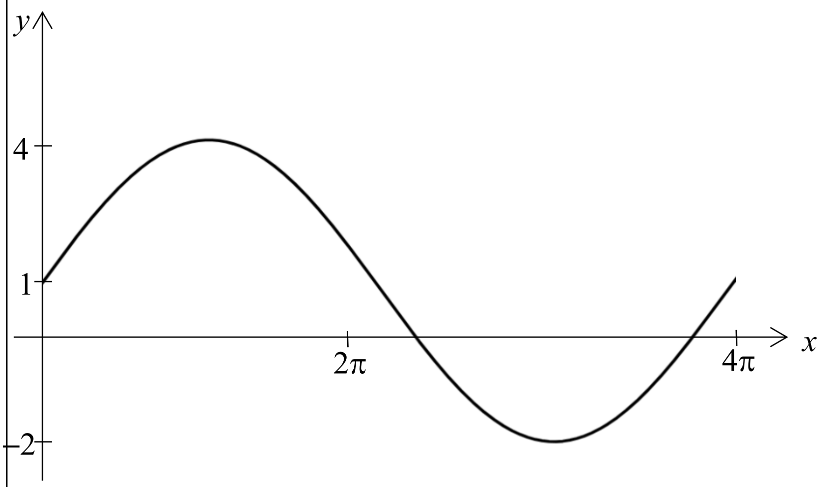
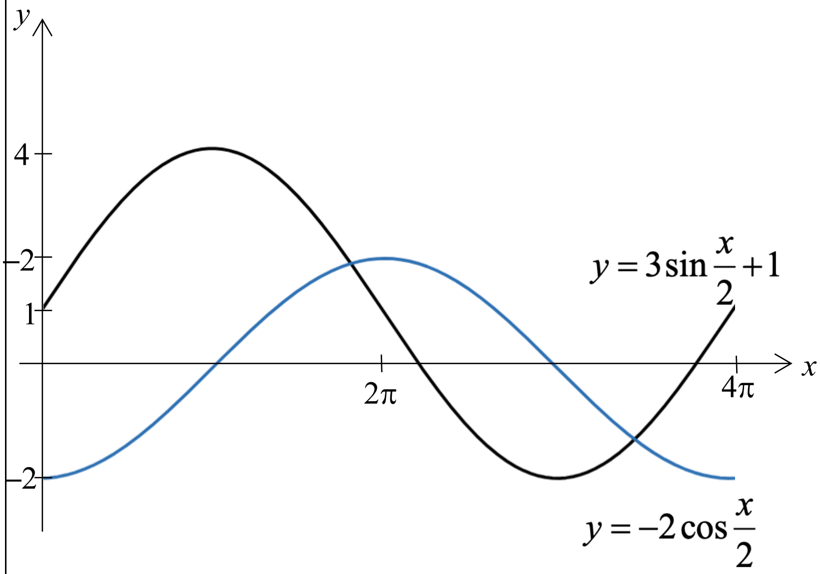
2025 A Math Prelim Paper 1 Solutions

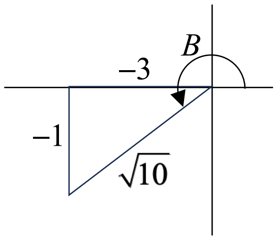
1. (a)	$V=\frac{1}{3}\pi r^2\left(\frac{3}{4}r\right)$ $=\frac{\pi}{4}r^3$
(b)	$\frac{\mathrm{d}v}{\mathrm{d}r}=\frac{3\pi}{4}r^2$ $\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}v}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$ $16=\frac{3\pi}{4}(8)^2\times\frac{\mathrm{d}r}{\mathrm{d}t}$ $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{1}{3\pi}=0.106$ cm/s
2.	$y=1-2x\ \ -----(1)$ $x^2-xy-y^2+5=0\ -----(2)$ Sub (1) into (2) $x^2-x(1-2x)-(1-2x)^2+5=0$ $x^2-x+2x^2-1+4x-4x^2+5=0$ $-x^2+3x+4=0$ $x^2-3x-4=0$ $(x-4)(x+1)=0$ $x=4$ or -1 $y=-7$ or 3 Length = $\sqrt{(4+1)^2+(-7-3)^2}$ $=\sqrt{125}$ $=5\sqrt{5}$ units

3(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(2x-3)e^{2x} = 2e^{2x} + (2x-3)2e^{2x}$ $= 2e^{2x}(1+2x-3)$ $= 4e^{2x}(x-1)$
(b)	$\int 4e^{2x}(x-1)\,\mathrm{d}x = (2x-3)e^{2x} + c$ $\int 4xe^{2x}\,\mathrm{d}x - \int 4e^{2x}\,\mathrm{d}x = (2x-3)e^{2x} + c$ $\int 4xe^{2x}\,\mathrm{d}x = (2x-3)e^{2x} + \int 4e^{2x}\,\mathrm{d}x + c$ $\int 4xe^{2x}\,\mathrm{d}x = (2x-3)e^{2x} + 2e^{2x} + c$ $\int 4xe^{2x}\,\mathrm{d}x = (2x-1)e^{2x} + c$ $\int_0^1 4xe^{2x}\,\mathrm{d}x = \left[(2x-1)e^{2x}\right]_0^1$ $= e^2 - (-1)$ $= e^2 + 1$
4(a)	$\angle RSU = \angle RUS$ $(RS = RU)$ $= \angle URQ$ (alt seg thm) $\angle URQ = \angle RUQ$ $\therefore \Delta RQU$ is isosceles
(b)	$\angle RQS = 2\angle QRU$ (ext $\angle$ of a triangle, ΔRQU is isosceles) $\angle RQS = \angle SPT$ (angles in the same seg) $\angle QRU = \angle QPT$ (alt seg theorem) $\therefore \angle SPT = 2\angle QPT$

5(a)	$2x^2-8x+3=1-4x$ $2x^2-4x+2=0$ $2(x-1)^2=0$ $x=1$ Since there is only one intersection point between the curve and the line, the line is a tangent to the curve. Or Discriminant = $(-4)^2-4(2)(2)=0$ $\therefore$ line is a tangent to the curve Coordinates = $(1, -3)$
(b)	$(m+1)x^2-8x+3m>5$ $(m+1)x^2-8x+3m-5>0$ $(-8)^2-4(m+1)(3m-5)<0$ and $m+1>0$ $64-12m^2+8m+20<0$ $\qquad$ $m>-1$ $12m^2-8m-84>0$ $3m^2-2m-21>0$ $(3m+7)(m-3)>0$ $m<-\frac{7}{3}$ or $m>3$ $\therefore$ $m>3$
6(a)	$$\begin{array}{r} x \quad -2 \\ 2x^2+1 \overline{\smash{\big)} 2x^3-4x^2+x-2} \\ \underline{-(2x^3 \qquad +x)} \\ -4x^2 \qquad -2 \\ \underline{-(-4x^2 \qquad -2)} \\ 0 \end{array}$$ $\therefore 2x^2+1$ is a factor since remainder = 0
(b)	$\frac{11x-5x^2-11}{(x-2)(2x^2+1)}=\frac{A}{x-2}+\frac{Bx+C}{2x^2+1}$ $11x-5x^2-11=A(2x^2+1)+(Bx+C)(x-2)$ Sub x = 2, $-9=9A \Rightarrow A=-1$ Compare x^2, $-5=2A+B \Rightarrow B=-3$ Compare constants, $-11=A-2C \Rightarrow C=5$

	$\therefore \dfrac{11x-5x^2-11}{(x-2)(2x^2+1)} = -\dfrac{1}{x-2} + \dfrac{5-3x}{2x^2+1}$
7(a)	Amplitude =3 Period = 4π or 720°
(b)	
(c)	 $3\sin\dfrac{x}{2} + 1 = -2\cos\dfrac{x}{2}$ Sketch $y = -2\cos\dfrac{x}{2}$ Number of solutions = 2

8(a)	Centre of $C_1 = (2, y)$ Grad of normal = $\frac{3}{4}$ $\frac{y-6}{2-6} = \frac{3}{4}$ $y = \frac{3}{4}(-4) + 6$ $= 3$ $\therefore$ Centre of $C_1 = (2, 3)$ Radius = 5 units Equation of C_1: $(x-2)^2 + (y-3)^2 = 25$
(b)	Let the centre $C_2 = (x, y)$ $\left(\frac{x+2}{2}, \frac{y+3}{2}\right) = (6, 6)$ $x = 10,\ y = 9$ Equation of C_2: $(x-10)^2 + (y-9)^2 = 25$
9(a)	$\sin 105^\circ = \sin(45^\circ + 60^\circ)$ $= \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ$ $= \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{1+\sqrt{3}}{2\sqrt{2}}$ $= \frac{1}{4}\left(\sqrt{2} + \sqrt{6}\right)$
(b)	$\frac{\cos(A+B)}{\cos(A-B)} = \frac{2}{7}$ $\frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B + \sin A \sin B} = \frac{2}{7}$ $7\cos A \cos B - 7\sin A \sin B = 2\cos A \cos B + 2\sin A \sin B$ $\frac{5\cos A \cos B}{9\sin A \sin B} = 1$ $\cot A \cot B = \frac{9}{5}$

(c)	$\tan B = \frac{1}{3}$  $\cos 2B = 2\cos^2 B - 1$ $= 2\left(-\frac{3}{\sqrt{10}}\right)^2 - 1$ $= \frac{4}{5}$ or $\cos 2B = 1 - 2\sin^2 B$ $= 1 - 2\left(-\frac{1}{\sqrt{10}}\right)^2$ $= \frac{4}{5}$ or $\cos 2B = \cos^2 B - \sin^2 B$ $= \left(-\frac{3}{\sqrt{10}}\right)^2 - \left(-\frac{1}{\sqrt{10}}\right)^2$ $= \frac{4}{5}$
10(a)	$V = \pi r^2 h$ $300 = \pi r^2 h$ $h = \frac{300}{\pi r^2}$
(b)	$A = 2\pi r^2 + 2\pi r\left(\frac{300}{\pi r^2}\right)$ $= 2\pi r^2 + \frac{600}{r}$ $C = 60\left(2\pi r^2\right) + 60\left(\frac{600}{r}\right)$ $= 120\pi r^2 + \frac{36000}{r}$ (Shown)

(c)	$C = 120\pi r^2 + \dfrac{36000}{r}$ $\dfrac{dC}{dr} = 240\pi r - \dfrac{36000}{r^2}$ $\dfrac{dC}{dr} = 0$ $\Rightarrow \quad 240\pi r - \dfrac{36000}{r^2} = 0$ $\Rightarrow \dfrac{36000}{r^2} = 240\pi r$ $\Rightarrow r^3 = \dfrac{36000}{240\pi}$ $\therefore r = 3.6278 = 3.63$ $\dfrac{dC}{dr} = 240\pi r - \dfrac{36000}{r^2}$ $\dfrac{d^2C}{dr^2} = 240\pi + \dfrac{72000}{r^3}$ When $r = 3.6278$, $C = 14884.9 = 14900$ $\dfrac{d^2C}{dr^2} > 0 \Rightarrow$ Minimum cost or When $r = 3.6278$, $C = 14884.9 = 14900$

x	3.63^-	3.63	3.63^+
$\dfrac{dC}{dr}$	$-$	0	$+$
slope	\	___	/

$\therefore$ minimum cost

11(a)	$L = 13 + 6 + AD + AB$ $= 19 + 6\cos\theta + 13\cos\theta + 6\sin\theta + 13\sin\theta$ $= 19 + 19\cos\theta + 19\sin\theta$
(b)	$R = \sqrt{19^2 + 19^2} = \sqrt{722}$ $\tan^{-1}(\alpha) = 1$ $\alpha = 45^\circ$ $L = 19 + \sqrt{722}\cos(\theta - 45^\circ)$ Max $L = \sqrt{722} + 19 = 45.9\,\text{m}$
(c)	$\sqrt{722}\cos(\theta - 45^\circ) + 19 = 45$ $\cos(\theta - 45^\circ) = \dfrac{26}{\sqrt{722}}$ Basic angle = $\cos^{-1}\left(\dfrac{26}{\sqrt{722}}\right)$ $= 14.620^\circ$ $\theta - 45^\circ = -14.620^\circ,\ 14.620^\circ$ $\therefore \theta = 30.4^\circ,\ 59.6^\circ$
12(a)	$y = \dfrac{1}{4}x^2 - \dfrac{1}{2}\ln\left(\dfrac{x}{3}\right)$ $\dfrac{dy}{dx} = \dfrac{x}{2} - \dfrac{1}{2}\left(\dfrac{1}{3}\right)\left(\dfrac{3}{x}\right)$ $= \dfrac{x}{2} - \dfrac{1}{2x}$ When $x = 3$, $\dfrac{dy}{dx} = \dfrac{4}{3}$ $y - 2.25 = \dfrac{4}{3}(x-3)$ $y = \dfrac{4}{3}x - \dfrac{7}{4}$ Hence $y = \dfrac{4}{3}x - 1$ is not a tangent to the curve at P.
(b)	$\dfrac{d}{dx}\left[x\ln\dfrac{x}{3} - x\right] = x\left(\dfrac{3}{x}\right)\left(\dfrac{1}{3}\right) + \ln\left(\dfrac{x}{3}\right) - 1$ $= \ln\left(\dfrac{x}{3}\right)$
(c)	Area = $\displaystyle\int_1^3 \left[\frac{1}{4}x^2 - \frac{1}{2}\ln\left(\frac{x}{3}\right)\right]dx$

	$= \left[\frac{x^3}{12}\right]_1^3 - \frac{1}{2}\left[x\ln\left(\frac{x}{3}\right) - x\right]_1^3$ $= \left[\frac{27}{12} - \frac{1}{12}\right] - \frac{1}{2}\left[(3\ln 1 - 3) - \left(\ln\frac{1}{3} - 1\right)\right]$ $= \frac{13}{6} - \frac{1}{2}\left(-2 - \ln\frac{1}{3}\right)$ $= \left(\frac{19}{6} + \frac{1}{2}\ln\frac{1}{3} \quad \text{or} \quad \frac{19}{6} - \frac{1}{2}\ln 3 \quad \text{or} \quad \frac{19}{6} - \ln\sqrt{3} \quad \text{or} \quad \frac{19}{6} + \ln\sqrt{\frac{1}{3}}\right)$ units^2