

Quantum Physics

Guiding Questions	Learning Objectives
1. How does light behave like particles?	(a) show an understanding that the existence of a threshold frequency in the photoelectric effect provides evidence that supports the particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence that supports its wave nature. (b) state that a photon is a quantum of electromagnetic radiation, and recall and use the equation $E = hf$ for the energy of a photon to solve problems, where h is the Planck constant.
2. How do particles display wave-like properties?	(c) show an understanding that while a photon is massless, it has a momentum given by $p = E/c$ and $p = h/\lambda$, where c is the speed of light in free space. (d) show an understanding that electron diffraction and double-slit interference of single particles provide evidence that supports the wave nature of particles. (e) recall and use the equation $\lambda = h/p$ for the de Broglie wavelength to solve problems.
3. How does a wavefunction explain where a particle can be found and why its energy is quantised?	(f) show an understanding that the state of a particle can be represented as a wavefunction ψ, e.g. for an electron cloud in an atom, and that the square of the wavefunction amplitude $\psi ^2$ is the probability density function (including calculation of normalisation factors for square and sinusoidal wavefunctions). (g) show an understanding that the principle of superposition applies to the wavefunctions describing a particle's position, leading to standing wave solutions for a particle in a box and phenomena such as single-particle interference in double-slit experiments. (i) show an understanding of standing wave solutions ψ_n for the wavefunction of a particle in a one-dimensional infinite square well potential. (j) solve problems using $E_n = \frac{h^2}{8mL^2} n^2$ for the allowed energy levels of a particle of mass m in a one-dimensional infinite square well of width L.

4. Why is the simultaneous measurement of position and momentum impossible?	(h) show an understanding that the Heisenberg position-momentum uncertainty principle $\Delta x \Delta p \gtrsim h$ relates to the necessity of a spread of momenta for localised particles, and apply this to solve problems.
5. How do spectral lines provide evidence for the quantisation of energy?	(k) show an understanding of the existence of discrete electronic energy levels for the electron's wavefunction in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to the observation of spectral lines. (l) distinguish between emission and absorption line spectra. (m) solve problems involving photon absorption or emission during atomic energy level transitions.



Nature of Science Moment – Why Quantum Physics?

For much of the 19th century, light was successfully described as a wave. This model explained reflection, refraction, diffraction, and interference so well that few scientists doubted it. However, experiments soon revealed behaviour that the wave model could not explain.

To account for this, physicists were forced to treat light as **quantised particles called photons**, each carrying a fixed amount of energy. This single idea resolved contradictions that classical physics could not.

Quantum physics did not arise because classical physics was wrong in everyday situations—it arose because classical ideas had limits. When observations no longer matched existing models, physicists did not ignore the data. Instead, they changed the model.

The study of quantum physics begins with a fundamental question:

What happens when our best ideas no longer match what we observe?

How does light behave like particles?

1. Light as a Particle

In the earlier topics on **Wave Motion and Superposition**, we saw strong experimental evidence that light behaves as a wave, with diffraction and interference patterns such as those observed in Young's double-slit experiment.

However, further experiments showed that the wave model of light could not explain all observed behaviour. To account for these results, physicists proposed that light, and more generally all electromagnetic radiation, transfers energy in **discrete packets rather than continuously**. This led to the idea that light can be treated as **quantised particles called photons**, each carrying a fixed amount of energy.

1.1 Energy of a Photon

Since each quantum of electromagnetic radiation behaves like a particle, the minimum amount of energy of electromagnetic radiation would be the energy of a single photon.



A **photon** is a quantum of electromagnetic radiation. The energy of one photon, E , is given by the equation

$$E = hf$$

where

h : Planck's constant, $h = 6.63 \times 10^{-34} \text{ J s}$

f : frequency of electromagnetic radiation

For electromagnetic radiation with larger amounts of energy, there will be multiple photons, and the total energy will be an integer multiple of hf .

Example 1.1

Determine the energy of a photon of wavelength 440 nm.

$$\begin{aligned} E &= hf \\ &= \frac{hc}{\lambda} \\ &= \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{440 \times 10^{-9}} \\ &= 4.520 \times 10^{-19} \\ &\approx 4.52 \times 10^{-19} \text{ J} \end{aligned}$$

1.2 The Electron Volt

From Example 1.1, it can be seen that the typical energies of a single photon (and other fundamental particles which we will encounter in Nuclear Physics) are very small when expressed in joules. The electron volt (eV) is a unit of energy which is more suitable for such contexts.

One electron volt (eV) is defined as the work done by an electric field to accelerate an electron across a potential difference of 1 V.

gain in kinetic energy of electron = loss in electric potential energy

$$W = q\Delta V$$

$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(1 \text{ V})$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

In Example 1.1, the photon with wavelength 440 nm would have $\frac{4.520 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.83$ eV of energy.

Example 1.2

(a) A source emits monochromatic light of wavelength λ at power P .

Given that h is Planck's constant and c is the speed of light, show that the rate of emission of photons is given by the expression $\frac{P\lambda}{hc}$.

(b) An FM radio transmitter has a power output of 150 kW and operates at a frequency of 99.7 MHz. Determine the number of photons emitted by the transmitter in one second.

Solution

$$(a) \quad \text{rate of emission} = \frac{\text{number of photons emitted}}{\text{time}} = \frac{N}{t}$$

$$\text{power} = \frac{\text{number of photons emitted} \times \text{energy of each photon}}{\text{time}}$$

$$P = \frac{Nhf}{t} = \left(\frac{N}{t}\right)\left(\frac{hc}{\lambda}\right)$$

$$\frac{N}{t} = \frac{P\lambda}{hc} \quad (\text{shown})$$

$$(b) \quad \text{From (a): } P = \frac{Nhf}{t}$$

$$150 \times 10^3 = \frac{N}{t} (6.63 \times 10^{-34}) (99.7 \times 10^6)$$

$$\frac{N}{t} = 2.27 \times 10^{30} \text{ photons per second}$$

1.3 The Photoelectric Effect

The photoelectric effect refers to electrons being emitted from a material when electromagnetic radiation is incident on the material. It was first observed experimentally in the late 19th century and did not fit with classical theory of light as a wave.

Einstein published his theory that light behaves like a particle, along with an equation to describe the photoelectric effect in 1905. Subsequently, experiments were carried out to test this theory. A diagram of an experimental setup is shown in Fig. 1.1 below.

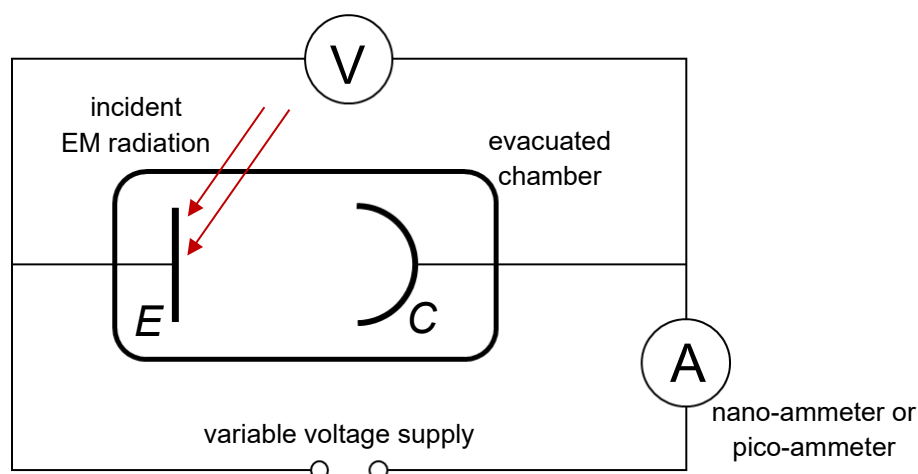


Fig. 1.1: A diagram of a possible experimental setup used to investigate the photoelectric effect. When EM radiation is incident on the emitter E , photoelectrons are emitted and travel to the collector C . This photocurrent will then be measured by the ammeter.

As shown in Fig. 1.1, a potential difference is maintained between the emitter E and the collector C . The magnitude and polarity of the potential difference can be adjusted using the variable voltage supply and measured using the voltmeter.

Monochromatic light (electromagnetic radiation of a single frequency) is incident on the emitter E . The ammeter used is highly sensitive, to measure the very small photocurrent due to any photoelectrons from the emitter E reaching the collector C . The emitter E and the collector C are placed in a vacuum chamber so that the photoelectrons can travel from E to C without losing energy due to collisions with air molecules.

1.3.1 Work Function and Threshold Frequency

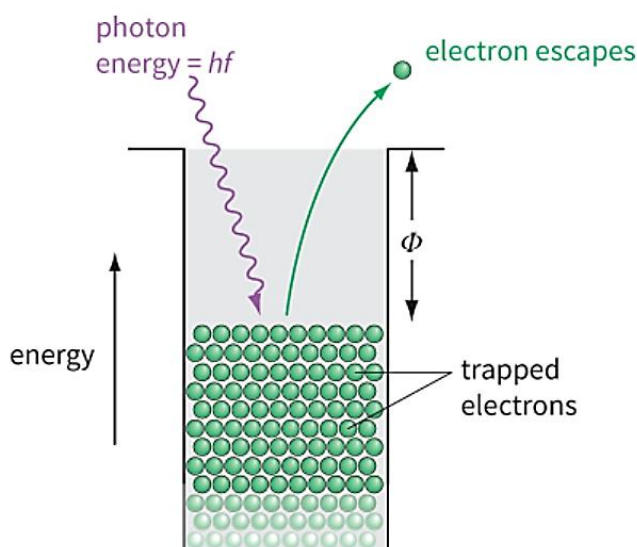
Electrons in a metal can escape from the metal if they have enough energy to overcome the forces of electrostatic attraction by the positively-charged ions.

An electron which absorbs a photon and is emitted is called a photoelectron, and the emission is called photoemission. In photoemission, since photons can only penetrate the emitting metal to a small depth, only electrons near the surface of the emitting metal can be emitted.

The electrons in the metal surface have varying initial energies. The electron with the largest initial energy (that is, the most energetic electron) will be the easiest to release from the metal surface, requiring the smallest amount of energy.



The **work function**, Φ , of a metal is the minimum energy to release an electron (the most energetic electron) from the surface of the emitter metal.



Source: Sang, D., Jones, G., Chadha, G., & Woodside, R. (2020). *Cambridge international As and A level physics coursebook*. Cambridge University Press.

Fig. 1.2: A single photon may interact with a single electron to release it.

In 1905, to explain the experimental observations of the photoelectric effect, Einstein proposed that the energy from electromagnetic radiation is transmitted in discrete quantised packets, and formulated the following equation to describe the photoelectric effect:

$$hf = \Phi + \frac{1}{2}mv_{\max}^2$$

where

hf : energy of each incident photon

Φ : work function of the metal

$\frac{1}{2}mv_{\max}^2$: maximum kinetic energy of each emitted photoelectron

This equation also suggests that there is a one-to-one interaction between the incident photons and the emitted photoelectrons – each photon either transmits all or none of its energy to an electron, and each electron can only absorb one photon. Note that this does not mean that every single photon will produce a photoelectron – many of the photons may not be absorbed, or the electron absorbing the photon may not be emitted if the initial energy of the electron was too low or the electron was too far from the surface of the metal.



Threshold frequency, f_0 , is the minimum frequency of incident electromagnetic radiation to release an electron from the surface of the metal.

Thus, we can relate the work function of a metal to its threshold frequency with the following equation:

$$\Phi = hf_0$$

where

f_0 : threshold frequency of the metal

Example 1.3

Radiation of frequency 18.0×10^{14} Hz falls on the cathode of a photocell. The material of the cathode has a threshold frequency of 4.0×10^{14} Hz.

Determine the maximum energy of the emitted electrons.

$$\begin{aligned} E &= hf \\ &= (6.63 \times 10^{-34}) (18.0 \times 10^{14}) \\ &= 1.193 \times 10^{-18} \text{ J or } 7.459 \text{ eV} \end{aligned}$$

$$\begin{aligned} \Phi &= hf_0 \\ &= (6.63 \times 10^{-34}) (4.0 \times 10^{14}) \\ &= 2.652 \times 10^{-19} \text{ J or } 1.658 \text{ eV} \end{aligned}$$

$$\begin{aligned} KE_{\max} &= hf - hf_0 \\ &= (1.193 \times 10^{-18}) - (2.652 \times 10^{-19}) \\ &= 9.28 \times 10^{-19} \text{ J or } 5.80 \text{ eV} \end{aligned}$$

1.3.2 Stopping Potential

The magnitude and polarity of the potential difference between the emitter E and the collector C can be adjusted using the variable voltage supply.

If the collector C is set to a higher potential than the emitter E , the photoelectrons will be accelerated towards the collector. This helps to ensure that all the photoelectrons which are emitted will arrive at the collector, and be measured as photocurrent by the ammeter.

If the collector C is set to a lower potential than the emitter E , the kinetic energy of the photoelectrons will be converted into electric potential energy and the electrons will decelerate as they approach the collector, due to the repulsive electric force. Only photoelectrons with sufficient kinetic energy will be able to reach the collector. As the potential difference increases, fewer photoelectrons will have sufficient kinetic energy to reach the collector, and the photocurrent measured by the ammeter will decrease.

Eventually, at a certain potential difference, even the most energetic photoelectrons will not be able to reach the collector, and the ammeter will read zero photocurrent. This potential difference is known as the **stopping potential**, V_s .

Thus, we can relate maximum kinetic energy of an emitted electron to the stopping potential with the following equation:

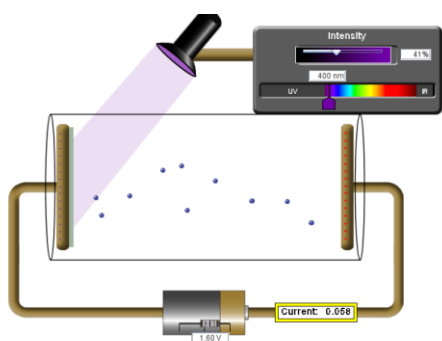
$$\frac{1}{2}mv_{\max}^2 = eV_s$$

where

V_s : stopping potential

Therefore, Einstein's photoelectric effect equation can also be written in the following variation:

$$hf = hf_0 + eV_s$$



Use this simulation to investigate the photoelectric effect by varying the frequency and intensity of light. Adjust the battery so that the emitter is at a higher potential than the collector and determine the stopping potential at which the photoelectric current just falls to zero.

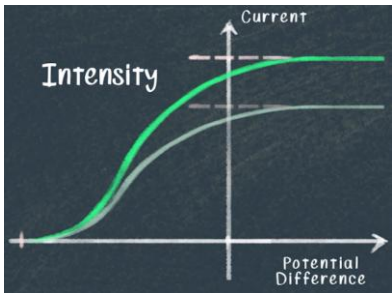
1.3.3 Key Observations of the Photoelectric Effect Experiment

According to the classical wave model of light, energy is transferred continuously from the electromagnetic wave to electrons in a metal. The amount of energy transferred depends on the intensity of the light, area of the metal and the time of exposure. Since the intensity of light is related to the square of the wave amplitude, the intensity can be increased independently of the frequency.

Based on this model, classical physics predicts that increasing the intensity of light—regardless of its frequency—should increase the energy of emitted electrons. However, experiments on the photoelectric effect produced results that contradicted these predictions, as shown in the following table.

	Observation	Inference	Why this supports light as particles
1	Presence of threshold frequency	No electrons are emitted below a certain frequency, regardless of intensity	Each photon must have energy minimally equal to the work function to eject an electron. Increasing intensity adds more photons per unit time, not higher-energy photons.
2	Increasing frequency increases stopping potential	Higher-frequency light produces photoelectrons with greater maximum kinetic energy	The energy of each photon depends on its frequency, so higher-frequency photons transfer more energy to individual electrons.
3	Increasing intensity without changing frequency increases current, not stopping potential	Greater intensity light produces more photoelectrons per unit time, but with the same maximum kinetic energy	Higher intensity means more photons per unit time, not more energy per photon.
4	No measurable time lag	Photoelectrons are emitted immediately when light shines on the metal	Energy is transferred in a single photon–electron interaction, not accumulated gradually as predicted by a wave model.

See Annex A for further elaboration on classical vs quantum predictions of the photoelectric effect.






Try it out on SLS! Watch the video and do the activity to see how changing frequency and intensity affects the I–V curve in the photoelectric effect.

Example 1.4

- (a) Light energy is incident on each square millimetre of the surface at a rate of $3.0 \times 10^{-22} \text{ J s}^{-1}$.
The frequency of the light is $1.5 \times 10^{15} \text{ Hz}$.

- (i) Calculate the energy of an incident photon.
- (ii) Calculate the number of photons incident per second on each square millimetre of the metal surface.

- (b) In the wave theory of light, electrons on the surface of a metal absorb energy from a small area of the surface.

- (i) The light striking the surface delivers energy to this small area at a rate of $3.0 \times 10^{-22} \text{ J s}^{-1}$.
The minimum energy required to liberate the electron is $6.8 \times 10^{-19} \text{ J}$.

Deduce the minimum time it would take for an electron to absorb this amount of energy if the wave theory is correct.

- (ii) In the photoelectric experiment, the time delay calculated in (b)(i) does not occur.
Explain how this experimental evidence was used to develop the particle model for the behaviour of light.

Solution

(a) (i) $E = hf$

$$= (6.63 \times 10^{-34}) (1.5 \times 10^{15})$$

$$= 9.95 \times 10^{-19} \text{ J or } 6.22 \text{ eV}$$

(ii) $P = \frac{N}{t} (hf)$

$$\frac{N}{t} = \frac{P}{hf}$$

$$= \frac{3.0 \times 10^{-22}}{9.95 \times 10^{-19}} = 3.02 \times 10^{-4} \text{ photons per second}$$

(b) (i) $P = \frac{E}{t}$

$$t_{\min} = \frac{E_{\min}}{P}$$

$$= \frac{6.8 \times 10^{-19}}{3.0 \times 10^{-22}} = 2.27 \times 10^3 \text{ s} = 37 \text{ min } 47 \text{ s}$$

- (ii) According to the model of light behaving like a particle, there is a fixed amount of energy in each photon. Low intensity corresponds to a low rate of incident photons, but does not affect the energy of each photon.

Instantaneous emission without the time delay in (b)(i) is possible because each photon is able to transfer energy to the electrons in a one-to-one interaction, so an electron can be emitted the moment the first photon is incident on the metal surface.

How do particles display wave-like properties?

2. Wave-Particle Duality

Classical experiments by Young on interference and diffraction indicated that light behaves like a wave. Yet, the photoelectric effect suggests that light behaves like a particle. Since both models have strong evidences supporting both sides, it seems that EM radiation is a wave as well as a particle. This two-sided identity of EM radiation has come to be known as **wave-particle duality**. The wave and particle aspects of light complement each other. Light has both the wave and particle natures, but they do not exhibit have both nature in the same experiment (Bohr's Theory of Complementarity).

2.1 De Broglie Wavelength

De Broglie (*pronounced as "de broy"*) proposed the following in 1924: Nature loves symmetry. Light is dualistic in nature, behaving in some situations like waves and in other situations like particles. If nature is symmetric, this duality should also hold for matter – electrons and protons (which are usually thought of as particles) should be able to exhibit wave-like behaviour in some situations, but not at the same time as particle-like behaviour.

The **de Broglie wavelength** λ of a particle with momentum p (or mv) is given by

$$\lambda = \frac{h}{p}$$

This means that a particle with momentum p , in situations where it behaves like a wave, will act as though it has a wavelength equal to the de Broglie wavelength λ .

Similarly, for light that we would normally describe as a wave having a wavelength λ , in situations where it behaves like a particle (a photon), we can also use the equation to determine the momentum p of the photon. Note that although photons have no rest mass, they still possess momentum because they are travelling at the speed of light.

In fact, we can categorise all particles into two categories – massless particles and massive particles. Table 2.1 below summarises the expressions for each category.

Table 2.1

	massless particles (e.g., photons)	massive particles (e.g. electrons)
wavelength	$\lambda = \frac{c}{f}$	$\lambda = \frac{h}{p}$
momentum	$p = \frac{h}{\lambda} = \frac{E}{c}$	$p = mv$
energy	$E = hf = \frac{hc}{\lambda} = pc$	$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Example 2.1

A parallel beam of violet light of wavelength 4.5×10^{-7} m and intensity 700 W m^{-2} is incident on a surface.

- (a) Calculate
- (i) the energy of a photon of violet light,
 - (ii) the number of photons incident per second on $1.0 \times 10^{-4} \text{ m}^2$ of the surface.
- (b) (i) State the de Broglie relation for the momentum p of a particle in terms of its associated wavelength λ .
- (ii) Use the equation in (i) to calculate the momentum of a photon of violet light.
- (c) (i) Use your answers to (a) and (b) to calculate the total change in momentum of all the photons incident on $1.0 \times 10^{-4} \text{ m}^2$ of the surface in one second. Assume that the photons are absorbed by the surface.
- (ii) Suggest why the quantity you have calculated in (i) is referred to as a 'radiation pressure'.

Solution

$$\begin{aligned} \text{(a) (i)} \quad E &= \frac{hc}{\lambda} \\ &= \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{4.5 \times 10^{-7}} = 4.42 \times 10^{-19} \text{ J or } 2.76 \text{ eV} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad I &= \frac{\text{Power}}{\text{Area}} = \frac{\text{Energy}}{\text{Area} \times \text{time}} = \frac{E}{At} = \frac{N hf}{t A} \\ \text{Number of photons incident on surface per second} \\ &= \frac{N}{t} = \frac{IA}{hf} = \frac{(700)(1.0 \times 10^{-4})}{4.42 \times 10^{-19}} = 1.58 \times 10^{17} \text{ s}^{-1} \end{aligned}$$

$$\begin{aligned} \text{(b) (i)} \quad p &= \frac{h}{\lambda} \\ \text{(ii)} \quad p_{\text{violet}} &= \frac{h}{\lambda_{\text{violet}}} \\ &= \frac{6.63 \times 10^{-34}}{4.5 \times 10^{-7}} = 1.47 \times 10^{-27} \text{ N s} \end{aligned}$$

$$\begin{aligned} \text{(c) (i)} \quad &\text{If a photon is absorbed by the metal surface, its final momentum } p_f = 0. \\ &\text{Taking incident direction of photons as +ve,} \\ \Delta p &= p_{\text{final}} - p_{\text{initial}} = 0 - (1.47 \times 10^{-27}) = -1.47 \times 10^{-27} \text{ N s} \\ \text{In 1 s, total } \Delta p &\text{ of all photons} = (\text{number of incident photons})(\Delta p \text{ for each photon}) \\ &= (1.58 \times 10^{17})(-1.47 \times 10^{-27}) \\ &= -2.33 \times 10^{-10} \text{ N s (opposite to incident direction)} \end{aligned}$$

- (ii) By Newton's 2nd law, the rate of change of the momentum of the photons corresponds to a force acting on the photons (by the metal surface).
By Newton's 3rd law, the photons exert a force on the metal surface that is equal in magnitude but opposite in direction to the force of the metal surface on the photons.
Pressure is defined as normal force per unit area; hence photons exert pressure on metal surface, called 'radiation pressure' because the EM radiation is the source of the pressure.

Example 2.2

Calculate the de Broglie wavelength of

- (a) a virus particle of mass 1.0×10^{-15} kg moving at a speed of 2.0 mm s^{-1} .
- (b) an electron with kinetic energy of 120 eV.

Hence explain why the wave behaviour of ordinary objects is not observable.

Solution

$$\begin{aligned} \text{(a)} \quad \lambda &= \frac{h}{p} \\ &= \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(1.0 \times 10^{-15})(2.0 \times 10^{-3})} = 3.32 \times 10^{-16} \text{ m} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad KE &= \frac{1}{2}mv^2 = \frac{p^2}{2m} \\ p &= \sqrt{KE(2m)} \\ &= \sqrt{(120)(1.60 \times 10^{-19})(2)(9.11 \times 10^{-31})} = 5.91 \times 10^{-24} \text{ N s} \\ \lambda &= \frac{h}{p} \\ &= \frac{6.63 \times 10^{-34}}{5.91 \times 10^{-24}} = 1.12 \times 10^{-10} \text{ m} \end{aligned}$$

For ordinary objects, the mass is very large, so the momentum is very large even at low velocities. Thus, the de Broglie wavelength would be a very small value, so the wave behaviour cannot be shown.

Example 2.3

If a person had a de Broglie wavelength of 1.0 m, they would undergo considerable diffraction when moving through a doorway.

- (a) Assuming the person has a mass of 60.0 kg, determine their speed for them to have a de Broglie wavelength 1.0 m.
- (b) For the speed calculated in (a), determine the number of years it would take to move 0.80 m (the approximate length of one step).
- (c) State whether any diffraction will occur when walking normally through a doorway.

Solution

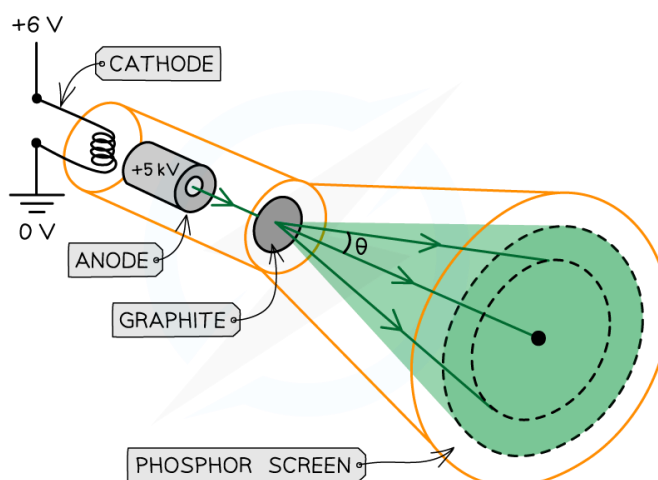
$$\begin{aligned} \text{(a)} \quad \lambda &= \frac{h}{p} = \frac{h}{mv} \\ v &= \frac{h}{m\lambda} \\ &= \frac{6.63 \times 10^{-34}}{(60.0)(1.0)} = 1.11 \times 10^{-35} \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad t &= \frac{\text{distance}}{v} \\ &= \frac{0.80}{1.11 \times 10^{-35}} \\ &= 7.24 \times 10^{34} \text{ s} = 2.3 \times 10^{27} \text{ years} \end{aligned}$$

- (c) No diffraction will occur. When walking normally, the velocity will be $\approx 10^{35}$ larger than the answer in (a), so the de Broglie wavelength will not be comparable to the size of the door.

2.2 Electron Diffraction

In this section, we will examine the wave-like behaviour of electrons in exhibiting the characteristic wave phenomena of diffraction and interference.



Source: *Electron Diffraction (OCR A Level Physics): Revision Note* — Save My Exams, *Electron Diffraction* revision page.

Fig. 2.2: Schematic diagram of apparatus used to observe electron diffraction

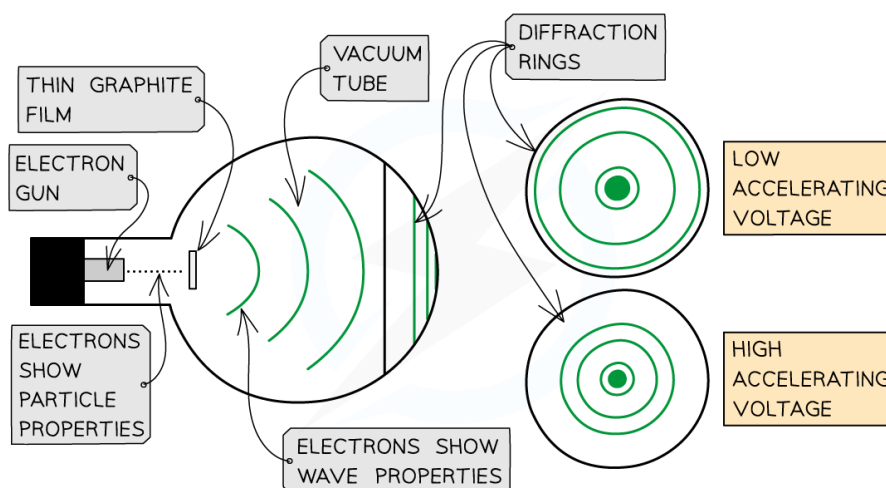
Experimental Procedure	
1.	The cathode is heated to a high temperature, thus emitting electrons (thermionic electrons).
2.	A potential difference is applied between the cathode (lower potential) and the anode (higher potential). Due to this high potential difference (usually about 4 kV), the electrons can accelerate and attain a high velocity (momentum).
3.	The beam of electron hits a thin film of graphite (a regular arrangement of carbon atoms).
4.	The beam of electron diffracts as it passes through the graphite film to form an interference pattern.
5.	When the electron hits the fluorescent coating on the phosphor screen, the screen glows.

For this experiment, classical theory would predict that electrons would behave as particles. Each electron would land on the fluorescent screen and impart its energy to the screen, causing a bright spot to appear where it landed. Since a beam of many electrons was used, a series of bright spots would be expected. As the accelerating potential difference increases, classical theory would predict that each electron would gain more kinetic energy and hence transfer more energy to the screen, resulting in each bright spot becoming brighter.

Instead, the pattern observed contains concentric rings of varying diameters, showing that the electrons diffract, and then undergo interference. With an appropriate accelerating potential difference, the electrons would have a de Broglie wavelength comparable to the separation between carbon atoms in the graphite film and would undergo significant diffraction, causing interference to take place. The 2-dimensional, regular arrangement of carbon atoms in the graphite film causes it to act like a diffraction grating.

As the accelerating potential difference increases, the spacing between the concentric rings will be observed to decrease. An increase in the accelerating potential difference will result in the velocity of each electron increasing, and hence the momentum of each electron will increase.

Applying $\lambda = \frac{h}{p}$, the de Broglie wavelength of the electron will decrease. Using the diffraction grating equation $d \sin \theta = n\lambda$, a decrease in the de Broglie wavelength will result in a decrease in the spacing of the concentric rings, which is indeed what is observed experimentally as shown in Fig. 2.3.



Source: *Electron Diffraction (OCR A Level Physics): Revision Note* — Save My Exams, *Electron Diffraction* revision page.

Fig. 2.3: Effect of accelerating voltage on the observed electron diffraction pattern

Example 2.4

In an electron diffraction experiment, calculate the de Broglie wavelength for the electron when the accelerating potential difference is 3.6 kV.

Assuming the gain in KE = loss in EPE:

$$\begin{aligned}
 \frac{p^2}{2m} - 0 &= q\Delta V \\
 p &= \sqrt{2mq\Delta V} \\
 \lambda &= \frac{h}{p} \\
 &= \frac{h}{\sqrt{2mq\Delta V}} \\
 &= \frac{6.63 \times 10^{-34}}{\sqrt{2(9.11 \times 10^{-31})(1.60 \times 10^{-19})(3.6 \times 10^3)}} \\
 &= 2.05 \times 10^{-11} \text{ m}
 \end{aligned}$$

2.3 Double-Slit Interference of Single Particles

In another experiment to demonstrate the wave nature of particles, a parallel beam of electrons is incident on a double slit. An electron detector screen is placed at a distance much greater than d , the separation of the slits. As electrons are continuously being sent towards the double-slit over an extended period of time, Fig. 2.4 shows the number of electrons detected at the screen per unit time.

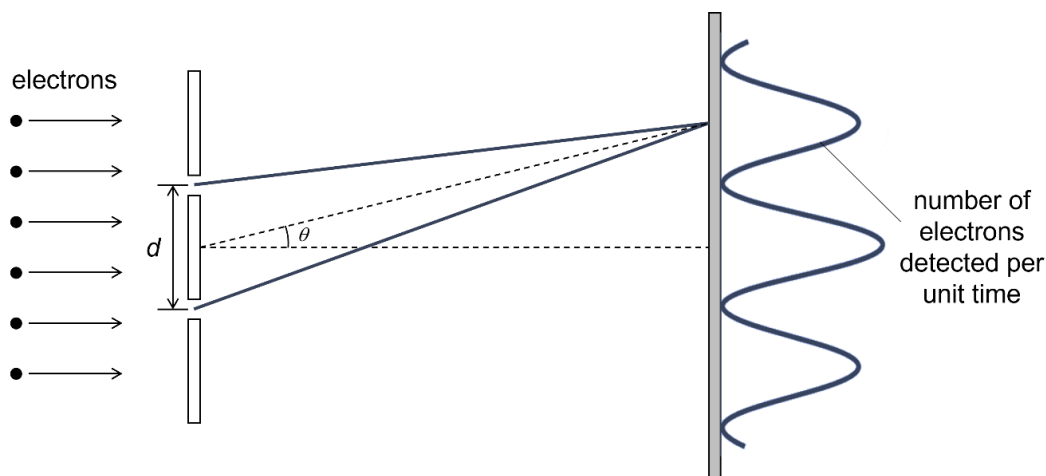


Fig. 2.4

The detection pattern on the screen is similar to the interference pattern when light is sent through a double-slit in Young's Double-Slit Experiment. If the electrons behaved as classical particles, one would expect the electrons to only arrive at positions on the detector directly aligned with the two slits. Therefore, this result shows that the electrons are interfering, a wave-like behaviour. Furthermore, the angles θ at which the maximum intensity of electrons arrives at the screen follow the exact same equation as that of light.

In 1989, Akira Tonomura led a team to show that the same pattern is still eventually achieved even if the electrons are sent through one at a time (Fig. 2.5). Therefore, the electrons cannot be colliding with one another to form this pattern. Each electron interacts with both slits simultaneously and wave components of itself interferes.

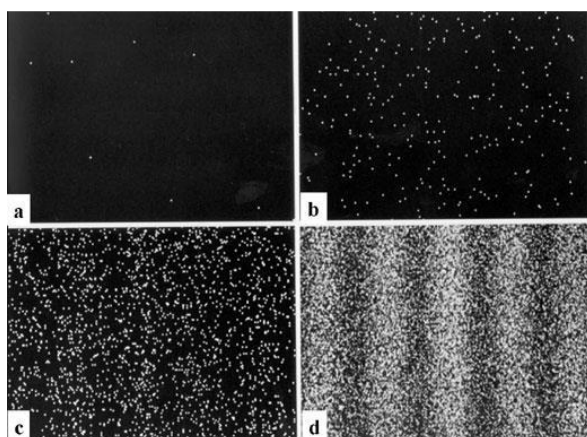


Fig. 2.5: Excerpts of Tonomura's data, showing the detector output after (a) 8 electrons, (b) 270 electrons, (c) 2000 electrons, and (d) 160,000 electrons. Watch how single electrons arrive as particles, yet together form a wave-like interference pattern.

How does a wavefunction explain where a particle can be found and why its energy is quantised?

3. The Wavefunction

In the earlier section, we learned about the wave-particle duality of both electromagnetic radiation and matter and, were introduced to the de Broglie wavelength of particles. While the wave properties of electromagnetic waves can be understood mathematically through electric field equations, the wave properties of a particle are described using the associated wavefunction ψ of the particle. The wavefunction contains within it all the information that can be known about the particle.

More commonly, the physical implication of the wavefunction of a particle is seen through the absolute square of the wavefunction $|\psi|^2$. $|\psi|^2$ is known as the **probability density function** and it is the relative probability per unit volume that the particle will be found at any given point in the volume.

3.1 Normalisation Condition

Consider a particle located along the x -axis. While it is not possible to exactly specify the position of the particle due to Heisenberg's uncertainty principle, it is possible to determine the probability of finding it in a region along the x -axis. This probabilistic nature of the wavefunction was first suggested by Max Born in 1928 and is fundamental to the approach in Quantum Mechanics. In order to calculate this probability, the boundaries of this region must be specified.

The probability P_{ab} of finding the particle in the interval $a \leq x \leq b$ is given by the following:

$$P_{ab} = \int_a^b |\psi|^2 dx$$

Graphically, the probability density function can be sketched in a $|\psi|^2$ against x graph as shown in Fig. 3.1. The probability P_{ab} is given by the area under the curve, bounded by the positions a and b .

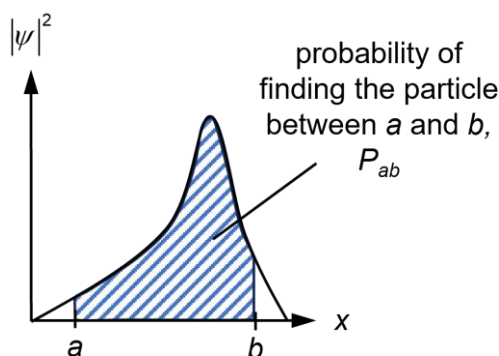


Fig. 3.1

Mathematically, the probability of finding a particle within a region must be between 0 and 1. For example, if the probability is 0.40, this implies that there is a 40% chance of finding that particle in the region. This necessarily means that if this interval were to be widened to everywhere along the x-axis, the probability of finding the particle will be 1 because if the particle exists, the probability of finding the particle somewhere along the x-axis must be 100%.

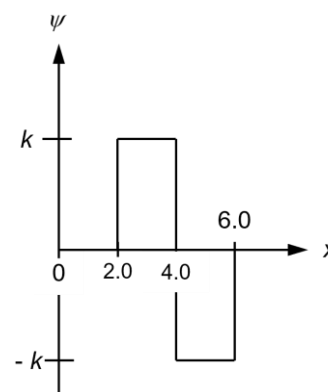
This normalisation condition is expressed as:

$$\text{total area under the } |\psi|^2 \text{ against } x \text{ graph, } \int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

Example 3.1

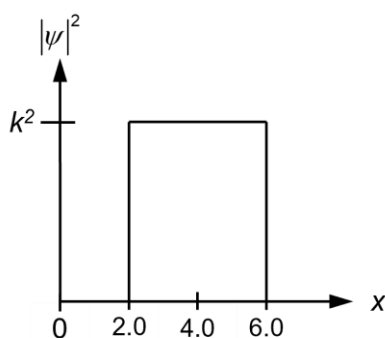
The wavefunction ψ of a quantum particle takes a value of k between $x = 2.0$ and $x = 4.0$, and a value of $-k$ between $x = 4.0$ and $x = 6.0$ as shown.

- (a) Given that ψ is normalised, determine the constant k .
(b) Determine the probability that the particle will be found between $x = 5.0$ and $x = 6.0$.



Solution

- (a) $\int_{-\infty}^{\infty} |\psi|^2 dx = \int_{2.0}^{6.0} k^2 dx$
= area under probability density function between $x = 2.0$ and $x = 6.0$



$$\begin{aligned} \int_{2.0}^{6.0} k^2 dx &= k^2 (6.0 - 2.0) \\ &= 4.0k^2 \end{aligned}$$

Since ψ is normalised, $4.0k^2 = 1$

$$\begin{aligned} k &= \sqrt{\frac{1}{4.0}} \\ &= 0.50 \end{aligned}$$

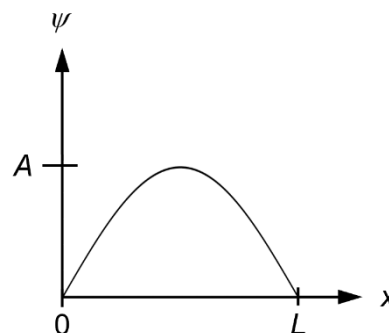
- (b) $P_{x=5 \text{ to } x=6} = \int_5^6 |\psi|^2 dx$
= $\int_5^6 |-0.50|^2 dx$
= area under probability density function between $x = 5.0$ and $x = 6.0$
= $\left(\frac{1}{4}\right) [k^2 (6.0 - 2.0)]$
= $\left(\frac{1}{4}\right) [0.25 (6.0 - 2.0)]$
= 0.25

Example 3.2

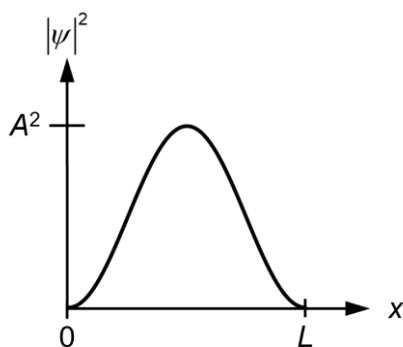
The normalised wavefunction of a particle confined in a box between $0 \leq x \leq L$ is given by $\psi = A \sin\left(\frac{\pi x}{L}\right)$ as shown.

Determine the value of A .

Solution

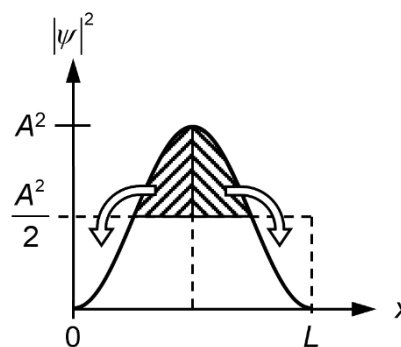


$$\begin{aligned} \int_{-\infty}^{\infty} |\psi|^2 dx &= \int_0^L A^2 \sin^2\left(\frac{\pi x}{L}\right) dx \\ &= \text{area under } A^2 \sin^2\left(\frac{\pi x}{L}\right) \text{ against } x \text{ graph between } 0 \text{ and } L \end{aligned}$$



Step 1: Sketch the modulus square of the wavefunction. We obtain a sine square graph.

$$|\psi|^2 = A^2 \sin^2\left(\frac{\pi x}{L}\right)$$



Step 2: The area under the sine square graph is equal to the area under the rectangle of length L and breadth $\frac{A^2}{2}$.

$$\text{area under } A^2 \sin^2\left(\frac{\pi x}{L}\right) \text{ against } x \text{ graph between } 0 \text{ and } L = \frac{A^2 L}{2}$$

$$\frac{A^2 L}{2} = 1 \Rightarrow A = \sqrt{\frac{2}{L}}$$

Hence, the normalised wave function is expressed as $\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$.

3.2 Particle in a Box

This section investigates the wavefunctions of a particle when it is confined in a box. This box has a base of length L and two rigid walls. The walls are impenetrable so there is zero probability of finding the particle outside the box. This is also seen by the fact that the potential energy U of the particle will tend to infinity if it is at the positions $x \leq 0$ and $x \geq L$. For $0 < x < L$, $U = 0$ as shown in Fig. 3.2.

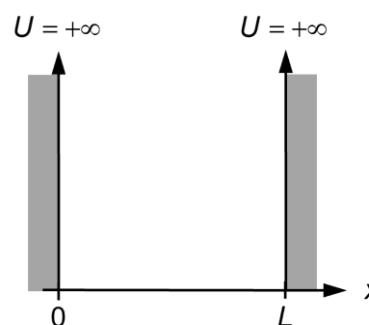


Fig. 3.2

Classically, if we put a ball in a box like this, the ball can bounce back and forth between $x = 0$ and $x = L$ without any constraints on its momentum and energy. However, when we study particles that are small enough such that it displays wave-like behaviour, only the wavefunctions that satisfy the boundary conditions can exist. This means that the particle can only have specific energies when it exists in the box. The boundary conditions refer to the conditions that the wavefunction is equal to zero at $x = 0$ and $x = L$. The analysis is similar to how only some stationary waves are allowed because the two ends are nodes.

3.2.1 Wavefunction for a Particle in a Box

The wavefunction for a particle in the box can be expressed as sinusoidal functions which satisfy the boundary conditions as shown in Fig. 3.3.

Let the general form of the wavefunction be $\psi = A\sin(kx) + B\cos(kx)$.

Boundary Condition at $x = 0$

$$\begin{aligned}\psi &= A\sin(kx) + B\cos(kx) \\ &= A\sin[k(0)] + B\cos[k(0)] \\ &= B\end{aligned}$$

In order to satisfy the boundary condition where $\psi = 0$ at $x = 0$, $B = 0$.

Boundary Condition at $x = L$

$$\begin{aligned}\psi &= A\sin(kx) \\ &= A\sin[k(L)]\end{aligned}$$

In order to satisfy the boundary condition at $x = L$, $A\sin(kL) = 0$. This equation is only valid if the values kL and hence k take on specific values.

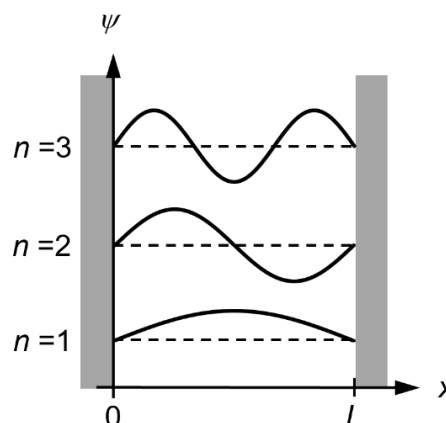


Fig. 3.3

Consider each case separately to determine k .

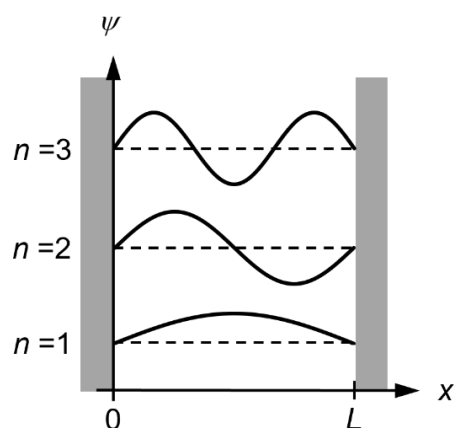


Fig. 3.3

n	number of wavelengths	kL	k
1	$\frac{1}{2}$	π	$\frac{\pi}{L}$
2	$1 = \frac{2}{2}$	2π	$\frac{2\pi}{L}$
3	$\frac{3}{2}$	3π	$\frac{3\pi}{L}$

By observing the trend, for the n^{th} wavefunction, there will be $\frac{n}{2}$ wavelengths and $k_n = \frac{n\pi}{L}$.

Hence, only certain wavelengths for the particle are allowed and each wavelength correspond to a quantum state for the system. n is the quantum number of the state and the n^{th} wavefunction is

$$\psi_n = A \sin\left(\frac{n\pi x}{L}\right), \text{ where } A \text{ is the normalisation constant.}$$

As shown in Example 3.2, A is found to be $\sqrt{\frac{2}{L}}$. Therefore, the normalised n^{th} wavefunction for the particle in a box is

$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), \text{ where } n = 1, 2, 3, \dots$$

3.2.2 Energy Levels for a Particle in a Box

Each wavefunction describes a possible state that the particle can exist in the box. As only specific wavelengths are allowed, the energy of each of these states are also constrained to specific values. Since the potential energy of the system is zero when the particle is inside the box, the energy of the system is the kinetic energy of the particle which is given by

$$E = \frac{p^2}{2m}.$$

The momentum of the particle p is related to the de Broglie wavelength λ of the particle by

$$p = \frac{h}{\lambda}.$$

Combining the two expressions,

$$E = \frac{h^2}{2m\lambda^2}.$$

This energy can be expressed in terms of L by considering the wavelength for the n^{th} wavefunction.

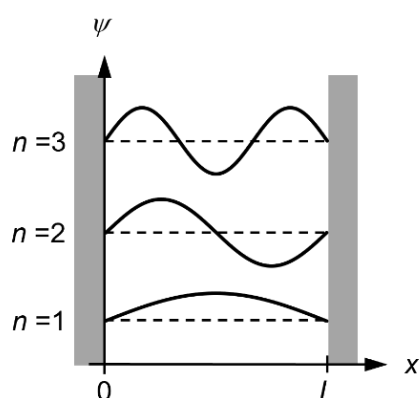


Fig. 3.3

n	number of wavelengths	λ
1	$\frac{1}{2}$	$\frac{1}{2}\lambda = L \Rightarrow \lambda = 2L$
2	$1 = \frac{2}{2}$	$\lambda = L \Rightarrow \lambda = L$
3	$\frac{3}{2}$	$\frac{3}{2}\lambda = L \Rightarrow \lambda = \frac{2}{3}L$

By observing the trend, for the n^{th} wavefunction, $\lambda_n = \frac{2}{n}L$.

Hence, the energy for the n^{th} state is given by

$$E_n = \frac{h^2}{2m \left(\frac{2}{n}L \right)^2}$$

$$E_n = \frac{n^2 h^2}{8mL^2}, \text{ where } n = 1, 2, 3, \dots$$

This shows that the particle is restricted to having only specific energies. The lowest allowed energy state corresponds to the *ground state*. This is the $n = 1$ state with energy $E_1 = \frac{h^2}{8mL^2}$. This implies that the particle can never be at rest since its energy cannot be zero according to quantum mechanics.

The higher energy states are known as the *excited states*. They correspond to $n = 2, 3, 4, \dots$ having energies of $E_n = n^2 E_1$. These energies are shown in the energy level diagram in Fig. 3.4.

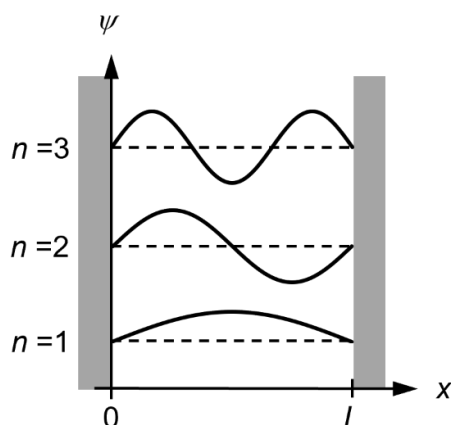


Fig. 3.3

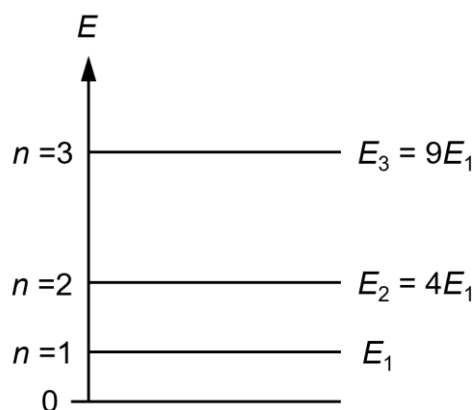


Fig. 3.4

Example 3.3

- (a) Determine the lowest energy level for a particle in a box if the particle is an electron in a box 2.5×10^{-10} m across.
- (b) Determine the speed of the electron in the $n = 1$ state.
- (c) A 0.500 kg baseball is confined between two walls of a stadium and is modelled as particle in a box of length 100 m. Calculate the minimum speed of the baseball.

Solution

$$(a) \quad E_1 = \frac{h^2}{8mL^2}$$

$$E_1 = \frac{(6.63 \times 10^{-34})^2}{8(9.11 \times 10^{-31})(2.5 \times 10^{-10})^2}$$

$$= 9.65 \times 10^{-19} \text{ J}$$

$$(b) \quad E = \frac{1}{2}mv^2$$

$$v_1 = \sqrt{\frac{2E_1}{m}}$$

$$v_1 = \sqrt{\frac{(2)9.65 \times 10^{-19}}{9.11 \times 10^{-31}}}$$

$$= 1.46 \times 10^6 \text{ m s}^{-1}$$

$$(c) \quad E_1 = \frac{h^2}{8mL^2}$$

$$E_1 = \frac{(6.63 \times 10^{-34})^2}{8(0.500)(100)^2}$$

$$= 1.10 \times 10^{-71} \text{ J}$$

$$v_1 = \sqrt{\frac{2E_1}{m}}$$

$$v_1 = \sqrt{\frac{(2)1.10 \times 10^{-71}}{0.500}}$$

$$= 6.63 \times 10^{-36} \text{ m s}^{-1}$$

For any particle confined in a box, the ground-state energy is non-zero according to quantum mechanics. Confinement forces a minimum momentum and hence a minimum kinetic energy. However, for macroscopic objects like the baseball, this energy is so small that the baseball can be treated as at rest. Quantum effects like energy quantisation are unobservable. Macroscopic objects obey quantum laws, but their quantum effects are too small to notice.

Some examples of particles being subjected to boundary conditions because it is confined to a region of space resulting in the quantisation of energy include:

- an electron in a hydrogen atom being restricted to stay near the nucleus of the atom.
- two atoms bound together forming a diatomic molecule.
- proton trapped in a nucleus.

You also may have learned from chemistry that electrons occupy orbitals around an atom. In fact, each orbital is an 'electron cloud', i.e. a 3D wavefunction that describes the most likely location of an atom. Each orbital state corresponds to a different energy of the electron. Fig. 3.5 shows 2D cross sections of the probability density functions of the energy states of hydrogen.

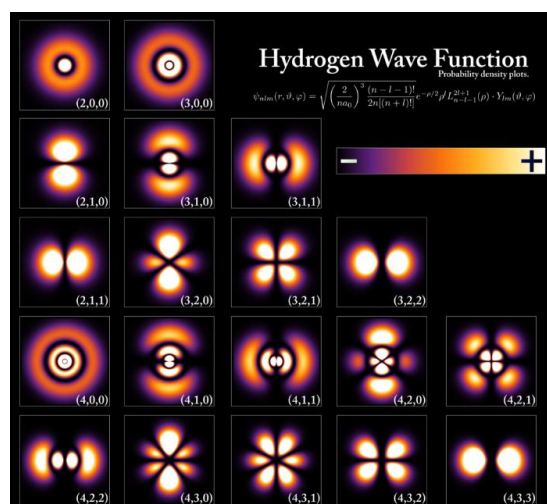
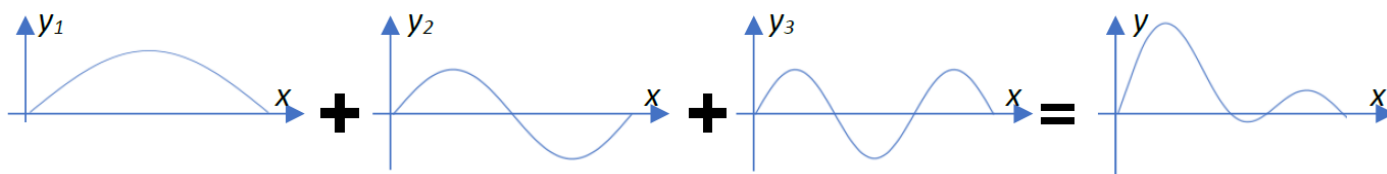


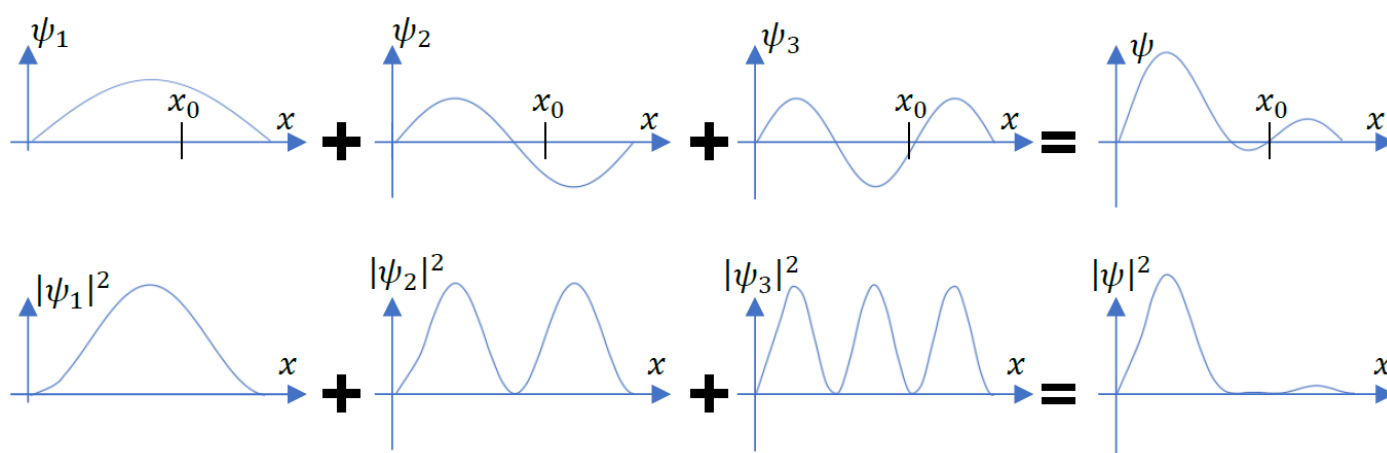
Fig. 3.5

3.2.3 Comparison between the Quantum Wavefunction and Classical Superposition

Typically, the sound waves produced by a musical instrument comprise of harmonics, which are integer multiples of a fundamental frequency. For example, the fundamental (y_1) and first two harmonics (y_2 and y_3) of a musical note (y) could be represented as shown:

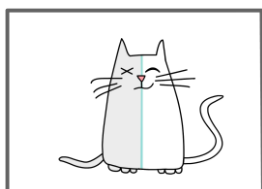


Similarly, in the quantum framework, particles are generally in a mixed state (ψ) comprising the summation of several pure states ($\psi_1, \psi_2, \dots$). For example, the wavefunctions of the lowest three energy states of a particle trapped in a box are as shown:



Just as the intensity of sound is proportional to its amplitude squared, the probability density function is proportional to the square of the amplitude of the quantum wavefunction and is always positive. For example, at position x_0 , none of the constituent states have $\psi_i = 0$. In other words, if the particle is in any of the 'pure states' ψ_1, ψ_2 or ψ_3 , it is possible to measure the particle's position as $x = x_0$. However, in the mixed state ψ , it is not possible for the particle to be found at $x = x_0$, because $\psi = \psi_1 + \psi_2 + \psi_3$ is equal to zero at that location.

This is an example of how the wave nature of a single particle can lead to interference effects. Similar to how the superposition of multiple waves may cause nodes or antinodes that differ from where the individual wave(s) have crests or troughs, the summation of a particle's various states may result in interference effects.



Watch this video to learn more about the famous Schrödinger's Cat thought experiment where the cat is in a superposition of being in two states: alive and dead.

Why is the simultaneous measurement of position and momentum impossible?

4. Heisenberg's Uncertainty Principle

4.1 Position-Momentum Uncertainty Principle

For a single particle in a vacuum, if we know the momentum of the particle with absolute certainty, then the wavefunction of the particle is described by a single wavelength according to the de Broglie relation (Fig. 4.1). Since this wavelength is uniform everywhere, we are completely unable to comment on the particle's position. Therefore, the uncertainty in the particle's position is large.

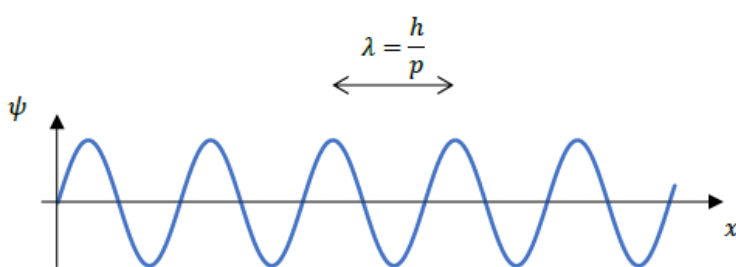


Fig. 4.1

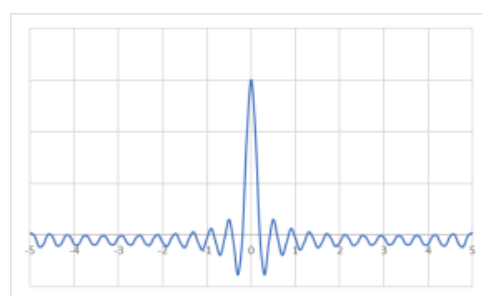


Fig. 4.2

However, if there is more than one possible momentum and therefore more than one de Broglie wavelength, the interference between these states causes the wavefunction begins to become more localised, forming a “wave packet” as shown in Fig. 4.2. Hence, as the uncertainty in momentum increases, the uncertainty in position decreases since we are more certain about the particle's position.

Heisenberg's uncertainty principle for position and momentum states that if the position and momentum of a particle are measured simultaneously, there is a minimum uncertainty in the two measurements, given by

$$\Delta x \Delta p \gtrsim h$$

where

Δx : uncertainty in the measurement of the position

Δp : uncertainty in the measurement of the momentum

In applying the position-momentum uncertainty principle, we are considering the measurement of position in only 1 dimension (e.g. x-direction), and the measurement of momentum is in the same direction as the measurement of the position (e.g. x-component of momentum).

If the position of the particle can be measured precisely on its own (i.e. $\Delta x \rightarrow 0$), the momentum of the particle cannot be measured precisely (i.e. $\Delta p \rightarrow \infty$), at the same time. In fact, making a precise measurement of the momentum of the particle will result in the position of the particle becoming uncertain, and vice versa, even if they were previously measured precisely.

Example 4.1

Absolute zero ($T = 0 \text{ K}$) is the temperature at which molecular motion is at an absolute minimum. Use the Heisenberg Uncertainty Principle to explain why there must be some molecular motion even at $T = 0 \text{ K}$.

Solution

If there is no molecular motion, every molecule is at rest ($p = 0$) at a fixed position ($x = 0$). This implies that the position and momentum of the molecule can simultaneously be precisely measured to be equal to zero, so $\Delta p = \Delta x = 0$. However, from the Heisenberg Uncertainty Principle, $\Delta x \Delta p \gtrsim h$, so there must be some molecular motion.

Example 4.2

An electron is confined within a region of width $1 \times 10^{-10} \text{ m}$.

- (a) Estimate the minimum uncertainty in the electron's momentum.
(b) If the electron has momentum with magnitude equal to the uncertainty found in (a), determine its kinetic energy.

Solution

- (a) Since the electron is confined within the region, but could be anywhere within the region, the uncertainty in the position of the electron is equal to the width of the region.

$$\Delta x \Delta p \geq h$$

$$\Delta p \geq \frac{h}{\Delta x}$$

$$\Delta p \geq \frac{6.63 \times 10^{-34}}{1 \times 10^{-10}} = 6.63 \times 10^{-24}$$

$$\Delta p_{\min} = 6.63 \times 10^{-24} \text{ N s}$$

$$\begin{aligned} \text{(b)} \quad KE &= \frac{1}{2}mv^2 \\ &= \frac{p^2}{2m} \\ &= \frac{(6.63 \times 10^{-24})^2}{2(9.11 \times 10^{-31})} = 2.41 \times 10^{-17} \text{ J or } 151 \text{ eV} \end{aligned}$$

Example 4.3

A golf ball has a mass of 50 g and travels horizontally with a velocity of 80 m s^{-1} .

If the uncertainty in measuring the velocity is 0.01 m s^{-1} , state the limit of uncertainty placed on a measurement of its position by the Heisenberg Uncertainty Principle, assuming the uncertainty in the mass of the golf ball is negligible.

Solution

$$\Delta x \Delta p \geq h$$

$$\Delta x \Delta(mv) \geq h$$

$$\Delta x (m \Delta v) \geq h$$

$$\Delta x \geq \frac{h}{m \Delta v} = \frac{6.63 \times 10^{-34}}{(0.050)(0.01)} = 1.326 \times 10^{-30} \text{ m}$$

$$\Delta x_{\min} = 1.4 \times 10^{-30} \text{ m (2 s.f.)}$$

How do spectral lines provide evidence for the quantisation of energy?

5. Energy Levels in Atoms

5.1 Energy Levels of Electrons in Atoms

A common visualisation of the atom is the “planetary model” where electrons orbit the proton mimicking the Earth orbiting the Sun. This is not the most up to date and useful model, but it gives a good introduction to the energy levels in the atom.

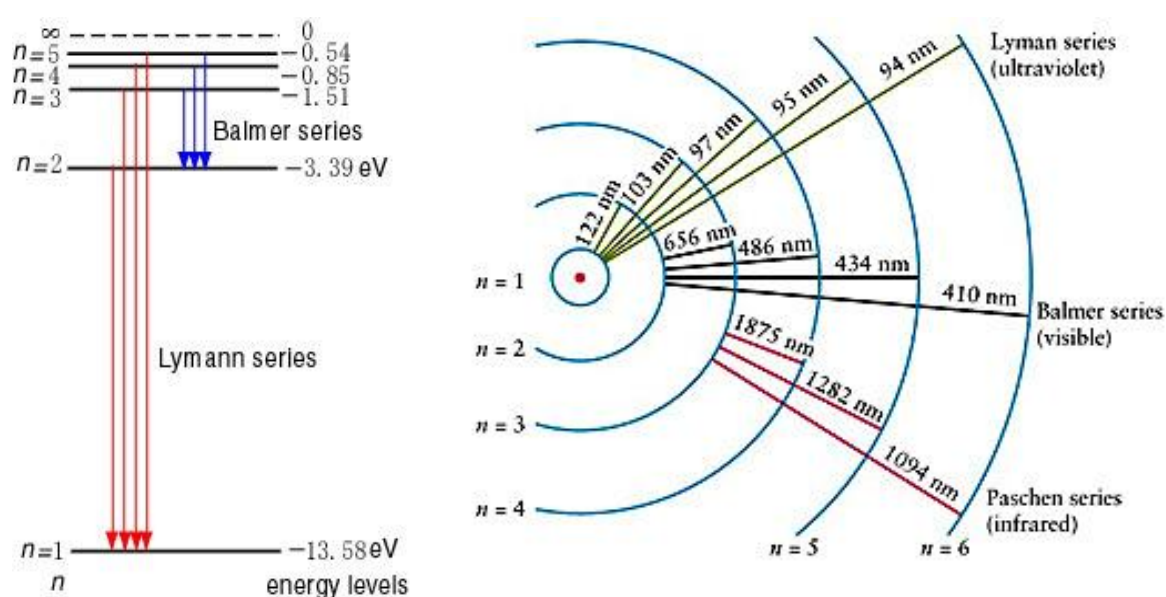


Fig. 5.1: An illustration of the different energy levels in an atom.

(Left) The transitions between energy levels are indicated by arrows.

(Right) Each transition is labelled with the corresponding photon wavelength.

Electrons in an isolated gas atom can only have certain specific energies. These energies are called the electron energy levels of the atom (discrete energy values). The energy levels may be represented as a series of horizontal lines against a vertical scale of energy, as shown in Fig. 5.1 (left). The separation between the levels decreases at higher energy values.

The electron in the hydrogen atom can have any of these energy levels, but cannot have energies between them. The lines are drawn such that moving from the bottom to the top, the difference between the energy levels are increasingly smaller.

Due to the small values of the energies at each energy level, the energies are typically given in electron volts (eV) to avoid the use of very small numbers. The energies are all negative, because the electrostatic force between the electron and the nucleus is attractive.

The energy levels are labelled with numbers n , with $n = 1$ corresponding to the lowest energy level which has the most negative energy. This is the most stable state.

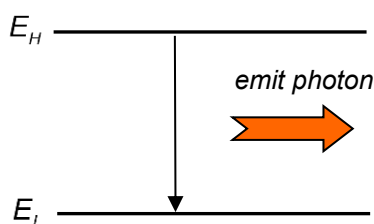
Table 5.2 below gives a summary of some of the common terms which relate to the discrete energy levels in atoms.

Table 5.2

Terms Used	Description
Ground State	At lower temperatures, electrons occupy the <u>lowest energy levels available</u> . The atom and its electrons is said to be in the ground state .
Excited State	If any bound electron absorbs energy (excitation energy) through heating, or by collision with a free electron, or by an incident photon, it can transit to a higher energy level. 1. The <u>energy of a photon</u> must be <u>exactly equal to</u> the difference in energy of the two levels. 2. The <u>energy of a bombarding electron</u> can be <u>equal to or higher than</u> the energy difference. The atom and its electrons is said to be in an excited state.
Ionised State	If the energy absorbed is sufficient for the most loosely bound <u>electron to escape from the atom</u> , the atom becomes ionised .
Excitation Energy	Excitation energy is the energy required to raise an atom from its ground state to an excited state.
Ionisation Energy	Ionisation energy is required to completely remove an electron (initially in the ground state) from an atom.

In non-isolated gas atoms or liquids and solids, electron energies are no longer discrete, but continuous.

5.2 Transition of Electrons between Energy Levels – Atomic Line Spectra

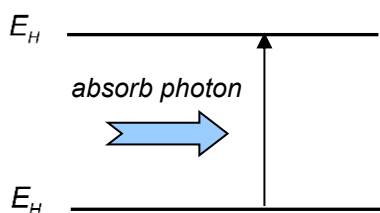


When an electron transits from a higher energy level (E_H) to a lower energy level (E_L), it loses energy by emitting a photon of frequency f .

$$E_H - E_L = hf$$

An electron in a higher energy level can transit to a lower energy level in an atom due to either:

1. **Spontaneous emission** - an electron in the higher energy level will lose its energy without any external stimulus. One photon will be emitted, with energy equal to the energy lost by the electron.
2. **Stimulated emission** - a photon whose energy is equal to the energy difference between 2 levels can “stimulate” an electron to transit downwards from a higher energy level to a lower energy level, producing a second photon with the same energy.



For an electron to transit from a lower energy level (E_L) to a higher energy level (E_H), one way is to absorb a photon of frequency f . Similarly,

$$E_H - E_L = hf$$

Ways for an electron to transit from a lower energy level to a higher energy level in an atom:

1. **Collisions between atoms** – transfer of energy to the electron by another energetic atom or fast moving electron
2. **Heating** to increase thermal energy that can be absorbed by the atoms and hence electrons
3. **Spontaneous absorption** - absorption of a photon whose energy is exactly equal to the energy difference between the levels

Example 5.1

Two energy levels in a hydrogen atom are -3.40 eV and -1.51 eV. Deduce

- (a) the energy of a photon emitted when an electron transitions between these two levels, and
- (b) the wavelength of the emitted light.

Solution

$$\begin{aligned} \text{(a)} \quad E &= E_H - E_L \\ &= -1.51 - (-3.40) \\ &= 1.89 \text{ eV or } 3.02 \times 10^{-19} \text{ J} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad E &= hf = \frac{hc}{\lambda} \\ \lambda &= \frac{hc}{E} \\ &= \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{3.02 \times 10^{-19}} = 6.58 \times 10^{-7} \text{ m} \end{aligned}$$

Example 5.2

The electron in a hydrogen atom is at the ground state, as shown in Fig. 5.3. Describe what happens to the electron when

- (a) an electron of 11 eV bombards the hydrogen atom, and
- (b) a photon of 11 eV is incident on the hydrogen atom.

Solution

- (a) The hydrogen atom is excited and its electron at the ground state (-13.6 eV) will transit to a higher energy level (-3.4 eV).
The bombarding electron, after transferring the excitation energy of 10.2 eV to the hydrogen atom, will still have 0.8 eV of energy remaining.
- (b) The energy of the photon does not match any of the energy level differences.

For the electron at ground state (-13.6 eV) to move to the next higher energy level (-3.4 eV), it will require a photon with energy exactly equal to $E_2 - E_1 = 10.2$ eV, since either all or none of the energy of the photon must be transferred to the electron.

Hence, the electron at the ground state will not absorb any energy from the photon.

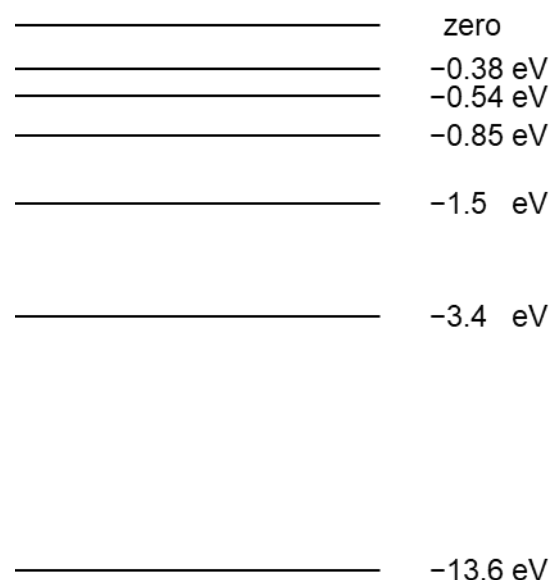


Fig. 5.3: Energy levels of a hydrogen atom.

We will look at the experimental evidence for the existence of energy levels within isolated gas atoms.

When isolated gas atoms are excited, their electrons are at higher energy levels, and both the atoms and the electrons are unstable. The excited electrons will spontaneously transit to lower energy levels, emitting photons in a range of frequencies. The different frequencies of the photons can be observed on the screen as sharply defined lines (spectra) using a diffraction grating.

Different elements have different sets of energy levels; the energy level differences are unique to each element. Consequently, each element produces unique, characteristic line spectra. There are 2 types of line spectra - emission and absorption spectra.

5.2.1 Emission Line Spectra

Emission line spectra are discontinuous coloured lines on a dark background (no light). The emission spectrum for the hydrogen atom is as shown:

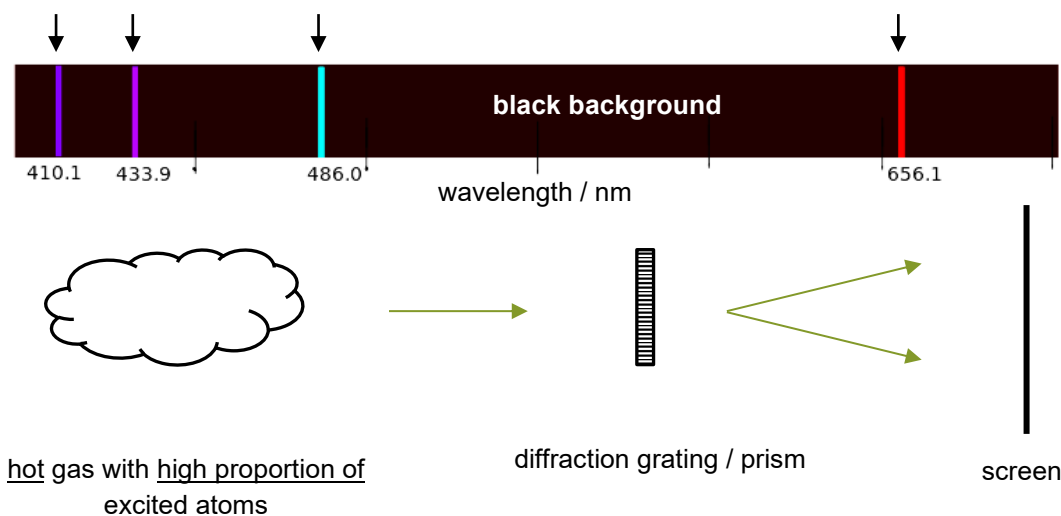


Fig. 5.4: (Top) The emission line spectrum of hydrogen. The arrows indicate where the coloured lines are. (Bottom) A typical experimental setup to observe an emission spectrum of a gas.

There are 4 basic requirements to obtain an emission spectra.

1. The gas has sufficiently high proportion of atoms in an excited state (i.e., electrons are in the higher energy level). This can be achieved by the following mechanisms:
 - Heating the gas, so that the atoms collide with each other frequently, causing each other to move to a higher energy level (excited state).
 - Bombarding the gas atoms with high energy electrons. This has the same effect as the first method.
 - Shining a continuous spectrum of light at the gas from one direction, and observing the photons emerging from another direction.
2. The gas must have low pressure so that the atoms are sufficiently far apart (isolated atoms) that different atoms do not affect the energy levels of each other's electrons. This ensures that the lines are discrete and separate.
3. A spectrum requires light to be separated into the different wavelengths to see the separated coloured lines. We can use the following methods:
 - diffraction grating –the diffraction angle is related to the wavelength of light, $d \sin \theta = n\lambda$ (as covered in Superposition).
 - prism - many transparent materials have refractive indices that vary with wavelength, which can also be used to separate the photons into different paths.
4. The spectrum must be observed or detected. This can be done in a few different ways – projection of the spectrum onto a screen, the use of light intensity sensors with data loggers, or direct observation with the eye.

In an emission line spectrum, the only photons being detected are those emitted by the hot gas as the electrons transition to lower energy levels. Thus, most of the spectrum will be dark, with a few coloured lines corresponding to the photons emitted from each of the possible energy level transitions.

5.2.2 Absorption Line Spectra

Absorption line spectra are discontinuous dark lines on a continuous spectrum background. The absorption spectrum for the hydrogen atom is as shown:

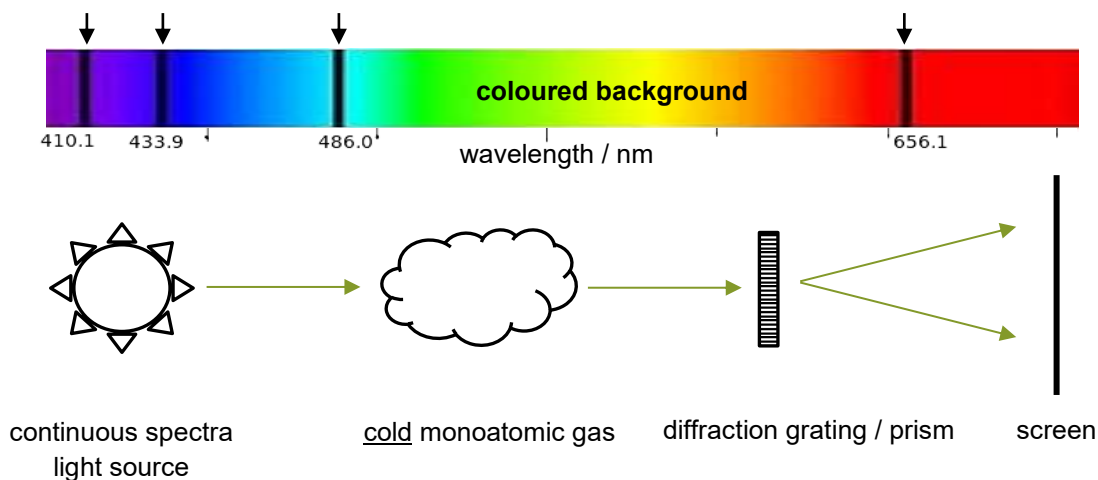


Fig. 5.5

Absorption line spectra are obtained in a similar way to the emission spectra, except for the following key differences:

1. The gas should have a large proportion of atoms in the lowest energy level (ground state). This is so that the gas is ready to absorb photons from an external source. This can be ensured by starting with a cold gas.
2. Similar to the emission spectrum, the gas must have low pressure.
3. Light with a continuous spectrum (e.g., incandescent lamp) is shone on the cool gas. Only the photons with energy exactly equal to the difference in any 2 of the atom's energy levels will be absorbed by the atom. Photons not absorbed will continue to travel on to the screen. However, the excited atoms are highly unstable and their electrons will return to a lower energy level by emitting photons of the same energy as those that were absorbed earlier. These emitted photons will travel in all directions so only a small number of photons will reach the screen. This results in dark lines at these wavelengths as the number of photons with these wavelengths will be much smaller than the other wavelengths, so the intensity will be much lower.

The observation of well-defined spectral lines (for both emission and absorption spectra) which are separated from each other provide experimental evidence for the existence of quantised energy levels in the atom.

5.2.3 Comparing the Emission and Absorption Line Spectra

As seen in Fig. 5.6 below, for the same gaseous element, the frequencies (colours) missing from its absorption spectra are those present in the emission line spectra.

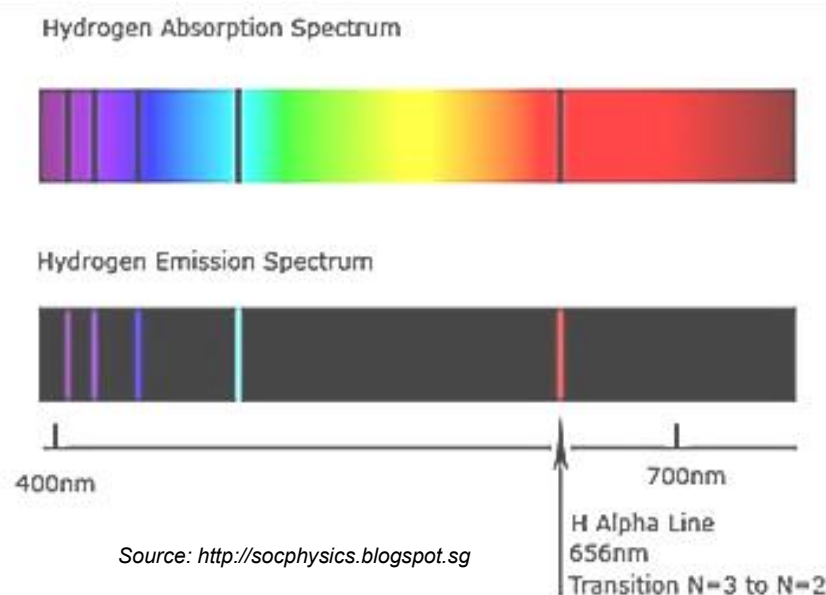
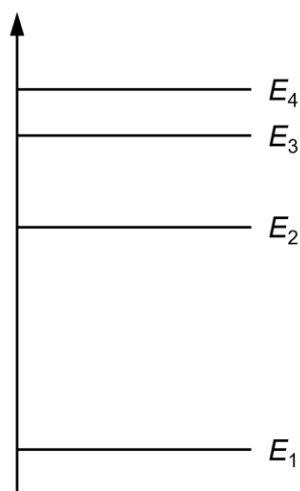


Fig. 5.6 – (Top) The absorption line spectrum of hydrogen.
(Bottom) The emission line spectrum of hydrogen.

Note that the bright lines on the emission spectrum exactly match the dark lines on the absorption spectrum.

Example 5.3

energy



The diagram represents the energy levels for an electron in a certain atom. The transition from E_3 to E_1 produces a green line.

What transition could give rise to a red line?

- A** E_4 to E_3
- B** E_4 to E_1
- C** E_3 to E_2
- D** E_2 to E_1

Example 5.4

White light from a tungsten filament lamp is passed through a sodium vapour and viewed through a diffraction grating. Which of the following best describes the spectrum which would be seen?

- A** coloured lines on a black background
- B** coloured lines on a white background
- C** dark lines on a coloured background
- D** dark lines on a white background

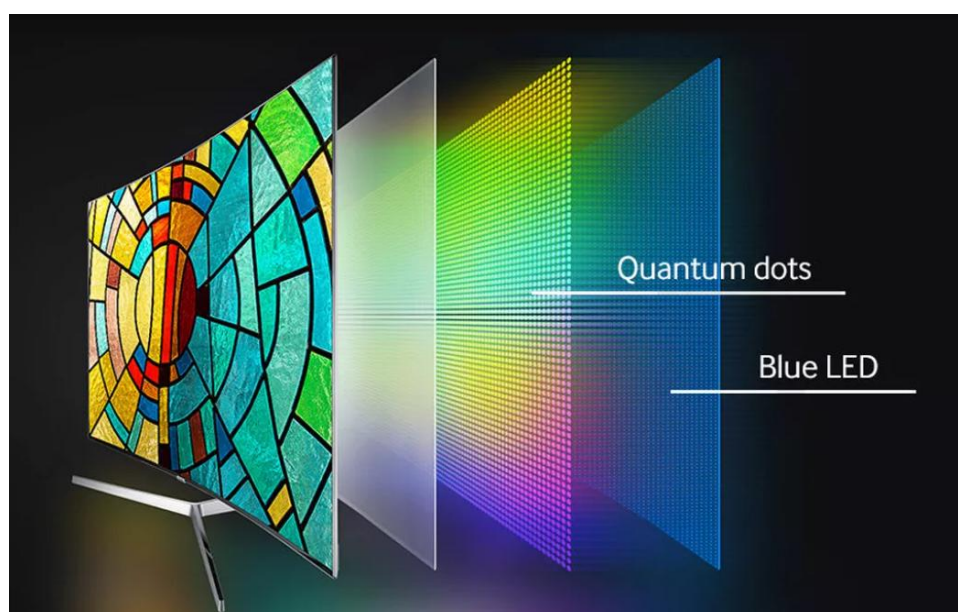


Nature of Science Moment – Tiny Dots, Brilliant Colours

Quantum LED (QLED) monitors get their brilliant, precise colours from quantum dots—nanometre-sized semiconductor crystals that glow when excited by light. In quantum physics, electrons in such tiny structures can only have specific, quantised energy levels. When an electron drops to a lower level, it emits a photon whose colour depends directly on the energy gap.

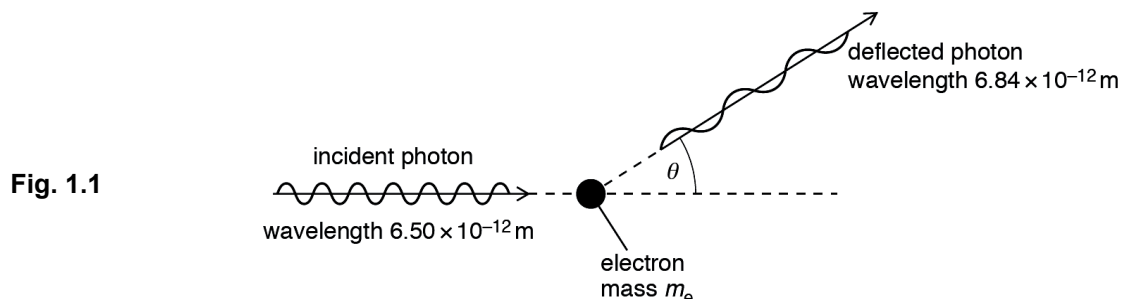
Here's the clever part: change the size of the quantum dot, and you change the allowed energy levels. Smaller dots emit higher-energy (bluer) light, while larger dots emit lower-energy (redder) light. This size–colour relationship lets engineers tune colours with remarkable precision—no dyes, just quantum mechanics in action.

Next time you see the intense reds and crisp blues on a QLED screen, remember: you're watching electrons jump between quantised energy levels and emit colour, one photon at a time. Want to find out how confinement leads to quantisation—and why this works so reliably? That's quantum physics lighting up your display.



6 Review Questions

- 1 (a) Explain what is meant by a *photon*.
- (b) An X-ray photon of wavelength 6.50×10^{-12} m is incident on an isolated stationary electron, as illustrated in Fig. 1.1.



The photon is deflected elastically by the electron of mass m_e . The wavelength of the deflected photon is 6.84×10^{-12} m.

- (i) On Fig. 1.1, draw an arrow to indicate a possible initial direction of motion of the electron after the photon has been deflected.
- (ii) Calculate
1. the change in energy of the deflected photon.
 2. the speed of the electron after the photon has been deflected.
- (c) Explain why the magnitude of the final momentum of the electron is not equal to the change in magnitude of the momentum of the photon.
- (d) The angle θ through which the photon is deflected is given by the expression

$$\Delta\lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

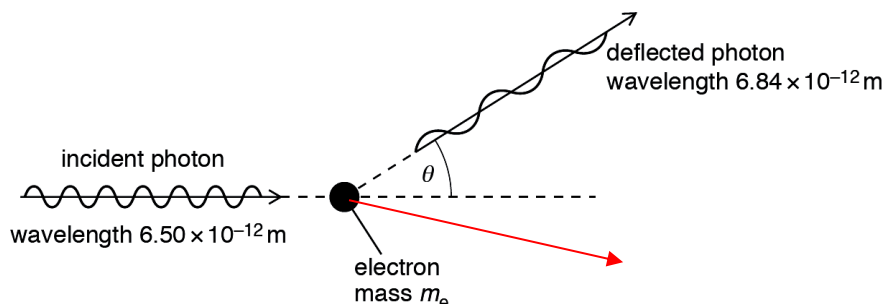
where $\Delta\lambda$ is the change in wavelength of the photon, h is the Planck constant and c is the speed of light in free space.

- (i) Calculate the angle θ .
- (ii) Use energy considerations to suggest why $\Delta\lambda$ must always be positive.

Solution:

- 1 (a) A photon is a quantum (i.e. a discrete amount) of energy of electromagnetic radiation, and its energy $E = hf$ where h is the Planck's constant and f the frequency of the radiation

(b) (i)



(ii) 1.

$$\begin{aligned}
 \text{loss in energy of photon} &= E_{\text{initial}} - E_{\text{final}} \\
 &= hc \left(\frac{1}{\lambda_{\text{initial}}} - \frac{1}{\lambda_{\text{final}}} \right) \\
 &= (6.63 \times 10^{-34}) (3.00 \times 10^8) \left(\frac{1}{6.50 \times 10^{-12}} - \frac{1}{6.84 \times 10^{-12}} \right) \\
 &= 1.52 \times 10^{-15} \text{ J}
 \end{aligned}$$

(ii) 2. gain in energy of electron = loss in energy of photon

$$\begin{aligned}
 \frac{1}{2} m v^2 &= 1.52 \times 10^{-15} \\
 v &= 5.78 \times 10^7 \text{ m s}^{-1}
 \end{aligned}$$

(c) momentum is a vector quantity

either must consider momentum in two directions

or direction changes so cannot just consider magnitude

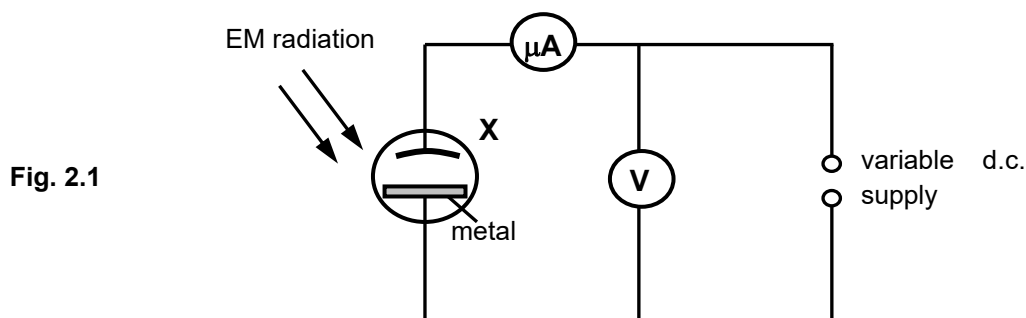
(d) (i)

$$\begin{aligned}
 6.84 \times 10^{-12} - 6.50 \times 10^{-12} &= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.0 \times 10^8} (1 - \cos \theta) \\
 \theta &= 30.7^\circ
 \end{aligned}$$

(ii) photon loses energy to the deflected electron

deflected photon has less energy, longer wavelength (so $\Delta\lambda$ always positive)

- 2 (a) (i) State what is meant by the photoelectric effect.
- (ii) Explain what a photon is.
- (b) In an experiment to demonstrate the photoelectric effect, a monochromatic light source of power 2.0 W and wavelength 450 nm is incident on a metal surface as shown in Fig 2.1.



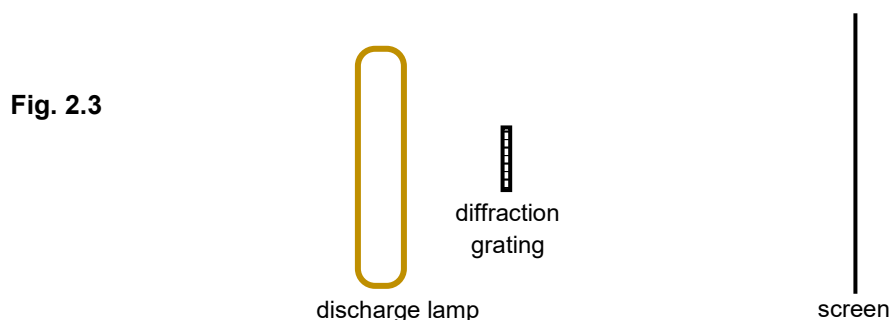
- (i) Calculate the number of photons incident on the metal surface per second.
- (ii) The current recorded on the micro-ammeter is $3.2 \mu\text{A}$. Calculate the ratio
- $$\frac{\text{number of electrons emitted per second}}{\text{number of photons incident per second}}$$
- (iii) By making reference to the work function energy of metal, suggest why, for frequencies of light slightly above the threshold frequency, the number of electrons emitted from the metal may be smaller than expected.
- (c) In another photoelectric experiment, the metal plate is illuminated by two lamps A and B. The wavelengths of light emitted by the two lamps are 400 nm and 600 nm respectively, as shown in Fig. 2.2.

Fig. 2.2

lamp	A	B
wavelength / nm	400	600

- (i) It is given that the current recorded when both lamps are switched on is $5.0 \mu\text{A}$. Assume the frequency of light from both lamps is above the threshold frequency.
Use your answer in (b)(ii) to show that the total number of photons incident on the metal surface in one second is 7.0×10^{18} .
- (ii) State an assumption made in the above calculation.
- (iii) When the intensity of lamp A is doubled, the current reading increases to $7.2 \mu\text{A}$. Calculate the total power dissipated by the two lamps before the intensity of lamp A is doubled.

- (d) Fig 2.3 shows an experimental set-up to produce the emission line spectrum of a hydrogen gas.



- (i) Explain how the line spectrum of hydrogen provides evidence for the existence of discrete electron energy levels in atoms.

Some of the lines of the emission spectrum of atomic hydrogen are shown in Fig 2.4, which is drawn to scale.

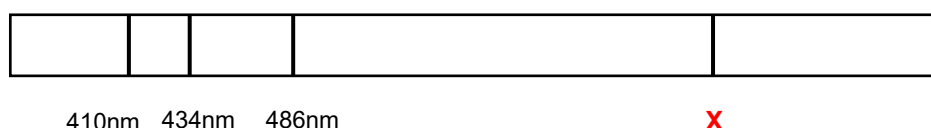
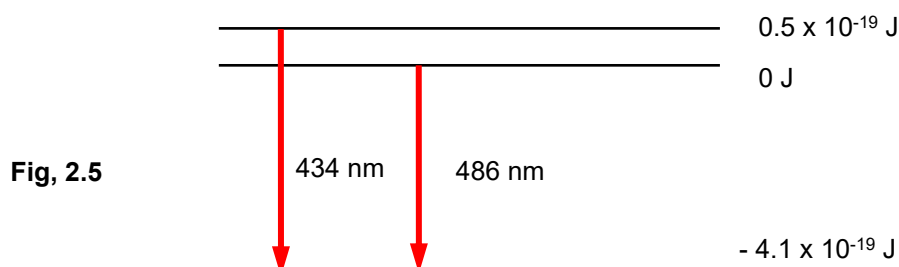


Fig. 2.4 (drawn to scale)

- (ii) Calculate the energy of the photon emitted for emission line labelled X.
- (iii) Suggest why the actual spacing of the lines in the emission spectrum is not proportional to the difference in the wavelengths of the photons emitted.
- (iv) Fig. 2.5 shows a partially completed diagram depicting the energy levels of a hydrogen atom. The energy changes corresponding to photon emissions of wavelength 486 nm and 434 nm are also shown.

Complete the energy level diagram based on the emission spectrum on **Fig 2.5**. Label with appropriate values.



- (v) Each accelerated electron in the discharge tube has a kinetic energy of 4.85×10^{-19} J. Suggest why the accelerated electrons do not cause excitation of the electrons of the cool hydrogen gas.

Solution

- 2 (a) (i) Photoelectric effect is the phenomenon of emission of electrons from a metal surface when EM radiation of sufficiently high frequency is radiated on it.
- (ii) A photon is a quantum of energy of electromagnetic radiation and is given by $E = hf$ (h and f must be defined)

(b) (i)

$$P = \frac{\text{total energy, } Nhf}{t}$$

$$\frac{N_{\text{ph}}}{t} = \frac{(2.0)(450 \times 10^{-9})}{(6.63 \times 10^{-34})(3.0 \times 10^8)}$$

$$= 4.5 \times 10^{18} \text{ s}^{-1}$$

(ii)

$$I = \frac{\text{total charge, } N_e e}{t}$$

$$\frac{N_e}{t} = \frac{3.2 \times 10^{-9}}{1.60 \times 10^{-19}}$$

$$= 2.0 \times 10^{13} \text{ s}^{-1}$$

$$\text{Ratio} = 4.44 \times 10^{-6}$$

- (iii) Since work function energy is the minimum energy for electron to be emitted from the metal surface, even more energy is required to bring other electrons to the surface before they escape.

Hence when frequency is slightly above threshold value, the photon will only have sufficient energy to allow those electrons at the metal surface to escape but not enough to allow other electrons below the surface to escape. Hence the number of emitted electrons will be smaller compared to those metal illuminated by light of frequency well above the threshold value.

(c) (i)

$$I = \frac{\text{total charge, } N_e e}{t}$$

$$\frac{N_e}{t} = \frac{5.0 \times 10^{-6}}{1.60 \times 10^{-19}} = 3.13 \times 10^{13}$$

$$\frac{N_{\text{ph}}}{t} = \frac{N_e / t}{4.44 \times 10^{-6}} = 7.0 \times 10^{18} \text{ s}^{-1}$$

- (ii) either the ratio of the rate of emission of electrons to the rate of photons incident on metal remains the same for different frequencies for light
or the frequency is well above the threshold value

(iii)

$$I = \frac{\text{total charge, } N_e e}{t}$$

$$\frac{N_{\text{ph},1} / t}{N_{\text{ph}} / t} = \frac{N_{e,1} / t}{N_e / t} = \frac{I_1}{I}$$

$$\frac{N_{\text{ph},1}}{t} = \frac{I_1}{I} \left(\frac{N_{\text{ph}}}{t} \right)$$

$$= \frac{7.2}{5.0} (7.0 \times 10^{18}) = 1.01 \times 10^{19} \text{ s}^{-1}$$

$$\text{after: } 2 \frac{N_{\text{ph, A}}}{t} + \frac{N_{\text{ph, B}}}{t} = 1.01 \times 10^{19}$$

$$\text{before: } \frac{N_{\text{ph, A}}}{t} + \frac{N_{\text{ph, B}}}{t} = 7.0 \times 10^{18}$$

Solving simultaneously,

$$\frac{N_{\text{ph, A}}}{t} = 3.1 \times 10^{18} \text{ s}^{-1}$$

$$\frac{N_{\text{ph, B}}}{t} = 3.9 \times 10^{18} \text{ s}^{-1}$$

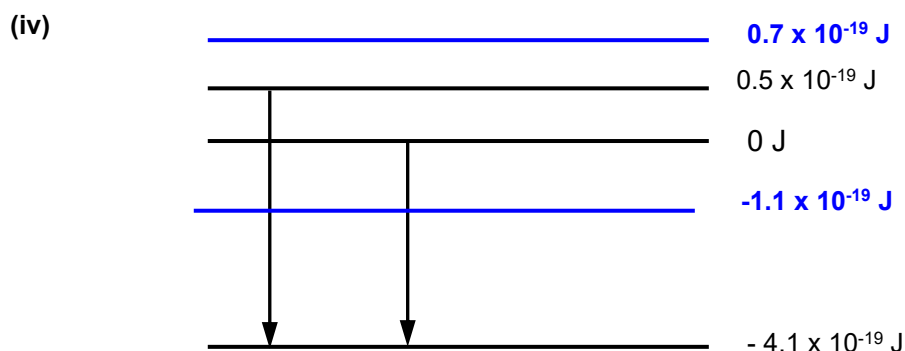
$$\begin{aligned} P &= \frac{N_{\text{ph, A}}}{t} \frac{hc}{400 \times 10^{-9}} + \frac{N_{\text{ph, B}}}{t} \frac{hc}{600 \times 10^{-9}} \\ &= 1.5 + 1.3 \\ &= 2.8 \text{ W} \end{aligned}$$

- (d) (i) When electrons get de-excited, they lose energy and photons are emitted.
Due to discrete energy levels of atom, only photons of specific energies are emitted. Energy equal to the difference of any two energy level of the atoms.
Hence only photons of specific frequencies/wavelengths are emitted as they have

- (ii) By measuring, wavelength = 656 nm

$$\text{Energy of photon } E = hf = h \frac{c}{\lambda} = 3.0 \times 10^{-19} \text{ J}$$

- (iii) Based on the diffraction grating formula, $d \sin \theta = n \lambda$. Hence the distance is not proportional to λ or $\Delta \lambda$ as the distance is not proportional to $\sin \theta$.



- (v) The lowest energy level shown in Fig 2.5 may not be the energy level at ground state.

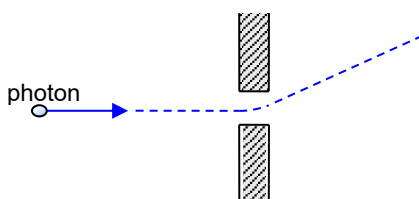
- 3 (a) Two beams of monochromatic light have similar intensities. The light in one beam has wavelength of 400 nm and the light in the other beam has wavelength of 700 nm. The two beams are incident separately on three different metal surfaces. The work function of each of these surfaces is shown in Fig. 3.1.

Fig. 3.1

metal	magnesium	potassium	caesium
work function / eV	3.68	2.26	1.88

- (i) Explain what is meant by the *work function* of the surface.
- (ii) Using appropriate calculation, explain which combination/s, if any, of monochromatic light and metal surface would give rise to photo-electric emission.
- (b) In another experiment, light is passed through a small opening. A photon changes direction as shown in Fig. 3.2 below.

Fig. 3.2



- (i) Using classical Physics, explain why the photon could not possibly have changed direction.
- (ii) Suggest how this phenomenon could be explained using Heisenberg's Uncertainty Principle.

Solution

- 3 (a) (i) The work function Φ of the surface is defined as minimum amount of the work necessary to remove a free electron from the surface of the material.

- (ii) By photoelectric equation, $\frac{hc}{\lambda} = \Phi + K_{\max}$

For any photo-electric emission to occur, $\frac{hc}{\lambda} \geq \Phi$

$$\text{For wavelength of 400 nm, } \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} (3.0 \times 10^8)}{400 \times 10^{-9}} = 4.97 \times 10^{-19} \text{ J} = 3.11 \text{ eV}$$

$$\text{For wavelength of 700 nm, } \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} (3.0 \times 10^8)}{700 \times 10^{-9}} = 2.84 \times 10^{-19} \text{ J} = 1.78 \text{ eV}$$

Since $3.11 \text{ eV} > 2.26 \text{ eV}$ and $3.11 \text{ eV} > 1.88 \text{ eV}$,
and $1.78 \text{ eV} < 1.88 \text{ eV} < 2.26 \text{ eV} < 3.68 \text{ eV}$

Therefore, the possible combinations of light and metal surface are:

- Wavelength of 400 nm and potassium
- Wavelength of 400 nm and caesium

- (b) (i) The photon does not experience any force acting on it. Hence, by Newton's 1st law, the photon should continue its state of motion without any change in direction.

Or

By conservation of momentum, the momentum of photon should remain the same since there is no resultant external force acting on the photon. Hence, photon should be undeflected.

Or

By Newton's 2nd law, since there is no resultant force acting on the photon, there is no change in photon's momentum. Hence, photon should be undeflected.

- (ii) By Heisenberg's Uncertainty Principle, as the photon passes through the small opening, the uncertainty of its possible position is reduced at the opening. Hence, the uncertainty of momentum of the photon will increase. This leads to the change in direction of the photon.

- 4 (a) Explain what is meant by a photon. State one way by which it is produced.
- (b) The wavelength λ_n of spectral lines produced by a hydrogen lamp due to electron transitions to energy level $m = 2$, i.e. the Balmer series, is given by this expression:

$$\frac{1}{\lambda_n} = R \left(\frac{1}{m^2} - \frac{1}{n^2} \right),$$

where R is a constant of value $1.097 \times 10^7 \text{ m}^{-1}$ and n is the quantum number of the upper energy level involved in the transition, i.e. $n = 3, 4, 5, \dots$ etc.

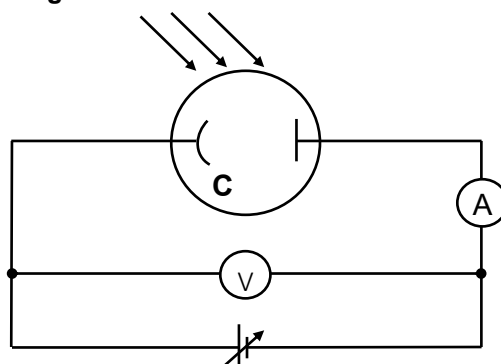
- (i) Complete **Fig. 4.1** for different values of n .

Fig. 4.1

n	λ_n / nm
4	
5	
6	410
7	397

- (ii) Show that the maximum energy of an emitted photon associated with the Balmer series is 3.41 eV.
- (iii) Using the concept of electron energy levels in atoms, describe and explain the spectrum which is emitted.
- (iv) Explain why it is not possible to see the entire spectrum of the Balmer series.
- (c) In a photoelectric experiment, light of wavelength 365 nm is incident on a metal electrode E of work function 2.60 eV as shown in **Fig. 4.2**.

Fig 4.2



- (i) State what is meant by the *photoelectric effect*.
- (ii) Describe how the photoelectric effect provides evidence for the particulate nature of electromagnetic radiation when it is interacting with matter.
- (iii) Calculate the maximum kinetic energy in eV of the emitted photoelectrons.

Solution

- 4 (a) A photon is a quantum of electromagnetic energy, and its energy is $E = hf$ where h is the Planck's constant and f the frequency of the radiation. It is emitted when an excited atom de-excites to a lower energy level.

Allow other methods, such as acceleration of charged particles, etc.

- (b) (i) $n = 4$, 486 nm
 $n = 5$, 434 nm

- (ii) Maximum energy of an emitted photon is due to transition from $n = \infty$.

$$\frac{1}{\lambda_4} = R \left(\frac{1}{2^2} - \frac{1}{\infty} \right)$$

$$\frac{1}{\lambda_{\infty}} = (1.097 \times 10^7) \left(\frac{1}{4} \right) \Rightarrow \lambda_{\infty} = 3.646 \times 10^{-7} = 364.6 \text{ nm}$$

$$E = \frac{hc}{\lambda_{\infty}}$$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{3.646 \times 10^{-7}} \text{ or } (6.63 \times 10^{-34})(3 \times 10^8)(1.097 \times 10^7) \left(\frac{1}{4} \right)$$

$$= 5.455 \times 10^{-19} \text{ J}$$

$$= \frac{5.455 \times 10^{-19}}{1.60 \times 10^{-19}} = 3.41 \text{ eV}$$

- (ii)
 - An isolated atom has discrete energy levels. Due to the transition of excited electrons from higher energy levels to that of level 2, photons of specific wavelengths are emitted.
 - The spectrum consists of coloured lines on a dark background.
 - At higher quantum numbers, the energy levels converge into a continuum (ie they are very close to one another), and hence the wavelengths of photons emitted due to these energy transitions result in closer spacing of the spectral lines as wavelength decreases.

OR

- The spectrum consists of coloured lines on a dark background.
 - At low quantum numbers, the energy difference between discrete energy levels (e.g. $n = 3$, $n = 4$) is large, and hence the emission lines are distinct.
 - At higher quantum numbers, the energy levels converge into a continuum, and hence the wavelengths of photons emitted are very close to each other, resulting in an emission band.
- (iv) It is because at higher quantum numbers ($n > 7$), the wavelengths of the photons emitted are in the ultraviolet range or not in the visible range.
- (c) (i) The photoelectric effect is a phenomenon in which electrons are emitted from a metal surface when light of sufficiently high frequency is incident on the surface.
- (ii)
 - There exists a minimum frequency of the incident electromagnetic (EM) radiation (threshold frequency) before photoemission take place, unlike wave theory which predicts that intensity determines the amount of energy absorbed by the electrons.
 - There is an (almost) immediate emission of photoelectrons from the metal surface even with low intensity EM radiation, as long as it is above the threshold frequency, whereas wave theory predicts that electrons would absorb energy over a period of time before it gains enough energy to escape.

- The maximum kinetic energy of photoelectrons is dependent on the frequency of the incident EM radiation and not on the intensity, unlike wave theory which predicts that higher intensity radiation (i.e. larger amount of energy striking the metal surface) produces electrons with higher the kinetic energy.
- These 3 observations of photoelectric effect provide evidence for the particulate nature of EM radiation which classical wave theory cannot explain.

$$\begin{aligned}
 \text{(iii)} \quad KE_{\max} &= hf - hf_0 \\
 &= \frac{hc}{\lambda e} - \Phi \\
 &= \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{(365 \times 10^{-9})(1.60 \times 10^{-19})} - 2.60 \\
 &= 3.41 - 2.60 \\
 &= 0.81 \text{ eV}
 \end{aligned}$$

Jewett, J. W., Serway, R. A., & Peroomian, V. (2014). *Physics for scientists and engineers with modern physics* (p. 1289). Brooks/Cole. / Q2

5 A quantum particle of mass m_1 is in a square well with infinitely high walls and length 3 nm.

Rank the situations (a) through (e) according to the particle's energy from highest to lowest, noting any cases of equality.

- (a) The particle of mass m_1 is in the ground state of the well.
- (b) The same particle is in the $n = 2$ excited state of the same well.
- (c) A particle with mass $2m_1$ is in the ground state of the same well.
- (d) A particle of mass m_1 in the ground state of the same well, and the uncertainty principle has become inoperative; that is, Planck's constant has been reduced to zero.
- (e) A particle of mass m_1 is in the ground state of a well of length 6 nm.

Solution

$$\begin{aligned}
 \text{5 (a)} \quad E_1 &= \frac{n^2 h^2}{8mL^2} \\
 &= \frac{(1)^2 h^2}{8m_1 (3 \text{ nm})^2} \\
 &= \left(\frac{1}{9 \text{ nm}^2} \right) \frac{h^2}{8m_1} \\
 \text{(b)} \quad E_2 &= \frac{n^2 h^2}{8mL^2} \\
 &= \frac{(2)^2 h^2}{8m_1 (3 \text{ nm})^2} \\
 &= \left(\frac{4}{9 \text{ nm}^2} \right) \frac{h^2}{8m_1} \\
 \text{(c)} \quad E_1 &= \frac{n^2 h^2}{8mL^2} \\
 &= \frac{(1)^2 h^2}{8(2m_1)(3 \text{ nm})^2} \\
 &= \left(\frac{1}{18 \text{ nm}^2} \right) \frac{h^2}{8m_1} \\
 \text{(d)} \quad E_1 &= \frac{n^2 h^2}{8mL^2} \\
 &= \frac{(1)^2 (0)^2}{8m_1 (3 \text{ nm})^2} \\
 &= 0 \\
 \text{(e)} \quad E_1 &= \frac{n^2 h^2}{8mL^2} \\
 &= \frac{(1)^2 h^2}{8m_1 (6 \text{ nm})^2} \\
 &= \left(\frac{1}{36 \text{ nm}^2} \right) \frac{h^2}{8m_1}
 \end{aligned}$$

In order of the particle's energy from the highest to the lowest:
(b), (a), (c), (e) and (d).

Annex A: Classical vs Quantum Predictions of the Photoelectric Effect

Presence of Threshold Frequency

Predictions		Experimental Observations
Classical (Wave)	Quantum (Particle)	
According to classical theory, the energy transferred to the surface depends only on the intensity. If high-frequency light of intensity I is able to transfer enough energy to overcome the work function of the metal and produce photoelectrons, any other frequency of light with the same intensity I should transfer the same amount of energy, and also produce photoelectrons. This means that <u>the same intensity of light should always produce photoelectrons, regardless of the frequency of the incident light.</u>	According to quantum theory, each photon of light consists of energy equal to hf, and each photon either transfers all of its energy to the electron or nothing at all. For photoelectrons to be emitted, each individual photon will need to transfer enough energy to overcome the work function of the metal. This means that <u>even if the incident intensity is very large, photoelectrons will not be produced if the frequency of the light is too low, because each individual photon will have energy smaller than the work function of the metal.</u>	When light below a specific frequency is incident on the surface, no photocurrent is observed, even if the incident intensity is very large. When light above that frequency is incident on the surface of the metal emitter, a photocurrent is observed, showing that photoelectrons are being emitted. This is true even if the incident intensity is smaller than the intensity of the low-frequency light. This shows that <u>there is a threshold frequency</u> – a minimum frequency of incident light that must be used for any photocurrent to be observed, regardless of intensity.

Variation of Maximum KE of Photoelectrons with Frequency

Predictions		Experimental Observations
Classical (Wave)	Quantum (Particle)	
According to classical theory, the energy transferred to the surface depends only on the intensity. The same intensity of light should transfer the same energy to the surface (and hence to the emitted photoelectrons), regardless of frequency. This means that <u>for the same intensity of light, the maximum KE of the photoelectrons should be the same, regardless of frequency.</u>	According to quantum theory, each photon of light consists of energy equal to hf, and each photon either transfers all of its energy to the electron or nothing at all. Light of higher frequency will have photons of higher energy, even if the overall incident intensity is the same. Thus, the energy transferred to each electron will be larger. This means that <u>for the same intensity of light, a higher frequency should result in a higher maximum KE of photoelectrons.</u>	For the same intensity of incident light, the higher-frequency light has a larger stopping potential, which means that the photoelectrons have a higher maximum KE. Thus, the <u>maximum KE of photoelectrons depends on the frequency of the incident light.</u>

Variation of Photocurrent and Maximum KE of Photoelectrons with Intensity

Predictions		Experimental Observations
Classical (Wave)	Quantum (Particle)	
According to classical theory, the energy transferred to the surface depends only on the intensity. A higher intensity of light should transfer more energy to the surface (and hence to the emitted photoelectrons), regardless of frequency. Thus, more photoelectrons should be emitted, and each photoelectron could have absorbed more energy. This means that <u>for the same frequency of light, both the photocurrent and the maximum KE of the photoelectrons should be larger if the incident intensity is higher.</u>	According to quantum theory, each photon of light consists of energy equal to hf, and each photon either transfers all of its energy to the electron or nothing at all. A larger intensity of light of the same frequency means that each photon has the same energy, but there are more incident photons, which should result in more photoelectrons. This means that <u>for the same frequency of light, the maximum KE of the photoelectrons should be the same, but the photocurrent should be larger if the incident intensity is higher.</u>	Source: University Physics by Young & Freedman Fig. 1.2 – Graph of photocurrent i against the potential difference V_{AC} The graph above shows that for the same frequency of incident light, an increase in the intensity causes the photocurrent to increase, but the stopping potential (and hence the maximum KE of the photoelectrons) remains constant. Thus, the <u>photocurrent depends on the intensity</u>, but the <u>maximum KE of photoelectrons is independent of the intensity of the incident light.</u>

Time Taken to Emit Photoelectrons

Predictions		Experimental Observations
Classical (Wave)	Quantum (Particle)	
According to classical theory, the energy transferred to the surface depends only on the intensity. If the intensity of the incident light is very low, each electron should take some time to accumulate enough energy from the incident light before it can escape. For the first electron to escape, this will be when the accumulated energy just exceeds the work function. This means that <u>when the incident intensity is low, there should be a time lag before the first emission of photoelectrons.</u>	According to quantum theory, each photon of light consists of energy equal to hf, and each photon either transfers all of its energy to the electron or nothing at all. If the intensity of the incident light is very low, there are much fewer incident photons, but each photon still has the same energy. The emission of photoelectrons is a result of one electron absorbing one photon, with the energy transfer taking place in an instant. As soon as photons of sufficient energy are incident on the surface, photoelectrons can be emitted. This means that <u>when the incident intensity is low, the first emission of photoelectrons should still take place almost instantly.</u>	As long as the frequency of the incident light is above the threshold frequency, there is no measurable time delay – a non-zero photocurrent is measured within $\sim 10^{-9}$ s from when the light is incident on the surface of the emitting metal. This is true no matter how low the incident intensity is.