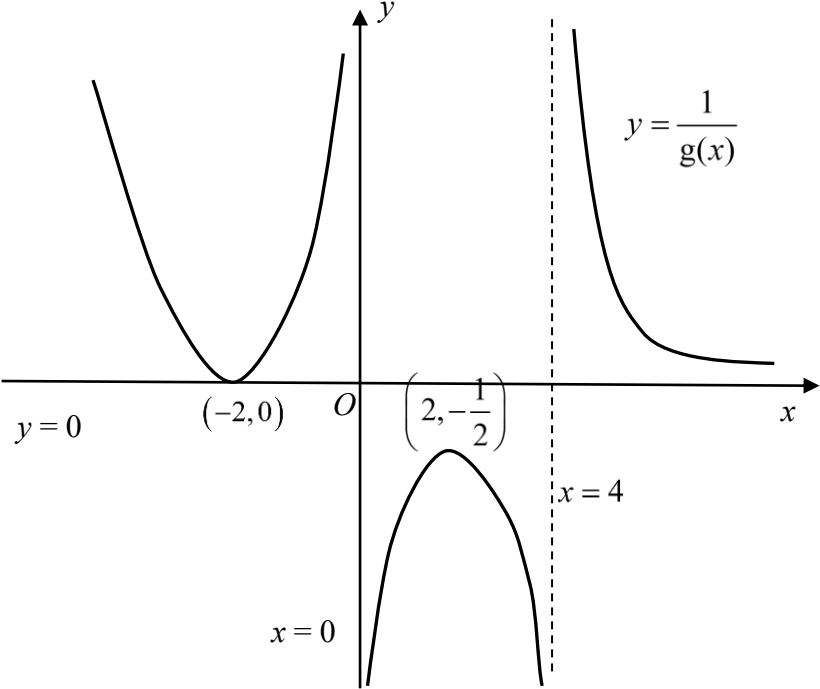
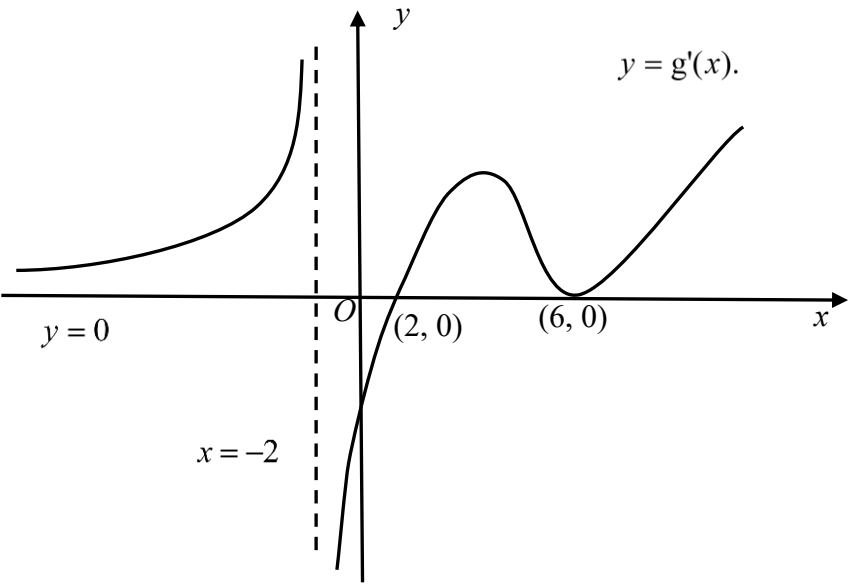


2026 ACJC H2 Math Preliminary Examination Paper 2 Solutions

Qn	Solution	Remarks
1(a)	 The graph shows a function $y = \frac{1}{g(x)}$ on a Cartesian coordinate system. The x-axis is labeled $y=0$ and the y-axis is labeled $x=0$. The origin is labeled O. The graph has a vertical asymptote at $x=4$. The function has a turning point at $(-2, 0)$ and a point at $(2, -\frac{1}{2})$.	There is a tendency for students to miss out the turning point $(-2, 0)$. They need to understand why $(-2, 0)$ exists.
1(b)	 The graph shows a function $y = g'(x)$ on a Cartesian coordinate system. The x-axis is labeled $y=0$ and the y-axis is labeled $x=-2$. The origin is labeled O. The graph has a vertical asymptote at $x=-2$. The function has x-intercepts at $(2, 0)$ and $(6, 0)$.	Since the gradient at $x=2$ and $x=6$ for $y=g(x)$ is zero, therefore $y=g'(x)$ has $(2, 0)$ and $(6, 0)$.

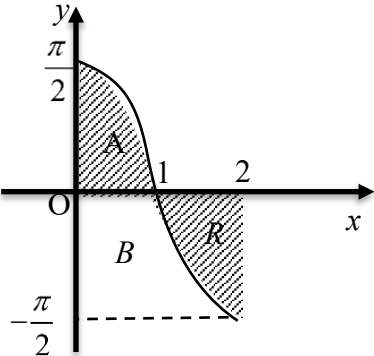
Qn	Solution	Remarks
2(a)	$y = \frac{(x-k)^2}{x+k}$ $y(x+k) = x^2 - 2kx + k^2$ $x^2 + (-y-2k)x + k^2 - yk = 0$ For x to be real, $(-y-2k)^2 - 4(k^2 - yk) \geq 0$ $y^2 + 4ky + 4k^2 - 4k^2 + 4ky \geq 0$ $y^2 + 8ky \geq 0$ $y(y+8k) \geq 0$ $y \leq -8k \text{ or } y \geq 0$ Range of values of y: $y \leq -8k \text{ or } y \geq 0$ Alternative method: $y = \frac{(x-k)^2}{x+k}$ Let $\frac{dy}{dx} = \frac{(x+k)2(x-k) - (x+k)^2}{(x+k)^2} = 0$ $2x^2 - 2k^2 - x^2 - 2kx - k^2 = 0$ $x^2 + 2kx - 3k^2 = 0$ $(x-k)(x+3k) = 0$ $x = k \text{ or } -3k$ when $x = k, y = 0$ when $x = -3k, y = -8k$ Range of values of y: $y \leq -8k \text{ or } y \geq 0$	Note that $D \geq 0$, not $D > 0$. Solving $y^2 + 8ky \geq 0$ was quite a challenge. Mistakes such as $y \geq -8k \text{ or } y \geq 0$ $y \leq -8k \text{ and } y \geq 0$ $y \leq -8k, y \geq 0$ are commonly seen. There should be working to indicate which coordinate refers to the maximum or minimum point.
2(b)	Let $y = \frac{(x-k)^2}{x+k} = f(x)$ ↓ $y = \frac{(x+k-k)^2}{x+k+k} = f(x+k)$ $y = \frac{x^2}{x+2k}$ ↓ $y = \frac{kx^2}{x+2k} = kf(x+k)$ Translation of C by k units in the negative x-direction followed by scaling by factor k parallel to the y-axis OR Scaling of C by factor k parallel to the y-axis followed by translation by k units in the negative x-direction	Words that are NOT allowed and will get penalised: - Transform - Shift - Left, right, up, down Key words like “factor of k” is commonly missing.

3 (a)	$\sum_{r=1}^n r(3r+1) = 3 \sum_{r=1}^n r^2 + \sum_{r=1}^n r$ $= 3 \left(\frac{n(n+1)(2n+1)}{6} \right) + \frac{n(n+1)}{2}$ $= \frac{n(n+1)(2n+1)}{2} + \frac{n(n+1)}{2}$ $= \frac{n(n+1)}{2} [(2n+1)+1]$ $= \frac{n(n+1)}{2} [2(n+1)] = n(n+1)^2 \text{ (Shown)}$	Note: $\sum_{r=1}^n r = \frac{n(n+1)}{2}$ can also be derived using AP Sn formula.
3 (b)	Replace k with $r+3$, $\sum_{k=4}^{n+3} \frac{(k-3)(3k-8)}{n^3} = \sum_{r+3=4}^{r+3=n+3} \frac{(r+3-3)(3(r+3)-8)}{n^3}$ $= \sum_{r=1}^n \frac{(r)(3r+1)}{n^3}$ $= \frac{1}{n^3} \sum_{r=1}^n (r)(3r+1)$ $= \frac{1}{n^3} (n(n+1)^2)$ $= \frac{1}{n^2} (n+1)^2$ $= \left(\frac{n+1}{n} \right)^2 = \left(1 + \frac{1}{n} \right)^2$	Replace k with $r-3$ are seen.
3 (c)	When $n \rightarrow \infty$, $\frac{1}{n} \rightarrow 0$. $\therefore a = \lim_{n \rightarrow \infty} \left[\sum_{k=4}^{n+3} \frac{(k-3)(3k-8)}{n^3} \right] = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^2 = 1.$ Hence, $S_{\infty} = \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}$.	Do NOT write $a \rightarrow 1$
3 (d)	Given $u_{n+1} = \frac{u_n + 1}{1 - u_n}$ and $u_1 = 2$, using GC, it is observed that the sequence has a periodic cycle of $2, -3, -\frac{1}{2}, \frac{1}{3}$.	Many students are able to observe that there's a periodic cycle and are able to evaluate the summation.

	$\sum_{r=1}^{4n} u_r = \binom{n}{r} \left(2 - 3 - \frac{1}{2} + \frac{1}{3} \right)$ $= \binom{n}{r} \left(-\frac{7}{6} \right)$ $= -\frac{7n}{6}$	
4(a)	$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ $\mathbf{n} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$ Scalar-product form: $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} = 13$ Cartesian form: $x + 5y + 3z = 13$ (shown).	As this is a shown question, using G.C to solve is not allowed. Note: $\overrightarrow{OA} \times \overrightarrow{OB}$ is NOT the normal vector.
4(b)	Line of intersection of π_1 and π_2: $\mathbf{r} = \begin{pmatrix} 1 \\ \frac{1}{3} \\ \frac{38}{15} \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ \frac{1}{3} \\ -\frac{2}{3} \\ 1 \end{pmatrix}, \text{ where } \mu \in \mathbb{R}$	This answer can be easily obtained using G.C app under “Simult Eqns”. Many various answers can be obtained manually. Students should note that substituting x, y or z to be zero is an easier way to solve for the common point of intersection.
4(c)	Let d be the length of projection of $\overrightarrow{AB}$ onto π_2, which is the same as length of projection of $\overrightarrow{AB}$ onto line of intersection found in (b),	Common misconception of formulae applied: $\left \overrightarrow{AB} \cdot \frac{\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{10}} \right$ $\left \overrightarrow{AB} \times \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \right$

	$d = \frac{\left \overrightarrow{AB} \times \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \right }{\sqrt{10}}$ $= \frac{1}{\sqrt{10}} \left \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \right $ $= \frac{1}{\sqrt{10}} \left \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} \right $ $= \frac{\sqrt{35}}{\sqrt{10}}$ $= \sqrt{\frac{7}{2}} \text{ or } 1.87 \text{ (to 3s.f.)}$ Or $d = \frac{\left \overrightarrow{AB} \cdot \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \right }{\sqrt{14}} = \frac{1}{\sqrt{14}} \left \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \right = \frac{7}{\sqrt{14}} \text{ or } 1.87 \text{ (to 3s.f.)}$	
4(d)	$l: \mathbf{r} = p\mathbf{i} - \mathbf{j} + \lambda(3\mathbf{i} + q\mathbf{j} + 4\mathbf{k})$ $= \begin{pmatrix} p \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ q \\ 4 \end{pmatrix}$ l does not intersect plane π_1. l is parallel to plane π_1 $\Rightarrow \begin{pmatrix} 3 \\ q \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} = 0 \Rightarrow 3 + 5q + 12 = 0 \Rightarrow q = -3$ l does not lie on plane π_1, hence point B does not lie on plane π_1 $\Rightarrow \begin{pmatrix} p \\ -1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} \neq 13 \Rightarrow p - 5 \neq 13 \Rightarrow p \neq 18$	Common misconception of formulae applied: $q \neq -3, p \neq 18$ $q = -3, p = 18$

4(e)	Let θ be acute angle between the line l and plane π_1. $l: \mathbf{r} = \begin{pmatrix} 18 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$ $\sin \theta = \frac{\left \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} \right }{\sqrt{25}\sqrt{35}}$ $= \frac{15}{5\sqrt{35}}$ $= \frac{3}{\sqrt{35}}$ $\theta = 30.470^\circ = 30.5^\circ \text{ (to 1 d.p.)}$	Common misconception of formula applied: $\bullet \sin \theta = \frac{\left \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \times \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} \right }{\sqrt{25}\sqrt{35}}$
5(a)(i)	$z = \sin^{-1}(1-x)$ $\sin z = 1-x$ $\cos z \frac{dz}{dx} = -1$ $x = 0, z = \sin^{-1} 1 = \frac{\pi}{2}$ $x = 1, z = \sin^{-1} 0 = 0$ $\int_0^1 \sin^{-1}(1-x) dx = \int_{\frac{\pi}{2}}^0 -z \cos z dz$ $= \int_0^{\frac{\pi}{2}} z \cos z dz$ $a = 0 \text{ and } b = \frac{\pi}{2}$ Method 2 $z = \sin^{-1}(1-x)$ $\frac{dz}{dx} = \frac{-1}{\sqrt{1-(1-x)^2}} = \frac{-1}{\sqrt{1-\sin^2 z}}$ $= \frac{-1}{\sqrt{\cos^2 z}} = \frac{-1}{\cos z}$ $x = 0, z = \sin^{-1} 1 = \frac{\pi}{2}$ $x = 1, z = \sin^{-1} 0 = 0$	As this is “show” question, students should not skip steps. Students need to show the replacement of x values to the corresponding z values in the integral.

	$\int_0^1 \sin^{-1}(1-x) dx = \int_{\frac{\pi}{2}}^0 -z \cos z dz$ $= \int_0^{\frac{\pi}{2}} z \cos z dz$ $a = 0 \quad \text{and} \quad b = \frac{\pi}{2}$	
(a)(ii)	$\int_0^{\frac{\pi}{2}} z \cos z dz$ $= [z \sin z]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin z dz$ $= \frac{\pi}{2} + [\cos z]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$	Students need to present the application of <i>integration by parts</i> formula clearly.
(b) (i)	 Since curve $y = \sin^{-1}(1-x)$ is symmetrical about $(1,0)$, area A = area R $\int_0^2 \sin^{-1}(1-x) dx = 2 \int_0^1 \sin^{-1}(1-x) dx$ $= 2 \left(\frac{\pi}{2} - 1 \right) = \pi - 2$	Students need to understand how to remove the modulus before integrating. Students DO NOT need to integrate $\sin^{-1}(1-x)$ again since the result from (a)(ii) can be used to evaluate $2 \int_0^1 \sin^{-1}(1-x) dx .$

(ii)

$$y = \sin^{-1}(1-x)$$

$$\sin y = 1-x$$

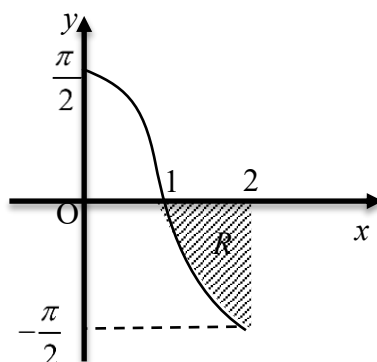
$$x = 1 - \sin y$$

when $x = 2$,

$$y = \sin^{-1}(1-2) = -\frac{\pi}{2}$$

when $x = 1$,

$$y = \sin^{-1}(1-1) = 0$$



Volume

$$= \pi(2)^2 \left(\frac{\pi}{2} \right) - \pi \int_{-\frac{\pi}{2}}^0 x^2 dy$$

$$= 2\pi^2 - \pi \int_{-\frac{\pi}{2}}^0 (1 - \sin y)^2 dy$$

$$= 2\pi^2 - \pi \int_{-\frac{\pi}{2}}^0 (1 + \sin^2 y - 2\sin y) dy$$

$$= 2\pi^2 - \pi \int_{-\frac{\pi}{2}}^0 \left(\frac{3}{2} - \frac{1}{2} \cos 2y - 2\sin y \right) dy$$

$$= 2\pi^2 - \pi \left[\frac{3y}{2} - \frac{1}{4} \sin 2y + 2\cos y \right]_{-\frac{\pi}{2}}^0$$

$$= 2\pi^2 - \pi \left[2 - \left(-\frac{3\pi}{4} \right) \right]$$

$$= 2\pi^2 - 2\pi - \frac{3\pi^2}{4}$$

$$= \frac{5\pi^2}{4} - 2\pi$$

Students cannot simply apply the formula, $\pi \int_{-\frac{\pi}{2}}^0 x^2 dy$, to find the volume generated about the y -axis.

6

Let the random variable X denote the delivery time of an order in minutes. $X \sim N(\mu, \sigma^2)$

$$P(X < 13) = 0.2$$

$$P\left(Z < \frac{13 - \mu}{\sigma}\right) = 0.2$$

$$\frac{13 - \mu}{\sigma} = -0.84162 \Rightarrow \mu - 0.84162\sigma = 13$$

$$P(X > 25) = 0.26$$

$$P\left(Z > \frac{25 - \mu}{\sigma}\right) = 0.26$$

$$\frac{25 - \mu}{\sigma} = 0.643345 \Rightarrow \mu + 0.643345\sigma = 25$$

Some students are confused between these two expressions for Z .

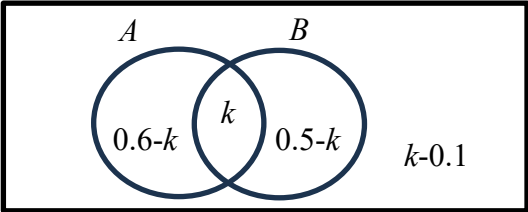
$$X \sim N(\mu, \sigma^2)$$

$$Z = \frac{X - \mu}{\sigma}$$

vs

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

	Using GC $\frac{25 - \mu}{\sigma} = 0.643345 \Rightarrow \mu + 0.643345\sigma = 25$ $\mu = 19.8$ $\sigma = 8.08$	
7 (a)	Given $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = k$. <u>Method 1:</u> $P(A \cap B) \leq P(B)$ Hence maximum of $P(A \cap B) = P(B) = 0.5$ $P(A \cap B) \geq P(A) + P(B) - 1$ Hence minimum of $P(A \cap B) = 0.6 + 0.5 - 1 = 0.1$ $\therefore 0.1 \leq k \leq 0.5$ <u>Method 2:</u>  Consider: $0 \leq 0.6 - k \leq 1 \Rightarrow 0 \leq k \leq 0.6$ AND Consider: $0 \leq 0.5 - k \leq 1 \Rightarrow 0 \leq k \leq 0.5$ AND Consider: $0 \leq k - 0.1 \leq 1 \Rightarrow 0.1 \leq k \leq 1$ Combing all results, $\therefore 0.1 \leq k \leq 0.5$	Some students gave the answer as strict inequality.
7 (b)	$P(A) \times P(B) = 0.6 \times 0.5 = 0.3$ $P(A \cap B) = 0.25$ Since $P(A \cap B) \neq P(A) \times P(B)$, hence A and B are not independent.	Students need to write down the values of BOTH the LHS and RHS in this part of the solution. $P(A \cap B) = P(A) \times P(B)$

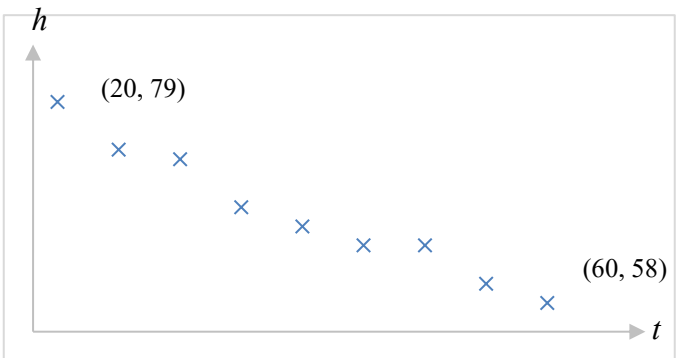
7 (c)	$P(A' \cap B' \cap C') = 1 - P(A \cup B \cup C)$ $= 1 - [P(A) + P(A' \cap B) + P(C \cap B')]$ $= 1 - [0.6 + (0.5 - 0.25) + (0.2 - 0.08)]$ $= 1 - 0.97$ $= 0.03$	
7 (d)	$P(B (A \cup C)) = \frac{P(B \cap (A \cup C))}{P(A \cup C)} = \frac{P(B \cap A) + P(B \cap C)}{P(A) + P(C)}$ $= \frac{0.25 + 0.08}{0.6 + 0.2} = \frac{33}{80} = 0.4125$	Do not round off exact answers to 3 sf.
8 (a)	<u>Method 1:</u> No. of ways = $3! \times {}^4C_1 \times ({}^3C_1 + {}^3C_2 + {}^3C_3)$ $= 6 \times 4 \times (3 + 3 + 1)$ $= 6 \times 4 \times 7$ $= 168$ Case 1: Room Y has 1 rapper and 1 vocalist. ${}^3C_1 \Rightarrow$ Choose 1 vocalist from the 3 remaining vocalists for Room Y. Hence Room Y has 1 rapper and 1 vocalist. Room Z has 2 vocalists. Case 2: Room Y has 1 rapper and 2 vocalists. ${}^3C_2 \Rightarrow$ Choose 2 vocalists from the 3 remaining vocalists for Room Y. Hence Room Y has 1 rapper and 2 vocalists. Room Z has 1 vocalist. Case 3: Room Y has 1 rapper and 3 vocalists. ${}^3C_3 \Rightarrow$ Choose 3 vocalists from the 3 remaining vocalists for Room Y. Hence Room Y has 1 rapper and 3 vocalists. Room Z has no vocalist. <u>Method 2:</u> Case 1: Room X (1R, 1V) & Room Y (1R, 1V) & Room Z (1R, 2V) No. of ways = $({}^3C_1 \times {}^4C_1)({}^2C_1 \times {}^3C_1)({}^1C_1 \times {}^2C_2) = 72$ Case 2: Room X (1R, 1V) & Room Y (1R, 2V) & Room Z (1R, 1V) No. of ways = $({}^3C_1 \times {}^4C_1)({}^2C_1 \times {}^3C_2)({}^1C_1 \times {}^1C_1) = 72$ Case 3: Room X (1R, 1V) & Room Y (1R, 3V) & Room Z (1R, 0V) No. of ways = $({}^3C_1 \times {}^4C_1)({}^2C_1 \times {}^3C_3)({}^1C_1) = 24$ No. of ways = $72 + 72 + 24 = 168$	Method 1 is an efficient method which is rarely observed. Common issue among those who use method 1 is $3! \times {}^4C_1 \times ({}^3C_1 {}^3C_2 {}^3C_3)$ instead of $3! \times {}^4C_1 \times ({}^3C_1 + {}^3C_2 + {}^3C_3)$. Method 2 is the more popular method. The following are the most common mistakes: Adding up the 3 scenarios within each case instead of multiplying them together. eg $({}^3C_1 \times {}^4C_1) + ({}^2C_1 \times {}^3C_1) + ({}^1C_1 \times {}^2C_2)$ [Note that adding up the 3 scenarios within each case is incorrect as it means the 3 scenarios may or may not occur concurrently within each case.]

	<u>Method 3:</u> No. of ways = $3! \times {}^4C_1 \times (2^3 - 1)$ $= 6 \times 4 \times 7$ $= 168$ $3! \Rightarrow$ The 3 rappers permute among the 3 rooms. ${}^4C_1 \Rightarrow$ Choose 1 vocalist from the 4 vocalist because Room X need one more person. $2^3 \Rightarrow$ For each of the remaining 3 vocalists, each of them has 2 choices: either join Room Y or Room Z. Each of them need to choose 1 out of these 2 choices. Hence, $2 \times 2 \times 2$ $-1 \Rightarrow$ Need to minus off one case from 2^3 whereby all the remaining 3 vocalists choose to join Room Z. This means that Room Y will only have 1 person and will not fulfill the condition that it need to have at least 2 people.	Permutating the members within each room which is not required. Eg. $({}^3C_1 \times {}^4C_1)({}^2C_1 \times {}^3C_1)({}^1C_1 \times {}^2C_2)(2!2!3!)$ [Note: We are only interested in the number of combinations of members for each room and not the number of ways they are permuted within each room.] Method 3 is also an efficient method but rarely observed.
8 (b)	<u>Method 1:</u> No. of ways = $(3! \times 2) \times (3-1)! \times ({}^3C_2 \times 2!)$ $= 12 \times 2 \times 6$ $= 144$ $(3! \times 2) \Rightarrow$ $3!$ ways to permute the 3 rappers followed by 2 ways to slot in the manager between the 3 rappers such that manager is in between 2 rappers. $(3-1)! \Rightarrow$ No. of ways to permute the block of [3 Rappers & Manager] and the 2 vocalists in a circle. ${}^3C_2 \times 2! \Rightarrow$ 3C_2 means choose 2 out of the 3 possible positions to insert the 2 vocalist so that they are separated. $2!$ Ways for the 2 vocalists to swap positions among themselves. <u>Method 2:</u> No. of ways = $({}^3C_2 \times 2! \times 2!) \times (3-1)! \times ({}^3C_2 \times 2!)$ $= 12 \times 2 \times 6$ $= 144$ ${}^3C_2 \times 2! \times 2! \Rightarrow$ 3C_2 means choose 2 rappers from the 3 rappers to sit beside the manager. $2!$ means the 2 chosen rappers can swap positions among themselves. $2!$ means the block of [Rapper,	Method 1 is the most common method observed. Note that 4C_2 is not needed as the two particular vocalists are already identified and hence there is no need to choose these 2 particular vocalists from the 4 vocalists. For those who use method 2 for the arrangement of the manager & the 3 rappers, the most common mistake is forgetting that the block of [Rapper, Manager, Rapper] and the 3rd unchosen Rapper can swap position. Hence having a missing $2!$ in their workings.

	Manager, Rapper] and the 3rd unchosen Rapper can swap position. $(3-1)! \Rightarrow$ No. of ways to permute the block of [3 Rappers & Manager] and the 2 vocalists in a circle. ${}^3C_2 \times 2! \Rightarrow$ 3C_2 means choose 2 out of the 3 possible positions to insert the 2 vocalist so that they are separated. 2! Ways for the 2 vocalists to swap positions among themselves. <u>Method 3: (Complement Method)</u> No. of ways = $[3! \times 2 \times (5-1)!] - [3! \times 2 \times (4-1)! \times 2!]$ $= 288 - 144 = 144$ $3! \times 2 \times (5-1)! \Rightarrow$ Total number of ways without restrictions to the 2 particular vocalists. $(3! \times 2) \Rightarrow 3!$ ways to permute the 3 rappers followed by 2 ways to slot in the manager between the 3 rappers such that manager is in between 2 rappers. $(5-1)! \Rightarrow$ No. of ways to permute the block of [3 Rappers & Manager] and the 4 vocalists in a circle. $3! \times 2 \times (4-1)! \times 2! \Rightarrow$ Total number of ways whereby the 2 particular vocalists are together. $(3! \times 2) \Rightarrow 3!$ ways to permute the 3 rappers followed by 2 ways to slot in the manager between the 3 rappers such that manager is in between 2 rappers. $(4-1)! \Rightarrow$ No. of ways to permute the block of [3 Rappers & Manager], the block of [2 particular vocalists together], and the 2 remaining vocalists. $2! \Rightarrow$ The 2 particular vocalists who are together can swap seats.	The complement method is not a very efficient method to adopt for this question. Those who attempted using this method need to be very thorough in their considerations to be successful.
9(a)	Let X be the random variable of the mass of Oolong tea leaves in grams. Test $H_0: \mu = 600$ against $H_1: \mu \neq 600$ μ represents the population mean mass of Oolong tea leaves.	The keyword “population” is often missing in the definition of μ.
(b)	Unbiased estimate of population mean, $\bar{x} = \frac{30685}{50} = 613.7$ (exact) Unbiased estimate of population variance, $s^2 = \frac{1}{50-1} \left(18925681 - \frac{30685^2}{50} \right) = \frac{188593}{98}$ or 1924.4	Note that $\mu \neq \bar{x}$. Note that $\sigma^2 \neq s^2$.

(c)	Under H_0, Since the sample size of 50 is large enough, $\bar{X} \sim N\left(600, \frac{1924.418}{50}\right) \text{ approx by Central Limit Theorem.}$ $z = 2.2083$ $p\text{-value} = 0.02722 < 0.05$ H_0 is rejected since $p\text{-value} < 0.05$. There is sufficient evidence at 5% level of significance that the population mean mass of Oolong tea packs is <u>not</u> 600g.	Note that $z = 2.2083$ and $p\text{-value} = 0.02722$ can be obtained from GC. Note that: $p\text{-value} = 2P(\bar{X} > 613.7)$ $p\text{-value} = 2P(z > 2.2083)$ Please ensure that your conclusion includes the key words and is also in context.
(c)	Let Y be the random variable of the mass of Oolong tea leaves in grams. Under H_0, Since the sample size of 47 is large enough, $\bar{Y} \sim N\left(600, \frac{49}{47}\right) \text{ approx by Central Limit Theorem.}$ $z = \frac{k - 600}{\sqrt{\frac{49}{47}}} = \frac{(k - 600)\sqrt{47}}{7}$ Since H_0 is not rejected, $-1.95996 < \frac{(k - 600)\sqrt{47}}{7} < 1.95996$ $-1.95996 < \frac{(k - 600)\sqrt{47}}{7} < 1.95996$ $597.999 < k < 602.001$ $\{k \in \mathbb{R} : 598 < k < 602\}$	Please ensure that you state the reason for using Central Limit Theorem. Common mistakes observed includes: $X \sim N\left(600, \frac{49}{47}\right)$ $\bar{Y} \sim N\left(600, \frac{49^2}{47}\right)$ $\bar{Y} \sim N\left(613.7, \frac{49}{47}\right)$ $\bar{Y} \sim N\left(600, \frac{1924.4}{47}\right)$ $\bar{Y} \sim N\left(600, \frac{49}{50}\right)$ $s^2 = \frac{47}{47-1}(49)$ - Wrong critical values Eg. ± 1.64485
10 (a)	- The probability that a drone is successfully intercepted by a missile is assumed to be constant at p. - Whether a drone is successfully intercepted by a missile is assumed to be independent of any other drones.	The most common <u>mistake</u> is stating that “The <u>probability</u> a drone is successfully intercepted by a missile is assumed to be <u>independent</u> of any other drones.”

10 (b)	Let random variable A be the number of successfully intercepted drones by Battery Alpha. $A \sim B(10, p)$ Let random variable D be the number of successfully intercepted drones by Battery Delta. $D \sim B(15, p)$ Let random variable W be the total number of successfully intercepted drones by both batteries. $\therefore W \sim B(25, p)$ $P(W \leq 1) = 2[P(A = 0)][P(D = 1)]$ $P(W = 0) + P(W = 1) = 2[P(A = 0)][P(D = 1)]$ ${}^{25}C_0 p^0 (1-p)^{25} + {}^{25}C_1 p^1 (1-p)^{24} = 2[{}^{10}C_0 p^0 (1-p)^{10}][{}^{15}C_1 p^1 (1-p)^{14}]$ $(1-p)^{25} + 25p(1-p)^{24} = 30p(1-p)^{10}(1-p)^{14}$ $(1-p)^{25} + 25p(1-p)^{24} = 30p(1-p)^{24}$ Since $0 < p < 1$, $(1-p) + 25p = 30p$ $1 + 24p = 30p$ $p = \frac{1}{6}$	Common misinterpretations of question are: $P(W \leq 1) = 2[P(A = 0)] + [P(D = 1)]$ $2P(W \leq 1) = [P(A = 0)][P(D = 1)]$
10 (c)	Let X be the number of successfully intercepted drones in a sector. $X \sim B(40, 0.15)$ $E(X) = (40)(0.15) = 6$ $\text{Var}(X) = (40)(0.15)(0.85) = 5.1$ Since $n = 60$ is large, by Central Limit Theorem, $\bar{X} = \frac{X_1 + X_2 + \dots + X_{60}}{60} \sim N\left(6, \frac{5.1}{60}\right) \text{ approx.}$ $P(5.5 < \bar{X} < 6.5) = 0.914 \text{ (3s.f.)}$	Common mistakes made include: • Not stating reason for applying Central Limit Theorem. • $E(X) = (60)(0.15)$ • $\text{Var}(X) = (60)(0.15)(0.85)$ • $X \sim N(6, 5.1) \text{ approx}$ • $\bar{X} \sim N\left(6, \frac{5.1}{40}\right) \text{ approx}$ • $P(5.5 < \bar{X} < 6.5) = P(\bar{X} = 6)$ • $P(5.5 < \mu < 6.5)$
11(a)	Let the random variable Y denote the number of patients attending their appointment, out of 25. $Y \sim B(25, 0.84)$ $P(Y \geq 21) = 1 - P(Y \leq 20)$ $= 0.629$	Note: $P(Y \geq 21) \neq 1 - P(Y \leq 21)$ $P(Y \geq 21) = 1 - P(Y < 21)$ $= 1 - P(Y \leq 20)$
(b)	$P(Y = 21 Y \geq 21) = \frac{P(Y = 21)}{P(Y \geq 21)}$ $= \frac{0.213027}{0.62933}$ $= 0.338$	Numerator must be the intersection and is $P(Y = 21)$ only.

(c)	Let the random variable M denote the number of patients attending their appointment, out of n. $M \sim B(n, 0.84)$ $P(M \leq 50) \geq 0.97$ <table border="1"><thead><tr><th>n</th><th>$P(M \leq 50)$</th></tr></thead><tbody><tr><td>53</td><td>0.99407</td></tr><tr><td>54</td><td>$0.98088 \geq 0.97$</td></tr><tr><td>55</td><td>$0.95239 < 0.97$</td></tr></tbody></table> Greatest number of appointments that can be made is 54.	n	$P(M \leq 50)$	53	0.99407	54	$0.98088 \geq 0.97$	55	$0.95239 < 0.97$	Some students did not understand the question and did not know what the question wants.
n	$P(M \leq 50)$									
53	0.99407									
54	$0.98088 \geq 0.97$									
55	$0.95239 < 0.97$									
(d)	Let the random variables G and C denote the duration of a general consultation and cosmetic consultation respectively. $G \sim N(30, 2.7^2)$ and $C \sim N(42, 0.8^2)$ Let $A = G_1 + G_2 + \dots + G_6$ $E(A) = E(G_1 + G_2 + \dots + G_6) = 180$ $\text{Var}(A) = \text{Var}(G_1 + G_2 + \dots + G_6) = 6(2.7^2) = 43.74$ $P(A > 170) = 0.935$	$\text{Var}(G_1 + G_2 + \dots + G_6)$ $= 6\text{Var}(G)$								
(e)	Let $L = 0.85(G_1 + G_2 + G_3 + G_4 - 3C)$ $E(L) = 0.85[E(G_1 + G_2 + G_3 + G_4) - E(3C)]$ $= 0.85[4E(G) - 3E(C)]$ $= -5.1$ $\text{Var}(L) = \text{Var}(0.85(G_1 + G_2 + G_3 + G_4 - 3C))$ $= 0.85^2 \text{Var}(G_1 + G_2 + G_3 + G_4 - 3C)$ $= 0.85^2 [\text{Var}(G_1 + G_2 + G_3 + G_4) + \text{Var}(-3C)]$ $= 0.85^2 [4\text{Var}(G) + (-3)^2 \text{Var}(C)]$ $= 25.2297$ $P(L \leq 10) = P(-10 \leq L \leq 10) = 0.834$	Students need to remember to include the factor of 0.85 when formulating L. The presence of the “modulus” sign in $P(L \leq 10)$ is due to the phrase “differs from” used in the question.								
12(a) (i)		Students must label both axes and indicate the maximum and minimum values for both axes.								
(a) (ii)	For (A) $h = a + b \ln t : r = -0.990$ (3 sf) For (B) $h = d + et^2 : r = -0.957$ (3 sf)	Students should check their r values have the negative sign (using (i) as a clue)								

(a) (iii)	The product moment correlation coefficient of the model (A) $h = a + b \ln t$ is closer to -1, hence, $h = a + b \ln t$ is a better model. $h = 134 - 18.3 \ln t$	Students should indicate which model and say closer to -1 and not 1. Equation must be presented to 3 s.f
(b) (i)	For every additional hour of screen time, the (18 year old) students' sleep duration decreases by 0.55 hours.	Students must be clear with which variable is increasing/decreasing with the value.
(b) (ii)	$\bar{x} = \frac{0.9 + 1.9 + 3.3 + 3.9 + 4.7}{5} = 2.94$ $\bar{y} = -0.55(2.94) + 8.72 = 7.103$ $p = 7.103 \times 5 - (8.2 + 6.9 + 6.6 + 6.1) = 7.715 = 7.7$	Students must use raw data to calculate the mean of x then sub into the equation to get the mean of y.
(b) (iii)	$y = -0.55(7) + 8.72 = 4.87$ The estimate is not reliable as it is an extrapolation since $x = 7$ lies outside the data range $0.9 \leq x \leq 4.7$.	The explanation must show the data range of x, not for y.
(b) (iv)	$y = -0.55x + 7.72$	Quite easily done if students recognise that it is a translation of the data points by 1 unit parallel to the y axis. Hence the gradient of the line remains unchanged.