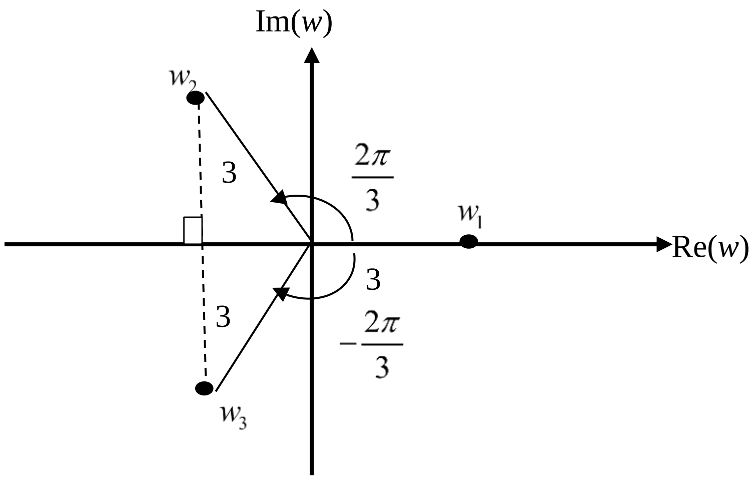
2018 H2 Mathematics 9758 Paper 2 Suggested Solutions

Section A (Pure Math)	
1(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\left(\frac{1}{3}y-15\right)^{\frac{1}{3}}$ $\int\left(\frac{1}{3}y-15\right)^{\frac{-1}{3}}\ \mathrm{d}y=\int 1\ \mathrm{d}x$ $\frac{\left(\frac{1}{3}y-15\right)^{\frac{2}{3}}}{\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)}=x+c$, where c is an arbitrary constant $\frac{9}{2}\left(\frac{1}{3}y-15\right)^{\frac{2}{3}}=x+c$ When $x = 0$, $y = 69$ $\frac{9}{2}\left(\frac{1}{3}(69)-15\right)^{\frac{2}{3}}=c$ $c = 18$ $\frac{9}{2}\left(\frac{1}{3}y-15\right)^{\frac{2}{3}}=x+18$ $\left(\frac{1}{3}y-15\right)^{\frac{2}{3}}=\frac{2}{9}x+4$ $\frac{1}{3}y-15=\left(\frac{2}{9}x+4\right)^{\frac{3}{2}}$ $\frac{1}{3}y=\left(\frac{2}{9}x+4\right)^{\frac{3}{2}}+15$ $y=3\left(\frac{2}{9}x+4\right)^{\frac{3}{2}}+45$
1(ii)	Since $\frac{\mathrm{d}y}{\mathrm{d}x}=4$, $\left(\frac{1}{3}y-15\right)^{\frac{1}{3}}=4$ $\frac{1}{3}y-15=64$ $y=237$ $\therefore 3\left(\frac{2}{9}x+4\right)^{\frac{3}{2}}+45=237$

	From GC, $x = 54$ Coordinates of the point is (54, 237)
2(a)	Since the coefficients of the equation are real, the complex roots occur in conjugate pairs. Therefore since $2 - 3\text{i}$ is a root, then $2 + 3\text{i}$ is also a root. So we have $4x^4 - 20x^3 + sx^2 - 56x + t = (x - 2 + 3i)(x - 2 - 3i)(4x^2 + ax + b)$ $= (x^2 - 4x + 13)(4x^2 + ax + b),$ where $a, b \in \mathbb{R}$ Comparing coefficients of x^3, $-20 = a - 16$ $a = -4$ Comparing coefficients of x, $-56 = -4b + 13a$ $4b = 56 + 13(-4)$ $b = 1$ Comparing constants, $t = 13b$ $= 13$ Comparing coefficients of x^2, $s = b - 4a + 52$ $= 1 - 4(-4) + 52$ $= 69$ $\therefore$ we have $4x^4 - 20x^3 + 69x^2 - 56x + 13 = (x^2 - 4x + 13)(4x^2 - 4x + 1)$ $= (x^2 - 4x + 13)(2x - 1)^2$ The other roots are $2 + 3\text{i}$ and $\frac{1}{2}$. Note: $x = \frac{1}{2}$ is a repeated real root.
(bi)	$w^3 = 27$ $w^3 - 27 = 0$ Since $w = 3$, $w^3 - 27 = (w - 3)(w^2 + aw + b)$, where $a, b \in \mathbb{R}$

	Comparing constants, $-27=-3b$ $b=9$ Comparing coefficients of w, $0=b-3a$ $a=\frac{b}{3}$ $=3$ $\therefore$ we have $w^3-27=(w-3)(w^2+3w+9)$ To find the other roots, let $w^2+3w+9=0$ $w=\frac{-3\pm\sqrt{9-4(1)(9)}}{2}$ $=\frac{-3\pm\sqrt{-27}}{2}$ $=\frac{-3\pm 3\sqrt{3}i}{2}$ The other roots are $\frac{-3}{2}+\frac{3\sqrt{3}i}{2}$ and $\frac{-3}{2}-\frac{3\sqrt{3}i}{2}$
(bii)	Let $w_1=3$, $w_2=\frac{-3}{2}+\frac{3\sqrt{3}i}{2}$ and $w_3=\frac{-3}{2}-\frac{3\sqrt{3}i}{2}$ $\lvert w_1\rvert=3$ $\lvert w_2\rvert=\sqrt{\left(\frac{-3}{2}\right)^2+\left(\frac{3\sqrt{3}}{2}\right)^2}=\sqrt{\frac{9}{4}+\frac{27}{4}}=3$ $\lvert w_3\rvert=\sqrt{\left(\frac{-3}{2}\right)^2+\left(\frac{-3\sqrt{3}}{2}\right)^2}=\sqrt{\frac{9}{4}+\frac{27}{4}}=3$ $\arg w_1=0$ $\arg w_2=\pi-\tan^{-1}\left\lvert\frac{\frac{3\sqrt{3}}{2}}{\frac{-3}{2}}\right\rvert=\frac{2\pi}{3}$ $\arg w_3=-\pi+\tan^{-1}\left\lvert\frac{\frac{-3\sqrt{3}}{2}}{\frac{-3}{2}}\right\rvert=-\frac{2\pi}{3}$

	$\therefore w_1 = 3(\cos 0 + i\sin 0)$, $w_2 = 3\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ and $w_3 = 3\left(\cos\frac{-2\pi}{3} + i\sin\frac{-2\pi}{3}\right)$ $= 3\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)$ 
(biii)	$w_1w_2w_3 = 3\left(3e^{\frac{2\pi}{3}i}\right)\left(3e^{\frac{-2\pi}{3}i}\right) = 27$ $w_1 + w_2 + w_3 = 3 + 3\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) + 3\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)$ $= 3 + 3\left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right) + 3\left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right)$ $= 0$
3(i)	$\overrightarrow{AD} = \overrightarrow{BC}$ $\overrightarrow{OD} - \overrightarrow{OA} = \overrightarrow{OC} - \overrightarrow{OB}$ $\overrightarrow{OD} = \overrightarrow{OC} - \overrightarrow{OB} + \overrightarrow{OA}$ $\overrightarrow{OD} = \begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix}$ $\therefore D(-5, -4, 3)$

(ii)	$\overrightarrow{EC} = \begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \\ -8 \end{pmatrix}$ $\overrightarrow{EB} = \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ -10 \end{pmatrix}$ $\overrightarrow{EC} \times \overrightarrow{EB} = \begin{pmatrix} -5 \\ 4 \\ -8 \end{pmatrix} \times \begin{pmatrix} 5 \\ 4 \\ -10 \end{pmatrix} = \begin{pmatrix} -8 \\ -90 \\ -40 \end{pmatrix}$ Normal vector of plane *BCE* is $-2\begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix}$ Therefore the equation of plane *BCE* is $\underset{\sim}{r} \bullet \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix} = \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} = 200$ $\therefore$ Cartesian equation of plane *BCE* is $4x + 45y + 20z = 200$
(iii)	$\overrightarrow{BC} = \begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ 2 \end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -1 \end{pmatrix}$ $\overrightarrow{BC} \times \overrightarrow{AB} = \begin{pmatrix} -10 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 8 \\ -1 \end{pmatrix} = \begin{pmatrix} -16 \\ -10 \\ -80 \end{pmatrix} = -2\begin{pmatrix} 8 \\ 5 \\ 40 \end{pmatrix}$ Normal vector of base of the pyramid is $\begin{pmatrix} 8 \\ 5 \\ 40 \end{pmatrix}$ Let the angle between the planes be θ $\cos\theta = \left\| \frac{\begin{pmatrix} 8 \\ 5 \\ 40 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix}}{\sqrt{1689}\sqrt{2441}} \right\| = \frac{1057}{\sqrt{1689}\sqrt{2441}}$ $\theta = 58.6^0$

(iv)	Let $\overrightarrow{OM}$ be the midpoint of AD $\overrightarrow{OM} = \dfrac{\begin{pmatrix}5\\-4\\1\end{pmatrix}+\begin{pmatrix}-5\\-4\\3\end{pmatrix}}{2} = \begin{pmatrix}0\\-4\\2\end{pmatrix}$ $\overrightarrow{ME} = \begin{pmatrix}0\\0\\10\end{pmatrix} - \begin{pmatrix}0\\-4\\2\end{pmatrix} = \begin{pmatrix}0\\4\\8\end{pmatrix}$ Distance from M to the face BCE $=$length of projection of $\overrightarrow{ME}$ on $\begin{pmatrix}4\\45\\20\end{pmatrix}$ $= \dfrac{\left\lvert \overrightarrow{ME} \bullet \begin{pmatrix}4\\45\\20\end{pmatrix} \right\rvert}{\sqrt{4^2+45^2+20^2}} = \dfrac{\left\lvert \begin{pmatrix}0\\4\\8\end{pmatrix} \bullet \begin{pmatrix}4\\45\\20\end{pmatrix} \right\rvert}{\sqrt{2441}} = \dfrac{340}{\sqrt{2441}} = 6.88$
4(i)	$\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \ldots + \dfrac{(-1)^r x^{2r}}{(2r)!} + \ldots$ $\cos 2x = 1 - \dfrac{(2x)^2}{2!} + \dfrac{(2x)^4}{4!} - \dfrac{(2x)^6}{6!} + ...$ $= 1 - 2x^2 + \dfrac{2}{3}x^4 - \dfrac{4}{45}x^6 + ...$ $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \ldots + \dfrac{(-1)^{r+1} x^r}{r} + \ldots$

	$\ln(\cos 2x) \approx \ln\left(1-2x^2+\frac{2}{3}x^4-\frac{4}{45}x^6\right)$ $\approx \left(-2x^2+\frac{2}{3}x^4-\frac{4}{45}x^6\right)-\frac{\left(-2x^2+\frac{2}{3}x^4-\frac{4}{45}x^6\right)^2}{2}+\frac{\left(-2x^2+\frac{2}{3}x^4-\frac{4}{45}x^6\right)^3}{3}$ $\approx -2x^2+\frac{2}{3}x^4-\frac{4}{45}x^6-\left(\frac{4x^4-\frac{4}{3}x^6-\frac{4}{3}x^6}{2}\right)+\left(\frac{-8x^6}{3}\right)$ $=-2x^2+\frac{2}{3}x^4-\frac{4}{45}x^6-2x^4+\frac{4}{3}x^6-\frac{8}{3}x^6$ $=-2x^2-\frac{4}{3}x^4-\frac{64}{45}x^6$ Expansion will be not valid for $\cos 2x=0$ since ln(0) will be undefined. $\cos 2x=0$ $2x=\frac{\pi}{2}$ $x=\frac{\pi}{4}$ Hence when $x=\frac{\pi}{4}$, expansion will not be valid.
(ii)	$\int \frac{\ln(\cos 2x)}{x^2}\,dx = \int \frac{-2x^2-\frac{4}{3}x^4-\frac{64}{45}x^6}{x^2}\,dx$ $=\int -2-\frac{4}{3}x^2-\frac{64}{45}x^4\,dx$ $=-2x-\frac{4}{9}x^3-\frac{64}{225}x^5+c$ $\int_0^{0.5} \frac{\ln(\cos 2x)}{x^2}\,dx = \left[-2x-\frac{4}{9}x^3-\frac{64}{225}x^5\right]_0^{0.5}$ ≈ -1.0644
(iii)	Using GC, $\int_0^{0.5} \frac{\ln(\cos 2x)}{x^2}\,dx \approx -1.0670$
Section (B) Statistics	
5(i)	Since the time to failure does not follow normal distribution, in order for the mean time to failure to follow normal distribution, sample size must be at least 30 for the manager to apply Central Limit Theorem. The fans should be randomly chosen. That means each fan in the population must have an equal chance of being selected and the fans must be selected independently of one another.

(ii)	Let X be the random variable "time to failure of a fan in hours" with population mean μ. $H_0 : \mu = 65000$ $H_1 : \mu < 65000$
(iii)	At the 5% level of significance. Under H_0, since n = 43 is sufficiently large, by Central Limit Thorem, $\overline{X} \sim N\left(65000, \frac{s^2}{43}\right)$ approximately Using a 1-tailed z-test, **Critical value**: $z = -1.64485$ In order not to reject, test stat value $z_{calc} = \frac{\overline{x} - \mu_0}{\sqrt{\frac{s^2}{n}}} > -1.644853626$ $\frac{64230 - 65000}{\frac{s}{\sqrt{43}}} > -1.64485$ $-770 > -1.64485\frac{s}{\sqrt{43}}$ $\frac{s}{\sqrt{43}} > 468.13$ $s > 3069.7$ $s^2 > 9423264.967$ $s^2 \geq 943000$ (3 significant figures)
6(i)	Let X be the random variable that represents the number of steps to the left out of 8 steps. $X \sim B(8, p)$ $P(\text{bug finished at } D) = P(X = 5)$ $= \binom{8}{5} p^5 q^3$ $= 56 p^5 q^3$ (shown) The bug has to make the decision to make the right or left turn at 8 different junctions, and at 5 of the 8 junctions, bug has to make a left turn, that's why $\binom{8}{5}$.

(ii)	Since $\mathrm{P}(X=5)$ is the highest, $X = 5$ is the modal number. Therefore $\mathrm{P}(X=5)>\mathrm{P}(X=4)$ and $\mathrm{P}(X=5)>\mathrm{P}(X=6)$ $\Rightarrow 56p^5q^3>\binom{8}{4}p^4q^4$ and $\Rightarrow 56p^5q^3>\binom{8}{6}p^6q^2$ $\Rightarrow 56p^5q^3>70p^4q^4$ and $\Rightarrow 56p^5q^3>28p^6q^2$ $\Rightarrow p>\frac{5}{4}q$ and $\Rightarrow p<2q$ $\Rightarrow p>\frac{5}{4}(1-p)$ and $\Rightarrow p<2(1-p)$ $\Rightarrow p>\frac{5}{9}$ and $\Rightarrow p<\frac{2}{3}$ Therefore the possible range of values of p is $\frac{5}{9}<p<\frac{2}{3}$.
(iii)	Let Y be the random variable that represents the number of steps out of 8 that ends up at the black hole. $Y \sim \mathrm{B}(8, 0.1)$ $\mathrm{P}(\text{reaching an endpoint}) = \mathrm{P}(Y=0)$ $= 0.430 \ \ (3\text{sf})$ <u>Alterative</u> $\mathrm{P}(\text{reaching an endpoint}) = \mathrm{P}(\text{not being swallowed by a black hole at all 8 junctions})$ $=(0.9)^8$ $=0.430$
7(i)	$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $=a+b-ab \qquad (\text{since } A, B \text{ are independent})$ $\mathrm{P}(A'\cap B')=1-\mathrm{P}(A\cup B)$ $=1-a-b+ab$ $\mathrm{P}(A')\times\mathrm{P}(B')=(1-a)(1-b)$ $=1-a-b+ab$ Since $\mathrm{P}(A'\cap B')=\mathrm{P}(A')\times\mathrm{P}(B')$, A', B' are independent events.

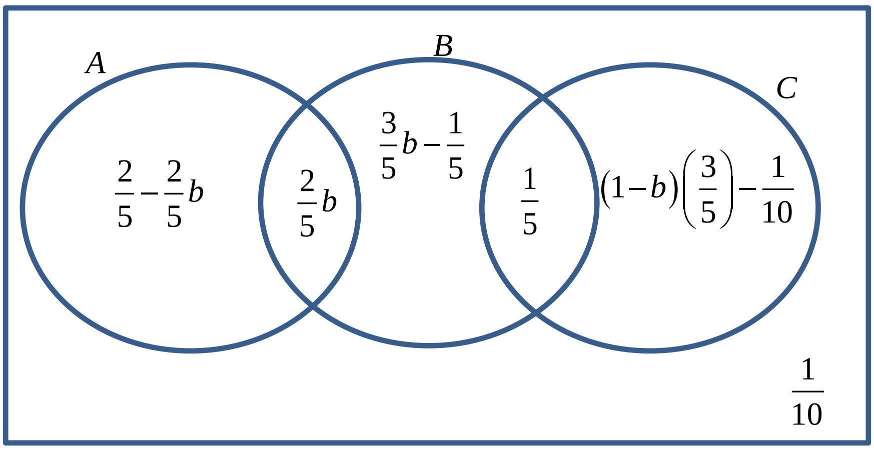
(ii)

$$\mathrm{P}(A'\cap C') = 1 - \mathrm{P}(A \cup C)$$
$$= 1-(a+c) \quad \text{since } A \text{ and } C \text{ are mutually exclusive}$$
$$= 1-a-c$$

If A' and C' are mutually exclusive , $\mathrm{P}(A'\cap C') = 0$

$$1-a-c=0$$
$$a+c=1$$

(iii)



Since A, B are independent , $\mathrm{P}(A\cap B) = \frac{2}{5}b$

$$\mathrm{P}(A\cap B') = \mathrm{P}(A) - \mathrm{P}(A\cap B) = \frac{2}{5} - \frac{2}{5}b$$

$$\mathrm{P}(C\cap B') = \mathrm{P}(A'\cap B') - \mathrm{P}(A'\cap B'\cap C')$$
$$= \left(1-\frac{2}{5}\right)(1-b) - \frac{1}{10}$$
$$= \frac{3}{5}(1-b) - \frac{1}{10}$$

$$\mathrm{P}(A'\cap B)\cap \mathrm{P}(C'\cap B) = \mathrm{P}(B) - \mathrm{P}(A\cap B) - \mathrm{P}(C\cap B)$$
$$= b - \frac{2}{5}b - \frac{1}{5}$$
$$= \frac{3}{5}b - \frac{1}{5}$$

Maximum value of b is obtained when

$$(1-b)\left(\frac{3}{5}\right)-\frac{1}{10}=0$$

$$1-b=\frac{1}{6}$$

$$b=\frac{5}{6}$$

$\therefore$ Maximum value of $\mathrm{P}(A\cap B)$ = $\frac{2}{5}b=\frac{2}{5}\left(\frac{5}{6}\right)=\frac{1}{3}$

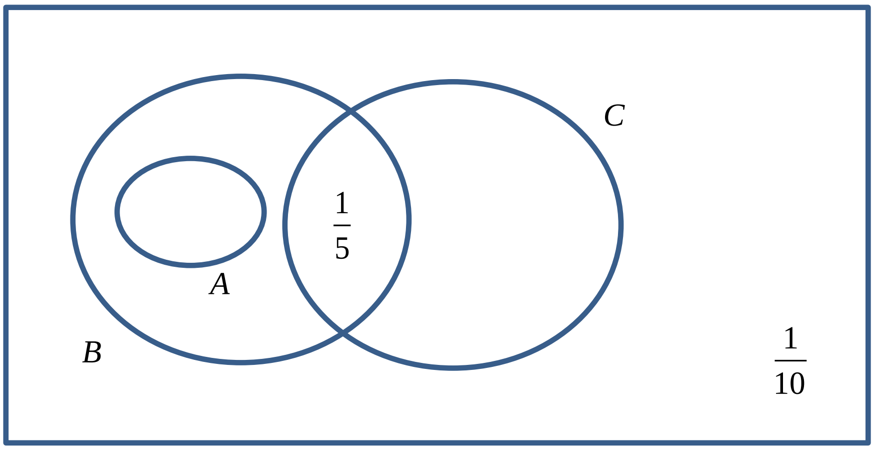
Minimum value of b is obtained when

$$\frac{3}{5}b-\frac{1}{5}=0$$

$$b=\frac{1}{3}$$

$\therefore$ Minimum value of $\mathrm{P}(A\cap B)$ = $\frac{2}{5}b=\frac{2}{5}\left(\frac{1}{3}\right)=\frac{2}{15}$

Note: If $A\subseteq B$, (NOT POSSIBLE)



$\mathrm{P}(A\cap B)=\mathrm{P}(A)=\frac{2}{5}=\frac{2}{5}\mathrm{P}(B)\Rightarrow \mathrm{P}(B)=1$ ($\mathrm{P}(B)$ cannot be 1)

<u>Alternative</u>

$$\mathrm{P}(A\cap B)=\frac{2}{5}b$$

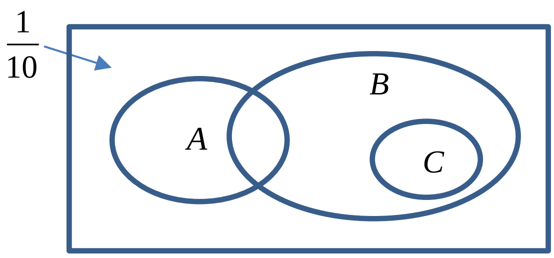
To find the maximum and minimum values of $\mathrm{P}(A\cap B)$, we need to find the maximum and minimum values of b.

Maximum value of b is obtained when

$$\mathrm{P}(A\cup B)=\frac{9}{10}$$

$$\Rightarrow \frac{2}{5}+b-\frac{2}{5}b=\frac{9}{10}$$

$$\Rightarrow b=\frac{5}{6}$$

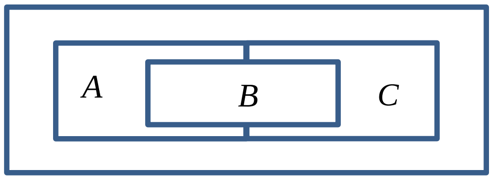


Minimum value of b is obtained when

$$B=(A\cap B)\cup(B\cap C)$$

$$b=\frac{2}{5}b+\frac{1}{5}$$

$$\Rightarrow b=\frac{1}{3}$$



Therefore the maximum and minimum values of $\mathrm{P}(A\cap B)$ are $\frac{2}{5}\times\frac{5}{6}=\frac{1}{3}$ and $\frac{2}{5}\times\frac{1}{3}=\frac{2}{15}$ respectively.

8(i) Let S be the random variable " the sum of the numbers on the two balls taken".

$$\mathrm{P}(S=6)=\left(\frac{2}{n+5}\right)\left(\frac{1}{n+4}\right)=\frac{2}{(n+5)(n+4)}$$

$$\mathrm{P}(S=7)=\left(\frac{2}{n+5}\right)\left(\frac{3}{n+4}\right)(2)=\frac{12}{(n+5)(n+4)}$$

$$\mathrm{P}(S=8)=\left(\frac{2}{n+5}\right)\left(\frac{n}{n+4}\right)(2)+\left(\frac{3}{n+5}\right)\left(\frac{2}{n+4}\right)=\frac{4n+6}{(n+5)(n+4)}$$

$$\mathrm{P}(S=9)=\left(\frac{3}{n+5}\right)\left(\frac{n}{n+4}\right)(2)=\frac{6n}{(n+5)(n+4)}$$

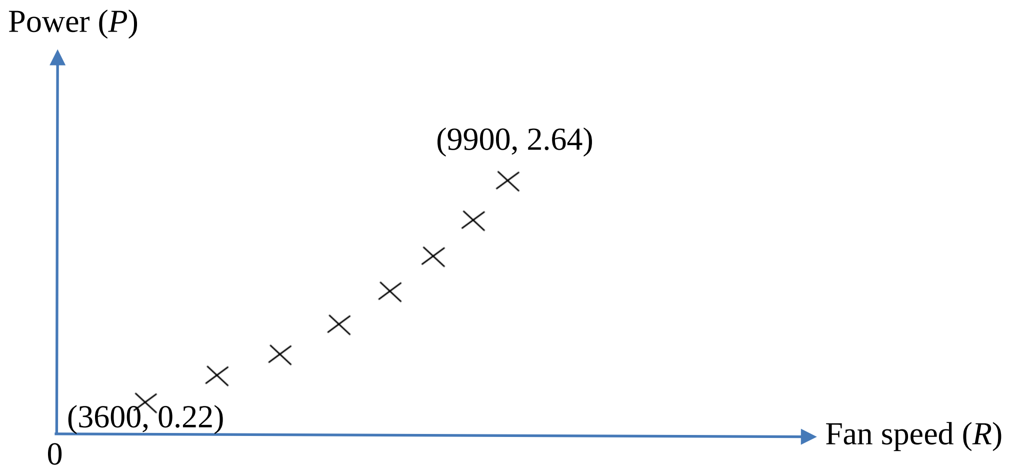
$$\mathrm{P}(S=10)=\left(\frac{n}{n+5}\right)\left(\frac{n-1}{n+4}\right)=\frac{n(n-1)}{(n+5)(n+4)}$$

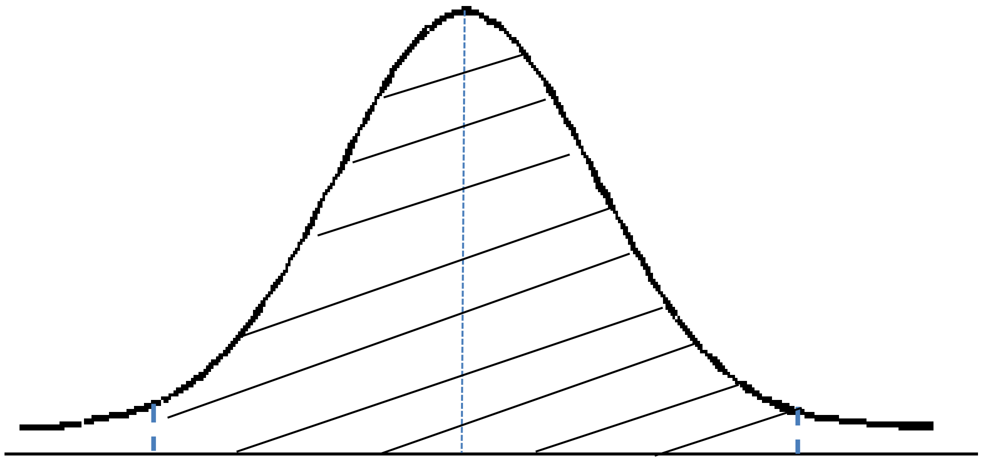
s	6	7	8	9
$\mathrm{P}(S=s)$	$\frac{2}{(n+4)(n+5)}$	$\frac{12}{(n+4)(n+5)}$	$\frac{6+4n}{(n+4)(n+5)}$	$\frac{6n}{(n+4)(n+5)}$
s	10			
$\mathrm{P}(S=s)$	$\frac{(n-1)(n)}{(n+4)(n+5)}$			

(ii) When n = 1, $\mathrm{P}(S=10)=0$

The probability of the sum of the two numbers being 10 is 0 since there is only one ball numbered 5 and the balls are taken without replacement.

(iii)	$\mathrm{E}(S)=\left[6\times\frac{2}{(n+4)(n+5)}\right]+\left[7\times\frac{12}{(n+4)(n+5)}\right]+\left[8\times\frac{6+4n}{(n+4)(n+5)}\right]$ $+\left[9\times\frac{6n}{(n+4)(n+5)}\right]+\left[10\times\frac{(n-1)(n)}{(n+4)(n+5)}\right]$ $=\frac{10n^2+76n+144}{(n+4)(n+5)}$ $=\frac{(n+4)(10n+36)}{(n+4)(n+5)}$ $=\frac{10n+36}{n+5}$ $\mathrm{E}(S^2)=\left[6^2\times\frac{2}{(n+4)(n+5)}\right]+\left[7^2\times\frac{12}{(n+4)(n+5)}\right]+\left[8^2\times\frac{6+4n}{(n+4)(n+5)}\right]$ $+\left[9^2\times\frac{6n}{(n+4)(n+5)}\right]+\left[10^2\times\frac{(n-1)(n)}{(n+4)(n+5)}\right]$ $=\frac{100n^2+642n+1044}{(n+4)(n+5)}$ $\mathrm{Var}(S)=\mathrm{E}\left(S^2\right)-\left[\mathrm{E}(S)\right]^2$ $=\frac{100n^2+642n+1044}{(n+4)(n+5)}-\left(\frac{10n+36}{n+5}\right)^2$ $=\frac{\left(100n^2+642n+1044\right)\left(n+5\right)-\left(n+4\right)\left(10n+36\right)^2}{(n+4)(n+5)^2}$ $=\frac{\left(100n^3+1142n^2+4254n+5220\right)-\left(n+4\right)\left(100n^2+720n+1296\right)}{(n+4)(n+5)^2}$ $=\frac{\left(100n^3+1142n^2+4254n+5220\right)-\left(100n^3+1120n^2+4176n+5184\right)}{(n+4)(n+5)^2}$ $=\frac{22n^2+78n+36}{(n+4)(n+5)^2}$ $\therefore g(n)=22n^2+78n+36$

9 (i)	 The relationship between P and R is most likely not well modelled by $P = aR + b$ is not a suitable model as the scatter diagram shows that P is increasing at an increasing rate.
(ii)	For $P = aR + b$, $r = 0.969$ (3sf) For $P = aR^2 + b$, $r = 0.993$ (3sf) Therefore $P = aR^2 + b$ is a better model as its correlation coefficient is closer to 1. Using GC, $P = \left(2.8457 \times 10^{-8}\right) R^2 - 0.282608$ $\approx \left(2.85 \times 10^{-8}\right) R^2 - 0.283$ (3sf)
(iii)	When $P = 0.9$, $0.9 = \left(2.8457 \times 10^{-8}\right) R^2 - 0.282608 \Rightarrow R = 6450$ (3sf) The estimation is reliable since $P = 0.9$ is within the data range and the correlation coefficient between P and R^2 is close to 1.
(iv)	When $R = 3300$, $P = \left(2.8457 \times 10^{-8}\right)(3300)^2 - 0.282608$ $= 0.0273$ (3sf) The estimate is not reliable as $R = 3300$ is outside the data range.
(v)	Replace R with $\frac{R}{60}$ $P = \left(2.8457 \times 10^{-8}\right)\left(\frac{R}{60}\right)^2 - 0.282608$ $= \left(7.90472 \times 10^{-12}\right) R^2 - 0.282608$ $\approx \left(7.90 \times 10^{-12}\right) R^2 - 0.283$ (3sf)

10(i)	Let X be the random variable that represents the mass of a light bulb in grams. $X \sim \mathrm{N}\left(50, 1.5^2\right)$  Since $\mathrm{P}(40 < X < 60) \approx 1$, the shaded area should be almost the entire area under the normal distribution curve. And also since mean is 50, $\mathrm{P}(X < 40) = \mathrm{P}(X > 60)$, the unshaded region at both ends must be symmetrical.
(ii)	$\mathrm{P}(X < 50.4) = 0.605$ (3sf)
(iii)	Let Y be the random variable that represents the mass of an empty box. $Y \sim \mathrm{N}\left(75, 2^2\right)$ $Y_1 + Y_2 + Y_3 + Y_4 \sim \mathrm{N}\left(300, 16\right)$ $\mathrm{P}\left(Y_1 + Y_2 + Y_3 + Y_4 > 297\right) = 0.773$ (3sf)
(iv)	$X + Y \sim \mathrm{N}\left(125, 6.25\right)$ $\mathrm{P}\left(124.9 < X + Y < 125.7\right) = 0.126$ (3sf)
(v)	Let W be the random variable that represents the total mass of a bulb with padding and an empty box, where $W = 1.3X + Y$. $W \sim \mathrm{N}\left(140, 7.8025\right)$ $P\left(W > k\right) = 0.9$ Using inverse normal, $k = 136$ (3sf)
(vi)	$W_1 + W_2 + W_3 + W_4 \sim \mathrm{N}\left(560, 31.21\right)$ $\mathrm{P}\left(W_1 + W_2 + W_3 + W_4 > 565\right) = 0.185$ (3sf)