NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME					
CT CLASS	2	4		Centre Number/ Index Number	/

MATHEMATICS 9758/01

Paper 1

2 September 2025

3 hours

Additional Materials: Printed Answer Booklet

List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

1 Curve C has equation $y=\dfrac{px^2+qx+r}{x-1}$, where p, q and r are real numbers. Given that C has a turning point at $(3,\ 7)$ and an oblique asymptote parallel to the line $y=\dfrac{3}{2}x+\sqrt{5}$, find p, q and r. [4]

2 An ornament consists of two identical solid right circular cones whose flat bases are separated by a solid sphere, such that the sphere is in contact with the two bases. The axis passes through the vertex of each cone and the centre of the sphere. It is given that the distance between the vertices of the cones is $2l$, and the radii of the bases of the cones are $\sqrt{\dfrac{3}{2}}\,l$, where l is a constant. Find the radius of the sphere in terms of l if the total volume of the ornament is to be a minimum.

[The volume of a sphere with radius r is given by $V=\dfrac{4}{3}\pi r^3$.] [6]

3 **(a)** A triangle ABC is such that $AB^2+BC^2=AC^2$.

By considering $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ and using the fact that $|\mathbf{v}|^2=\mathbf{v}\bullet\mathbf{v}$ for any vector $\mathbf{v}$, prove that $\angle ABC$ is a right-angle. [4]

(b) In a triangle ABC, the point D divides AC in the ratio $\lambda:1-\lambda$, where $0<\lambda<1$.

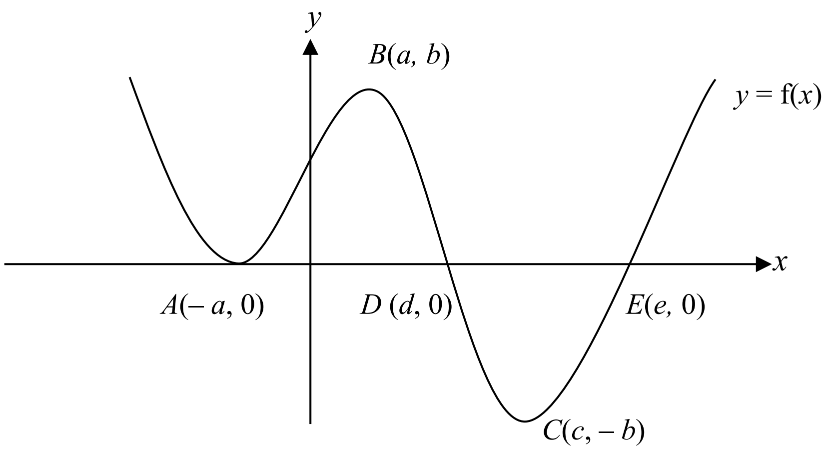
Let the position vectors of A, B, C and D be denoted by $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ respectively.

Show that the area of triangle ABD is given by

$$k\left|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}\right|$$

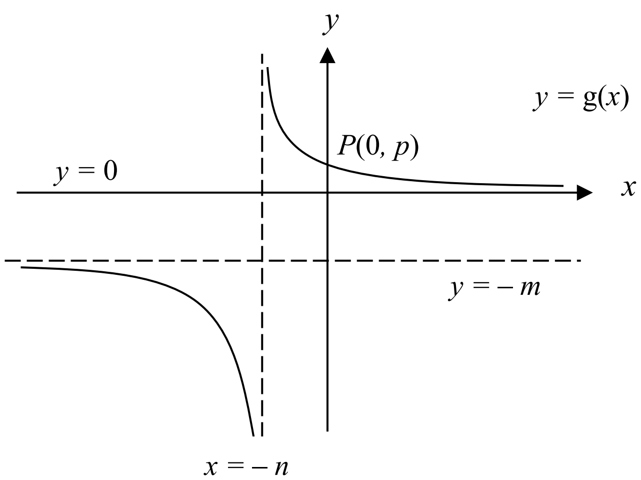
where k is to be determined in terms of λ. [4]

4 **(a)** The graph of $y = \mathrm{f}(x)$ has turning points at $A(-a, 0)$, $B(a, b)$ and $C(c, -b)$ and intersects the x-axis at the points $D(d, 0)$ and $E(e, 0)$. The graph of $y = \mathrm{f}(x)$ is as shown in the diagram below.



Sketch the graph of $y = \mathrm{f}(a - x)$, labelling the coordinates of the corresponding points of *A, B, C, D,* and *E* clearly. [3]

(b) The graph of $y = \mathrm{g}(x)$ has asymptotes $y = 0$, $y = -m$ and $x = -n$, and intersects the y-axis at $P(0, p)$, where $p < m$. The graph of $y = \mathrm{g}(x)$ is as shown in the diagram below.



Sketch on separate diagrams the graphs of

(i) $y = |\mathrm{g}(x)|$ and [3]

(ii) $y = \mathrm{g}(|x|)$, [2]

labelling clearly the asymptote(s) and coordinates of P on each graph.

5 The nth term of a series G is given by $g_n = 3n^2 - 17n + 17$, where $n \geq 1$.

(a) Given that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$, find $\sum_{r=m+1}^{n} g_r$. (You need not simplify your answer.) [4]

The nth term of a series H is given by $h_n = 7(5^{n-3}) + 10$, where $n \geq 1$.

(b) State the smallest number that can be found in both series H and G. [1]

The sum of the first n terms of a series J is given by $\frac{b}{2}(3^n - 1)$, where b is a positive odd integer.

(c) By finding the nth term of series J, explain why each of the terms in series J is a term of the arithmetic progression with first term 1 and common difference 2. [3]

6 The points P, Q and R representing the complex numbers p, q and r on an Argand diagram are such that $\arg(p) = \alpha$ and $\arg(q) = \beta$, where $0 < \alpha < \beta < \frac{\pi}{2}$, $\beta > 2\alpha$ and $r = p + q$.

(a) If $|p| = |q|$, describe the shape of the quadrilateral of $OPRQ$. Hence find $\arg(r)$ in terms of α and β. [3]

(b) The point Q', representing the complex number q', is the reflection of the point Q in OP. State the angle POQ'. [1]

By leaving your answers in terms of α, β and $|q|$ where applicable, hence, or otherwise,

(i) find the argument of the complex number q', [1]

(ii) find the real and imaginary parts of q' and write down q' in $a + \mathrm{i}b$ form. [3]

7 **(a)** Use the substitution $x = \sin^2 \theta$, where $0 \leq \theta \leq \frac{\pi}{2}$, to find $\int_0^{\frac{1}{2}} \sqrt{\frac{16x}{1-x}}\, \mathrm{d}x$ exactly. [4]

(b) Find $\int_a^b \frac{|x-1|}{\sqrt{\frac{1}{2}x^2 - x + 1}}\, \mathrm{d}x$ in terms of a and b, where $a < 1 < b$. [4]

8 A sequence is such that $u_{n+1} = u_n + 1.5$, for $n = 1, 2, 3, \ldots$.

(a) Given that $u_{50} = 99u_1$, show that $u_1 = 0.75$. [2]

(b) Find the least value of n such that $u_2 + u_4 + u_6 + \ldots + u_{2n}$ is more than 2025. [2]

(c) It is given that u_{25}, u_k and u_5 are the first three terms of a geometric progression.

(i) Find k. [2]

(ii) Find the sum of the first 13 terms of the geometric series, giving your answer to 3 decimal places. [2]

9 The function f is defined by

$$\mathrm{f}(x)=\frac{x+1}{2x-3} \quad \text{for } x\in\mathbb{R},\ x\neq k.$$

(a) State the value of k and explain why this value has to be excluded from the domain of f. [2]

(b) Determine, with a reason, if f^2 exists. [2]

(c) Find $\mathrm{f}^{-1}(x)$. [2]

(d) Hence find $\mathrm{h}(x)$ for which $\mathrm{fh}(x)=\dfrac{3x-7}{2x-1}, x\neq\dfrac{1}{2}$. [3]

10 In the question you may use expansions from the List of Formulae (MF27).

(a) Find the Maclaurin expansion of $\cos(t^3)$ in ascending powers of t, up to and including the term in t^{12}.

Hence, find the Maclaurin series of $\displaystyle\int_0^x t^2\cos(t^3)\,\mathrm{d}t$ up to and including the term in x^{15} given that x is small. [4]

(b) Use your expansion from part **(a)** to find an approximate value for $\displaystyle\int_0^{0.1} t^2\cos(t^3)\,\mathrm{d}t$, correct to 5 decimal places. [1]

(c) Find $\displaystyle\int_0^x t^2\cos(t^3)\,\mathrm{d}t$ in terms of x. Hence evaluate $\displaystyle\int_0^{0.1} t^2\cos(t^3)\,\mathrm{d}t$, correct to 5 decimal places. [3]

(d) Comparing your answers to parts **(b)** and **(c)**, and with reference to the value of x, comment on the accuracy of your approximations. [1]

11 The curve C has equation $x^2-y^2=16$, where $y\geq 0$. The line L has equation $y=-\dfrac{1}{2}x+\dfrac{11}{2}$.

(a) Find the area enclosed by C, L and the line $x=8$. [3]

(b) For $x>0$, the region bounded by C, L and the x-axis is rotated about the x-axis through 2π radians. Find the exact volume generated. [4]

(c) The region bounded by C, the line $x=5$ and the x-axis is rotated about the y-axis through 2π radians. Find the exact volume generated. [4]

12 Scientists are studying the growth of a newly discovered biomass that entered Earth's atmosphere together with an asteroid. A sample of 100 grams of the biomass was collected initially. It was found that on the next day, the sample grew to 110 grams. The scientists observed that the rate of growth of biomass was proportional to the amount of biomass present. The amount of biomass at time t days is denoted by B grams.

(a) Write down a differential equation relating B and t. [1]

(b) Solve the differential equation obtained in part **(a)**, expressing B in terms of t. [4]

(c) Sketch the graph of B against t, and explain what would happen if the biomass was left on its own. [2]

To limit the growth of the biomass, the scientist collected a second sample of 100 grams of the biomass and introduced a nutrient depletion serum to the sample the moment it was collected. The rate of growth of the biomass for this second sample is modelled by the differential equation

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{mB}{3 + 2t^2}$$

where m is a constant.

After 1 day, the second sample grew to 101 grams.

(d) For this second sample, find B in terms of t. [5]

(e) Determine the amount of the second biomass sample after a long time, leaving your answer to 5 decimal places. [1]

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