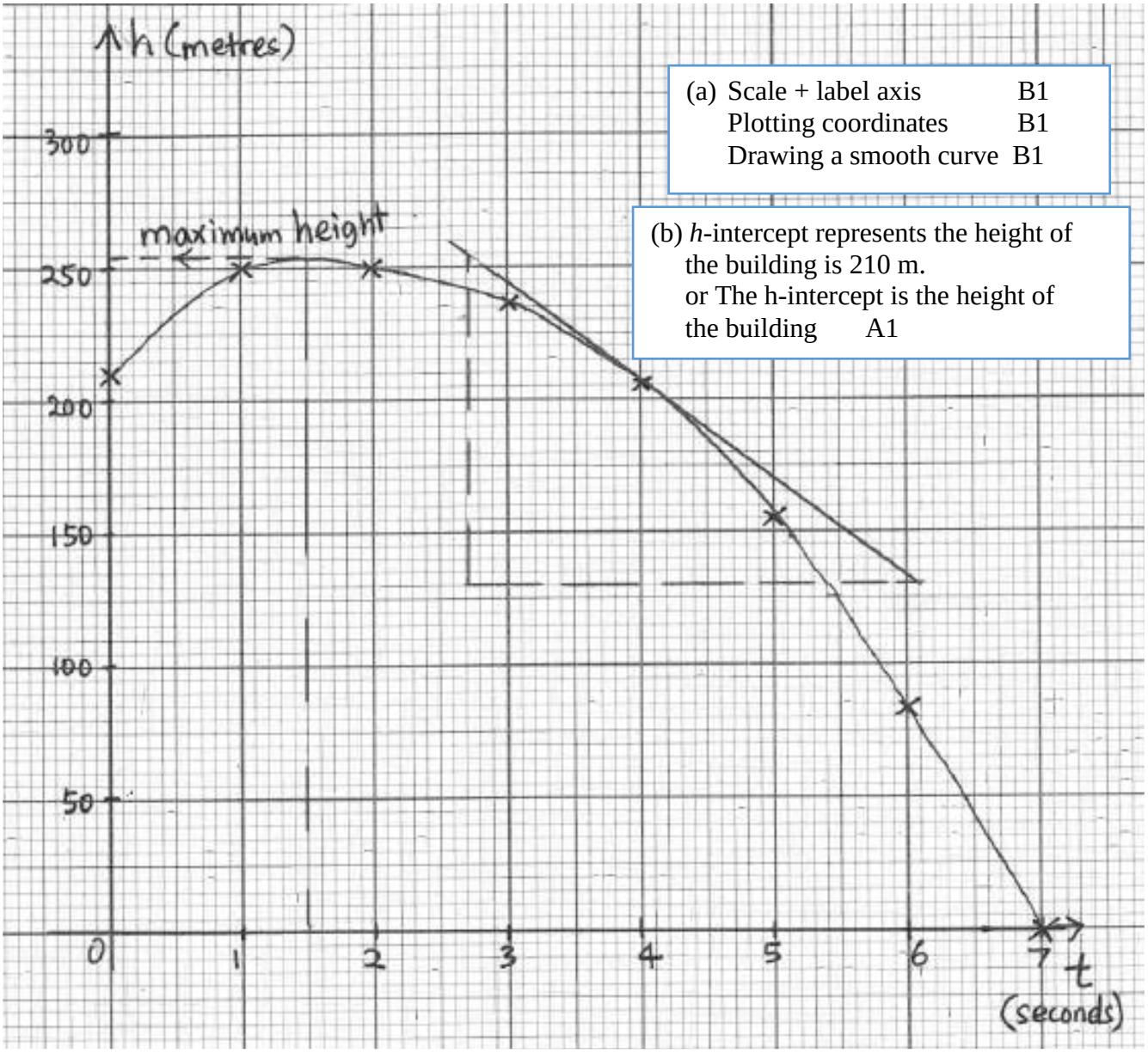
(c) Use your graph to estimate

(i) the maximum height of the ball, [1]

(ii) the time taken for the ball to reach its maximum height. [1]

(d) **(i)** By drawing a tangent, find the gradient of the curve at (4, 206). [2]

(ii) Use your answer in **(d)(i)** to explain what was happening to the ball at $t = 4$. [1]



(a) Scale + label axis B1
Plotting coordinates B1
Drawing a smooth curve B1

(b) *h*-intercept represents the height of the building is 210 m.
or The h-intercept is the height of the building A1

(c) (i) maximum height
= 2.55 m A1
(ii) time taken = 1.5 s A1

(d) (i) Drawing tangent at x = (4, 206) B1

$$\text{gradient} = \frac{125}{3.4}$$

$$= -36.8 \quad \text{A1}$$

(ii) The ball is falling at a speed of 36.8 m /s. A1

7 **(a)** A shopkeeper mixed 30 kg of Brand *A* tea, which he bought at \$32 per kg, with 20 kg of Brand *B* tea, which he bought at \$35 per kg.
He sold all the mixture at \$40 per kg.
Determine whether the shopkeeper made a gain or loss from this transaction. Show your working clearly. [2]

(b) Mrs Tan bought some packets of coffee for \$800. Each packet of coffee costs \$*x*.

(i) Write down an expression, in terms of *x*, for the number of packets of coffee bought. [1]

It was found that 2 packets were damaged and had to be thrown away.
Mrs Tan then sold each of the remaining packets of coffee for \$2 more than what she had paid for.

(ii) Write down an expression, in terms of *x*, for the total sum received from the sale of the packets of coffee. [1]

(iii) Given that Mrs Tan made a profit of \$99 from the sale of the packets of coffee, form an equation in *x* and show that it reduces to

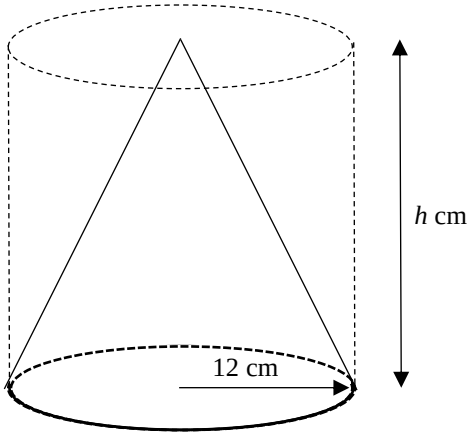
$$2x^2 + 103x - 1600 = 0 .$$ [3]

(iv) Solve the equation $2x^2 + 103x - 1600 = 0$. [3]

(v) Find the number of packets of coffee sold. [1]

(a)	$\frac{30\times 32 + 20\times 35}{30+20}$ \$33.20 per kg \$33.20 < \$40 $\therefore$ Gain Alternative method For 50 kg, cost is \$1660 Selling price is \$2000 \$1660 < \$2000	M1 A1	method for finding cost per kg of mixure Answer of 'gain'
(b)	(i) $\frac{800}{x}$	A1	
(ii)	$\left(\frac{800}{x} - 2\right)(x+2)$	A1	
(iii)	$\left(\frac{800}{x} - 2\right)(x+2) - 800 = 99$ $800 + \frac{1600}{x} - 2x - 4 - 800 = 99$ $\frac{1600}{x} - 2x - 103 = 0$ $-2x^2 - 103x + 1600 = 0$ $2x^2 + 103x - 1600 = 0$	M1 M1 B1	
(iv)	$2x^2 + 103x - 1600 = 0$ $(2x-25)(x+64) = 0$ $\therefore x = 12.5,\ x = -64$	M1 DA1, DA1	
(v)	$\frac{800}{12.5} - 2 = 62$	A1	

8



The diagram shows a solid cone of radius 12 cm and height h cm cut from a solid cylindrical steel block of the same radius and height.

(a) The cylinder has a volume of 4320π cm^3. Find the value of h. [2]

(b) Find the total surface area of the cone. [3]

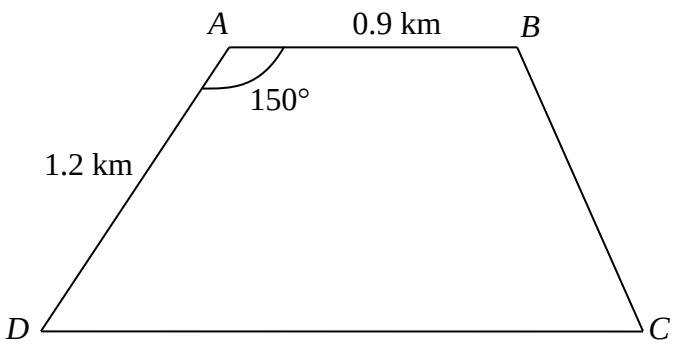
(c) After the cone is cut from the steel block, the remaining steel is melted down and made into a solid sphere.

(i) Find the radius of the sphere. [3]

(ii) Find the surface area of the sphere. [1]

8(a)	$\pi(12)^2 h = 4320\pi$ $h = \frac{4320}{144} = 30$	B1 A1	 units overall – 1
(b)	Slant height $= \sqrt{12^2 + 30^2}$ $= \sqrt{1044}$ $= 32.31\,\text{cm}$ Total surface area $= \pi(12)^2 + \pi(12)(32.31)$ $= 1670\,\text{cm}^2$ (3 s.f)	M1√ M1 A1	
(c)(i)	Volume of remaining steel $= 4320\pi - \frac{1}{3}\pi(12^2)30$ or $\frac{2}{3}\pi(12)^2(30)$ $= 2880\pi$ cm^3 $\frac{4}{3}\pi r^3 = 2880\pi$ $r^3 = 2160$ $r = 12.9\,\text{cm}$ (3 s.f)	M1 B1√ A1	
(ii)	Surface area of sphere $= 4\pi(12.92)^2$ $\approx 2\,097.6$ $= 2100\,\text{cm}^2$ (3 s.f.)	 A1	

9



The diagram shows four towns *A*, *B*, *C* and *D* on a piece of horizontal land.
ABCD is a trapezium.
AB = 0.9 km, *AD* = 1.2 km and angle *BAD* = 150°.

(a) Calculate the distance between Town *B* and Town *D*. [2]

(b) Calculate the value of angle *BDC*. [2]

(c) A tower is standing at Town *B*.
The greatest angle of elevation of the top of the tower, *T*, from the path *CD* is 18°.
Find the height of the tower in metres. [3]

9(a)	$BD^2 = (0.9)^2 + (1.2)^2 - 2(0.9)(1.2)\cos 150°$ BD = 2.0299 = 2.03 km (3 s.f.)	M1 A1	
(b)	$\frac{\sin\angle ABD}{1.2} = \frac{\sin 150°}{2.0299}$ $\sin\angle ABD = \frac{1.2\sin 150°}{2.0299}$ $\angle ABD = 17.19°$ = 17.2° (to 1 d.p.) $\angle BDC = \angle ABD$ (alt ∠s, *AB* // DC) = 17.2°	M1√ A1	accept no mention of angle property
(c)	Let the shortest distance from *B* to *CD* be *d* km. $\sin 17.19° = \frac{d}{2.0299}$ *d* = 0.5999 km Let *x* m be the height of the tower. $\frac{x}{0.5999} = \tan 18°$ $x = 0.5999 \tan 18°$ = 0.1949 km = 195 m (to 3 s.f.)	M1√ B1√ A1	

10 (a) A chicken farmer fed 15 new-born chicks with a new variety of grain.

The stem-and-leaf diagram shows the weight gains of the chicks after three weeks.

37	8			
38	1	9		
39	0	5	6	
40	2	3	7	9
41	8	9		
42	5	7		
43	9			

Key: 37 | 8 means 378 grams

(i) Find the median weight gain. [1]

(ii) Find the interquartile range. [2]

(iii) Calculate

(a) the mean of the weight gain, [1]

(b) the standard deviation. [2]

Chicks fed on the standard variety of grain had weight gains after three weeks. The mean of these weight gains was 392 grams while the standard deviation 12 grams.

(c) State briefly how the new variety of grain compares to the standard variety. [2]

10 (a)	(i) median weight gain = 403 grams	A1	
	(ii) interquartile range = 419 − 390 = 29 grams	M1 A1	
	(iii)(a) mean weight gain= $\frac{6078}{15}$ = 405.2 grams	 A1	
	(iii)(b) standard deviation = $\sqrt{\frac{2\,467\,210}{15} - \left(\frac{6078}{15}\right)^2}$ = 17.135 538..... = 17.1 grams (to 3 s.f.)	M1 A1	
	(iii)(c) 392 < 405.3, chicks had more weight gain when fed with the new variety of grain. 12 < 17.1, the weight gain from the new variety of grain shows more spread / more variation / less consistent results.	A1 A1	

(b) Box *A* contains 6 red cards, 4 blue cards and 2 green cards.
Box *B* contains 3 red cards and 5 blue cards.
A card is drawn at random from Box *A* and put into Box *B*.
Next, a card is drawn at random from Box *B*.

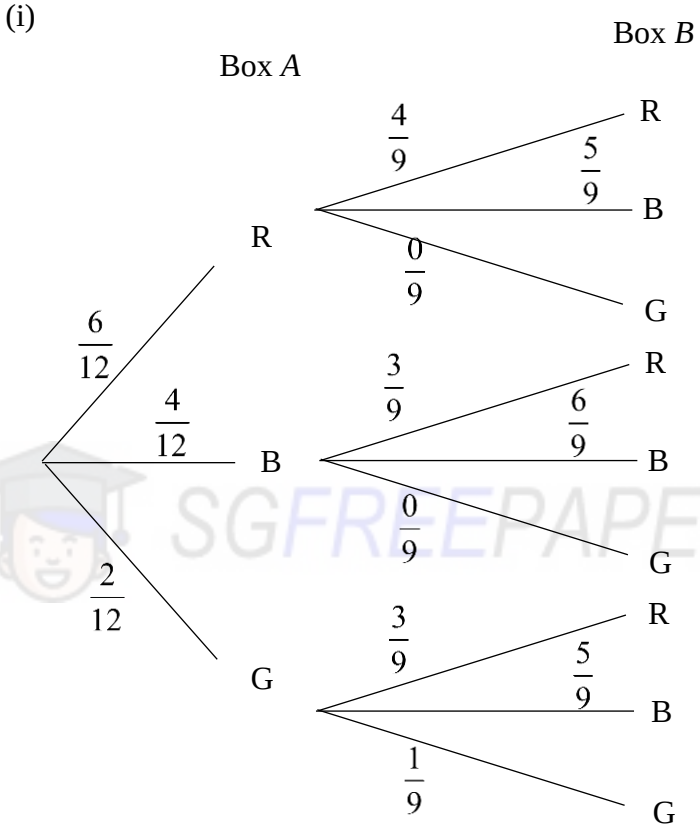
(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that

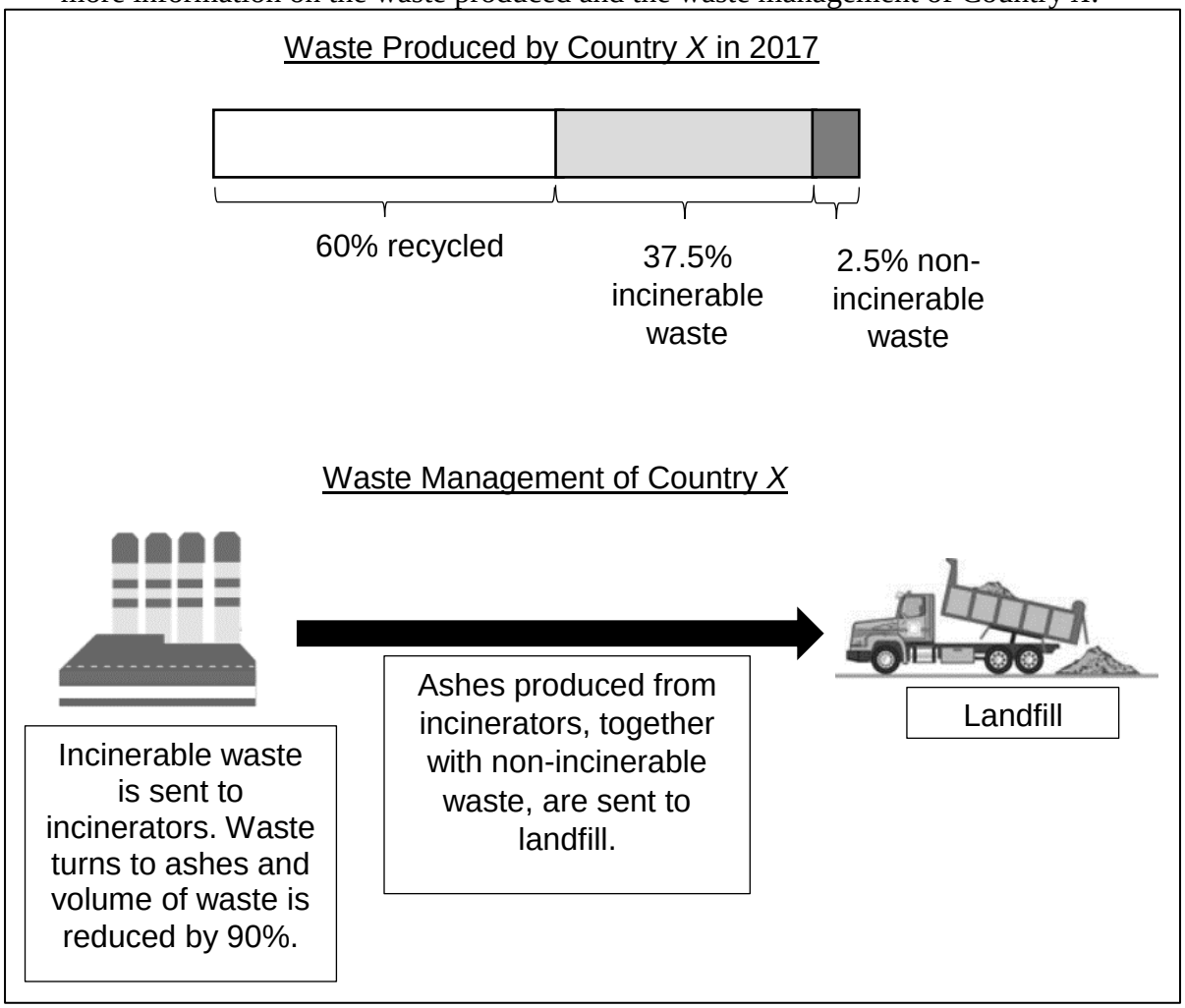
(a) two green cards are drawn, [1]

(b) neither of the cards is green, [1]

(c) the two cards are of different colours. [2]

10(b)	(i)  	A1 A1	1 mark for $\frac{6}{12}$, $\frac{4}{12}$ $\frac{2}{12}$ (fractions can be in lowest terms too) 1 mark for the prob of the draws from Box *B*.
10(b)	(ii)(a) P(GG) = $\frac{2}{12} \times \frac{1}{9}$ or $\frac{1}{6} \times \frac{1}{9}$ = $\frac{1}{54}$	A1	
	(ii)(b) P(neither is G) = $\frac{6}{12} + \frac{4}{12} = \frac{5}{6}$	A1	
	(ii)(c) P(different colour) = P(RR′) + P(BB′) + P(GG′) $= \frac{6}{12} \times \frac{5}{9} + \frac{4}{12} \times \frac{3}{9} + \frac{2}{12} \times \frac{8}{9}$ = $\frac{58}{108}$ = $\frac{29}{54}$	M1 A1	1 mark for any of the P(RR′), P(BB′), P(GG′)

11 Country *X* produced 3 million tonnes of waste in 2017. The infographic below shows more information on the waste produced and the waste management of Country *X*.



Images from: https://www.dreamstime.com/illustration/dumptruck.html, https://www.mewr.gov.sg/topic/landfill

(a) Given that the density of waste is 125 kg/m^3 and 1 tonne = 1000 kg, calculate the volume of waste, in m^3, that was incinerated in 2017.
Give your answer in standard form. [3]

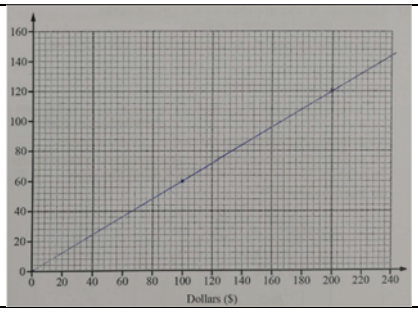
The landfill used by Country *X* has a total capacity of 42 000 000 m^3.
By the end of 2017, 30% of the landfill has already been used.
A news article claims it will take another 32 years before the landfill is completely used.

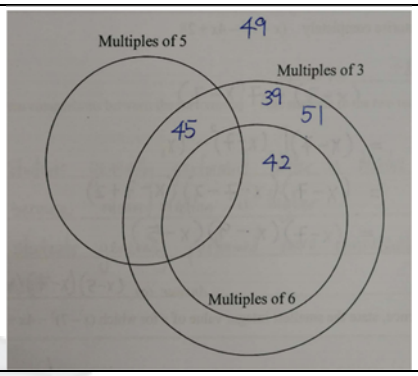
(b) Is the news article correct?
Justify your decision with calculations. [6]

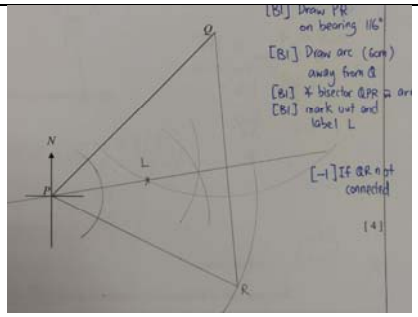
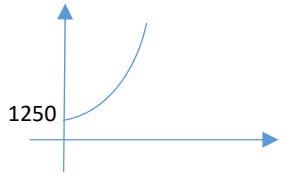
(c) State one assumption you made in your calculations in **(b)**. [1]

10	(a) Mass of incinerable waste = 3 000 000 × 37.5 % tonnes = 1 125 000 tonnes = 1.125×10^9 kg Volume of incinerable waste = $\frac{1.125\times10^9}{125}$ m^3 = 9 000 000 m^3	B1 M1√ A1	
	(b)Volume of non-incinerable waste = $\frac{3\,000\,000\times1000\times2.5\%}{125}$ = 600 000 m^3 Volume of ashes from incinerated waste = 9 000 000 × 10% = 900 000 m^3 Total volume of waste to be landfilled = 600 000 + 900 000 = 1500 000 m^3 Volume of landfill left = 42 000 000 × 70% = 29 400 000 m^3 Years left = $\frac{29\,400\,000}{1\,500\,000}$ = 19.6 No, the news article is incorrect.	M1 M1√ M1√ B1 A1 DA1	√ vol of incin waste √ vol of non-incin and incin waste
	(d) Amount of incinerated and non-incinerable waste remains the same every year. Or The percentage breakdown of waste remains the same every year.	A1	

Sec 4 Mathematics Paper 1 - Prelims 2018

Qn No.		Solutions	Marks	Remarks
1		-5.25	B1	o.e. $-5\frac{1}{4}$
				Total : 1 mark
2		$2 \leq \frac{f}{7} < 6$	B1	o.e. $14 \leq \frac{f}{7} < 42$
				Total : 1 mark
3		5:2 = 22.5: 9 31.5	M1 A1	Find the max. amt of red paint needed
				Total : 2 marks
4		$\frac{21y-7}{14} - \frac{4y+10}{14} + \frac{14}{14}$ $= \frac{17y-3}{14}$	M1 A1	Combine fraction (All terms with common denominator)
				Total : 2 marks
5	a	1.49×10^8	B1	
	b	116	B1	
				Total : 2 marks
6		21, 23, 26, 27, 28	B1 B1	26 in the middle position seen
				Total : 2 marks
7		$\left(\frac{4}{6}\right)^3$ 1.60	M1 A1	Find Vol ratio Reject 1.6
				Total : 2 marks
8		Basic angle = 46.30 133.7	M1 A1	Find basic angle Accept 2.33 soi
				Total : 2 marks
9	a		B1	
	b	132	B1	DM from graph (up to $220)
				Total: 2 marks
10	a	$\begin{pmatrix} 20 & 6 & 4 \\ 25 & 0 & 5 \end{pmatrix}$	B1	
	b	$\begin{pmatrix} 58 \\ 65 \end{pmatrix}$	B1	
	c	7	√B1	Reject negative
				Total : 3 marks
11	a	$2^2 \times 3^3 \times 5$	B1	Accept $2 \times 2 \times 3 \times 3 \times 3 \times 5$
	b	54, 60	B1, B1	
				Total: 3 marks

Qn No.		Solutions	Marks	Remarks
12	a	$\frac{x}{20}$	B1	
	b	$\frac{x+10}{30} = 2\left(\frac{x}{20}\right)$ $x = 5$	M1 A1	Form equation
				Total : 3 marks
13	a	$\frac{x^4}{16y^{12}}$	B1	
	b	$2^p \times 2^{-1} \times 3^p = 1$ $6^p = 4$	M1 A1	$2^{p-1} = 2p \div 2$
				Total : 3 marks
14	a	32	B1	
	b		B1 B1	45 and 49 39, 42 and 51
				Total : 3 marks
15		$\frac{3+1.4}{2} = 2.2$ 0.5(1.4+2.2) 1.2 × 2.5 5.4	M1 M1 A1	Find the parallel side Area of trapezium × 2.5
				Total : 3 marks
16	a	$(x-7)^3 - 4(x-7)$ $=(x-7)[(x-7)^2 - 4]$ $=(x-5)(x-7)(x-9)$	 M1 M1 A1	 Factorise $(x-7)$ Apply diff of 2 sq
	b	6	√B1	
				Total : 4 marks
17	a	25%	B1	
	b	78.5	B1	
	c	Students *performed better* in Maths because higher median/mean marks. Students performed *more consistently* for Maths because of smaller range.	B1 B1	
				Total : 4 marks

Qn No.		Solutions	Marks	Remarks
18	a	I disagree because $\frac{360}{55} = 6.545 \neq$ *whole number*/integer	B1	Each ext. angle not a whole number.
	b	144 Angle CAD= 180 – 144 – 60 = 156 n = 360/24 = 15	M1 M1 A1	each int. angle of decagon each in.t angle of n-sided polygon
				Total : 4 marks
19			B1 B1 B1 B1	Draw PR on bearing 116° ±1° Construction ΔPQR Draw arc 6 cm away from Q ($\pm$ 1mm) Angle bisector QPR with arc Mark out L
				Total : 4 marks
20	a	$1490.60	B1	cao
	b	% change = $\frac{their\ (a) - 1250}{1250} \times 100\%$ = 19.3	M1 A1	 Accept 19.2, 19.3
	c	4.5%	B1	
	d		B1 B1	Shape Vertical intercept
				Total : 6 marks
21	a	−0.5	B1	o.e
	b	(4, 4)	B1	
	c	2	√B1	
	d	(6, 3) Sub in (6,3) to find y-intercept $y = 2x - 9$	M1 M1 A1	Find the point the line cuts $y - 3 = 2(x - 6)$ o.e.
				Total: 6 marks
22	a	$\begin{pmatrix} -4 \\ 5 \end{pmatrix}$	B1	
	b	$(2k)^2 + (-k)^2 = 180$ $k = \pm 6$	M1 A1	Use magnitude
	c(i)	$\begin{pmatrix} -7 \\ -1 \end{pmatrix}$	B1	
	c(ii)	(−2, −5)	B1 B1	
				Total : 6 marks

Qn No.		Solutions	Marks	Remarks
23	a	$A = \pi a^2 + 2\pi ab$ $2\pi ab = A - \pi a^2$ $b = \frac{A-\pi a^2}{2\pi a}$	M1 A1	Isolate $2\pi ab$
	b(i)	$\pi(5)^2 h = \frac{2}{3}\pi r^3$ $r^3 = \frac{75h}{2}$ $r = \sqrt[3]{\frac{75h}{2}}$	B1 M1 A1	Equate the 2 volumes Isolate r^3
	b(ii)	90	B1	
				Total : 6 marks
24	a	Small pipe: $\frac{1}{120}$ Big pipe: $\frac{1}{80}$ 30 mins	B1 B1	Find rate of tank filled in one min. for at least one pipe
	b(i)	$(30 - d)^2 + 27.5^2 = 30^2$ $(30 - d)^2 = 143.75$ $d = 18.0$	M1 M1 A1	Form equation Solve equation
	b(ii)	42.0	√B1	60 – their b(i)
				Total: 6 marks

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2018
Secondary 4

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS 4048/01

Paper 1 **Thu 16 August 2018**

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Curved surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Solve the equation $6 - \frac{4}{3}x = 13$.

Answer $x =$ ..... [1]

2 Nurul works part-time in a supermarket.
In one week, Nurul works f hours at the supermarket.
Write down an inequality for the statement below.

Nurul must work at least 2 hours and less than 6 hours in a day.

Answer ..... [1]

3 Purple paint is made by mixing red paint and blue paint in the ratio 5 : 2.
Irene has 30 litres of red paint and 9 litres of blue paint.

What is the maximum amount of purple paint she can make?

Answer ..... litres [2]

4 Simplify $\frac{3y-1}{2} - \frac{2y+5}{7} + 1$.

Answer ..... [2]

5 **(a)** The distance between the Sun and Earth is approximately 149 million km. Convert this number to standard form.

Answer (a) ..... km [1]

(b) The radius of the Sun and Earth is approximately 695 000 km and 6 000 000 m respectively.

Complete the sentence, leaving your answer to the nearest integer.
Answer (b)

The diameter of the sun is ..... times the diameter of Earth. [1]

6 5 different integers between 19 and 30 were written.
The mean is 25 and the median is 26. They have a range of 7.

Write down the five integers.

Answer ....., ..., ..., ..., ... [2]

7 A cafe sells two sizes of cupcakes that are geometrically similar. The large cupcake is 6 cm wide at the base and the small cupcake is 4 cm wide at the base.

6 cm

4 cm

The price of a cupcake is proportional to its mass.
If the large cupcake is sold at \$5.40, what is the price of the small cupcake?

Answer \$ ..... [2]

8 The sine of an obtuse angle in a triangle is 0.723.

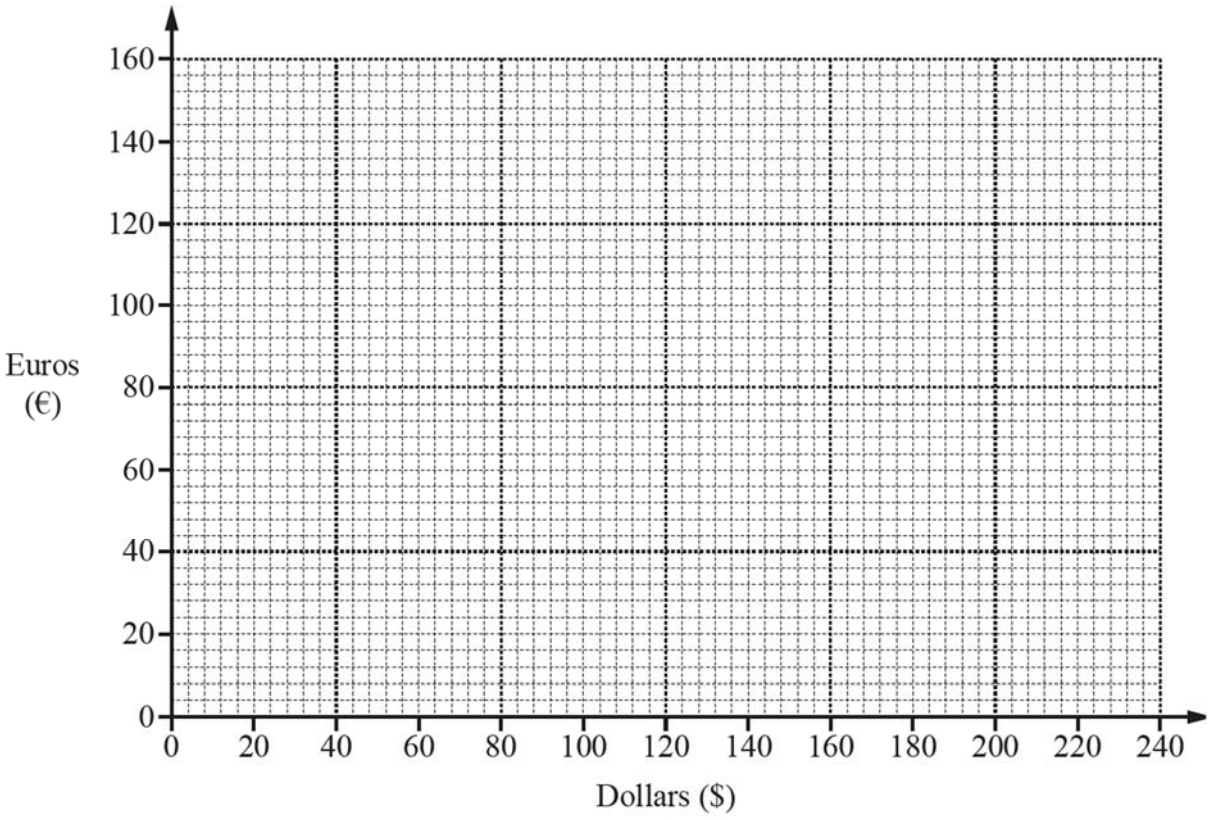
Find the angle.

Answer ..... [2]

9 The exchange rate between Euros (€) and Dollars ($) is €60 = $100.

(a) On the grid, draw a graph that represents the exchange rate.

Answer (a)



[1]

(b) Use your graph to change $220 to euros.

Answer (b) € ..... [1]

10 The point system of a soccer league is given:

- 3 points awarded for each game won
- 1 point awarded for each game drawn
- 2 points deducted for each game lost

The points system can be represented $\mathbf{P} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$.

(a) In 2017, Tagore soccer club played in the league of 30 games.
It won 20, drew 6 and lost the remaining games.

In 2018, the club played in the league of 30 games.
It won 25 and lost 5 games.

Represent this information in a 2×3 matrix, **E**.

Answer (a) **E** = [1]

(b) Evaluate the matrix **T** = **EP**.

Answer (b) **T** = [1]

(c) Find the difference between total points scored in 2017 and 2018.

Answer (c) ..... [1]

11 **(a)** Express 540 as the product of its prime factors.

Answer (a) ..... [1]

(b) Find two numbers, both smaller than 100, that have a lowest common multiple of 540 and a highest common factor of 6.

Answer (b) ..... [2]

12 A jar contains 20 coloured marbles of which x are red marbles.
A marble is removed at random from the jar.

(a) Write down, in terms of x, the probability that the marble will be red.

Answer (a) ..... [1]

A bowl contains 30 coloured marbles of which $(x + 10)$ are red marbles.

The probability that a red marble will be taken at random from this bowl is twice the probability that a red marble will be taken at random from the jar.

(b) Find the value of x.

Answer (b) $x =$ ..... [2]

13 **(a)** Simplify $\left(\frac{xy^{-3}}{2}\right)^4$.

Answer (a) ..... [1]

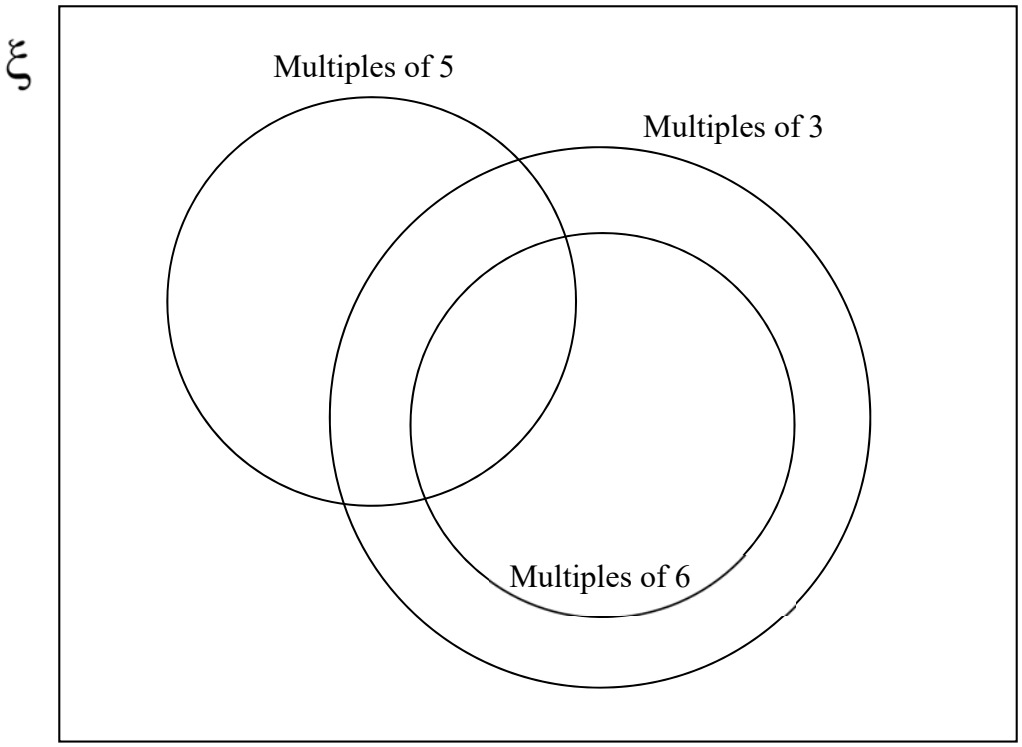
(b) Given that $2^{p-1} \times 3^p = 1$, find the value of 36^p.

Answer (b) ..... [2]

14 **(a)** Write down the smallest whole number that leaves a remainder 2 when divided by 3, 5 and 6.

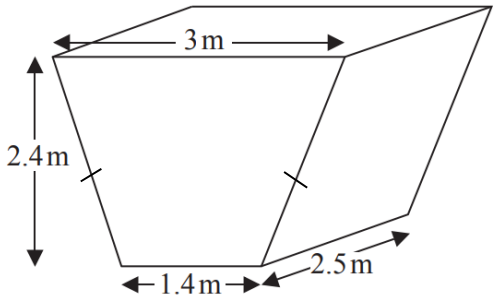
Answer (a) ..... [1]

(b) Place each of the five numbers 39, 42, 45, 49 and 51 in the correct position in the Venn diagram below.



[2]

15 The diagram shows a tank in the shape of a prism.
The cross section of the prism is an isosceles trapezium.



The tank is filled with water up to a depth of 1.2 m.

Find the volume of water.

Answer ..... m^3 [3]

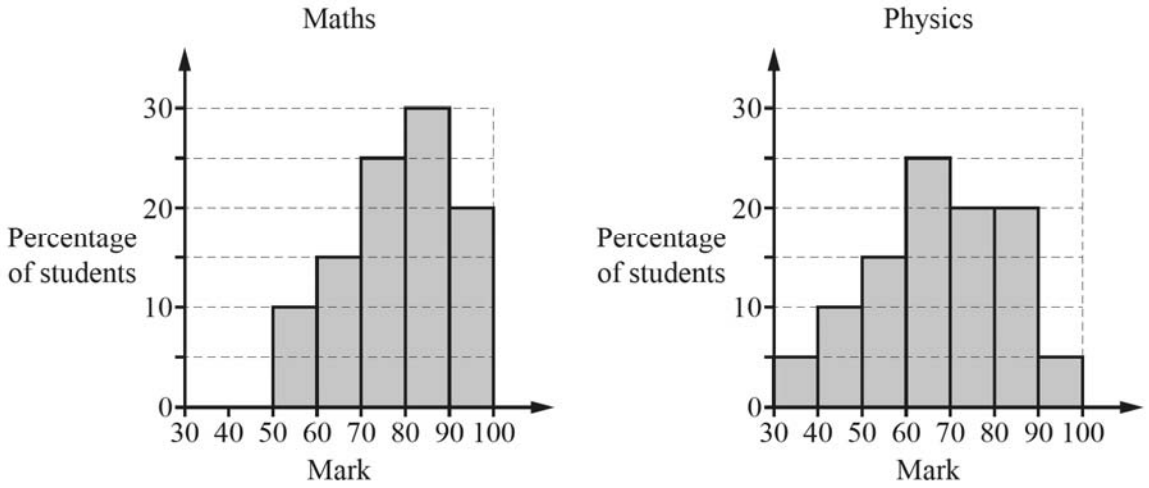
16 **(a)** Factorise completely $(x - 7)^3 - 4x + 28$.

Answer (a) ..... [3]

(b) Hence, state the smallest integer value of x for which $(x - 7)^3 - 4x + 28 > 0$.

Answer (b) $x =$ ..... [1]

17 The graphs show the distributions of the examination results, in Maths and Physics, of 20 students.



(a) What percentage of the students scored below 70 marks in Maths?

Answer (a) ..... % [1]

(b) Calculate an estimate of the mean mark for Maths.

Answer (b) ..... [1]

(c) Make **two** comparisons between the performances of the students in the two tests.

1 .....

.....

2 .....

.....

[2]

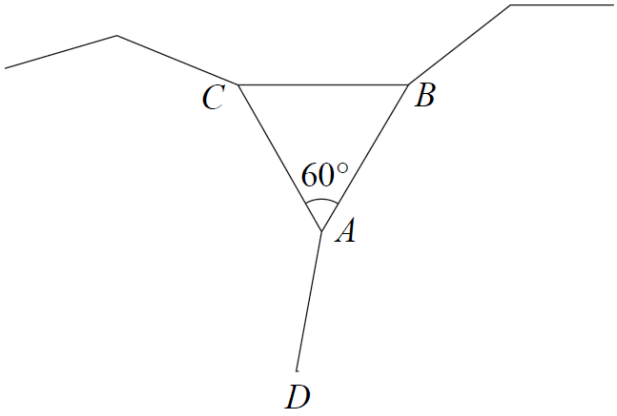
18 (a) It is possible to draw a regular polygon with an exterior angle of 50°.
Do you agree? Explain.

Answer (a) .....

..... [1]

(b) The sides of an equilateral triangle *ABC* and two regular polygons meet at *A*.
AB and *AD* are adjacent sides of a regular decagon.
AC and *AD* are adjacent sides of a regular *n*-sided polygon.

Find the value of *n*.



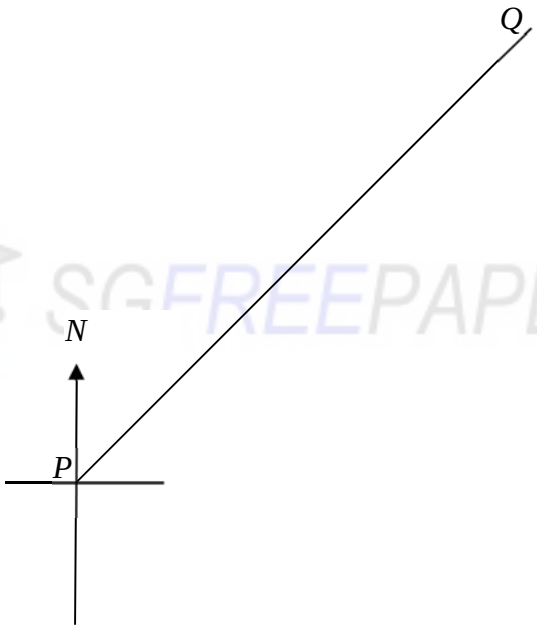
Answer (b) n = ... [3]

19 A boat starts from a point *P* and sails along the sides of a triangular circuit, *PQR*.

(a) The path *PQ* has been drawn.
The scale is 1 cm to 2 km.
Construct a scale drawing to represent the triangular circuit given that *R* is 15 km from *P* on a bearing of 116°. [1]

(b) A lighthouse, L is to be located within the circuit.
It is 12 km away from *Q* and equidistant from *PQ* and *PR*.
Mark and label the lighthouse on your diagram. [3]

Answer (a) and *(b)*



20 Ain invests some money into an account which pays a fixed rate of compound interest each year. The balance, \$*A*, of the account after *t* years is given by the formula

$$A = 1250 \times 1.045^t.$$

(a) Calculate *A* when $t = 4$.
Give your answer correct to the nearest ten cents.

Answer (a) $A =$ ..... [1]

(b) Using the answer in (a), calculate the percentage increase in Ain's savings after 4 years.

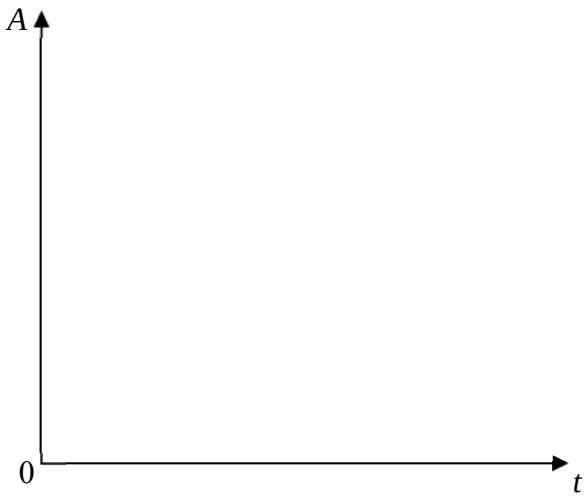
Answer (b) ..... % [2]

(c) Find the compound interest rate.

Answer (c) ..... % [1]

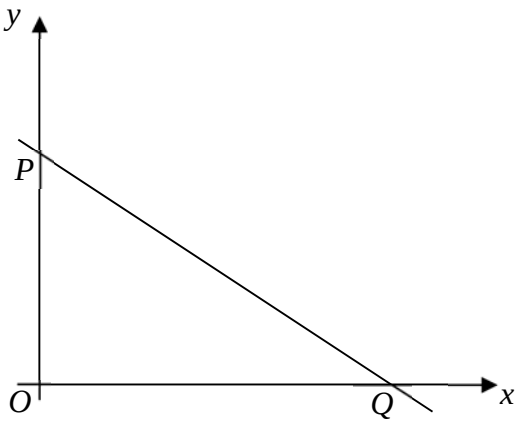
(d) Sketch the graph that represents the growth in Ain's account.

Answer (d)



[2]

21 The equation of the line $x + 2y = 12$ cuts the y- and x-axes at P and Q.



(a) Write down the gradient of PQ.

Answer (a) ..... [1]

(b) A point K lies on the line and is equidistant from x- and y-axes.
Find the coordinates of K.

Answer (b) ..... [1]

(c) **(i)** Find the gradient of the line that is perpendicular to PQ.

Answer (c)(i) ..... [1]

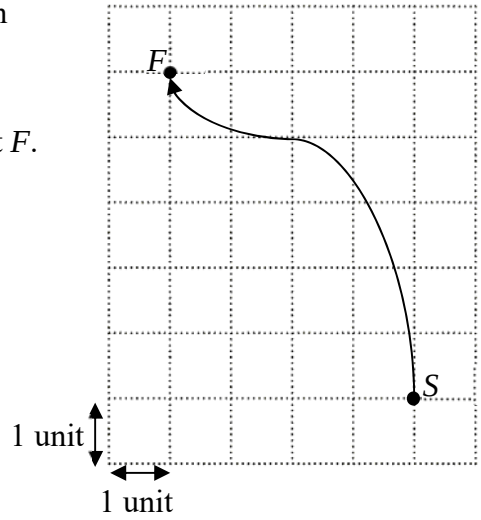
(ii) The line l is the perpendicular bisector of PQ.
Find the equation of the line l.

Answer (c)(ii) ..... [3]

22 **(a)** The diagram shows the map of part of an orienteering course.

Simon runs from the start, S, to the point F.

Write $\overrightarrow{SF}$ as a column vector.



Answer (a) $\overrightarrow{SF}$ = ... [1]

(b) $\overrightarrow{CD} = \begin{pmatrix} 2k \\ -k \end{pmatrix}$ and $|\overrightarrow{CD}| = \sqrt{180}$.
Find the possible values of k.

Answer (b) k = ... [2]

(c) Given that A is the point (1, 2), $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -5 \\ -4 \end{pmatrix}$, find

(i) $\overrightarrow{BC}$,

Answer (c)(i) $\overrightarrow{BC}$ = ... [1]

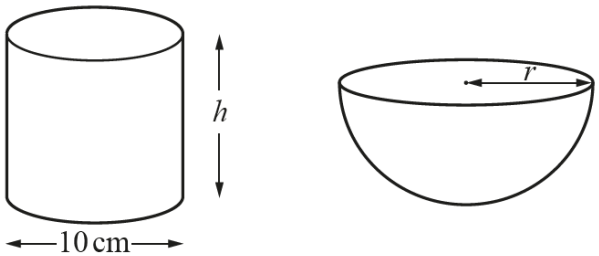
(ii) the coordinates of the point P such that $ABPC$ is a parallelogram.

Answer (c)(ii) ..... [2]

23 **(a)** The surface area of a solid is given by $A = \pi a(a+2b)$.
Make b the subject of the formula.

Answer (a) ..... [2]

(b) **(i)** The diagram shows a cylinder and a hemisphere.
The cylinder has diameter 10 cm and height h cm
and the hemisphere has radius r.



The volumes of the cylinder and hemisphere are equal.
Find r in terms of h.

Answer(b)(i) $r =$ ..... [3]

(ii) Find the smallest positive integer value of h such that r is a perfect cube.

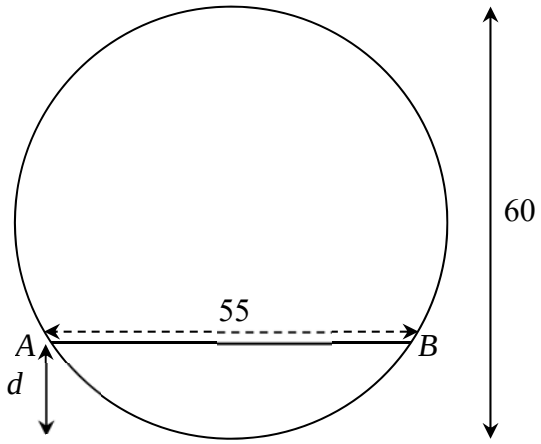
Answer(b)(ii) $h =$ ..... [1]

24 **(a)** A small pipe can fill up a tank in 120 minutes.
A large pipe can fill up the same tank in 80 minutes.

Find the time taken if the tank is to be filled up by one small and two large pipes at the same time.

Answer (a) ..... mins [2]

(b) Water flows through a horizontal water pipe of diameter 60 cm.
The surface width, *AB*, of the water is 55 cm.



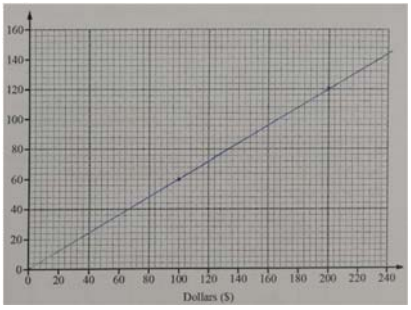
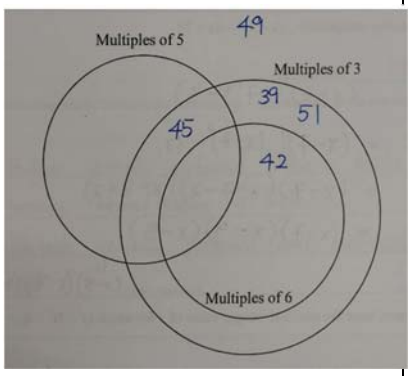
(i) Form an equation and calculate the depth, *d*, of the water in the pipe.

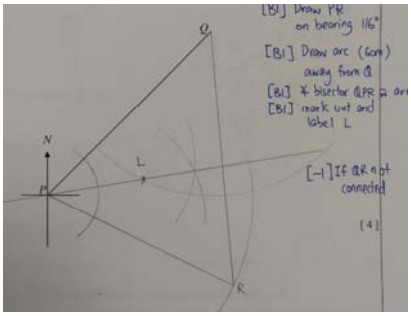
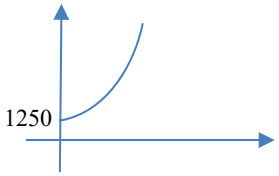
Answer (b)(i) d = ... cm [3]

(ii) Hence, state another depth of water that would give the same surface width.

Answer (b)(ii) ..... cm [1]

End of Paper

Qn No.		Answers
1		−5.25
2		$2 \leq \frac{f}{7} < 6$
3		31.5
4		$\frac{17y - 3}{14}$
5	a	1.49×10^8
	b	116
6		21, 23, 26, 27, 28
7		1.60
8		133.7
9	a	
	b	132
10	a	$\begin{pmatrix} 20 & 6 & 4 \\ 25 & 0 & 5 \end{pmatrix}$
	b	$\begin{pmatrix} 58 \\ 65 \end{pmatrix}$
	c	7
11	a	$2^2 \times 3^3 \times 5$
	b	54, 60
12	a	$\frac{x}{20}$
	b	$x = 5$
13	a	$\frac{x^4}{16y^{12}}$
	b	4
14	a	32
	b	
15		5.4
16	a	$(x - 5)(x - 7)(x - 9)$
	b	6
17	a	25%
	b	78.5
	c	Students *performed better* in Maths because higher median/mean marks. Students performed *more consistently* for Maths because of smaller range.
18	a	I disagree because $\frac{360}{55} = 6.545 \neq$ *whole number*/integer
	b	15

Qn No.		Answers
19		
20	a	$1490.60
	b	19.3
	c	4.5%
	d	
21	a	−0.5
	b	(4, 4)
	c	2
	d	$y = 2x - 9$
22	a	$\begin{pmatrix} -4 \\ 5 \end{pmatrix}$
	b	$k = \pm 6$
	c(i)	$\begin{pmatrix} -7 \\ -1 \end{pmatrix}$
	c(ii)	(−2, −5)
23	a	$b = \frac{A - \pi a^2}{2\pi a}$
	b(i)	$r = \sqrt[3]{\frac{75h}{2}}$
	b(ii)	90
24	a	30 mins
	b(i)	18.0
	b(ii)	42.0

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2018
Secondary 4

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS 4048/02

Paper 2 **Monday 27 August 2018**

2 hours 30 minutes

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This document consists of **11** printed pages and 1 blank page.

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r\ell$

Curved surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Solve the equation $\frac{2}{x^3} = -\frac{1}{32}$. [2]

(b) Simplify $\frac{8q - 12p + 2pq - 3p^2}{p^2 + 8p + 16}$. [3]

(c) Express $x^2 - 16x + 20$ in the form $(x + a)^2 + b$.
Hence, solve the equation $x^2 - 16x + 20 = 0$. [3]

(d) Given that $6x^2 - xy = 7y^2$, $x > 0$ and $y > 0$. Find the value of $\frac{12x}{y}$. [3]

2 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = 5x - 3 + \frac{1}{2x}.$$

The table below shows some values of x and the corresponding values of y correct to 2 decimal places.

x	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7
y	7.25	2.50	0.50	0.17	0.25	a	0.83	1.21

(a) Calculate the value of a. [1]

(b) Using a scale of 2 cm to represent 0.1 unit, draw a horizontal x-axis for $0 < x \leq 0.7$. Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $0 \leq y \leq 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

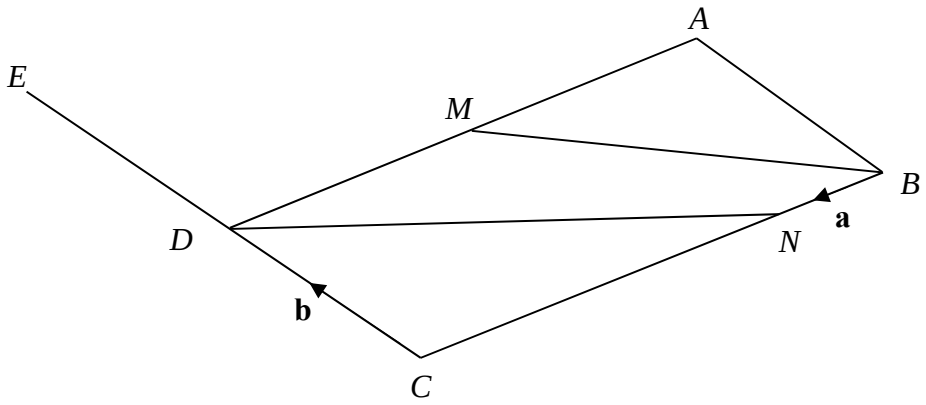
(c) By drawing a tangent, find the gradient of the curve at $(0.2, 0.5)$. [2]

(d) Use your graph to find the solutions of $10x^2 - 8x + 1 = 0$ in the range $0 < x \leq 0.7$. [2]

(e) Write down the coordinates of the points when the line $y = 4x + 2$ intersects the curve. [2]

(f) The equation $5x - 3 + \frac{1}{2x} = kx$ has only one solution in the range $0 < x \leq 0.7$.
Explain how the value of k can be obtained from your graph. [2]

3



ABCD is a parallelogram and *E* lies on *CD* produced such that *CD* = *DE*.
M is the midpoint of *AD*. *N* is a point on *BC* such that *BN* : *NC* = 1 : 3.
Given $\overrightarrow{BN}$ = **a** and $\overrightarrow{CD}$ = **b**,

(a) express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{AM}$, [1]

(ii) $\overrightarrow{BM}$, [1]

(iii) $\overrightarrow{BE}$. [1]

(b) State 2 facts about *B*, *M*, and *E*. [2]

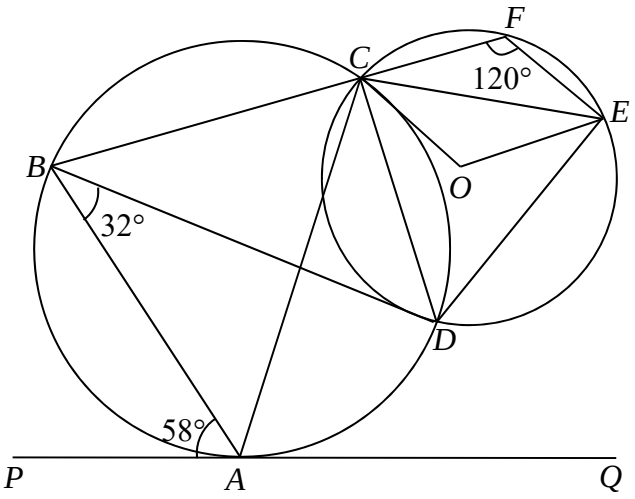
(c) Find the numerical value of

(i) $\dfrac{\text{area of } \Delta AMB}{\text{area of } \Delta DCN}$, [1]

(ii) $\dfrac{\text{area of } \Delta EDM}{\text{area of } DMBN}$. [1]

(d) Prove that triangles *EDM* and *ECB* are similar. [2]

4 **(a)** In the diagram, PAQ is a tangent to the circle $ABCD$ at A.
O is the centre of the circle $CDEF$ and BCF is a straight line.
It is given that $\angle PAB = 58°$, $\angle ABD = 32°$ and $\angle CFE = 120°$.



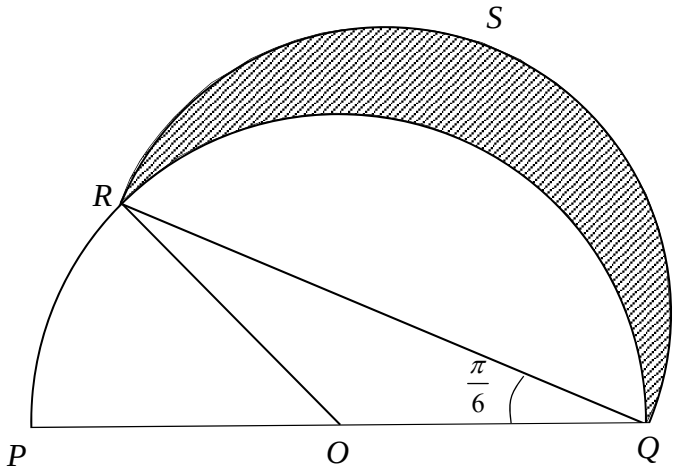
(i) Find, giving reason(s) for each answer,

(a) angle ACD, [1]

(b) angle ACB. [1]

(ii) Given that $FC = FE$, show that triangle CDE is equilateral. [3]

(b) The figure shows a semicircle, PRQ, with centre O and diameter PQ is 12 cm.
The chord QR makes an angle $\frac{\pi}{6}$ radian with the diameter PQ.
A second semicircle, RSQ is drawn, with QR as the diameter.



(i) Show that $QR = 6\sqrt{3}$ units. [2]

(ii) Find the perimeter of the shaded region. [3]

5 Alex plans to cycle from point A to point C which is 56 km apart.

He travels for 50 km, at a constant speed of x km/h until he reaches the point B, where he rested momentarily. The journey from A to B took y hours.

(i) Write down an equation in x and y, to represent the time taken to cycle from A to B. [1]

Alex then continues the remaining 6 km from B to C at a constant speed which is 16 km/h slower than his speed from A to B.

(ii) Given that the total time taken for the journey from A to C is 5 hours, form another equation in x and y and show that it simplifies to

$$y = \frac{5x - 86}{x - 16}.$$ [2]

(iii) Find the value(s) of x, correct to 2 decimal places. [4]

(iv) Calculate the time taken for Alex to cycle from point A to C, if he had completed the whole journey at the slower speed.
Give your answer in hours and minutes, correct to the nearest minute. [2]

6 The first four terms in the sequence of numbers are given below.

$$P_1 = 0^2 + 4 = 4$$

$$P_2 = 1^2 + 7 = 8$$

$$P_3 = 2^2 + 10 = 14$$

$$P_4 = 3^2 + 13 = 22$$

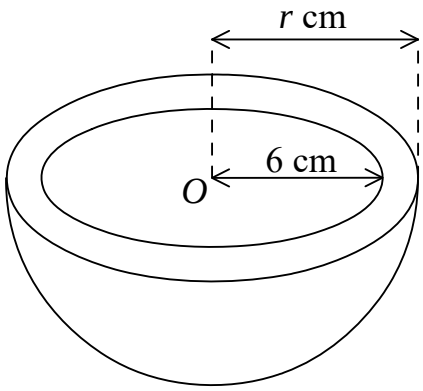
(a) State the value of P_5 and P_6. [2]

(b) The nth term of the sequence is P_n.
Find the expression of P_n in terms of n. [2]

(c) Explain why the value of P_n will never be an odd number for all values of n. [1]

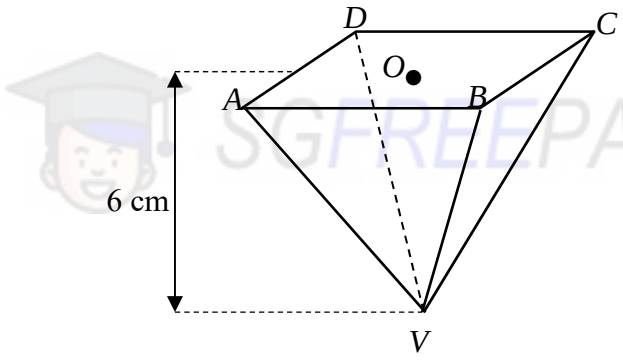
(d) P_n and P_{n+1} are two consecutive terms in the sequence.
Show that $P_{n+1} - P_n$ can be expressed into $2n + 2$. [2]

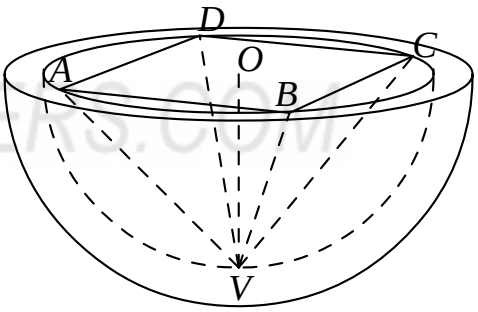
7 The diagram shows a hemispherical clay bowl with centre O.
The inner radius of the bowl is 6 cm and the outer radius is r cm



(a) Find the internal volume of the hemisphere with radius 6 cm. [2]

(b) Find the value of r if 408 cm^3 of clay is used to make the bowl. [2]





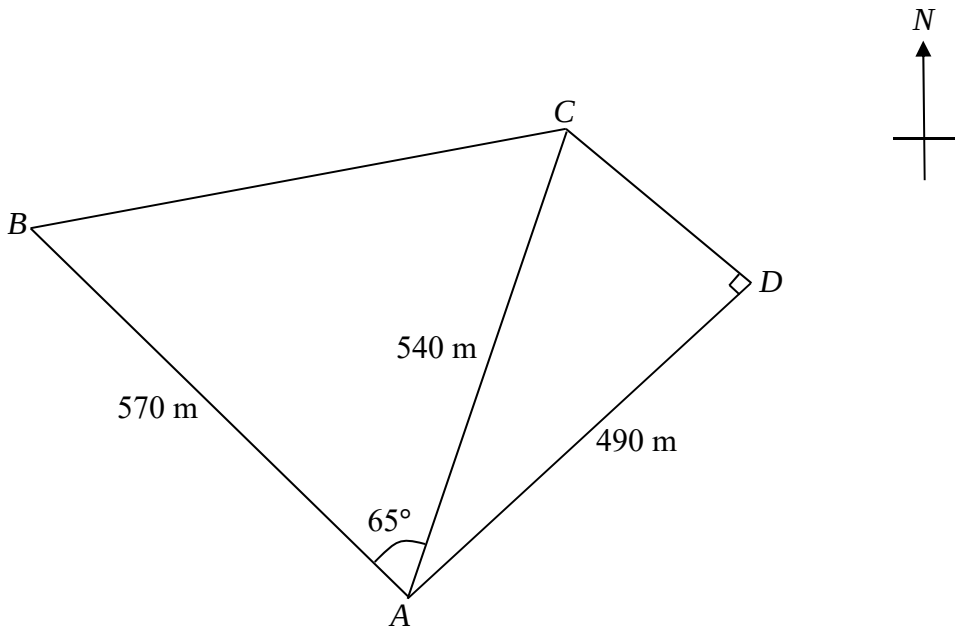
A solid pyramid with square base $ABCD$ and height OV, 6 cm, is placed in the bowl.
The points V, A, B, C and D touch the inner surface of the hemispherical bowl.

(c) Show that $AB = 6\sqrt{2}$ cm. [2]

Water is poured into the bowl to fill up the space between the pyramid and the clay bowl.
The pyramid is then removed from the bowl.

(d) Joe said that the height of the water in the bowl can be easily calculated by comparing volumes of similar solids.
Explain whether you agree or disagree with Joe. [2]

8 The diagram shows a field $ABCD$ on horizontal ground, crossed by a path AC.
$AB = 570$ m, $AC = 540$ m and $AD = 490$ m.
$B\widehat{A}C = 65°$, $C\widehat{D}A = 90°$ and the bearing of C from B is 079°.



(a) Find

(i) BC, [3]

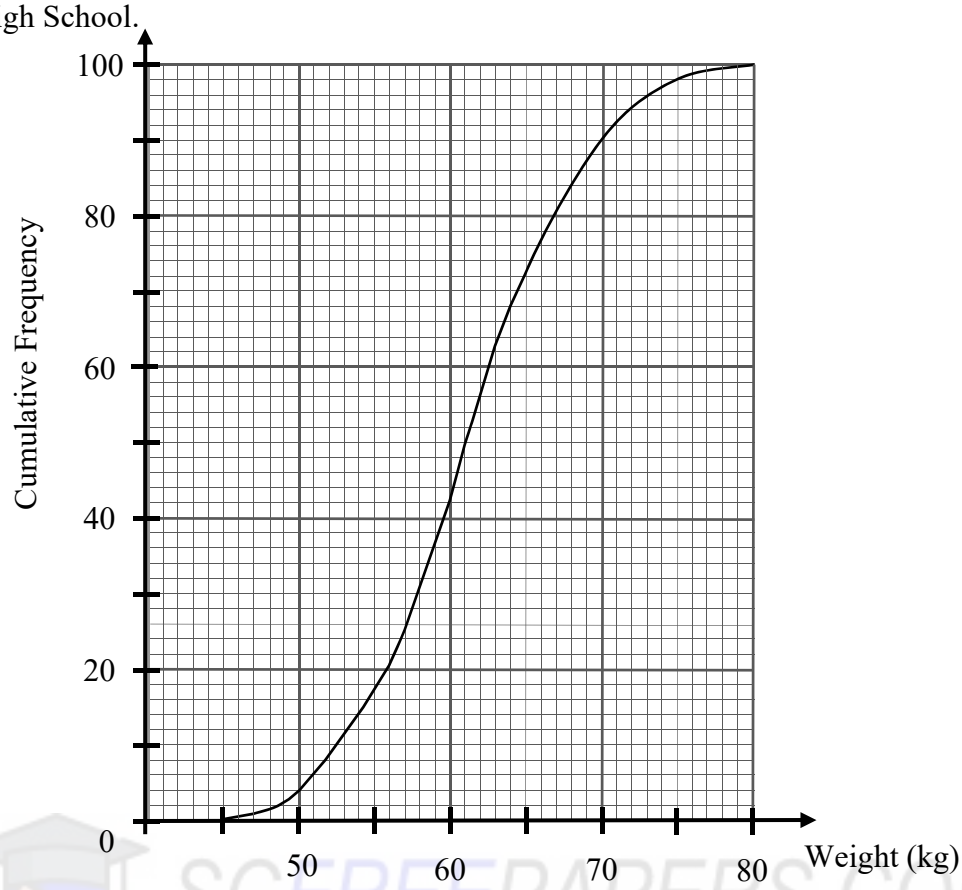
(ii) $B\widehat{C}A$, [2]

(iii) the bearing of A from C. [2]

(b) A drone is hovering vertically above point D.
The angle of depression of A from the drone is 2.6°.
Find the angle of depression of C from the drone. [4]

(c) The land is valued at \$45 000 per hectare.
Given that 1 hectare = 10 000 square metres, calculate the value of the field. [3]

9 The cumulative frequency curve below illustrates the weights of 100 students in Senoko High School.



(a) Use the graph to find

(i) the median weight of the students, [1]

(ii) the interquartile range. [2]

(b) The weights of 100 students in Changi High School have a higher median but a smaller interquartile range.
Describe how the cumulative frequency curve for Changi High School will differ from the curve for Senoko High School. [2]

(c) The table shows the distribution of ages for 200 students from both Senoko and Changi High School.

Age (x years)	$13 \leq x < 14$	$14 \leq x < 15$	$15 \leq x < 16$	$16 \leq x < 17$
Senoko High School	32	14	24	30
Changi High School	27	20	31	22

(i) One of the students is selected at random.
Find, as a fraction in its lowest terms, the probability that the student is
(a) a student from Changi High School who is aged 15 or more, [1]
(b) aged under 14. [1]

(ii) Two of the students are selected at random.
Find the probability that both are from Senoko High School aged under 16. [2]

10 Peter plans to buy a new car. He must successfully obtain the Certificate of Entitlement (COE) through bidding before he can own a car. Information on the current COE prices, quota and bids received for the different category of cars are in **Table 1**.

Table 1:

CAT A Cars up to 1600cc and 130bhp [View Past CAT A Results]

Quota Premium	Change	Quota	Bids Received
$25,000	⬇ $9,110	1,435	1,626

CAT B Cars above 1600cc or 130bhp [View Past CAT B Results]

Quota Premium	Change	Quota	Bids Received
$31,000	⬇ $2,900	1,288	1,637

Peter has shortlisted two cars. The specification and price details are in the **Table 2**.

Table 2:

Brand of car	**Phantom Series X**	**Sky Hawk V**
Engine capacity (cc)	1496	1598
Fuel type	Diesel (Euro V)	Petrol
Power (bph)	114	165
Fuel consumption (km/l)	23.8	17.8
CO_2 emission (g/km)	110	130
Car Price (S$) **excludes VES rebate / surcharge*	152, 888	147, 999
OMV (S$)	31, 410	26, 239
Road Tax per 6 months (S$) **excludes Special Tax if any*	372	372

(i) Peter said that he has a higher chance of obtaining COE for brand Phantom as compared to Sky Hawk. Do you agree? Explain why. [1]

A special tax is levied on diesel cars and is payable in addition to the Road Tax of the vehicle. The charge is S$0.20 per cc for 6 months.

(ii) Find the total amount of tax payable for 6 months for brand Phantom. [2]

Car buyers can either be granted rebates or imposed surcharge based on how clean the vehicle's emissions are. Vehicle Emission Scheme (VES) is based on a vehicle's carbon dioxide (CO_2) emissions, plus emissions of other pollutants.

Table 3:

Bands	CO_2 (g/km)	HC (g/km)	CO (g/km)	NO_x (g/km)	Rebate/ surcharge(-/+) for cars ($)
A1	A1 ≤90	A1 ≤0.020	A1 ≤0.150	A1 ≤0.007	-20,000
A2	90< A2 ≤125	0.020< A2 ≤0.036	0.150< A2 ≤0.190	0.007< A2 ≤0.013	-10,000
B	125< B ≤160	0.036< B ≤0.052	0.190< B ≤0.270	0.013< B ≤0.024	0
C1	160< C1 ≤185	0.052< C1 ≤0.075	0.270< C1 ≤0.350	0.024< C1 ≤0.030	+10,000
C2	C2 >185	C2 >0.075	C2 >0.350	C2 >0.030	+20,000

(iii) Use **Tables 2** and **3** to determine the price of brand Phantom car, **including** the VES rebate / surcharge, if any. [1]

Peter decided to take a 5-year bank loan for purchase of the car.
The interest rate is at 2.78% per annum.

The Maximum Loan Amount will be dependent on the Open Market Value (OMV) of the car.

Cars with OMV exceeding $20,000 will be entitled to a maximum loan value of 60% of car price with minimum 40% down payment.

(iv) Calculate the minimum down payment Peter has to pay if he decides to buy brand Phantom. [2]

(v) Peter decides to take a 60% loan. Suggest which car Peter should buy. Justify the decision you make and show your calculations clearly. [5]

END OF PAPER

Qn		Solutions
1	a	x = - 4
	b	$\frac{(2q-3p)}{(p+4)}$
	c	x = 14.6 or 1.37
	d	$\frac{12x}{y} = 14$
2	a	a = 0.5
	b	All points correctly plotted Graph is smooth
	c	Tangent line drawn Gradient = -11.5 (~ -7 to -14)
	d	When y = 1, x = 0.17 or 0.65 Accept 0.165~1.7 and 0.6~0.65
	e	Draw line y = 4x + 2 (0.1, 2.45)
	f	The value of k is obtained by finding the gradient of the line that passes through the origin and that cuts the curve once.
3	a(i)	$\overrightarrow{AM} = 2a$
	a(ii)	$\overrightarrow{BM} = 2a + b$
	a(iii)	$\overrightarrow{BE} = 4a + 2b$
	b	B, M and E are collinear 2BM = BE
	c(i)	$\frac{2}{3}$
	c(ii)	$\frac{2}{3}$
	d	Since ABCD is a parallelogram, Angle EDM = angle ECB (corr angles) As CD = DE, $\frac{ED}{EC} = \frac{1}{2}$ As M is midpoint AD, $\frac{DM}{CB} = \frac{1}{2}$ ∴ΔEDM is similar to ΔECB
4	a(i)a	∠ACD = 32° (∠ in same seg)
	a(i)b	∠ACB = 58° (∠ in alt seg)
	a(ii)	∠CDE = (180 – 120)° (∠ in opp seg) = 60° ∠BCD = (58+32) ° = 90° ∠FCE = (180 – 120)° ÷ 2 (isos Δ) = 30°

Qn		Solutions
		∠DCE = (90 – 30) ° = 60° ∴ $\angle CED = 60°$ ∴ ΔCDE is equilateral
	b(i)	∠PRQ = 90° (∠ in semicircle) cos $\frac{\pi}{6} = \frac{RQ}{12}$ $\frac{\sqrt{3}}{2} = \frac{RQ}{12}$ RQ = $6\sqrt{3}$ (shown)
	b(ii)	28.89 cm
5	(i)	y = $\frac{50}{x}$
	(ii)	5 – y = $\frac{6}{x-16}$ 5 – $\frac{6}{x-16}$ = y $\frac{5x-80-6}{x-16}$ = y $\frac{5x-86}{x-16}$ = y (shown)
	(iii)	x= 18.59 or 8.60 (2dp)
	(iv)	21 h 34 mins
6	(a)	$P_5 = 4^2 + 16 = 32$ $P_6 = 5^2 + 19 = 44$
	(b)	$P_n = n(n+1) + 2$
	c	For all values of n, n(n+1) is an even value.
	d	$P_n = n(n+1) + 2$ $P_{n+1} = (n+1)(n+2) + 2$ $P_{n+1} - P_n$ = (n+1)(n+2) + 2 - n(n+1) – 2 = (n+1)(n+2-n) = (n+1)(2) = 2n + 2 (shown)
7	(a)	452.389 cm^3
	(b)	r = 7.4338
	(c)	OV = OA = 6cm OA = OB $6^2 + 6^2 = AB^2$ AB = $\sqrt{72}$ = $6\sqrt{2}$
	(d)	Disagree. The volume of water in the bowl is not is a shape of a hemisphere (h ≠ r) or The volume of water and the volume of the bowl are not similar figures since h

Qn		Solutions
		$\neq r$.
8	a(i)	BC =596.939 cm
	(ii)	BCA = 59.929°
	(iii)	199.07°
	b	$\theta = 5.6°$
	c	\$878 000 (3sf)
9	a(i)	Median = 62.5kg
	a(ii)	IQR = 65.5 – 57 = 8.5 kg
	b	The curve will shift to the right of the curve for Senoko as the median is higher. The middle 50% of the curve will be steeper than for Senoko as the IQR is smaller.
	c(i)a	$\frac{53}{200}$
	c(i)b	$\frac{59}{200}$
	c(ii)	$\frac{70}{200} \times \frac{69}{199}$ $= \frac{483}{3980}$
10	(i)	P(brand Phantom) = $\frac{1435}{1626} \times 100\% = 88.3\%$ P(brand Sky Hawk) = $\frac{1288}{1637} \times 100\% = 78.7\%$ I agree with Peter.
	(ii)	\$671.20
	(iii)	\$142 888
	(iv)	\$571 55. 20
	(v)	<u>For Phantom:</u> Interest = 0.6 x their (iii) x 2.78 x 5 = \$11 916.86 Total repayment (balance + interest) = 0.6 x their (iii) + \$11 916.86 = \$97 649.66 Monthly instalment = \$1627.50 For Sky Hawk: Interest = 0.6 x 147 999 x 2.78 x 5 = \$12 343.12 Total repayment (balance + interest) = 0.6 x 147 999 + \$12 343.12 = \$101 142.52 Monthly instalment = \$1685.71 Since the monthly instalment is lower / total repayment amount with interest is lower, brand Phantom is a better buy.

Qn		Solutions	Marks	Remarks
1	a	$\frac{2}{x^3} = -\frac{1}{32}$ $64 = -x^3$ $x = -4$	 B1 B1	
	b	$\frac{8q - 12p + 2pq - 3p^2}{p^2 + 8p + 16}$ $= \frac{2q(4+p) - 3p(4+p)}{(p+4)^2}$ $= \frac{(2q - 3p)(4+p)}{(p+4)^2}$ $= \frac{(2q - 3p)}{(p+4)}$	 M1 M1 A1	Factorise denominator Factorise numerator
	c	$x^2 - 16x + 20 = (x-8)^2 - 44$ $(x-8)^2 - 44 = 0$ $x - 8 = \pm\sqrt{44}$ $x = 14.6$ or 1.37	B1 M1 A1	Both correct answer
	d	$6x^2 - xy = 7y^2$ $6x^2 - xy - 7y^2 = 0$ $(6x - y)(x + y) = 0$ Since $x + y > 0$, $6x = 7y$ $\frac{x}{y} = \frac{7}{6}$ $\frac{12x}{y} = 14$	 M1 M1 A1	Factorise quadratic Show ratio of $\frac{x}{y}$
			11m	
2	a	$a = 0.5$	B1	
	b	All points correctly plotted Graph is smooth	P2 G1	P1 if 1 or more point is missing / wrongly plotted
	c	Tangent line drawn Gradient = -11.5	T1 B1	Accept range from -7 to -14.6
	d	$10x^2 - 8x + 1 = 0$ $5x - 4 + \frac{1}{2x} = 0$ $5x - 3 + \frac{1}{2x} = 1$ When $y = 1$, $x = 0.17$ or 0.65	 B1 B1	
	e	Draw line $y = 4x + 2$ $(0.1, 2.45)$	L1 B1	
	f	The value of k is obtained by finding the gradient of the line that cuts the curve once.	B1 B1	"use gradient" "intersect the curve"
			12m	

Qn		Solutions	Marks	Remarks
3	a(i)	$\overrightarrow{AM} = 2a$	B1	
	a(ii)	$\overrightarrow{BM} = 2a + b$	B1	
	a(iii)	$\overrightarrow{BE} = 4a + 2b$	B1	
	b	$\overrightarrow{BE} = 2(2a + b)$ $\overrightarrow{BE} = 2\overrightarrow{BM}$ B, M and E are collinear 2BM = BE	 B1 B1	
	c(i)	$\frac{2}{3}$	B1	
	c(ii)	$\frac{2}{3}$	B1	
	d	Since ABCD is a parallelogram, Angle EDM = angle ECB (corr angles) As CD = DE, $\frac{ED}{EC} = \frac{1}{2}$ As M is midpoint AD, $\frac{DM}{CB} = \frac{1}{2}$ ∴ΔEDM is similar to ΔECB	 B1 B1	
			9m	
4	a(i)a	∠ACD = 32° (∠ in same seg)	B1	
	a(i)b	∠ACB = 58° (∠ in alt seg)	B1	
	a(ii)	∠CDE = (180 – 120)° (∠ in opp seg) = 60° ∠BCD = (58+32) ° = 90° ∠FCE = (180 – 120)° ÷ 2 (isos Δ) = 30° ∠DCE = (90 – 30) ° = 60° ∴ $\angle CED = 60°$ ∴ ΔCDE is equilateral	B1 B1 B1	
	b(i)	∠PRQ = 90° (∠ in semicircle) $\cos\frac{\pi}{6} = \frac{RQ}{12}$ $\frac{\sqrt{3}}{2} = \frac{RQ}{12}$ RQ = $6\sqrt{3}$ (shown)	B1 B1 CAG	soi $\frac{\sqrt{3}}{2}$ seen
	b(ii)	∠ROQ = $\pi - 2(\frac{\pi}{6})$ = $\frac{2\pi}{3}$ Perimeter = $6(\frac{2\pi}{3}) + \frac{1}{2}\pi(6\sqrt{3})$ = 28.891 cm	 M1, B1 A1	Find Arc length RQ and RSQ
			10m	

Qn		Solutions	Marks	Remarks
5	(i)	$y = \frac{50}{x}$	B1	o.e.
	(ii)	$5 - y = \frac{6}{x-16}$ $5 - \frac{6}{x-16} = y$ $\frac{5x-80-6}{x-16} = y$ $\frac{5x-86}{x-16} = y$ (shown)	B1 B1 CAG	o.e. combine fraction
	(iii)	$\frac{5x-86}{x-16} = \frac{50}{x}$ $50x - 800 = 5x^2 - 86x$ $5x^2 - 136x + 800 = 0$ $x = \frac{-(-136) \pm \sqrt{(-136)^2 - 4(5)(800)}}{2(5)}$ x= 18.59 or 8.60 (2dp)	B1 √ M1 M1 A1	Equate (i) and (ii) Remove fraction Any method to solve seen
	(iv)	Slower speed = 18.59 -16 = 2.59599 Time taken = 56 ÷ 2.59599 = 21. 57 hrs = 21 h 34 mins	 M1 A1	
			9m	
6	(a)	$P_5 = 4^2 + 16 = 32$ $P_6 = 5^2 + 19 = 44$	B1 B1	
	(b)	$P_n = (n-1)^2 + 3n + 1$ $= n^2 - 2n + 1 + 3n + 1$ $= n^2 + n + 2$ $= n(n+1) + 2$	B1, B1	$(n-1)^2$ seen & 3n + 1 seen
	c	For all values of n, n(n+1) is an even value.	B1	
	d	$P_n = n(n+1) + 2$ $P_{n+1} = (n+1)(n+2) + 2$ $P_{n+1} - P_n$ $= (n+1)(n+2) + 2 - n(n+1) - 2$ $= (n+1)(n+2-n)$ $= (n+1)(2)$ $= 2n + 2$ (shown)	 B1 B1	o.e. Leading to CAG
			7m	
7	(a)	Vol hemisphere $= \frac{1}{2}(\frac{4}{3}\pi 6^3)$ $= 144\pi$ $= 452.389$ cm^3	B1 B1	
	(b)	$\left(\frac{r}{6}\right)^3 = \frac{144\pi + 408}{144\pi}$ r = 7.4338	M1 A1	
	(c)	OV = OA = 6cm OA = OB $6^2 + 6^2 = AB^2$ $AB = \sqrt{72} = 6\sqrt{2}$	B1 B1 CAG	
	(d)	Disagree. The volume of water in the bowl is not is a shape of a hemisphere ($h \neq r$) or The volume of water and the volume of the bowl are not similar figures since $h \neq r$.	B1 B1	
			8m	

Qn		Solutions	Marks	Remarks
8	a(i)	$BC^2 = 570^2 + 540^2 - 2(570)(540)\cos 65°$ BC =596.939 cm	M2 A1	
	(ii)	$\frac{\sin BCA}{570} = \frac{\sin 65°}{596.939}$ BCA = 59.929°	M1 A1	
	(iii)	360° – 59.929° – (180 – 79) ° = 199.07°	M1 A1	
	b	tan 2.6° = $\frac{HD}{490}$ HD = 22.2508 cm $CD^2 = 540^2 - 490^2$ CD = 226.936 cm Tan $\theta = \frac{22.2508}{226.936}$ $\theta = 5.6°$	M1 A1 B1 A1	
	c	Area of land = 0.5(490)(226.936) + 0.5(570)(540)sin65 = 195080 sq meter = 19.5 hectare Value of land = 19.5 x 45 000 = $877 860	M1 M1 A1	
			13m	
9	a(i)	Median = 62.5kg	B1	
	a(ii)	IQR = 65.5 – 57 = 8.5 kg	M1 A1	soi
	b	The curve will shift to the right of the curve for Senoko as the median is higher. The middle 50% of the curve will be steeper than for Senoko as the IQR is smaller.	B1 B1	
	c(i)a	$\frac{53}{200}$	B1	
	c(i)b	$\frac{59}{200}$	B1	
	c(ii)	$\frac{70}{200} \times \frac{69}{199}$ $= \frac{483}{3980}$	M1 A1	Accept 0.121
			9m	

Qn		Solutions	Marks	Remarks
10	(i)	P(brand Phantom) = $\frac{1435}{1626} \times 100\% = 88.3\%$ P(brand Sky Hawk) = $\frac{1288}{1637} \times 100\% = 78.7\%$ I agree with Peter.	B1	
	(ii)	\$372 + (0.20 x 1496) = \$671.20	B1 B1	0.2 x 1496 seen
	(iii)	\$152 888 - \$10 000 = \$142 888	B1	
	(iv)	0.4 x 142 888 = \$571 55. 20	B1 B1	0.4 x their (iii)
	(v)	<u>For Phantom:</u> Interest = 0.6 x their (iii) x 2.78 x 5 = \$11 916.86 Total repayment (balance + interest) = 0.6 x their (iii) + \$11 916.86 = \$97 649.66 Monthly instalment = \$1627.50 For Sky Hawk: Interest = 0.6 x 147 999 x 2.78 x 5 = \$12 343.12 Total repayment (balance + interest) = 0.6 x 147 999 + \$12 343.12 = \$101 142.52 Monthly instalment = \$1685.71 Since the monthly instalment is lower / total repayment amount with interest is lower, brand Phantom is a better buy.	B1 B1 B1 B1 B1	Find interest Find total (balance + interest) Find interest Find total (balance + interest) Conclusion with justification.
			11m	

NAME:		( )	CLASS:	

YISHUN TOWN SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
MATHEMATICS PAPER 1 (4048/01)

DATE : **3 August 2018** **DAY** : **Friday**

DURATION : **2 h** **MARKS:** **80**

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.
Write your name, class and class index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/ tape.
Write your answers on the writing papers provided.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

80

This question paper consists of **20** printed pages including this cover page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $3-4(a-6)$.

Answer ..... [1]

(b) Factorise completely $2x^2-18$.

Answer ..... [1]

---

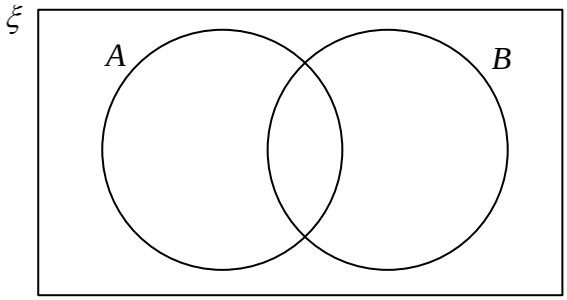
2 Factorise completely $6bd-9ad+3ac-2bc$.

Answer ..... [2]

---

3 $\xi = \{\text{integers } x : 1 \le x \le 16\}$
$A = \{\text{factors of } 12\}$
$B = \{\text{prime numbers}\}$

(a) On the Venn diagram, shade the region which represents $A' \cap B$.



[1]

(b) List the elements in $A' \cap B$.

Answer ..... [1]

---

4 If $(a-b)^2 = 19$ and $(a+b)^2 = 36$, find the value of $8ab$.

Answer ..... [2]

---

5 Kelly has 300 one-centimetre cubes.
She arranges all of the cubes into a cuboid.
The perimeter of the top of the cuboid is 18 cm.
Each side of the cuboid has a length greater than 3 cm.

Find the height of the cuboid.

Answer .....cm [2]

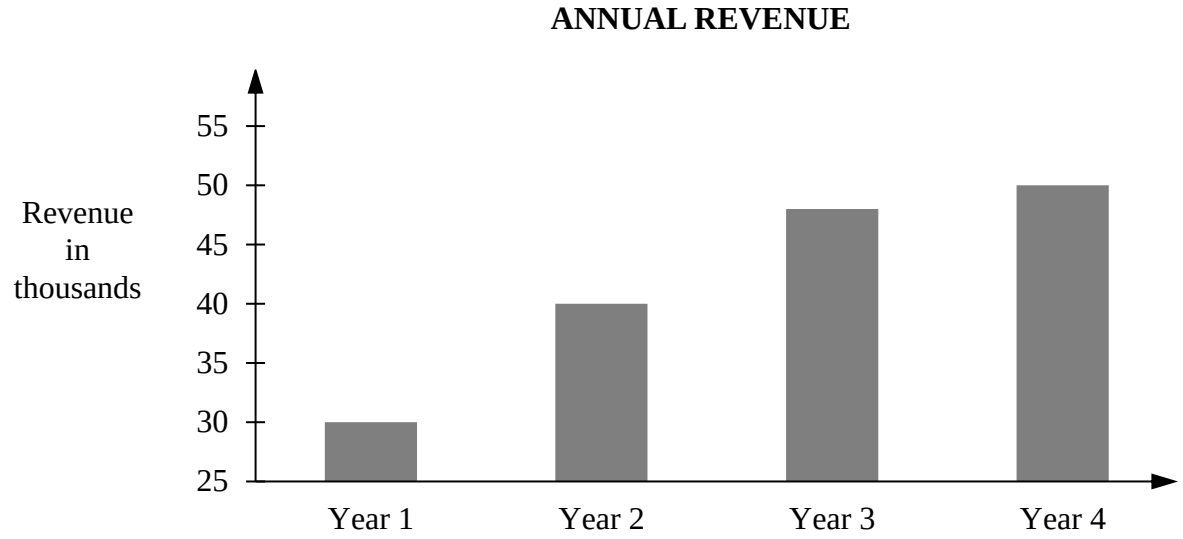
---

6 Write as a single fraction in its simplest form $\frac{7}{(3x-1)^2} - \frac{5}{1-3x}$.

Answer ..... [2]

---

7 The bar chart shows the revenue of a company in its first four years of operation.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer……

……

……[2]

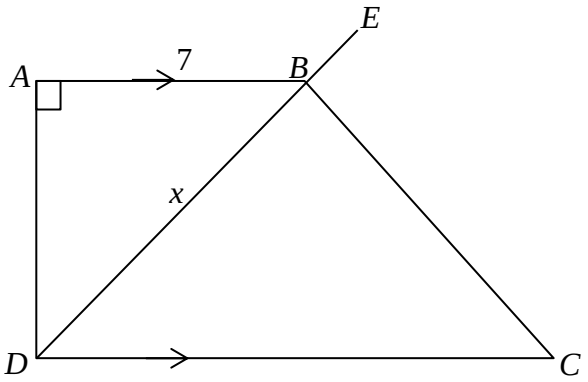
---

8 A quadratic graph cuts the x-axis at $x = -2$ and $x = 3$.
The quadratic graph also passes through a point with coordinates $(-3, 5)$.

Find the equation of the quadratic graph.

Answer ..... [2]

9



ABCD is a trapezium.
Angle $BAD = 90°$.
DBE is a straight line.
AB = 7 cm and *DB* = *x* cm.

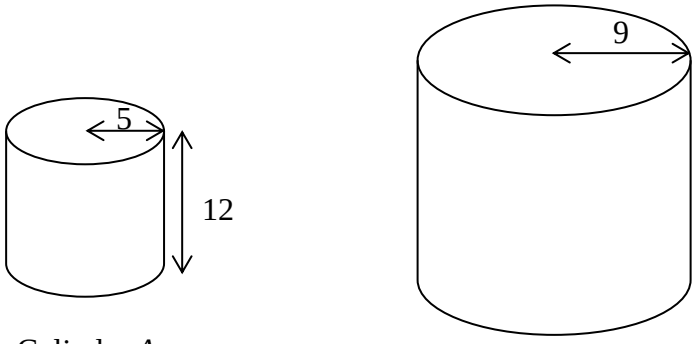
(a) Write down an expression, in terms of *x*, for $\cos A\hat{B}E$.

Answer $\cos A\hat{B}E =$ …….. [1]

(b) The area of the trapezium *ABCD* is 3 times the area of the triangle *ABD*.
Find the length of *DC*.

Answer ……cm [2]

10



Cylinder *A* Cylinder *B*

The diagram shows two similar cylinders, Cylinder *A* and Cylinder *B*.
These cylinders are made with the same material.
The radius of Cylinder *A* is 5 cm and the radius of Cylinder *B* is 9 cm.
The height of Cylinder *A* is 12 cm.

(a) Find the height of Cylinder *B*.

Answer ……cm [1]

(b) If the mass of Cylinder *A* is 250 g, find the mass of Cylinder *B*.

Answer ……...g [2]

11 Peter plans to save $25 000 in a bank for 5 years.
The bank offers him two investment plans.

Plan A: The bank pays 5.5% simple interest per year for 5 years.
Plan B: The bank pays 5% interest compounded yearly for 5 years.

Which plan should Peter choose?
You must show your calculations.

Answer Peter should choose Plan ……because ……
……[3]

---

12 The safe speed, v m/s, at which a train can turn at a circular bend is directly proportional to the square root of the radius, r m of the circular bend. The safe speed of the train is 22 m/s when it turns a circular bend of radius 121 m.

(a) Calculate the safe speed for a circular bend of radius 81 m.

Answer .....m/s [2]

(b) Calculate the radius of the circular bend if the safe speed is 28 m/s.

Answer ……..m [1]

---

13 **(a)** Express 462 as a product of its prime factors.

Answer ……...…….. [1]

(b) Written as a product of its prime factors, $1512 = 2^3 \times 3^3 \times 7$.

(i) Find the smallest whole number n for which 462n is a multiple of 1512.

Answer $n =$…… [1]

(ii) Explain why 1512 is not a perfect cube.

Answer ……

…… [1]

---

14 A map of an airport has a scale of 1: 50 000.
The length of the airport runway on the map is 2.5 cm.

(a) Calculate the actual length, in metres, of the airport runway.

Answer …… m [2]

(b) The actual area of the airport is 3.75 km^2.
Calculate the area, in square centimetres, of the airport on the map.

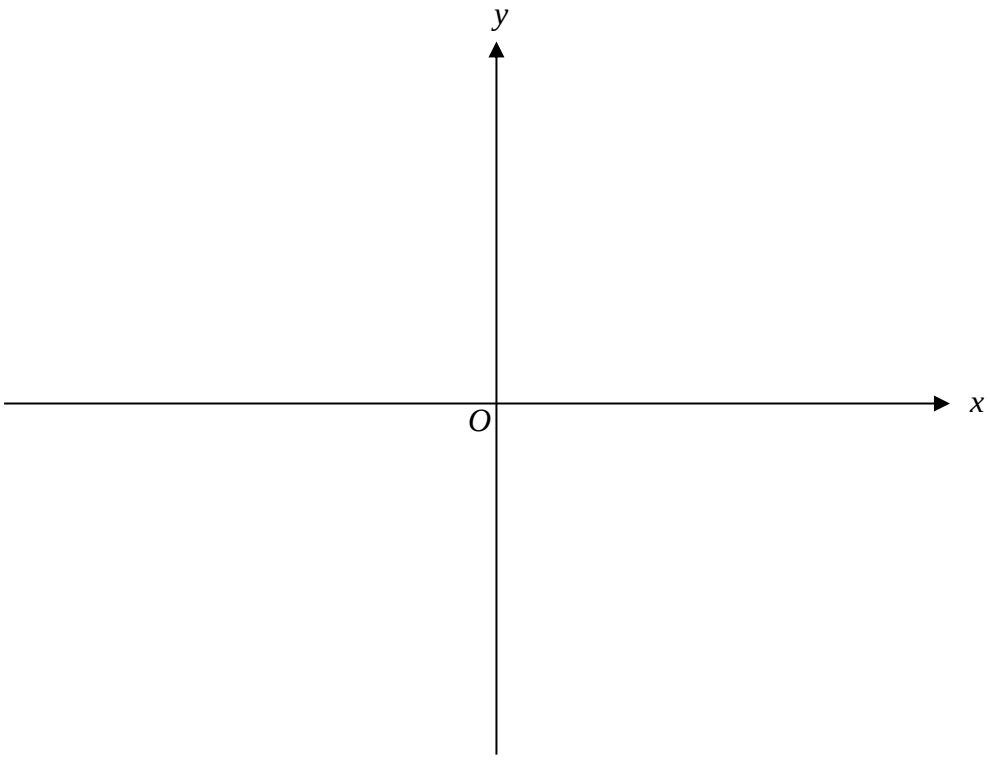
Answer …… cm^2 [2]

---

15 **(a)** Express $x^2 - 4x - 7$ in the form $(x - p)^2 - q$.

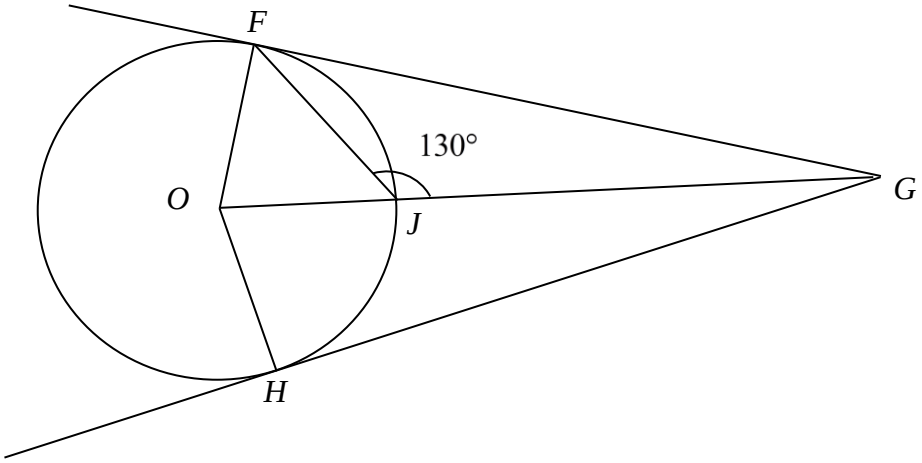
Answer ..... [2]

(b) **Sketch** the graph of $y = x^2 - 4x - 7$.
Indicate clearly the coordinates of the turning point and the points where the graph crosses the *x*- and *y*- axes.



[2]

16



In the diagram, *FG* and *HG* are tangents to the circle with centre *O* and *OJG* is a straight line.
Angle *FJG* = 130°.

Show your working and give reasons.

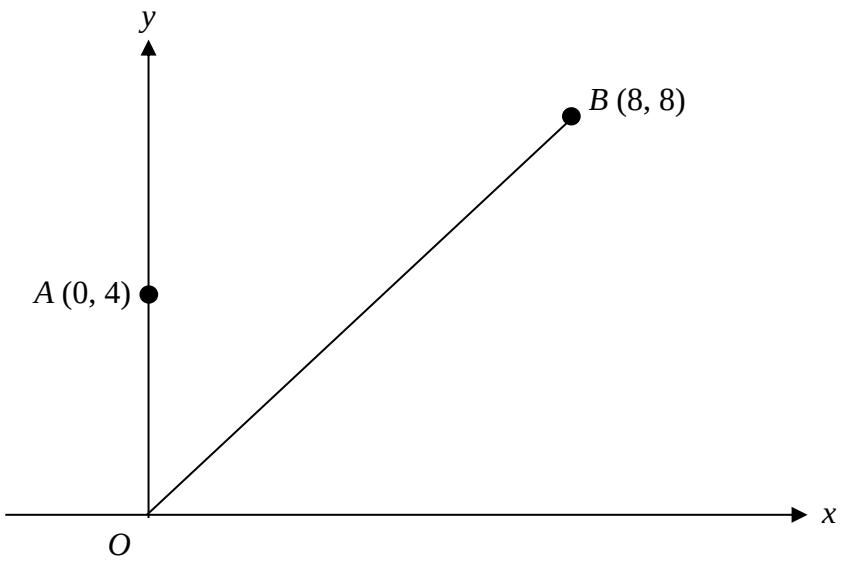
(a) Find angle *JFG*.

Answer .....……...…….° [3]

(b) Find angle *FGH*.

Answer .....……...…….° [2]

17 A is the point (0, 4) and B is the point (8, 8).



(a) Find the equation of line AB.

Answer ……. [2]

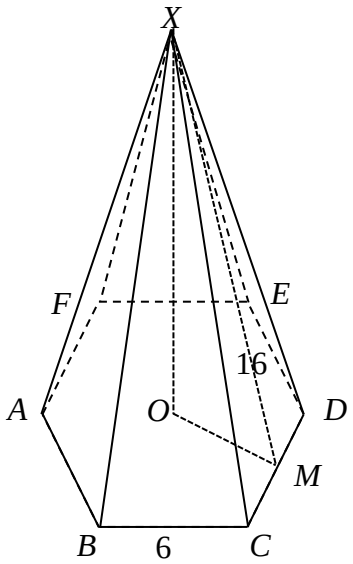
(b) Given that $ABCO$ is a kite and AC is a diagonal of the kite, find the coordinates of point C.

Answer (...…… , ……...……) [1]

(c) Find the area of kite $ABCO$.

Answer .....……..units^2 [2]

18



A regular hexagon *ABCDEF* of sides 6 cm forms the base of a crystal pyramid.
M is the midpoint of *DC* and *O* is the centre of the hexagon.
The vertex, *X*, is directly above *O*.
The slant height, *MX*, of the pyramid is 16 cm.

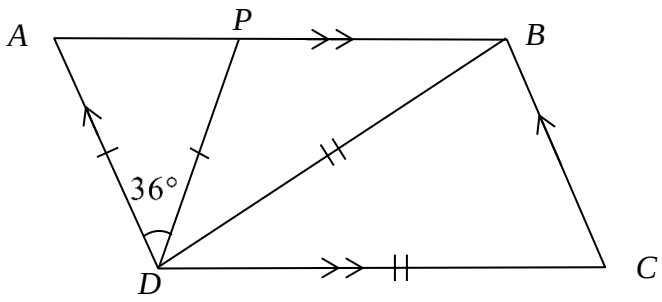
(a) Find the height, *OX*, of the pyramid.

Answer $OX =$……..cm [2]

(b) The mass of 1 cubic centimetre of crystal is 3.2 grams.
The price of 1 gram of the crystal is $9.75.
Calculate the price of the crystal pyramid.
Give your answer to the nearest dollar.

Answer $……... [4]

19

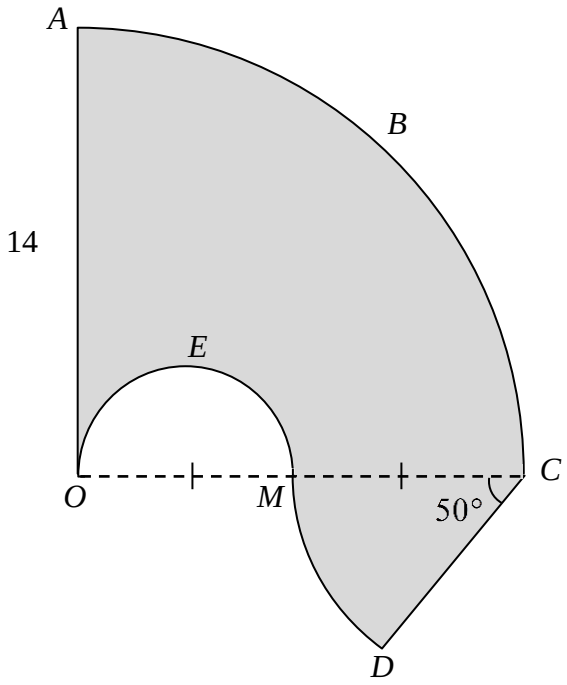


ABCD is a parallelogram with *DB* = *DC*.
P is a point on *AB* such that *DA* = *DP*.
Angle *ADP* = 36°.

Find angle *BDP*.
Give a reason for each step of your working.

Answer angle *BDP* =..... ° [4]

20 The diagram shows a company logo *OABCDME* in which *OABC* is a quadrant of a circle, centre *O* and radius 14 cm. *M* is the midpoint of *OC* and a semicircle *OEM* is drawn with *OM* as diameter. *CDM* is a sector of a circle with centre *C*, radius *CM* and angle *DCM* = 50°.



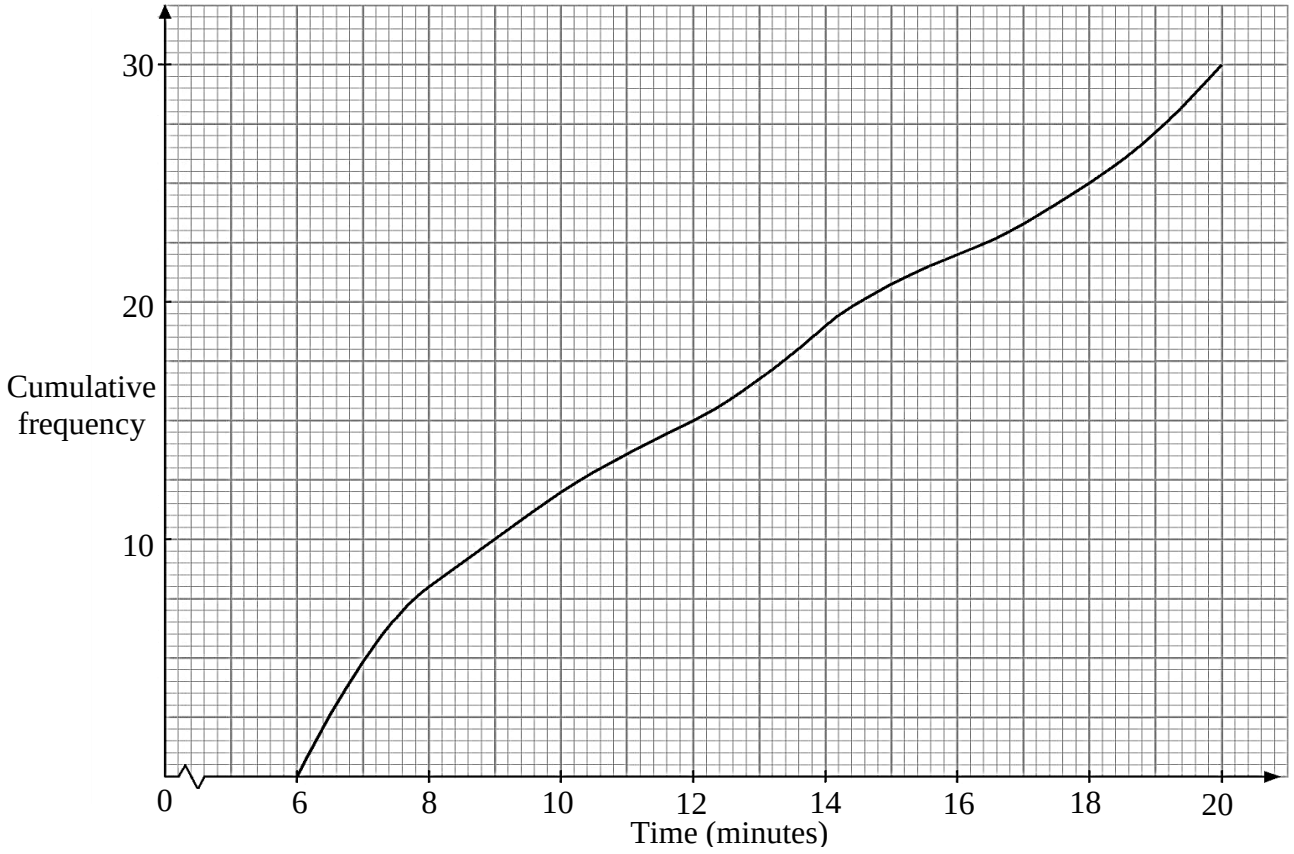
(a) Find the length of arc *DM*.

Answer *DM* =.....cm [1]

(b) Calculate the area of the logo *OABCDME*.

Answer …….. cm^2 [3]

21 The times taken by 30 students in Class Alpha to run 2.4 km in their fitness test are recorded. The cumulative frequency curve below shows the distribution of their times.



(a) Use the curve to estimate the interquartile range of the times.

Answer ……..minutes [2]

(b) The maximum number of points a student can obtain in the test is 5 points.
The students who take less than 9 minutes to run 2.4 km are given 5 points for the test.
Find the probability that a student, chosen at random, receives less than 5 points.

Answer ……...……..... [1]

(c) The times taken by 30 students from Class Beta to run 2.4 km in their fitness test had the same median as Class Alpha's times but a higher interquartile range.

Describe how the cumulative frequency curve for Class Beta may differ from the curve for Class Alpha.

Answer ……

…… [1]

22 XYZ is a triangular field.
$XZ = 78$ m. The bearing of Z from Y is $335°$.

(a) Construct a scale drawing of the field XYZ.
Use a scale of 1 cm to 10 m. [2]
The line XY has already been drawn.

(b) **(i)** Construct the perpendicular bisector of XZ. [1]

(ii) Construct the bisector of angle ZXY. [1]

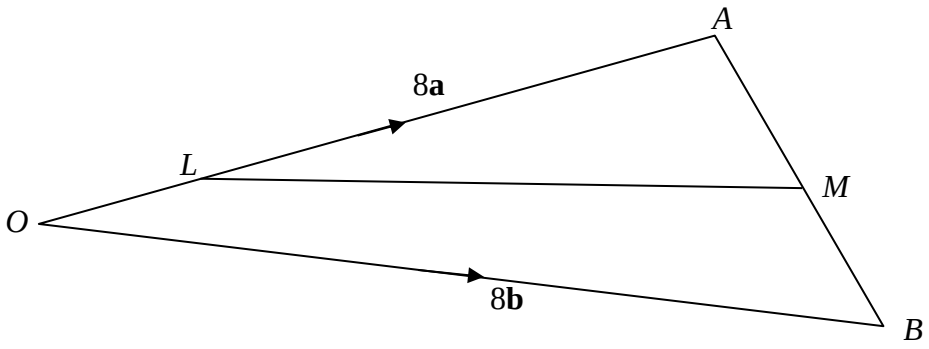
(iii) The point A is nearer to X than to Z and nearer to YX than to ZX.
Shade the region where point A can possibly be. [1]

(c) An accident occurred at point Y.
An ambulance is travelling in a straight line along XZ.

Find the point along XZ where the distance between the ambulance and point Y is the shortest, and measure this actual distance.

Answer .....m [1]

23



$\overrightarrow{OA} = 8\mathbf{a}$ and $\overrightarrow{OB} = 8\mathbf{b}$.

$\overrightarrow{OL} = \frac{1}{4}\overrightarrow{OA}$ and $BM = MA$.

(a) Write each of the following in terms of **a** and **b**.
Give your answers in their simplest form.

(i) $\overrightarrow{BM}$.

Answer ……...…….. [1]

(ii) $\overrightarrow{OM}$.

Answer ……...…….. [1]

(iii) $\overrightarrow{LM}$.

Answer ……...…….. [1]

(b) Find $\overrightarrow{OP}$ such that $\overrightarrow{LP} = 3\overrightarrow{LM}$.

Answer ……...…….. [1]

(c) Explain why $\overrightarrow{OP}$ is not parallel to $\overrightarrow{OM}$.

Answer ……

…… [1]

(d) Find $\overrightarrow{ON}$ such that *LMBN* is a parallelogram.

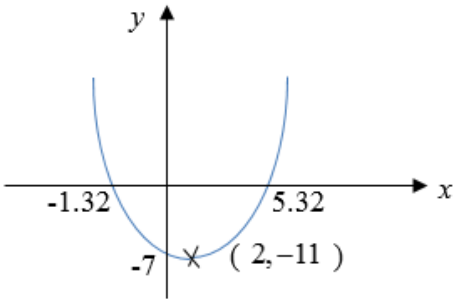
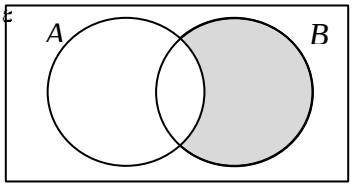
Answer ……..…….. [2]

END OF PAPER

BLANK PAGE

YISHUN TOWN SECONDARY SCHOOL
2018 Prelim Examination
Secondary Four Express/ Five Normal
MATHEMATICS

Answer Scheme

Qn	Answer	Qn	Answer
1(a)	$27-4a$	15(a)	$(x-2)^2-11$
1(b)	$2(x-3)(x+3)$	15(b)	
2	$(3d-c)(2b-3a)$		
3(a)			
3(b)	$A' \cap B = \{5, 7, 11, 13\}$	16(a)	40°
4	34	16(b)	20°
5	15	17(a)	$y=\frac{1}{2}x+4$
6	$\frac{2+15x}{(3x-1)^2}$	17(b) 17(c)	C (4, 0) 32 units square
7	The vertical axis does not start from 0, making it seems that there are thrice the number of students who choose Year 2 than Year 1.	18(a) 18(b)	15.1 cm $14720
8	$y=\frac{5}{6}(x+2)(x-3)$	19	36°
9(a)	$\cos\angle ABE=-\frac{7}{x}$	20(a)	6.11 cm
9(b)	14	20(b)	156 cm^2
10(a)	21.6	21(a)	8.6
10(b)	1458 g	21(b)	2/3
11	Peter should choose Plan B because Plan B offers a higher interest/ amount of money at the end of 5 years.	21(c)	The curve will have wider spread with gentler gradient.
12(a)	18	23(a)(i)	4**a**- 4**b**
12(b)	196	23(a)(ii)	4**b**+4**a**
13(a)	$462=2\times3\times7\times11$	23(a)(iii)	2**a**+4**b**
13(b)(i))	36	23(b)	12**b**+8**a**
13(b)(ii)	The powers of 7 is not multiples of 3.	23(c)	$\overrightarrow{OM}$ is not a scalar multiple of $\overrightarrow{OP}$, hence OP is not parallel to OM.
14(a)	1250 m	23(d)	-2**a**+4**b**
14(b)	15 cm^2		

NAME:		( )	CLASS:	

YISHUN TOWN SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
MATHEMATICS PAPER 2 (4048/02)

DATE : 14 Aug 2018 **DAY : Tues**

DURATION : 2 h 30 min **MARKS: 100**

ADDITIONAL MATERIALS

Writing Paper (8 sheets)
Mathematics Cover Sheet (1 sheet)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.

Write your name, class and class index number on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/ tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This question paper consists of **13** printed pages including this cover page

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** It is given that $p = \frac{4q^2}{3} + 5$.

(i) Find p when $q = -3$. [1]

(ii) Express q in terms of p. [2]

(b) Solve the equation $\frac{x}{5} - \frac{2x-3}{6} = 2$. [2]

(c) Solve these simultaneous equations.

$$2x - 3y = 15$$
$$3x - 7y = 27.5$$ [3]

(d) Simplify $\frac{4(x-2)}{x^2-4} - \frac{2(3x-1)}{3x^2+5x-2}$. [3]

---

2 The total number of visitors who visited a newly opened Theme Park on weekdays and weekends is given by the table below.

	Categories		
	Child	Adult	Senior citizen
Weekdays	600	1500	400
Weekends	1600	3500	1000

(a) Represent the number of visitors in a 2 × 3 matrix **T**. [1]

(b) The admission tickets are priced at \$12 per child, \$20 per adult and \$16 per senior citizen.

Represent the prices of tickets in a 3 × 1 column matrix **P**. [1]

(c) Evaluate the matrix **R** = **TP**. [1]

(d) State what the elements of **R** represent. [1]

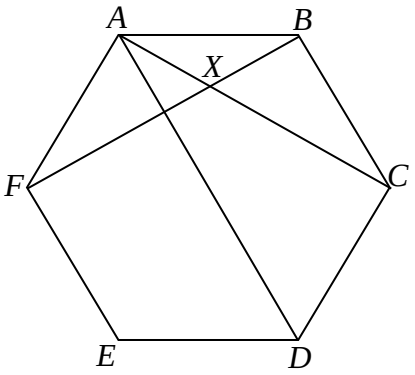
(e) The Theme Park decides to increase earnings by increasing the prices of all admission tickets by 35%.
This causes the total number of visitors in each category on both weekdays and weekends to drop by 20% due to the increase in prices.

Determine the difference in earnings and state whether it is an increase or decrease. [3]

---

3 **(a)** Name a quadrilateral with four equal sides and unequal diagonals. [1]

(b) The diagram shows a regular hexagon *ABCDEF*.
The diagonals *FB* and *AC* intersect at point *X*.



(i) Find the interior angle of the regular hexagon *ABCDEF*. [1]

(ii) Explain why triangles *FAB* and *CBA* are congruent. [2]

(iii) Find obtuse angle *FXC*. [2]

(iv) Given that diagonal *AD* bisects angle *FAB*, determine if *FE* and *AD* are parallel. [2]

4 The first four terms in a sequence of numbers are given below.

$$T_1 = 2^0 - 1^2 + 2^2 = 4$$
$$T_2 = 2^1 - 2^2 + 3^2 = 7$$
$$T_3 = 2^2 - 3^2 + 4^2 = 11$$
$$T_4 = 2^3 - 4^2 + 5^2 = 17$$

(a) Find T_5. [1]

(b) Show that the nth term of the sequence, T_n, is given by $2^{n-1} + 2n + 1$. [2]

(c) By first finding an expression for $T_{n+1} - T_n$, show that the difference between any 2 consecutive terms is always even for $n \geq 2$. [2]

(d) Find p if $T_{p+1} - T_p = 1026$. [2]

---

5 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = x^3 - 3x^2 + 2 .$$

Some corresponding values of x and y are given in the table below.

x	–2	–1	0	1	2	3	4
y	p	–2	2	0	–2	2	18

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-2 \le x \le 4$.
Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for $-20 \le y \le 20$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the solutions to the equation $x^3 - 3x^2 = -2$ in the range $-2 \le x \le 4$. [2]

(d) By drawing a tangent, find the gradient of the curve at $(3, 2)$. [2]

(e) **(i)** On the same axes, draw the line with gradient 3 that passes through the point with coordinates (0, 1). [1]

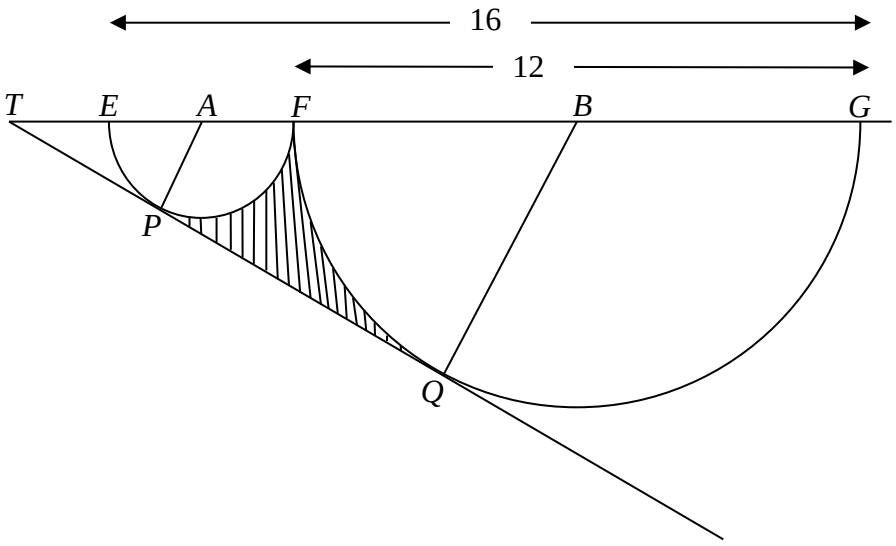
(ii) Write down the equation of this line. [1]

(iii) Write down the coordinates of the points where the line intersects the curve. [2]

(iv) These values of x are solutions of the equation $x^3 - 3x^2 + Ax + B = 0$.

Find the value of A and the value of B. [2]

6



The diagram shows a smaller semicircle *EPF* with centre *A* and a larger semicircle *FQG* with centre *B*.
TPQ is a tangent to both the smaller and larger semicircle.
The points *T*, *E*, *A*, *F*, *B* and *G* lie on the same straight line.
GE = 16 cm and *GF* = 12 cm.

(a) Show that triangles *ATP* and *BTQ* are similar.
Give a reason for each statement you make. [2]

(b) Show that *TE* = 2 cm. [2]

(c) Find the ratio of the area of triangle *ATP* to the area of quadrilateral *BAPQ*. [2]

(d) Calculate the area of the shaded region. [4]

7 A tank has a capacity of 1080 litres.

(a) Tap *A* fills the tank at a rate of *x* litres per minute.

Write an expression, in terms of *x*, the time taken in minutes, by Tap *A* to fill up the tank completely. [1]

(b) Tap *B* fills the tank at a rate of 2 litres per minute slower than Tap *A*.

Write an expression, in terms of *x*, the time taken in minutes, by Tap *B* to fill up the tank completely. [1]

(c) The difference in time taken by Tap *A* and Tap *B* to fill the tank completely is 40 minutes 36 seconds.

Write down an equation in *x* to represent this information and show that it reduces to

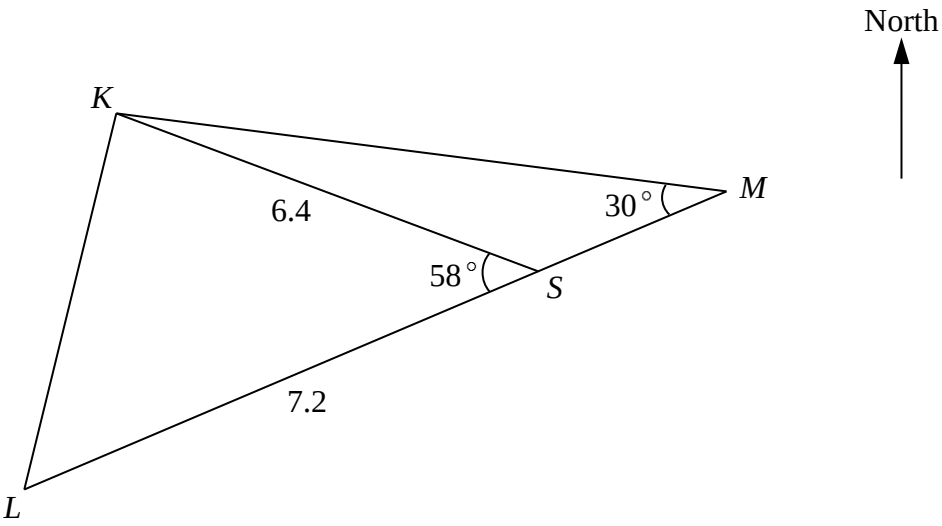
$$203x^2 - 406x - 10800 = 0 .$$ [3]

(d) Solve the equation $203x^2 - 406x - 10800 = 0$, giving your solutions correct to 2 decimal places. [3]

(e) Tap *A* and Tap *B* are turned on together to fill the tank when it is empty.

Find the time taken for the tank to be completely filled.
Give your answer in minutes and seconds, correct to the nearest second. [2]

8



The diagram shows a triangular garden *KLM* on horizontal ground.
A sprinkler, *S*, lies on *LM*.
LS = 7.2 m, *KS* = 6.4 m, angle *KSL* = 58° and angle *KML* = 30°.

The bearing of *M* from *K* is 095°.

(a) Calculate the bearing of *L* from *M*. [2]

(b) Calculate *KL*. [3]

(c) Calculate *MS*. [3]

(d) Calculate the area of the triangular garden *KLM*. [2]

(e) A treehouse of height 3 metres stands at *K*.
Calculate the angle of depression of *S* when seen from the top of the treehouse. [2]

(f) The owner of the garden moves the sprinkler at *S* to *T* along *LM* such that *T* is nearest to *K*.
Find the distance *LT*. [2]

9 **(a)** The table shows the ages of 50 staff in Company *A*. **[Turn over**

Age (x years)	Frequency
$20 \le x < 30$	15
$30 \le x < 40$	12
$40 \le x < 50$	10
$50 \le x < 60$	8
$60 \le x < 70$	5

(i) Calculate an estimate of the mean age. [1]

(ii) Calculate an estimate of the standard deviation. [1]

(iii) The standard deviation of the ages of 50 staff in Company *B* is 11 years.
Make one comparison between the ages of the staff in Company *A* and Company *B*. [1]

(b) 30 students sat for a Mathematics test.
The mean mark was 12.8 and the median mark was 12.
One student was late and sat for the test on the next day.
The new mean mark was 13.

(i) Calculate the marks scored by the student who was late. [2]

(ii) Copy and complete the sentence below with the correct phrase from the list.
The new median …

is definitely bigger than 12
is definitely smaller than 12
might be bigger than 12 but might still be 12
might be smaller than 12 but might still be 12
is still 12

[1]

(c) There are some coloured cards in a bag.

The probability of drawing a red card is $\frac{1}{6}$ and that of drawing a green card is $\frac{1}{5}$.

5 points are awarded when a red card is drawn and 3 points are awarded when a green card is drawn. No points are awarded when other coloured cards are drawn.

After each card is drawn, it is then put back into the bag before the next card is drawn.

(i) Charles draws a card at random from the bag.

Find the probability that he will not be awarded any points. [1]

(ii) Charlene draws 2 cards at random from the bag.

Find the probability that she will be awarded less than 8 points. [2]

10 Lim, Xian and Hui use smartphones as part of their everyday routine.
Lim and Xian sign up for the same mobile plan for 2 years as shown. **[Turn over**

Mobile Plan Contract Details (Bundled Plan and Extra Surcharges are subjected to 7% GST)			
Bundled Plan			
Phone Price	$1115	Local Free SMS	1000
Monthly Subscription	$42	Local Free Data Bundle	3GB
Local Free Voice Calls	200 mins		
One Time Registration Charge (not subjected to GST nor service charge)			$10.70
Extra Surcharges (Exceed Bundled Plan)			
Per global SMS	$0.15	Per local SMS	$0.0535
Excess local voice call usage billed per second			0.2675 cents/sec
Excess local mobile data charged at $10.70 per GB or part thereof* and final capped at $188.32 per month. Any additional charges on top of final capped amount will be $2.14 per MB. (1GB = 1000 MB) *For example, if user uses excess of 1.1GB, $21.40 will be charged.			

Lim bought a phone together with the bundled plan.

(a) How much does Lim need to pay in **total** for using the bundled plan for 2 years if he does not incur any extra surcharges? [2]

Xian sends 1100 local SMS, makes 205 minutes 40 seconds of local voice calls and uses 6.3GB of data locally in a particular month.

(b) How much **extra surcharge** does Xian need to pay in that month? [3]

Hui is looking for a new mobile plan contract and saw an advertisement from Eunonia Telecommunications Pte Ltd. The advertisement is on page 13.

Assume Hui's usage of SMS, calls, mobile data and global SMS is constant every month as shown below.

Local SMS: 500	Local Mobile Data: 8.5GB
Local Voice Calls: 980 minutes	Global SMS: 10

(c) Suggest the mobile plan that Hui should choose from Eunonia Telecommunications Pte Ltd. Justify the decision you make by showing your calculations clearly. [5]

Eunonia Telecommunications Pte Ltd

With packages inclusive of 7% GST and discounts,

I can save more and use more!

Mobile Plan Name	XS	S	M
Monthly Subscription**	$48	$68	$88
Discount	30% off monthly subscription		
Local Free Mobile Data	3GB	4GB	5GB
DataJump***	Additional 3GB local data at $6/month!		
Super DataJump $10/month!***		+5GB local data!	+ 10GB local data!
Local Free Voice Calls (Mins)	200	400	Unlimited!
Local Free SMS/MMS**	100	$0.0535 per SMS	

**Prices are based on the mobile plan featured. All prices are inclusive of 7% GST.
*** Optional and all prices quoted are inclusive of 7% GST.
Not allowed to combine DataJump and Super DataJump and/or multiple DataJumps and/or multiple SuperDataJumps.

Eunonia Telecommunications Pte Ltd Extra Surcharges (inclusive of 7% GST) (Exceed Bundled Plan)			
Per global SMS	$0.20	Per local SMS	$0.06
Excess local voice call usage billed per second			0.2675 cents/sec
Excess local mobile data charged at $10.70 per GB or part thereof* and final capped at $238 per month. Any additional charges on top of final capped amount will be 0.0107 cents per KB. (1MB = 1000 KB) *For example, if user uses excess of 1.1GB, $21.40 will be charged.			

End of Paper

BLANK PAGE

YISHUN TOWN SECONDARY SCHOOL
2018 Prelim Examination
Secondary 4E5N
MATHEMATICS Paper 1 and 2

Answer Key for Paper 2

Qn	Answer	Qn	Answer
1ai	$p = 17$	7a	$\frac{1080}{x}$ min
1aii	$q = \pm\sqrt{\frac{3p-15}{4}}$	7b	$\frac{1080}{x-2}$ min
1b	$x = -11.25$	7c	$203x^2 - 406x - 10800 = 0$ (shown)
1c	$x = 4.5$, $y = -2$	7d	$x = 8.36$ or -6.36 (2 d.p.)
1d	$\frac{2}{x+2}$	7e	73 min 21 seconds (nearest second)
2a	$\mathbf{T} = \begin{pmatrix} 600 & 1500 & 400 \\ 1600 & 3500 & 1000 \end{pmatrix}$	8a	245°
2b	$\mathbf{P} = \begin{pmatrix} 12 \\ 20 \\ 16 \end{pmatrix}$	8b	$KL = 6.63\,\text{m}$ (3 s.f.)
2c	$\mathbf{R} = \mathbf{TP} = \begin{pmatrix} 43600 \\ 105200 \end{pmatrix}$	8c	$MS = 6.01\,\text{m}$ (3 s.f.)
2d	The theme park earned \$ 43600 and \$105200 from the sale of admission tickets on weekdays and weekends respectively. or The amount of money collected by the theme park on weekdays and weekends respectively.	8d	$35.8\,\text{m}^2$ (3 s.f.)
2e	The Theme Park increases their earnings by \$11904.	8e	25.1°
3a	Rhombus	8f	$LT = 3.81\,\text{m}$ (3 s.f.)
3bi	120°	9ai	Mean age = 40.2 years old
3bii	By SAS Congruency Test, triangles *FAB* and *CBA* are congruent.	9aii	Standard deviation = 13.3 years old (3 s.f.)
3biii	$\angle FXC = 120°$	9aiii	Since the standard deviation by the Company *B* is lower, the age of Company *B* of 50 staff is more consistent than the Company *A*.

3biv	Since Angle *FAD* + Angle *AFE* $=180°$, then sum of interior angles show that sides *EF* and *DA* are parallel.	9bi	19
4a	$T_5 = 2^4 - 5^2 + 6^2 = 27$	9bii	The new median might be bigger than 12 but might still be 12
4b	$T_n = 2^{n-1} + 2n + 1$	9ci	$\frac{19}{30}$
4c	$2(2^{n-1} + 1 - 2^{n-2})$	9cii	$\frac{163}{180}$
4d	$p = 11$	10a	\$2282. 31
6a	By AA Similarity Test, Triangle *ATP* and Triangle *BTQ* are similar.	10b	\$52.49 (2 d.p.)
6b	*TE* = 2 cm	10c	**Eunonia Telecommunications Pte Ltd** XS: \$222.89/month $\frac{70}{100}(\$48)$ + \$6 + 3(\$10.70) + 780(\$0.1605) + 10($\frac{20}{100}$) + 400(\$0.06) Mthly Sub Data Jump Extra Data Voice call G SMS Local SMS S: \$179.44/month $\frac{70}{100}(\$68)$ + \$10 + 580(\$0.1605) + 10($\frac{20}{100}$) + 500(\$0.0535) Mthly Sub S Data Jump Voice call G SMS Local SMS **M: \$100.35/month** $\frac{70}{100}(\$88)$ + \$10 + 10($\frac{20}{100}$) + 500(\$0.0535) Mthly Sub S Data Jump G SMS Local SMS I would suggest with **Plan M at \$100.35/month mobile plan** for him to suit his needs which is the **least cost per month**.
6c	The ratio area of triangle *ATP* : area of quadrilateral *BAPQ* is 1 : 8.		
6d	4.67 cm^2		

ZHONGHUA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018
SECONDARY 4E/4N/5N

Candidate's Name	Class	Register Number

MATHEMATICS 4048/01

PAPER 1

27 Aug 2018
2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is **80**.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

For Examiner's Use
80

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Calculate $\frac{3+\sqrt{-4^2+2\times11}}{5}$.

Write down the first 6 digits on your calculator display.

Answer ..... [1]

(b) Write your answer to part **(a)** correct to 3 decimal places.

Answer ..... [1]

---

2 These are the first five terms of a sequence.

2018 2011 2004 1997 1990

(a) Write down the tenth term in the sequence

Answer ..... [1]

(b) Write down an expression, in terms of n, for the nth term in the sequence.

Answer ..... [1]

(c) Explain why the number 3 does not appear in the sequence.

Show your working clearly.

Answer .....

.....

..... [2]

---

3 **(a)** Factorise completely $48xy - 8y$.

Answer ..... [1]

(b) Factorise completely $x^4 + 3x^3 - 4x^2$.

Answer ..... [2]

---

4 Solve the equation $x(x-3) = 5(x^2 - 9)$.

Answer ..... [3]

---

5 Calculate the interior angle of a regular 10-sided polygon.
Show your working clearly.

Answer ..... ° [2]

---

6 **(a)** Given that $6^m \div 6^{-3} = 6^2$, find the value of m.

Answer $m =$ ..... [1]

(b) Arrange the following numbers in increasing value.
Show your working clearly.

0.0037×10^6 3.7×10^5 370×10^{-3} 37

Answer ....., ..., ..., ... [2]
smallest largest

(c) Simplify $\sqrt[3]{8x^6} \times \frac{1}{3y^{-5}}$. Leave your answer in positive index form.

Answer ..... [2]

---

7 Ken invested \$12 000 into a fund which pays compound interest of 4% per annum compounded half-yearly.
Calculate the total interest earned in 5 years.

Answer \$ ..... [3]

---

8 **(a)** Express $x^2 - 8x + 11$ in the form $(x - a)^2 + b$.

Answer ..... [2]

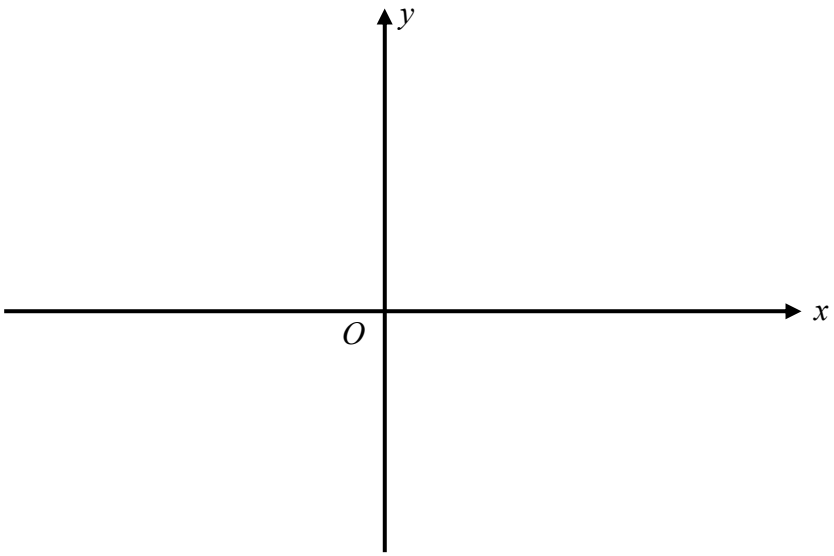
(b) **Hence** solve the equation $x^2 - 8x + 11 = 0$, giving your answers correct to two decimal places.

Answer $x =$ ..... or $x =$ ..... [2]

(c) Sketch the graph of $y = x^2 - 8x + 11$.

Indicate the y-intercept and the turning point of the graph clearly.

Answer

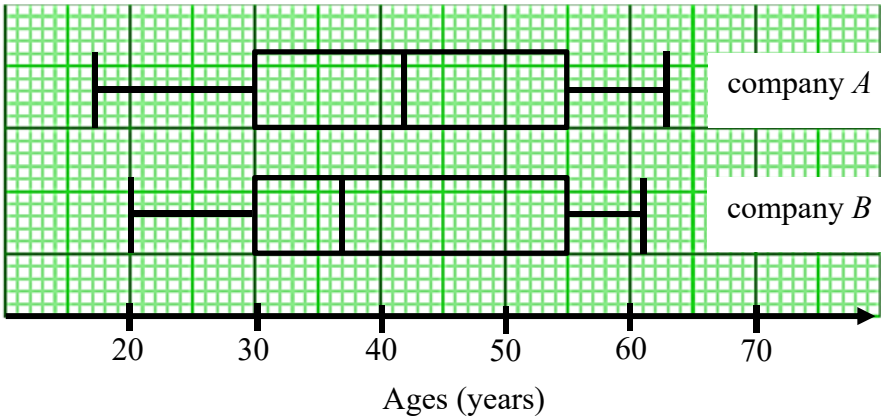


[2]

(d) Write down the equation of line of symmetry of the graph of $y = x^2 - 8x + 11$.

Answer ..... [1]

9 These box plots show the age distributions of the 200 employees in each of company *A* and company *B*.



(a) Find the range of the employee's age of company *A*.

Answer ..... [1]

(b) For each the following statements, write whether you agree or disagree.
Give a statistical reason which you use to support your decision.

(i) On average, company *A* has older employees than company *B*.

Answer ..... because .....
.....
..... [1]

(ii) There are more employees who are below 30 years old in company *A* than in company *B*.

Answer ..... because .....
.....
..... [1]

(c) An employee is randomly selected from company *B*.
Find the probability that the employee ages between 30 and 55 years old.

Answer ..... [1]

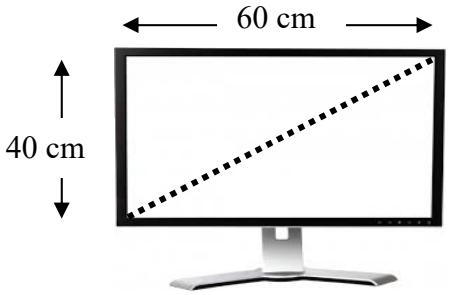
10 The size of television (TV) screens are measured diagonally in inches.

(1 inch = 2.54 cm)

(a) A TV screen has dimension 40 cm by 60 cm.

Find the size of the TV.

Give your answer correct to the nearest inch.



Answer .....inches [2]

(b) An electronic store offers 20% discount storewide.

Ali wishes to buy a new 45-inch TV which costs $2298.

All TV sets have an additional y % off after a storewide discount.

He received a receipt with poor print quality and some numbers missing.

45-inch TV		$ 2298
After 20% off		$ x
After y% off		$ 1562.64
Amount Paid		$

Find the value of x and y.

Answer x = ...

y = ... [2]

11 **(a)** $\xi = \{\text{integers } x : 2 \leq x \leq 10\}$

$F = \{\text{factors of } 18\}$

$G = \{\text{prime numbers}\}$

(i) List the elements in F'.

Answer ..... [1]

(ii) State the number of elements in $F \cup G$.

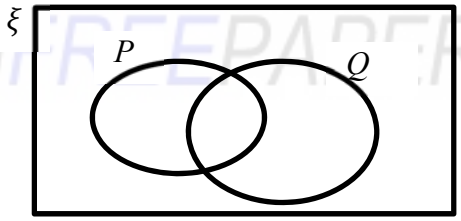
Answer ..... [1]

(iii) Explain why $2 \in (F \cap G)$.

Answer .....

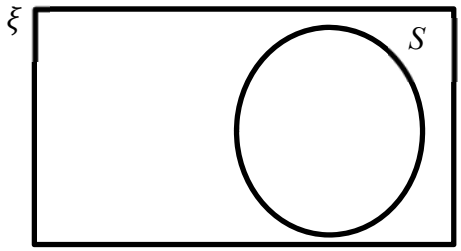
..... [1]

(b) On the Venn diagram shown below, shade the set $P \cap Q'$.



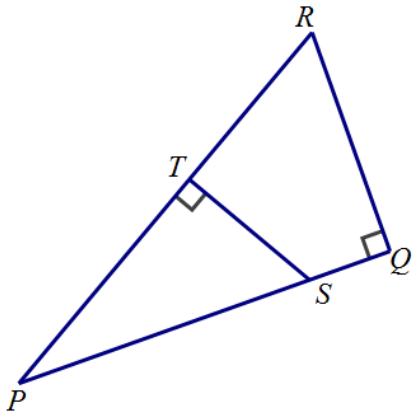
Answer On the diagram [1]

(c) On the diagram below, draw the set R such that $R \cap S = \emptyset$.



Answer On the diagram [1]

12 **(i)** PQR forms a right-angled triangle such that angle $PQR = 90°$.
ST is perpendicular to PR.



Name a triangle which is similar to triangle PQR.

Answer Triangle..... [1]

(ii) Given further that $PQ = 8$ cm , $QR = 6$ cm and $PR = 10$ cm, find the length of ST where T is the midpoint of PR.

Answer $ST =$ ..... cm [2]

---

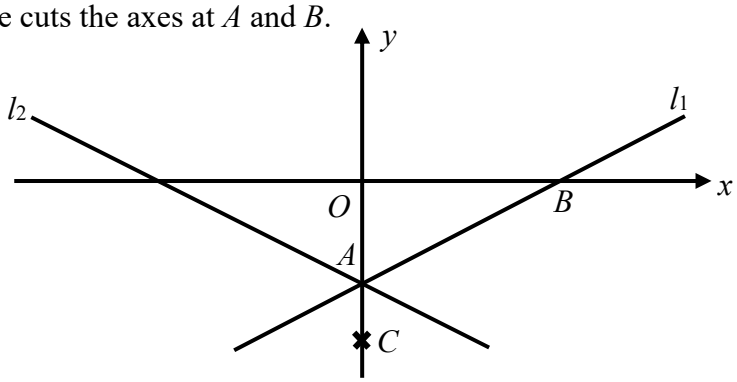
13 The radius of a cylinder is increased by 25% and its height is decreased by 50%.
Calculate the percentage decrease in the volume of the cylinder.
Give your answer correct to 1 decimal place.

Answer .....% [2]

---

14 The following diagram shows a sketch of the line $l_1 : y = \frac{3}{4}x - 3$.

The line cuts the axes at A and B.



(a) Find the coordinates of A and B.

Answer A (……, ……)

B (……, ……) [2]

(b) The line l_2 is a reflection of the line l_1 along the y-axis.

Write down the equation of the line l_2.

Answer ..... [1]

(c) C is a point on y-axis, as shown on the diagram.

Calculate the exact value of $\cos \angle BAC$.

Answer ..... [1]

(d) Another line $l_3 : y = \frac{3}{4}x + 2$ can be drawn on the same axes.

Explain why the lines l_1 and l_3 do not meet.

Answer .....

.....

.....

.....

..... [2]

15 Two geometrically similar containers have the following specifications.

	Container *A*	**Container *B***
Height (m)	p	50
Cost of painting the base ($)	120	480
Time taken to completely fill the container with water (to the nearest minute)	123 minutes	q hours r minutes

Find the values of p, q and r.

Answer $p =$ .....

$q =$ .....

$r =$ ..... [5]

---

16 Given that y varies as x^n, write down the value of n in each of the following cases:

(a) y is the volume of a sphere of radius x,

Answer $n =$ ..... [1]

(b) y and x are the sides of a rectangle of given area.

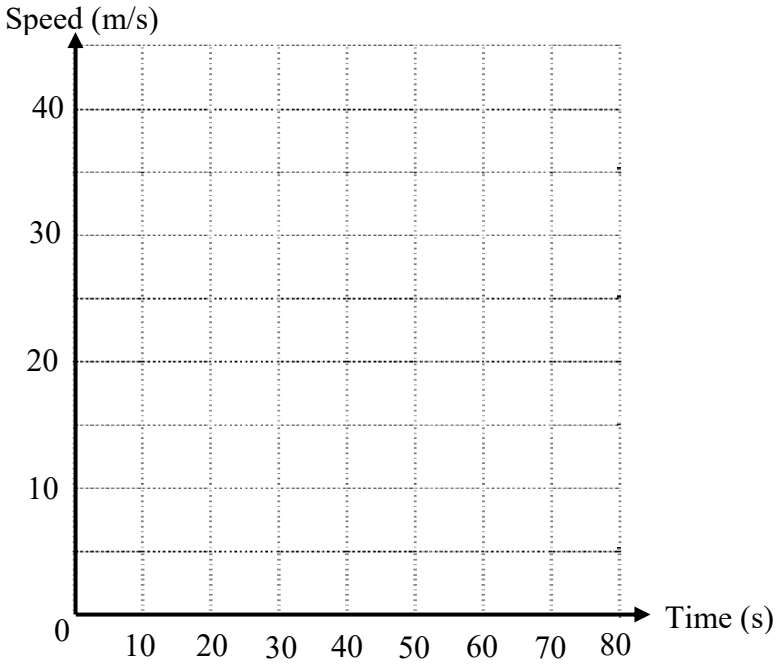
Answer $n =$ ..... [1]

---

17 A train travels at a constant speed of 40 m/s for 50 seconds.

It then slows down at a constant rate until it comes at rest in 20 seconds.

(a) On the axes, draw the speed-time graph for the journey.



Answer On the diagram [1]

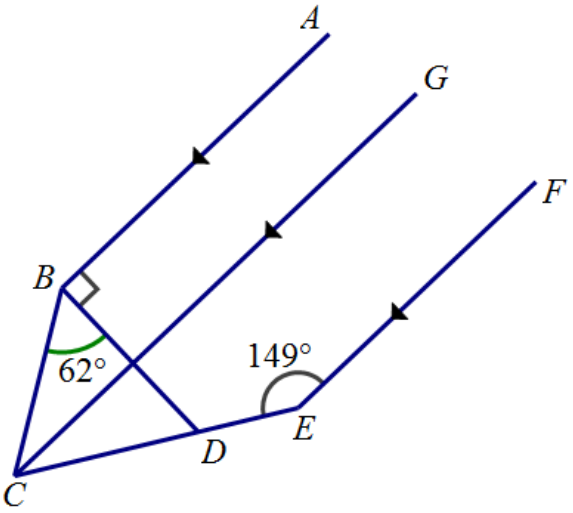
(b) Calculate the distance travelled by the train during the first 60 seconds.

Answer .....m [2]

18 In the figure, AB is parallel to GC and FE.

Angle $CEF = 149°$ and angle $CBD = 62°$.

D is a point on CE such that angle $ABD = 90°$.



Find, stating the reasons clearly,

(a) angle BCD,

Answer ..... ° [3]

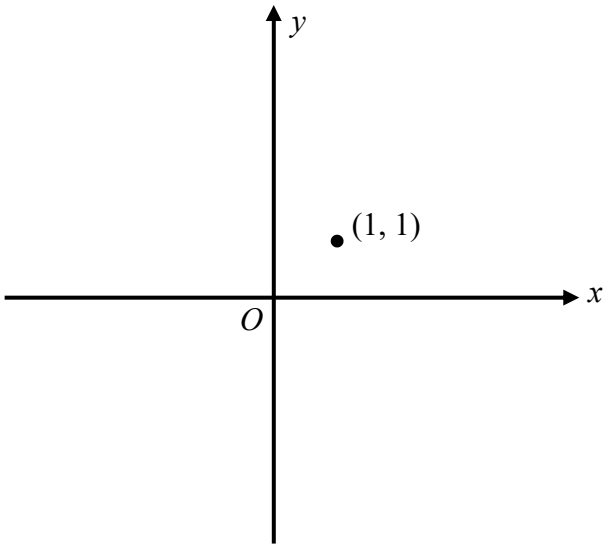
(b) angle BDE.

Answer ..... ° [2]

19 Sketch the graph of each of the following equations.

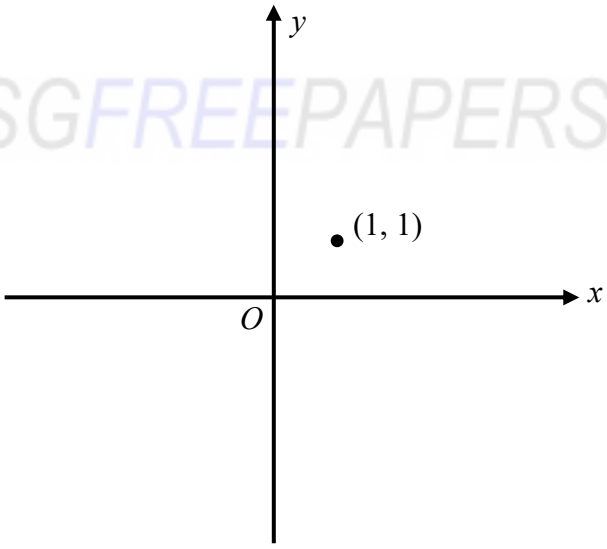
The point (1, 1) is plotted on each diagram.

(a) $y = 2^x$



Answer On the diagram [1]

(b) $y = \frac{1}{x^2}$



Answer On the diagram [1]

20 The line AB is drawn below.

(a) Construct triangle ABC where angle $ABC = 60°$ and $BC = 8$ cm. [2]

(b) Construct the perpendicular bisector of BC. [1]

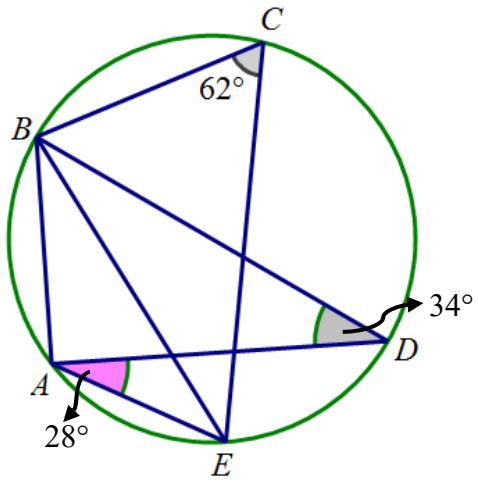
(c) From C, construct a line that is equidistant from the lines CA and CB. [1]

(d) Find the reflex angle ACB.

Answer ..... ° [1]

21 A circle passes through A, B, C, D and E.

It is given that angle $ADB = 34°$, angle $DAE = 28°$ and angle $BCE = 62°$.



By stating the reasons clearly,

(a) show that BD is a diameter of the circle,

Answer

[3]

(b) show that BE bisects angle ABD.

Answer

[2]

~~~ End of Paper ~~~

Answers

1a) 1.08989

1b) 1.090

2a) 1955

2b) $2025 - 7n$

3a) $8y(6x - 1)$

3b) $x^2(x + 4)(x - 1)$

4) x = 3 or $x = \frac{15}{4}$

5) 144°

6a) $m = -1$

6b) 370×10^{-3}, 37, 0.0037×10^6, 3.7×10^5

6c) $\frac{2}{5}x^2y^3$

7) \$2627.93

8a) $(x - 4)^2 - 5$

8b) $x = 6.24$ or $x = 1.76$

8d) $x = 4$

9a) 46 years

9b) agree, due to Company A has higher median age than Company B

9c) disagree, both companies have equal lower quartile and therefore they have equal number of employees whose age is below 30.

10a) 28

10b) x = 1838.40, y = 15

11ai) {4, 5, 7, 8, 10}

11aii) 6

11aiii) Because 2 is a factor of 18 AND it is also a prime number

12i) triangle PTS

12ii) 3.75 cm

13 21.9%

14a) A (0, –3), B (4, 0)

14b) $y = -\frac{3}{4}x - 3$

14c) $-\frac{3}{5}$

14d) Both lines have equal gradient, therefore they are parallel. As both lines are parallel and have different y-intercept, both lines do not meet.

15 p = 25, q = 16, r = 24

16a) n = 3

16b) n = –1

17b) 2300

18a) 59°

18b) 121°

ZHONGHUA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018
SECONDARY 4E/4N/5N

Candidate's Name	Class	Register Number
MARKING SCHEME		

MATHEMATICS 4048/01

PAPER 1

27 Aug 2018
2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is **80**.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

For Examiner's Use
80

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Calculate $\frac{3+\sqrt{-4^2+2\times11}}{5}$.

Write down the first 6 digits on your calculator display.

Answer 1.08989 [B1] [1]

(b) Write your answer to part **(a)** correct to 3 decimal places.

Answer 1.090 [B1] [1]

---

2 These are the first five terms of a sequence.

2018 2011 2004 1997 1990

(a) Write down the tenth term in the sequence.

Answer 1955 [B1] [1]

(b) Write down an expression, in terms of n, for the nth term in the sequence.

Answer $2025 - 7n$ or [1]

$2018 - 7(n - 1)$ [B1]

(c) Explain why the number 3 does not appear in the sequence.

Show your working clearly.

$2025 - 7n = 3$

$2022 = 7n$

$n = \frac{2022}{7}$ [B1]

Answer Since $n = \frac{2022}{7}$ is not a positive integer or whole number [A1], the number 3 does not appear in the sequence.[2]

---

3 **(a)** Factorise completely $48xy - 8y$.

Answer $8y(6x - 1)$ [B1] [1]

(b) Factorise completely $x^4 + 3x^3 - 4x^2$.

$x^4 + 3x^3 - 4x^2 = x^2(x^2 + 3x - 4)$[M1 - factorise x^2]

$= x^2(x + 4)(x - 1)$

Answer $x^2(x + 4)(x - 1)$ [A1] [2]

---

4 Solve the equation $x(x-3) = 5(x^2 - 9)$.

Method 1:

$x(x - 3) = 5(x - 3)(x + 3)$[M1]

$(x - 3)(x - 5(x + 3)) = 0$[M1]

$(x - 3)(-4x - 15) = 0$

Method 2:

$x^2 - 3x = 5x^2 - 45$[M1]

$-4x^2 - 3x + 45 = 0$

$(-4x - 15)(x - 3) = 0$[M1]

Answer $x = 3$ or $x = -\frac{15}{4}$ [A1] [3]

---

5 Calculate the interior angle of a regular 10-sided polygon.

Show your working clearly.

$\text{int. angle} = \frac{10 - 2}{10} \times 180°$[M1]

$= 144°$

Answer 144° [A1] [2]

---

6 **(a)** Given that $6^m \div 6^{-3} = 6^2$, find the value of m.

Answer $m = -1$ [B1] [1]

(b) Arrange the following numbers in increasing value.

Show your working clearly.

0.0037×10^6 $\quad$ 3.7×10^5 $\quad$ 370×10^{-3} $\quad$ 37

$= 3.7 \times 10^3$ $\quad$ $= 3.7 \times 10^{-1}$ $\quad$ $= 3.7 \times 10^1$

[M1 – conversion to multiplication of 3.7]

[A1 – correct order]

Answer 370×10^{-3}, 37, 0.0037×10^6, 3.7×10^5 [2]

smallest largest

(c) Simplify $\sqrt[3]{8x^6} \times \frac{1}{3y^{-5}}$. Leave your answer in positive index form.

$\sqrt[3]{8x^6} \times \frac{1}{3}y^5 = 2x^2 \times \frac{1}{3}y^5$ [B1 – $2x^2$ seen]

$= \frac{2}{3}x^2y^5$

Answer $\frac{2}{3}x^2y^5$ [A1] [2]

---

7 Ken invested \$12 000 into a fund which pays compound interest of 4% per annum compounded half-yearly.

Calculate the total interest earned in 5 years.

Total interest $= 12000\left(1 + \frac{4\%}{2}\right)^{10} - 12000$ [B1 – $\frac{4\%}{2}$ or 2% seen]

[B1 – to the power of 10]

$= 2627.93$

Answer \$ 2627.93 [A1] [3]

---

8 **(a)** Express $x^2 - 8x + 11$ in the form $(x-a)^2 + b$.

[B1 – for value of a] [B1 – for value of b]

Answer $(x-4)^2 - 5$ [2]

(b) **Hence** solve the equation $x^2 - 8x + 11 = 0$, giving your answers correct to two decimal places.

$x^2 - 8x + 11 = 0$

$(x-4)^2 - 5 = 0$

$x - 4 = \pm\sqrt{5}$[M1]

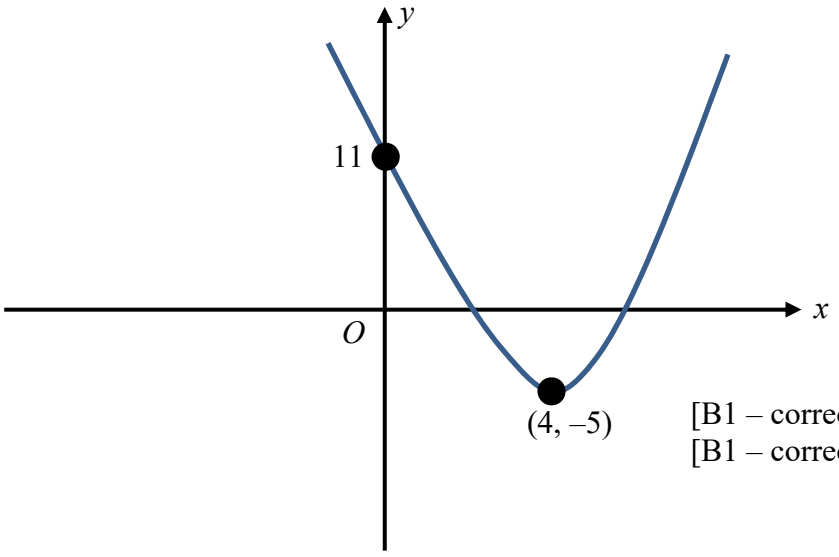
[A1 – for both values of x]

Answer $x = 6.24$ or $x = 1.76$ [2]

(c) Sketch the graph of $y = x^2 - 8x + 11$.

Indicate the y-intercept and the turning point of the graph clearly.

Answer



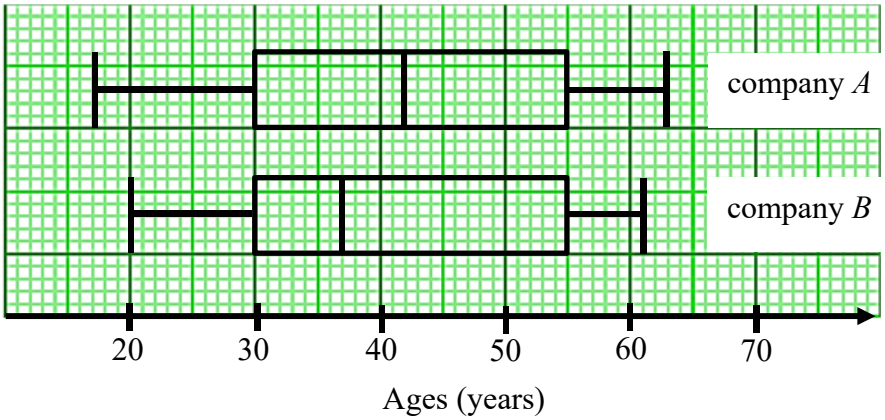
[B1 – correct y-intercept]
[B1 – correct turning pt]

[2]

(d) Write down the equation of line of symmetry of the graph of $y = x^2 - 8x + 11$.

Answer $x = 4$ [B1] [1]

9 These box plots show the age distributions of the 200 employees in each of company *A* and company *B*.



(a) Find the range of the employee's age of company *A*.

Answer 46 years [B1] [1]

(b) For each the following statements, write whether you agree or disagree.
Give a statistical reason which you use to support your decision.

(i) On average, company *A* has older employees than company *B*.

Answer Agree because the median age of employees in company *A* is higher than the median age of employees in company *B*. [1]

(ii) There are more employees who are below 30 years old in company *A* than in company *B*.

Answer Disagree because employees in both companies have equal lower quartile, therefore there are equal number of employees who are below 30 years old in both companies. [1]

(c) An employee is randomly selected from company *B*.
Find the probability that the employee ages between 30 and 55 years old.

Answer 0.5 [B1] [1]

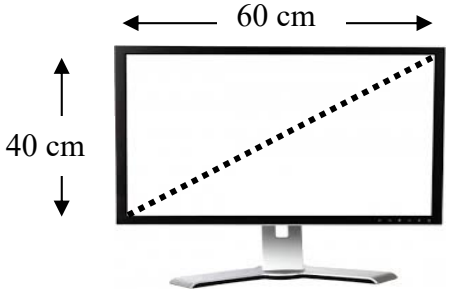
10 The size of television (TV) screens are measured diagonally in inches.

(1 inch = 2.54 cm)

(a) A TV screen has dimension 40 cm by 60 cm.

Find the size of the TV.

Give your answer correct to the nearest inch.



$$\text{size} = \sqrt{\left(\frac{40}{2.54}\right)^2 + \left(\frac{60}{2.54}\right)^2} \text{ [M1]}$$

$$= 28.390$$

Answer 28 [A1] inches [2]

(b) An electronic store offers 20% discount storewide.

Ali wishes to buy a new 45-inch TV which costs $2298.

All TV sets have an additional y% off after a storewide discount.

He received a receipt with poor print quality and some numbers missing.

45-inch TV		$ 2298
After 20% off		$ x
After y% off		$ 1562.64
Amount Paid		$

Find the value of x and y.

Answer $x = 1838.40$ [B1]

$y = 15$ [B1] [2]

11 **(a)** $\xi = \{\text{integers } x : 2 \leq x \leq 10\}$

$F = \{\text{factors of } 18\}$

$G = \{\text{prime numbers}\}$

(i) List the elements in F'.

Answer $\{4, 5, 7, 8, 10\}$ [B1] [1]

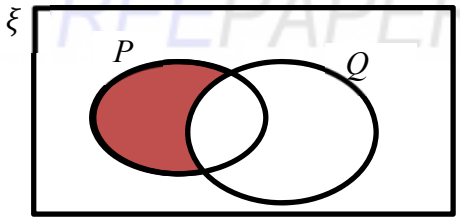
(ii) State the number of elements in $F \cup G$.

Answer 6 [B1] [1]

(iii) Explain why $2 \in (F \cap G)$.

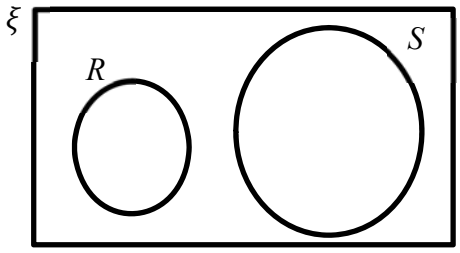
Answer Because 2 is a factor of 18 **AND** also a prime number. [B1]

(b) On the Venn diagram shown below, shade the set $P \cap Q'$.



Answer On the diagram [1]

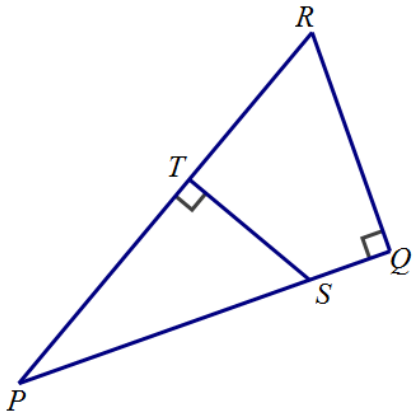
(c) On the diagram below, draw the set R such that $R \cap S = \emptyset$.



Answer On the diagram [1]

12 **(i)** *PQR* forms a right-angled triangle such that angle *PQR* = 90°.

ST is perpendicular to *PR*.



Name a triangle which is similar to triangle *PQR*.

Answer Triangle *PTS* [B1] [1]

(ii) Given further that *PQ* = 8 cm , *QR* = 6 cm and *PR* = 10 cm, find the length of *ST* where *T* is the midpoint of *PR*.

$\frac{PQ}{PT} = \frac{QR}{TS}$ [M1]

$\frac{8}{5} = \frac{6}{TS}$

$TS = 3.75$

Answer *ST* = 3.75 [A1] cm [2]

---

13 The radius of a cylinder is increased by 25% and its height is decreased by 50%.

Calculate the percentage decrease in the volume of the cylinder.

Give your answer correct to 1 decimal place.

Let the radius and the height of the cylinder be *r* and *h* respectively.

$\%\text{ change in volume} = \frac{\pi(1.25r)^2(0.5h) - \pi r^2 h}{\pi r^2 h} \times 100\%$[B1 - $1.25r$ or $0.5h$ seen]

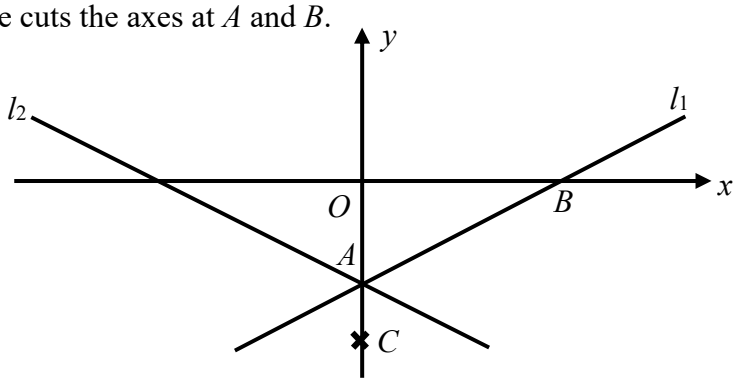
$= -21.875\%$

Answer 21.9 [A1] % [2]

---

14 The following diagram shows a sketch of the line $l_1 : y = \frac{3}{4}x - 3$.

The line cuts the axes at A and B.



(a) Find the coordinates of A and B.

Answer $A\ (0, -3)$ [B1]

$B\ (4, 0)$ [B1] [2]

(b) The line l_2 is a reflection of the line l_1 along the y-axis.

Write down the equation of the line l_2.

Answer $y = -\frac{3}{4}x - 3$ [B1] [1]

(c) C is a point on y-axis, as shown on the diagram.

Calculate the exact value of $\cos \angle BAC$.

Answer $-\frac{3}{5}$ [B1] [1]

(d) Another line $l_3 : y = \frac{3}{4}x + 2$ can be drawn on the same axes.

Explain why the lines l_1 and l_3 do not meet.

Answer The line l_1 and l_3 have **equal gradient**. Therefore, they are **parallel**. [B1]

As both lines are parallel and have **different *y*-intercepts**, both lines do not meet. [B1]

..... [2]

15 Two geometrically similar containers have the following specifications.

	Container *A*	**Container *B***
Height (m)	p	50
Cost of painting the base ($)	120	480
Time taken to completely fill the container with water (to the nearest minute)	123 minutes	q hours r minutes

Find the values of p, q and r.

$\left(\frac{p}{50}\right)^2 = \frac{120}{480}$ [M1]

$p = 25$

Let the time taken to completely fill container B be T minutes.

$\left(\frac{25}{50}\right)^3 = \frac{123}{T}$ [M1]

$T = 984$

$T = 16 \text{ hours } 24 \text{ minutes}$

Answer $p = 25$ [A1]

$q = 16$ [A1]

$r = 24$ [A1] [5]

---

16 Given that y varies as x^n, write down the value of n in each of the following cases:

(a) y is the volume of a sphere of radius x,

Answer $n = 3$ [B1] [1]

(b) y and x are the sides of a rectangle of given area.

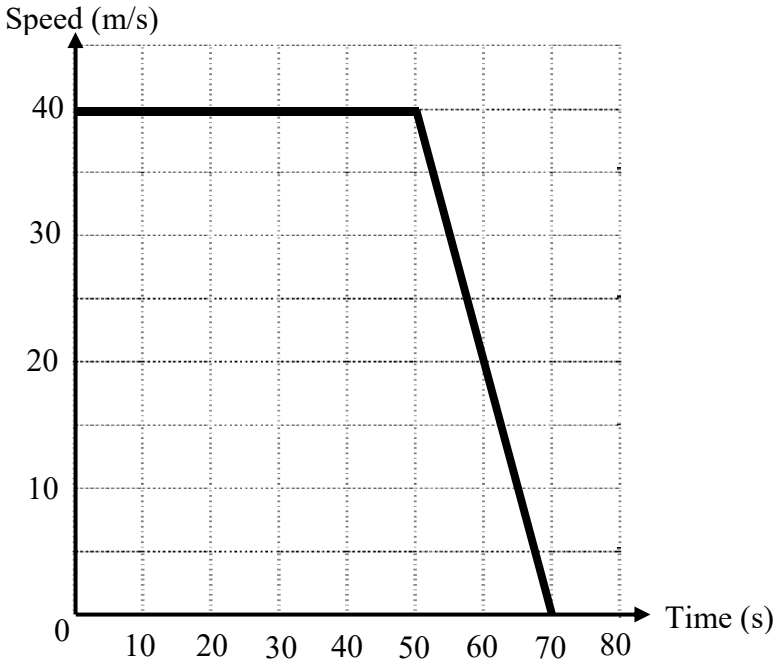
Answer $n = -1$ [B1] [1]

---

17 A train travels at a constant speed of 40 m/s for 50 seconds.

It then slows down at a constant rate until it comes at rest in 20 seconds.

(a) On the axes, draw the speed-time graph for the journey.



Answer On the diagram [1]

(b) Calculate the distance travelled by the train during the first 60 seconds.

distance travelled = area under graph

$$= (60 \times 40) - \frac{1}{2} \times 10 \times 20 \text{[M1]}$$

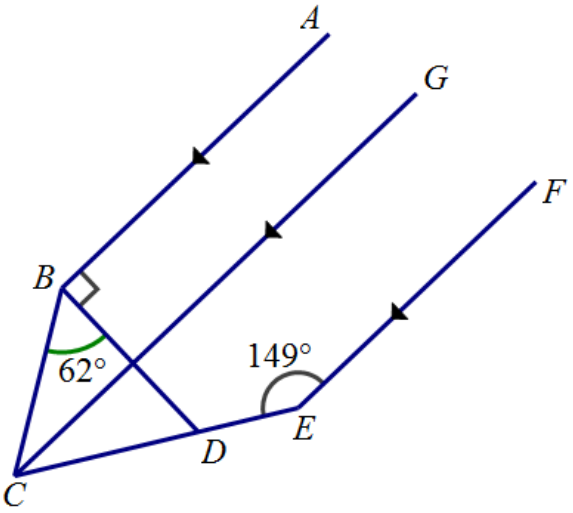
$$= 2300 \text{ m}$$

Answer 2300 [A1] m [2]

18 In the figure, AB is parallel to GC and FE.

Angle $CEF = 149°$ and angle $CBD = 62°$.

D is a point on CE such that angle $ABD = 90°$.



Find, stating the reasons clearly,

(a) angle BCD,

$\angle GCE + \angle CEF = 180°$ (sum of int. $\angle$s $= 180°, GC \parallel FE$)[B1 − reason]

$\angle GCE + 149° = 180°$

$\angle GCE = 31°$

Let T be the intersection of GC and BD.

$\angle BTC = \angle ABT = 90°$(alt. $\angle s, GC \parallel AB$)

$\angle BCG + \angle TBC = 90$ (complementary angles)[M1]

$\angle BCG + 62° = 90°$

$\angle BCG = 28°$

Therefore, $\angle BCD = 59°$

Answer 59 [A1] ° [3]

(b) angle BDE.

$\angle BDE = \angle CBD + \angle BCD$(sum of 2 int. $\angle$s = exterior $\angle$ of a triangle) [B1]

$= 62° + 59°$

$= 121°$

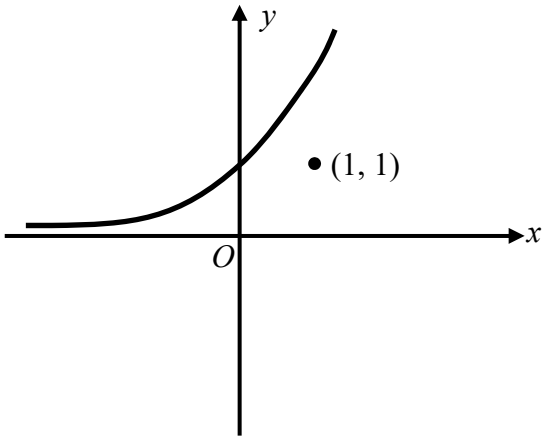
Answer 121 [A1] ° [2]

---

19 Sketch the graph of each of the following equations.

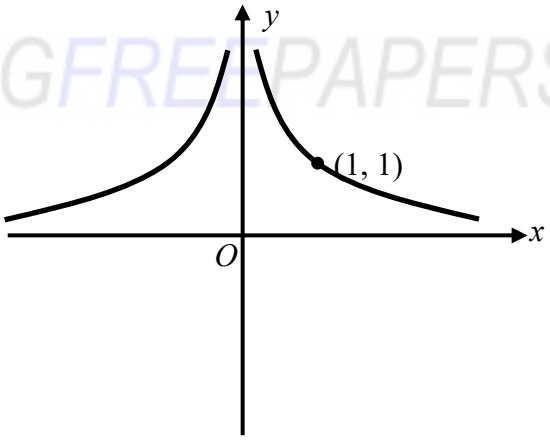
The point (1, 1) is plotted on each diagram.

(a) $y = 2^x$



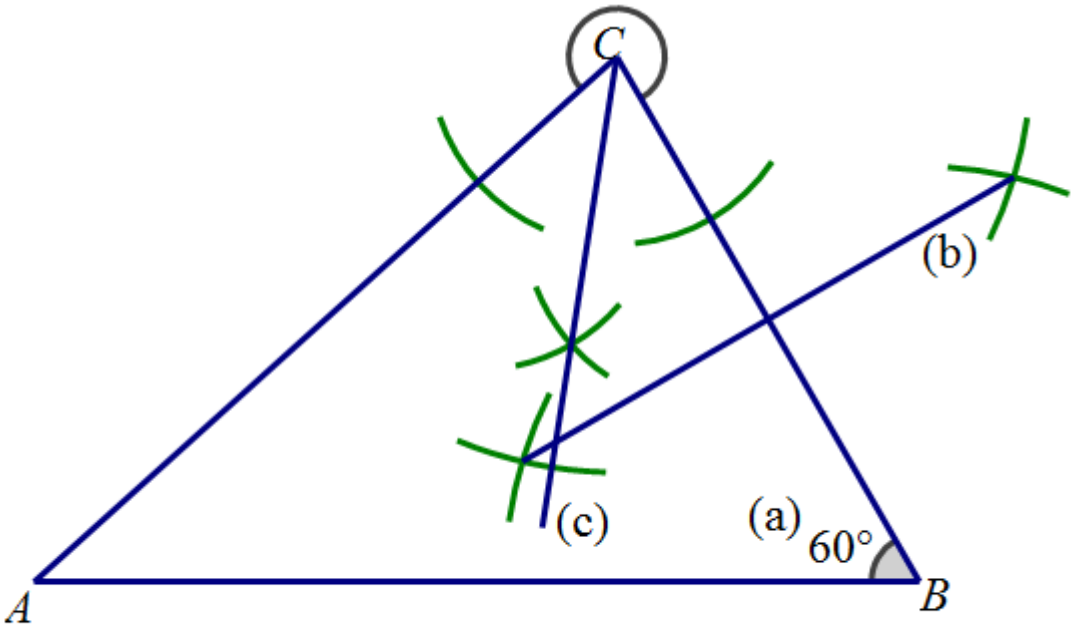
Answer On the diagram [1]

(b) $y = \frac{1}{x^2}$



Answer On the diagram [1]

20 The line AB is drawn below.



reflex $\angle ACB = 281.98° \pm 2$

(a) Construct triangle ABC where angle $ABC = 60°$ and $BC = 8$ cm. [2]

(b) Construct the perpendicular bisector of BC. [1]

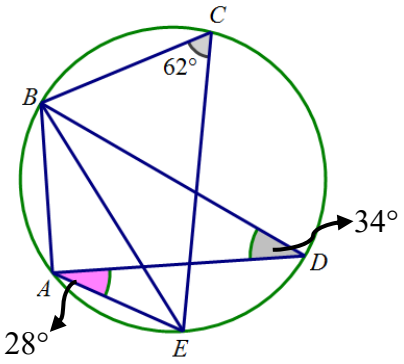
(c) From C, construct a line that is equidistant from the lines CA and CB. [1]

(d) Find the reflex angle ACB.

Answer ..... ° [1]

21 A circle passes through A, B, C, D and E.

It is given that angle $ADB = 34°$, angle $DAE = 28°$ and angle $BCE = 62°$.



By stating the reasons clearly,

(a) show that BD is a diameter of the circle,

Answer

$\angle BCE + \angle BAE = 180°$(∠s in opp. segments are supplementary) [B1]

$62° + \angle BAD + 28° = 180°$

$\angle BAD = 90°$[A1]

Since BD is a chord and $\angle BAD = 90°$, BD is a diameter of a circle. [AG]

(angle in a semicircle) [B1]

[3]

(b) show that BE bisects angle ABD.

Answer

$\angle DBE = \angle DAE$(∠s in the same segment are equal)[B1- reason]

$= 28°$

$\angle BEA = \angle BDA$(∠s in the same segment are equal)

$= 34°$

$\angle ABE = 180° - \angle BAE - \angle BEA$(∠ sum of a triangle $= 180°$)

$= 180° - (90° + 28°) - 34°$

$= 28°$[A1]

Since $\angle ABE = \angle DBE = 28°$, BE bisects angle ABD. [AG]

[2]

~~~ End of Paper ~~~

ZHONGHUA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018

SECONDARY 4E/4N/5N

Candidate's Name	Class	Register Number

MATHEMATICS 4048/02

PAPER 2

29 Aug 2018

2 hours and 30 minutes

Additional Materials: Writing paper, Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is **100**.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

For Examiner's Use
100

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $\frac{5x^3}{7y^3} \div \frac{25x}{49y^4}$ [2]

(b) Simplify $\frac{9x^2-1}{6x^2+x-1}$ [3]

(c) Solve the inequality $2x - 1 < 3x \leq \frac{2+3x}{3}$ [3]

(d) **(i)** Express as a single fraction in its simplest form

$$\frac{5}{x-7} - \frac{1}{x+1}.$$ [2]

(ii) Solve the equation

$$\frac{5}{x-7} - \frac{1}{x+1} = \frac{4}{5}.$$ [4]

---

2 **(a)** Carol and Pei are at a flower shop.
Carol buys three pots of mint and two pots of rosemary for $17.85.

Pei buys five pots of mint and four pots of rosemary.
She pays with a $50 note and receives change of $17.25.

(i) Write down a pair of simultaneous equations to represent this information.
Use m to represent the cost, in dollars, of a pot of mint
and r to represent the cost, in dollars, of a pot of rosemary. [2]

(ii) Solve your simultaneous equations to find m and r. [2]

(iii) Work out the cost of two pots of mint and three pots of rosemary. [1]

(b) Factorise $4x^2 + 4xy - x - y$. [2]

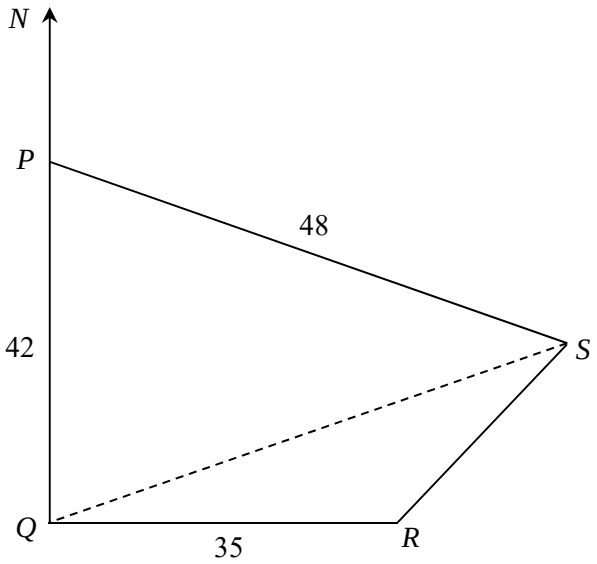
(c) **(i)** Express 3850 as the product of its prime factors. [1]

(ii) Given that $\frac{3850}{k}$ is a perfect square, find the smallest possible integer value of k. [1]

(iii) The product of two 2-digit numbers is 3850.
The highest common factor of these two numbers is 5.

Find the two numbers. [2]

3



The diagram shows a field, *PQRS*.
P, *Q*, *R* and *S* are on level ground such that *R* is due east of *Q*.
The bearing of *S* from *P* is 105°.
QS is a straight path across the field.
PQ = 42 m, *PS* = 48 m, *QR* = 35 m.

(a) Calculate *QS*. [3]

(b) Find the bearing of *S* from *Q*. [3]

(c) Calculate the area of the field *PQRS*. [3]

(d) A drone flies in a straight line from *Q* to *S* at a height of 20 m above ground.

A man standing at *R* looks up at the drone.
Calculate the largest angle of elevation of the drone from the man. [3]

4 **(a)** The daily dietary requirements differ slightly for school-going children and adolescents, based on their age-range.
The table summarises their respective daily dietary requirements of fruit and vegetables by number of servings.

Age	Fruit	Vegetables
3-6 years old	1	1
7-12 years old	2	2
13-18 years old	2	2

(i) Represent the above information in a 3 × 2 matrix **M**. [1]

(ii) One serving of fruit weighs 145g and one serving of vegetables weighs 150g.
Represent the weight for one serving in a 2 × 1 column matrix **N**. [1]

(iii) Evaluate the matrix **T** = **MN**. [1]

(iv) State what the elements of **T** represent. [1]

(b) The recommended period of time spent on physical activities for adolescents is one hour daily. 17-year-old Janet plots a route for her daily walk.

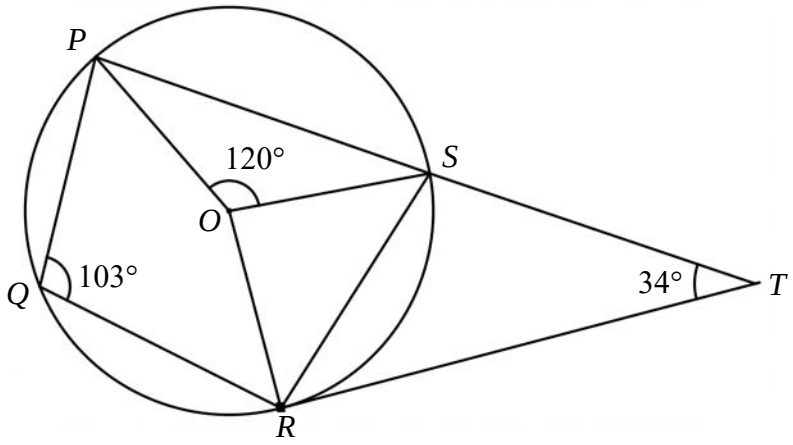
(i) The length of her route on a map is 8.2 cm.
The scale of the map is 1 : 50 000.

Work out the number of kilometres Janet walks each day. [1]

(ii) If Janet walks at a speed of 1.2 metres per second, will she be able to complete her route in an hour? Justify your answer with figures. [2]

---

5 (a)



O is the centre of the circle *PQRS*.
RT is a tangent to the circle, and when produced, the line *PS* meets the tangent at *T*.
Angle *PQR* = 103°, angle *SOP* = 120° and angle *STR* = 34°.

(i) Stating your reasons clearly, find

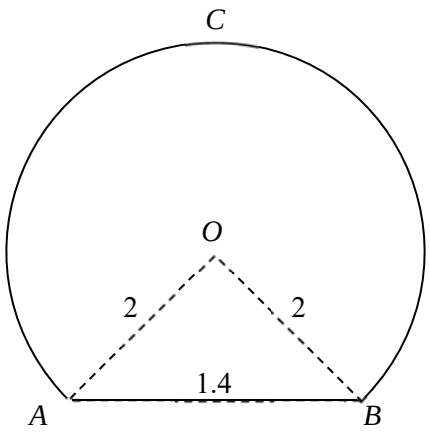
(a) angle *POR*, [2]

(b) angle *OST*, [2]

(ii) Show that angle *OPQ* + angle *ORQ* = 103°. [2]

(iii) Determine if quadrilateral *OSTR* is a trapezium.
Justify your answer with appropriate reason(s). [1]

(b)



The diagram shows a major segment, *ACB*, of radius 2 m with *AB* = 1.4 m.

(i) Calculate angle *AOB* in radians. [2]

(ii) Calculate the perimeter of the major segment *ACB*. [2]

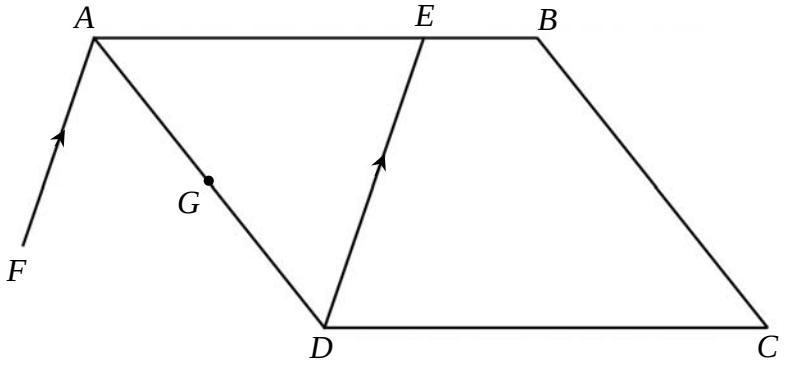
6 **(a)** P is the point (5, 12). Q is the point (–5, 0).

(i) Write down the column vector $\overrightarrow{PQ}$. [1]

(ii) Find $\left|\overrightarrow{PQ}\right|$. [2]

(iii) R is the point such that $\overrightarrow{PQ} = 2\overrightarrow{QR}$.
Find the coordinates of R. [2]

(b)



$ABCD$ is a parallelogram.
$\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$.
E is a point on AB such that $AE = 3EB$.
G is the midpoint of AD.
FA is parallel to DE such that $FA : DE = 4 : 5$.

(i) Express each of the following, as simply as possible, in terms of **a** and/or **b**.

(a) $\overrightarrow{AE}$, [1]

(b) $\overrightarrow{DE}$, [1]

(c) $\overrightarrow{FA}$. [1]

(ii) Write down the value of $\frac{\text{area of } \Delta AFD}{\text{area of } \Delta ADE}$. [1]

(iii) Determine if points F, G, and B lie on a straight line.
Justify your answer using vectors. [3]

7 **(a)** The number of goals scored by France in the 2018 World Cup is shown in the table below.

Number of goals	0	1	2	3	4
Frequency	1	2	2	0	2

(i) Is mode an appropriate measure of average for this set of data?
Justify your answer. [1]

(ii) Calculate

(a) the mean number of goals scored per game, [1]

(b) the standard deviation. [1]

(iii) France played 7 games at the 2006 World Cup.
The results are summarised below.

Mean	1.29 goals per game
Standard Deviation	0.88 goals per game

Make two comparisons between the number of goals scored per game by France in the two World Cups. [2]

(b) A packet of mixed nuts contains three different kinds of nuts: macadamia, almond, peanut.
There are a total of 10 macadamia nuts, 12 almond nuts, and 21 peanuts.
Mary picks two nuts at random without replacement.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that

(a) the first nut taken is a macadamia nut, [1]

(b) both nuts are peanuts, [1]

(c) one almond nut and one macadamia nut was picked, [2]

(d) both nuts are different. [2]

8 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = 4x + \frac{25}{x^2}.$$

Some corresponding values of x and y are given in the table below.

x	1	1.25	1.5	2.0	2.5	3.0	3.5	4.0
y	p	21.0	17.1	14.3	14.0	14.8	16.0	17.6

(a) Calculate the value of p, to 1 decimal place. [1]

(b) Using a scale of 2 cm to represent 0.5 unit, draw a horizontal x-axis for $0 \leq x \leq 4$.
Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for $0 \leq y \leq 30$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the solution(s) of the equation $4x + \frac{25}{x^2} = 25$ for $0 \leq x \leq 4$ [2]

(d) By drawing a tangent, find the gradient of the curve at $x = 1.5$. [2]

(e) By drawing a suitable straight line graph, solve $3x^3 + 10x^2 - 25 = 0$. [3]

9 Ivy runs an online business delivering goods by post.
The local postage rates are shown below.

Weight-Step Up to	Standard Regular (C5, C6 & DL size envelope)	Standard Large (Up to C4 size envelope)	Non-Standard
20g	$0.30	$0.60	$0.60
40g	$0.37		
100g			$0.90
250g		$0.90	$1.15
500g		$1.15	$1.70
1kg		$2.55	
2kg		$3.55	

[Extracted from https://www.singpost.com/sites/default/files/PostageRates-MailingGuidelines.pdf]

(a) **(i)** For a particular delivery, Ivy used a C4 size envelope for her paper-based goods which weighed 150g. Write down the cost of posting this package. [1]

(ii) However, when Ivy attempted to deliver the package, she was informed that her package is considered a non-standard mail as it contained merchandise.

Calculate the additional amount she has to pay to deliver the package. [2]

Ivy would like to expand her business to delivering to overseas customers.
The airmail rates for packages are shown below.

Destination	**Small Packages***	
	Weight-Step Up To (max weight: 2kg)	**Postage Rate**
Zone 1 Malaysia and Brunei	100g	$2.50
	250g	$3.90
	500g	$5.20
	per additional 100g	$1.10
Zone 2 Countries in the Asia & The Pacific (except Australia, Japan & New Zealand)	100g	$3.20
	250g	$6.80
	500g	$12.00
	per additional 100g	$2.50
Zone 3 Countries in the rest of the world, including Australia, Japan, New Zealand, Africa, The Americas, Europe & The Middle East	100g	$4.70
	250g	$9.85
	500g	$17.00
	per additional 100g	$3.50

*Small Packages are mail containing goods or merchandise that are up to 2kg in weight. The largest dimensions should not exceed 60 cm, with length, width and height combined not exceeding 90 cm.

[Extracted from https://www.singpost.com/sites/default/files/PostageRates-MailingGuidelines.pdf]

(b) **(i)** Calculate the cost of sending a package weighing 562g to Australia. [1]

(ii) Write down a possible set of dimensions of a small package, giving your answer in cm. [1]

(iii) Ivy observed that the packages usually weigh from 210g to 270g. For ease of charging, Ivy would like to implement a fixed delivery charge, regardless of destination.

Assume that an equal amount of packages is delivered to each zone, and considering the range of weights of packages, determine a reasonable fixed delivery charge that Ivy should implement. Justify your answer with appropriate working.

[4]

ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATIONS
MATHEMATICS PAPER 2 (4048/2)
ANSWER KEY

1a	$\frac{7}{5}x^2y$	5aiii	Since $\angle ROS + \angle TRO = 86° + 90° \neq 180°$, OR is not parallel to RT. Therefore, $OSTR$ is not a trapezium as it does not have a pair of parallel sides.
1b	$\frac{(3x+1)}{(2x+1)}$		
1c	$-1 < x \leq \frac{1}{3}$	5bi	0.715 radians
1di	$\frac{4x+12}{(x-7)(x+1)}$	5bii	12.5m
1dii	$x = 12.7 \quad or \quad -1.73$ (3 sig fig)	6ai	$\begin{pmatrix}-10\\-12\end{pmatrix}$
2ai	$3m + 2r = 17.85$ $5m + 4r = 32.75$	6aii	15.6 units
		6aiii	(–10, –6)
2aii	$m = 2.95$, $r = 4.5$	6bia	$\overrightarrow{AE} = \frac{3}{4}\mathbf{a}$
2aiii	\$19.40		
2b	$(4x-1)(x+y)$	6bib	$\overrightarrow{DE} = -\mathbf{b} + \frac{3}{4}\mathbf{a}$
2ci	$2 \times 5^2 \times 7 \times 11$		
2cii	154	6bic	$\overrightarrow{FA} = -\frac{4}{5}\mathbf{b} + \frac{3}{5}\mathbf{a}$
2ciii	55 *and* 70		
3a	55.0 m	6bii	$\frac{4}{5}$
3b	057.5°		
3c	1490m^2	7ai	No, because the highest frequency occurs 3 times.
3d	46.7°		
4ai	$\begin{pmatrix}1&1\\2&2\\2&2\end{pmatrix}$	7aiia	2 goals
		7aiib	1.41 goals (3 sig. fig.)
		7aiii	As the mean number of goals was smaller in 2006 than in 2018, France scored better at the 2018 World Cup. As the standard deviation of goals was smaller in 2006 than in 2018, France scored more consistently at the 2006 World Cup.
4aii	$\begin{pmatrix}145\\150\end{pmatrix}$		
4aiii	$\begin{pmatrix}295\\590\\590\end{pmatrix}$		
4aiv	The elements of **T** represent the total weight of fruits and vegetables consumed by school-going children and adolescents, based on their age-range.	7bi	1st selection: $\frac{10}{43}, \frac{12}{43}, \frac{21}{43}$ 2nd selection: $\frac{9}{42}, \frac{12}{42}, \frac{21}{42}, \frac{10}{42}, \frac{11}{42}, \frac{21}{42}, \frac{10}{42}, \frac{12}{42}, \frac{20}{42}$
		7biia	$\frac{10}{43}$
4bi	4.1 km	7biib	$\frac{10}{43}$
4bii	Yes.		
5aia	154°	7biic	$\frac{40}{301}$
5aib	150°		
5aii	$\angle POR = 154°$ (from (a)(i)(a)) $\angle OPQ + \angle ORQ + 154° + 103° = 360°$ (angle sum of quadrilateral) $\angle OPQ + \angle ORQ = 103°$ (shown)	7biid	$\frac{194}{301}$

8a	29.0
8b	Graph
8c	$x = 1.1$
8d	Gradient = –10.814 (exact)
8e	$x = 1.325$
9ai	\$0.90
9aii	\$0.25
9bi	\$20.50
9bii	e.g. 20cm by 30cm by 40cm
9biii	\$9.20

ZHONGHUA SECONDARY SCHOOL
4E/4N/5N
PRELIMINARY EXAMINATIONS (2018)

Marking Scheme

Qn [14m]		Answer	Mark Allocated
1	(a)	$\frac{5x^3}{7y^3} \div \frac{25x}{49y^4}$ $= \frac{5x^3}{7y^3} \times \frac{49y^4}{25x}$ $= \frac{7}{5}x^2y$	M1 A1
	(b)	$\frac{9x^2-1}{6x^2+x-1}$ $= \frac{(3x-1)(3x+1)}{(3x-1)(2x+1)}$ $= \frac{(3x+1)}{(2x+1)}$	B1 – factorisation of $9x^2-1$ B1 – factorisation of $6x^2+x-1$ A1
	(c)	$2x-1 < 3x \leq \frac{2+3x}{3}$ $2x-1 < 3x$ and $3x \leq \frac{2+3x}{3}$ $2x-1 < 3x$ and $9x \leq 2+3x$ $x > -1$ and $x \leq \frac{1}{3}$ $\therefore -1 < x \leq \frac{1}{3}$	B1, B1 – each correct inequality B1
	(d)(i)	$\frac{5}{x-7} - \frac{1}{x+1}$ $= \frac{5(x+1)-(x-7)}{(x-7)(x+1)}$ $= \frac{5x+5-x+7}{(x-7)(x+1)}$ $= \frac{4x+12}{(x-7)(x+1)}$	M1 – taking common denominator A1
	(d)(ii)	$\frac{5}{x-7} - \frac{1}{x+1} = \frac{4}{5}$ $\frac{4x+12}{(x-7)(x+1)} = \frac{4}{5}$ $\frac{x+3}{(x-7)(x+1)} = \frac{1}{5}$ $5x+15 = x^2-6x-7$	M1 – ft from their (d)(i)

		$x^2 - 11x - 22 = 0$ $x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(-22)}}{2(1)}$ $x = 12.7 \quad or \quad -1.73$ (3 sig fig)	M1 – reduce to quadratic M1 – substitution of values seen A1 – both roots

Qn [11m]		Answer	Mark Allocated
2	**(a)(i)**	$3m + 2r = 17.85$ $5m + 4r = 32.75$	B1 B1
	(a)(ii)	$3m + 2r = 17.85$ ---① $5m + 4r = 32.75$ ---② ① × 2: $6m + 4r = 35.7$ ---③ ③ – ②: $m = 2.95$ $r = 4.5$	 B1 B1
	(a)(iii)	\$19.40	B1 – B0 if not written to 2 d.p.
	(b)	$4x^2 + 4xy - x - y$ $= 4x(x + y) - (x + y)$ $= (4x - 1)(x + y)$	 M1 A1
	(c)(i)	$2 \times 5^2 \times 7 \times 11$	B1
	(c)(ii)	154	B1
	(c)(iii)	55 *and* 70	B1, B1

Qn [12m]		Answer	Mark Allocated
3	(a)	$\angle QPS = 180° - 105° = 75°$ (adj. ∠s on a st. line) By cosine rule, $QS^2 = 42^2 + 48^2 - 2(42)(48)\cos 75°$ $QS^2 = 3024.44161$ $QS = 54.9949 = 55.0$ m (3 sig. fig.)	M1 – seen or implied M1 – applying Cosine rule A1
	(b)	By sine rule, $\frac{sin\angle PQS}{48} = \frac{sin75°}{54.9949}$ $sin\angle PQS = 0.84306$ $\angle PQS = 57.4655$ Bearing of *S* from *Q* $= 057.5°$ (1 dec.pl.)	M1 – applying Sine rule A1 – finding $\angle PQS$ A1 – answer statement must be seen
	(c)	Area of $\Delta PQS = \frac{1}{2} \times 42 \times 48 \times \sin 75° = 973.653\text{m}^2$ Area of ΔQRS $= \frac{1}{2} \times 54.9949 \times 35 \times \sin(90 - 57.4655)°$ $= 517.591\text{m}^2$ Area of field *PQRS* = 1491.24 = 1490m^2 (3 sig. fig.)	M1 M1 – for $(90 - their\ \angle PQS)$ A1

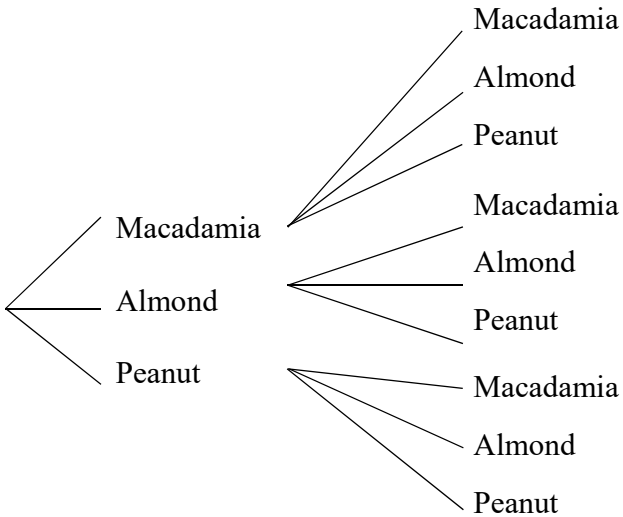
(d)	Let h be the shortest distance from R to QS. $\frac{1}{2} \times QS \times h = 517.59$ or $\sin \angle SQR = \frac{h}{35}$ $h = 18.823 = 18.8$ m (3 sig. fig.) Let angle of elevation be θ. $\tan\theta = \frac{20}{18.823}$ $\theta = 46.736 = 46.7°$ (1 dec. pl.) Largest angle of elevation = 46.7° (1 dec. pl.)	M1 (or trigo ratio) M1 – calculating angle of elevation A1 – answer statement

Qn [7 m]		Answer	Mark Allocated
4	(a)(i)	$\begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 2 & 2 \end{pmatrix}$	B1
	(a)(ii)	$\begin{pmatrix} 145 \\ 150 \end{pmatrix}$	B1
	(a)(iii)	$\mathbf{T} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 145 \\ 150 \end{pmatrix}$ $= \begin{pmatrix} 295 \\ 590 \\ 590 \end{pmatrix}$	B1
	(a)(iv)	The elements of **T** represent the total weight of fruits and vegetables consumed by school-going children and adolescents, based on their age-range.	B1
	(b)(i)	No. of kilometers Janet walks $= 8.2 \times 0.5km$ $= 4.1km$	B1
	(b)(ii)	Janet's walking speed in km/h = $1.2 \times \frac{3600}{1000} = 4.32km/h$ Since Janet walks at a speed of 4.32km in 1 hour, and 4.32km > 4.1km, she will be able to complete her route in 1 hour. OR Speed = 1.2 m/s Distance = 4.1 km = 4100 m Time taken = $\frac{4100}{1.2}$ seconds $= \frac{4100}{1.2} \div 60$ minutes = 56.94 min Since Janet took 56.94min which is < 60 min (1 hr), she will be able to complete her route in 1 hour.	M1 – conversion to km/h A1 – concluding statement seen M1 – conversion to min A1 – concluding statement seen

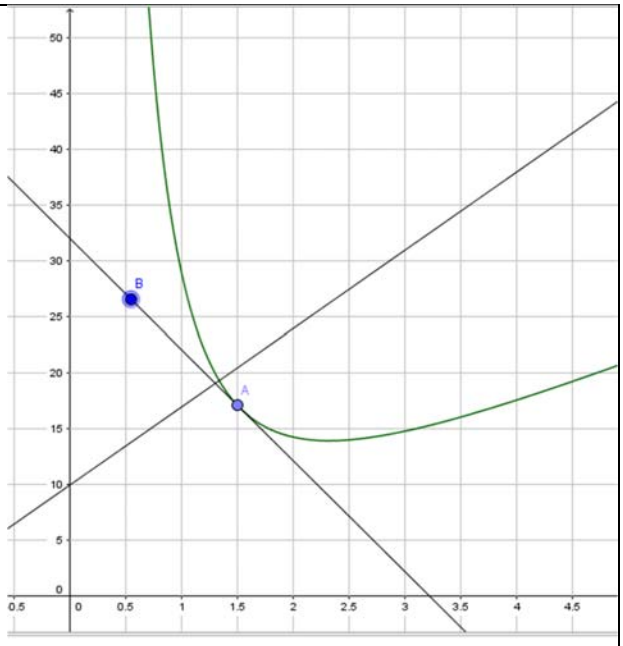
Qn [11m]		Answer	Mark Allocated
5	(a)(i)(a)	Reflex $\angle POR = 103° \times 2 = 206°$ (angle at centre = twice of angle at circumference) $\angle POR = 360° - 206° = 154°$ (angles at a point)	M1 A1
	(a)(i)(b)	$\angle ORT = 90°$ (tangent perpendicular to radius) $\angle ROS = 206° - 120° = 86°$ $\angle OST = 360° - 86° - 90° - 34° = 150°$ **OR** $\angle OSP = (180° - 120°) \div 2 = 30°$ ($\angle$ sum of triangle, base angles of isosceles triangle) $\angle OST = 180° - 30° = 150°$	M1 – reason stated A1 M1 – reason stated A1
	(a)(ii)	$\angle OPS = (180° - 120°) \div 2 = 30°$ $\angle ORS = (180° - 86°) \div 2 = 47°$ $\angle QPS + \angle QRS = 180°$ (angles in opposite segment) $\angle OPQ + \angle OPS + \angle ORQ + \angle ORS = 180°$ $\angle OPQ + 30° + \angle ORQ + 47° = 180°$ $\angle OPQ + \angle ORQ = 103°$ (shown) **OR** $\angle POR = 154°$ (from (a)(i)(a)) $\angle OPQ + \angle ORQ + 154° + 103° = 360°$ (angle sum of quadrilateral) $\angle OPQ + \angle ORQ = 103°$ (shown)	B1 – for both angles B1 – reason stated, leading to conclusion B1 – reason stated B1
	(a)(iii)	Since $\angle ROS + \angle TRO = 86° + 90° \neq 180°$, OR is not parallel to RT. Therefore, $OSTR$ is not a trapezium as it does not have a pair of parallel sides.	B1
	(b)(i)	By cosine rule, $\cos \angle AOB = \frac{2^2 + 2^2 - 1.4^2}{2(2)(2)} = \frac{151}{200}$ $\angle AOB = 0.71514 = 0.715$ radians (3 sig. fig.)	M1 – correct application of cosine rule A1
	(b)(ii)	Reflex $\angle POQ = 2\pi - 0.71514 = 5.5680$ Perimeter of major segment = 2(5.5680) + 1.4 = 12.5m (3 sig. fig.)	M1 – seen or implied, their (bi) A1

Qn [12m]		Answer	Mark Allocated
6	(a)(i)	$\overrightarrow{PQ} = \begin{pmatrix}-10\\-12\end{pmatrix}$	B1
	(a)(ii)	$\left\|\overrightarrow{PQ}\right\| = \sqrt{(-12)^2 + (-10)^2}$ = 15.6 units (3 sig. fig.)	M1 – ft from their (i) A1 – correct answer only
	(a)(iii)	$\overrightarrow{QR} = \frac{1}{2}\overrightarrow{PQ} = \begin{pmatrix}-5\\-6\end{pmatrix}$ $\overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{QR}$ $\overrightarrow{OR} = \begin{pmatrix}-5\\0\end{pmatrix} + \begin{pmatrix}-5\\-6\end{pmatrix} = \begin{pmatrix}-10\\-6\end{pmatrix}$ Coordinates of R = (–10, –6)	M1 – ft from their (i) A1 – coordinates must be stated
	(b)(i)(a)	$\overrightarrow{AE} = \frac{3}{4}\mathbf{a}$	B1
	(b)(i)(b)	$\overrightarrow{DE} = \overrightarrow{DA} + \overrightarrow{AE} = -\mathbf{b} + \frac{3}{4}\mathbf{a}$	B1
	(b)(i)(c)	$\overrightarrow{FA} = \frac{4}{5}\left(-\mathbf{b} + \frac{3}{4}\mathbf{a}\right) = -\frac{4}{5}\mathbf{b} + \frac{3}{5}\mathbf{a}$	B1
	(b)(ii)	$\frac{\text{area of } \Delta AFD}{\text{area of } \Delta ADE} = \frac{4}{5}$	B1
	(b)(iii)	$\overrightarrow{FB} = \overrightarrow{FA} + \overrightarrow{AB} = -\frac{4}{5}\mathbf{b} + \frac{3}{5}\mathbf{a} + \mathbf{a}$ $= \frac{8}{5}\mathbf{a} - \frac{4}{5}\mathbf{b}$ $= \frac{8}{5}\left(\mathbf{a} - \frac{1}{2}\mathbf{b}\right)$ $\overrightarrow{GB} = \overrightarrow{GA} + \overrightarrow{AB} = -\frac{1}{2}\mathbf{b} + \mathbf{a}$ $\overrightarrow{FG} = -\frac{3}{10}\mathbf{b} + \frac{3}{5}\mathbf{a}$ Since $\overrightarrow{FB} = \frac{8}{5}\overrightarrow{GB}$, $\overrightarrow{FB}$ is parallel to $\overrightarrow{GB}$, B is a common point, therefore F, G, and B lie on a straight line.	M2 – find any two vectors A1 – concluding statement

Qn [13m]		Answer	Mark Allocated
7	(a)(i)	No, because the highest frequency occurs 3 times.	B1
	(a)(ii)(a)	2 goals	B1
	(a)(ii)(b)	1.41 goals (3 sig. fig.)	B1
	(a)(iii)	As the mean number of goals was smaller in 2006 than in 2018, France scored better at the 2018 World Cup. As the standard deviation of goals was smaller in 2006 than in 2018, France scored more consistently at the 2006 World Cup.	B1 B1

	(b)(i)	 –1 mark if legend/key is missing	B2 – Correct tree diagram with all probabilities correct B1 – for branches for first selection correct 1st selection: $\frac{10}{43}, \frac{12}{43}, \frac{21}{43}$ 2nd selection: $\frac{9}{42}, \frac{12}{42}, \frac{21}{42}, \frac{10}{42}, \frac{11}{42}, \frac{21}{42}, \frac{10}{42}, \frac{12}{42}, \frac{20}{42}$
	(b)(ii)(a)	$\frac{10}{43}$	B1
	(b)(ii)(b)	$\frac{21}{43} \times \frac{20}{42} = \frac{10}{43}$	B1
	(b)(ii)(c)	$\frac{12}{43} \times \frac{10}{42} \times 2$ $= \frac{40}{301}$	M1 – ft from tree diagram A1
	(b)(ii)(d)	$1 - \left(\frac{10}{43} \times \frac{19}{42}\right) - \left(\frac{12}{43} \times \frac{11}{42}\right) - \left(\frac{21}{43} \times \frac{20}{42}\right)$ $= \frac{194}{301}$	M1 A1

Qn [11m]		Answer
8	(a)	$p = 29.0$
	(b)	P2 – All 8 points plotted (P1 – at least 6 points plotted correctly) C1 – Smooth curve drawn through plotted points, dependent on P1 Minus 1 mark if scale is not adhered to
	(c)	$x = 1.1$
	(d)	Gradient = –10.814 (exact) M1 – tangent line drawn at $x = 1.5$ A1 – gradient estimated and calculated between –9 to –11
	(e)	$3x^3 + 10x^2 - 25 = 0$ $7x + 10 = 4x + \frac{25}{x^2}$ Draw $y = 7x + 10$ M2 for straight line $y = 7x + 10$ drawn. (M1 if only $y = 7x + 10$ is seen) $x = 1.325$ [A1]



Qn [9m]		Answer	Mark Allocated
9	(a)(i)	\$0.90	B1
	(a)(ii)	Cost of mailing non-standard mail = \$1.15 Additional amount = \$1.15 – \$0.90 = \$0.25	M1 A1
	(b)(i)	\$17.00 + \$3.50 = \$20.50	B1
	(b)(ii)	Any set of 3 **reasonable** measurements, each not exceeding 60cm, total not exceeding 90cm e.g. 20cm by 30cm by 40cm [B1] e.g. 1cm by 1 cm by 1cm [B0]	B1
	(b)(iii)	Mean cost of packages from 210g up to 250g = (\$3.90 + \$6.80 + \$9.85) ÷ 3 = \$6.85 OR Median cost of packages from 210g up to 250g = \$6.80 Mean cost of packages heavier than 250g up to 270g = (\$5.20 + \$12.00 + \$17.00) ÷ 3 = \$11.40 OR Median cost of packages heavier than 250g up to 270g = \$12.00 Mean cost across weight categories = (\$6.85 + \$11.40) ÷ 2 = \$9.125 OR Median cost across weight categories = (\$6.80 + \$12.00) ÷ 2 = \$9.40 Ivy could charge \$9.20 as it is an average of the cost of airmailing an average package across two weight categories.	M1 – select costs across all 3 zones M1 – select costs across weight categories M1 – using mean or median A1 – conclusion, justified by calculations, answer correct to 2 d.p.