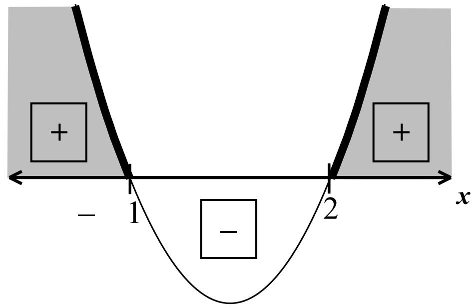
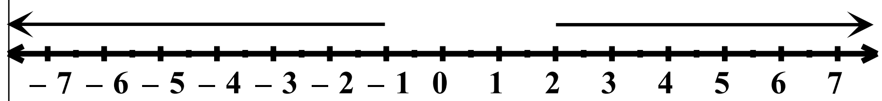
SEMBAWANG SECONDARY SCHOOL
SEC 4NA PRELIMINARY EXAMINATION 2025
ADDITIONAL MATHEMATICS PAPER 2 MARKING SCHEME

Qn	Solution
1	$8a^3+125b^3$ $=(2a+5b)\left[(2a)^2-(2a)(5b)+(5b)^2\right]$ $=(2a+5b)(4a^2-10ab+25b^2)$
2	$3x^2-5x+1=x-7$ $3x^2-6x+8=0$ b^2-4ac $=(-6)^2-4(3)(8)$ $=-60<0$ Since the discriminant is less than zero, the line and the curve does not meet
3	$\cos(y-30°)=2\sin(y-60°)$ $\cos y\cos 30°+\sin y\sin 30°=2(\sin y\cos 60°-\cos y\sin 60°)$ $\frac{\sqrt{3}}{2}\cos y+\frac{1}{2}\sin y=2\left(\frac{1}{2}\sin y-\frac{\sqrt{3}}{2}\cos y\right)$ $\frac{\sqrt{3}}{2}\cos y+\frac{1}{2}\sin y=\sin y-\sqrt{3}\cos y$ $\frac{3\sqrt{3}}{2}\cos y=\frac{1}{2}\sin y$ $\frac{3\sqrt{3}}{2}\div\frac{1}{2}=\frac{\sin y}{\cos y}$ $3\sqrt{3}=\tan y$ $\tan y=3\sqrt{3}$
4(a)	$x^2-2kx+k+2=0$ $[a=1, b=-2k, c=k+2]$ Real roots: $b^2-4ac\geq 0$ $(-2k)^2-4(1)(k+2)\geq 0$ $4k^2-4k-8\geq 0$ $(\div 4)$ $k^2-k-2\geq 0$ (Shown)

4(b)	$k^2 - k - 2 \geq 0$ $(k-2)(k+1) \geq 0$ [Leading to critical points: $k = 2$, $k = -1$]  From graph:  $\Rightarrow k \leq -1$ or $k \geq 2$

5(a)

Area of triangle XYZ

$=\frac{1}{2}\times(YZ)\times(XY)$

$\times$	$4\sqrt{3}$	$+1$
4	$16\sqrt{3}$	$+4$
$-\sqrt{3}$	$-4(3)$	$-\sqrt{3}$

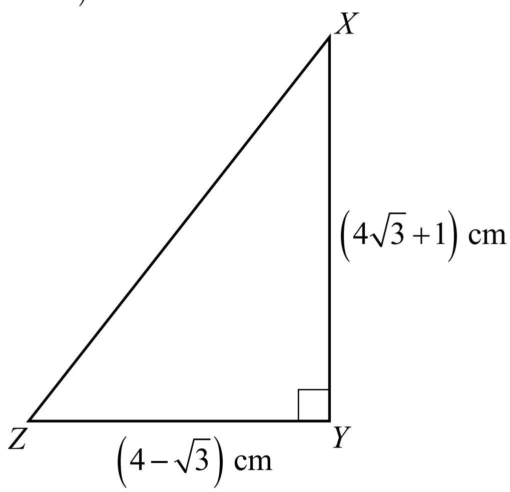
$= \frac{1}{2} \times \left(4-\sqrt{3}\right) \times \left(4\sqrt{3}+1\right)$

$= \frac{1}{2} \times \left(16\sqrt{3}+4-12-\sqrt{3}\right)$

$= \frac{1}{2} \times \left(15\sqrt{3}-8\right)$

$= \left(\frac{15}{2}\sqrt{3}-4\right)$ cm^2

$\left(a\sqrt{3}-b\right)$



5(b)

By Pythagoras' Theorem,

$(XZ)^2 = (YZ)^2 + (XY)^2$

$= \left(4-\sqrt{3}\right)^2 + \left(4\sqrt{3}+1\right)^2$

$=$

$\left[(4)^2 - 2(4)\left(\sqrt{3}\right) + \left(\sqrt{3}\right)^2\right] + \left[\left(4\sqrt{3}\right)^2 + 2\left(4\sqrt{3}\right)(1) + (1)^2\right]$

$= \left[16-8\sqrt{3}+3\right] + \left[(16)(3)+8\sqrt{3}+1\right]$

$= \left[19-8\sqrt{3}\right] + \left[48+8\sqrt{3}+1\right]$

$= 19+48+1-8\sqrt{3}+8\sqrt{3}$

$(XZ)^2 = 68$

$XZ = \sqrt{68} = \sqrt{4\times17} = \sqrt{4}\ \ \sqrt{17}$

$\therefore XZ = 2\sqrt{17}$ cm

6(a)	$\cos 60^{\circ} = \frac{QA}{x}$ $\frac{1}{2} = \frac{QA}{x}$ $QA = \frac{x}{2} = RB$ $2x + 2\left(\frac{x}{2}\right) + 2PS = 100$ $3x + 2PS = 100$ $PS = \frac{100-3x}{2}$
6(b)	$\sin 60^{\circ} = \frac{PA}{x}$ $\frac{\sqrt{3}}{2} = \frac{PA}{x}$ $PA = \frac{\sqrt{3}}{2}x$ $A = \frac{\sqrt{3}}{2}x\left(\frac{100-3x}{2}\right)$ $= \frac{\sqrt{3}}{4}\left(100x - 3x^2\right)$
6(c)	$A = \frac{\sqrt{3}}{4}\left(100x - 3x^2\right)$ $\frac{dA}{dx} = \frac{\sqrt{3}}{4}(100-6x)$ Let $\frac{dA}{dx} = \frac{\sqrt{3}}{4}(100-6x) = 0$ $100 - 6x = 0$ $6x = 100$ $x = \frac{50}{3} = 16\frac{2}{3}$ $\frac{d^2A}{dx^2} = -\frac{3\sqrt{3}}{2} < 0$ Therefore A is a maximum value.

7(a)	$\dfrac{dy}{dx}=3x^2+2hx+3$ .... B1 sub $x=-3$ and $\dfrac{dy}{dx}=0$ $3(-3)^2+2h(-3)+3=0$ ....M1 $27-6h+3=0$ $-6h=-30$ $h=5$ A1
7(b)	$y=x^3+5x^2+3x+k$ Sub $(-3,0)$ $(-3)^3+5(-3)^2+3(-3)+k=0$ $-27+45-9+k=0$ $k=-9$ B1
7(c)	Let $\dfrac{dy}{dx}=0$ $3x^2+10x+3 = 0$ .... M1 $(x+3)(3x+1)=0$ $x=-3\text{(given)}$ or $x=-\dfrac{1}{3}$ A1
7(d)	$\dfrac{dy}{dx}=3x^2+10x+3$ $\dfrac{d^2y}{dx^2}=6x+10$ .... B1√ When $x=-3, \dfrac{d^2y}{dx^2}=-8<0$. It is a maximum point. .... B1 When $x=-\dfrac{1}{3}, \dfrac{d^2y}{dx^2}=8>0$. It is a minimum point . ...B1 Accept the first derivative test using change in the gradients.
8(a)	$x^2+y^2-4x-3y-10=0$ $(x-2)^2-2^2+(y-1.5)^2-1.5^2-10=0$ .... M1 (completing of square) $(x-2)^2+(y-1.5)^2=16.25$ centre = (2,1.5) .... A1 radius = $\dfrac{\sqrt{65}}{2}$ or $\sqrt{16.25}$ units or 4.03 units................ A1

8(b)	Sub (6,1) $6^2+1^2-4(6)-3(1)-10$ $=0$ yes, (6,1) lies on the circle. B1
8(c)	$(2,1.5)\quad(4,-2)$ gradient $=\dfrac{1.5+2}{2-4}=-\dfrac{7}{4}$.............. M1 Sub into $y=\dfrac{4}{7}x+c$ $-2=\dfrac{4}{7}(4)+c$ .... M1 $c=-\dfrac{30}{7}$ equation of tangent is $y=\dfrac{4}{7}x-\dfrac{30}{7}$ A1
8(d)	centre $=(2,1.5)\quad A(4,-2)$ Let $B=(a,b)$ $\dfrac{a+4}{2}=2$ and $\dfrac{b-2}{2}=1.5$ .... M1(concept of midpoint) $a+4=4\qquad b-2=3$ $a=0\qquad b=5$ $B=(0,5)$ A1
9(a)	$6\sin t+10\cos t=R\sin(t+\alpha)$ $=R\sin t\cos\alpha+R\cos t\sin\alpha$ $R\cos\alpha=6$ and $R\sin\alpha=10$ $R=\sqrt{6^2+10^2}$ ....M1 $R=\sqrt{136}$ or $2\sqrt{34}$ $\tan\alpha=\dfrac{10}{6}$ .... M1 $\alpha=1.0303$ radians $6\sin t+10\cos t=\sqrt{136}\sin(t+1.03)$ A1
9(b)(i)	Since maximum value of $\sin(t+1.03)=1$, maximum value of $x=\sqrt{136}=11.661$, so it is not possible for the mass to reach a distance of 18 mm. …………………………… B1√

9(b)(ii)	Let $\sin(t+1.03)=1$ .... M1 $t+1.03=1.5707$ $t=0.541$ s (to 3sf) A1
9(c)	Let $\sqrt{136}\sin(t+1.03)=8$ .... M1√ $\sin(t+1.03)=0.68599$ $t+1.03=0.75596,\ 2.38563$ ....M1 (angles in 1st and 2nd quadrants) $t=-0.274$(rejected) , 1.36 s (to 3sf) A1
10(a)	$x^2-6x+11=-x+7$ $x^2-5x+4=0$ $(x-1)(x-4)=0$ $x=1$ or $x=4$ $y=6$ or $y=3$ $A(1,6)$ $B(4,3)$
10(b)	$\int_1^4 (x^2-6x+11)\mathrm{d}x$ $=\left[\frac{x^3}{3}-\frac{6x^2}{2}+11x\right]_1^4$ $=\frac{4^3}{3}-\frac{6(4)^2}{2}+11(4)-\frac{1}{3}+3-11$ $=9$ Area of trapezium $=\frac{1}{2}(6+3)\times 3$ $=13.5$ Shaded area: $13.5-9$ $=4.5$ units^2