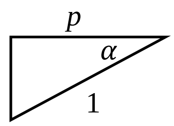
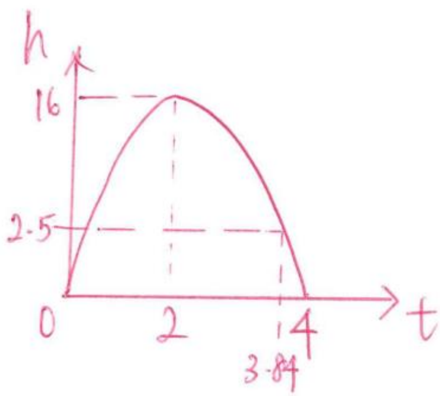
Solutions to 2022 AMKSS Prelim AMP1

Qn	Solution	Marks
1i [2]	Opp side = $\sqrt{1-p^2}$ 3rd quadrant $\sin A = \frac{-\sqrt{1-p^2}}{1} = -\sqrt{1-p^2}$  	M1 for opp side A1
1ii [2]	$\tan(A-\pi)$ $= \frac{\tan A - \tan\pi}{1+\tan A\tan\pi}$ $= \frac{\tan A - 0}{1}$ $= \tan A$ (in 3rd quad) $= \frac{-\sqrt{1-p^2}}{p}$	M1 for use of formula A1 or B2
2 [4]	$-x^2+(h-6)x-3=-2$ $x^2-(h-6)x+1=0$ No real roots as curve does not touch line $b^2-4ac<0$ $(h-6)^2-4(1)(1)<0$ $h^2-12h+36-4<0$ $h^2-12h+32<0$ $(h-4)(h-8)<0$ $4<h<8$	M1 form equation =0 M1 for < 0 M1 form correct discriminant A1
3 [4]	$\frac{dy}{dt} = -0.05$ $\frac{dy}{dx} = -8x^{-3}$ Sub $x=2, \frac{dy}{dx} = \frac{-8}{(2)^3} = -1$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $-0.05 = -1 \times \frac{dx}{dt}$ $\frac{dx}{dt} = 0.05$ units/s OR $\frac{1}{20}$	 M1 correct differentiation M1 find value M1 correct relation only if -0.05 used A1

4i [1]	Sub $t=0$, $V=1.2+0.015=\$1.215$ million	B1
4ii [2]	$1.3=1.2+0.015e^{2k}$ $\frac{0.1}{0.015}=e^{2k}$ Take ln on both sides $\ln\left(\frac{20}{3}\right)=\ln e^{2k}$ $\ln\left(\frac{20}{3}\right)=2k\ln e$ $\ln\left(\frac{20}{3}\right)=2k$ $k=0.949$ (to 3sf)	M1 for simplify to e^{2k} A1
4iii [2]	$2\times1.215=1.2+0.015e^{\frac{\ln\left(\frac{20}{3}\right)}{2}t}$ $\frac{2.43-1.2}{0.015}=e^{\frac{\ln\left(\frac{20}{3}\right)}{2}t}$ Take ln on both sides $\ln 82=\ln e^{\frac{\ln\left(\frac{20}{3}\right)}{2}t}$ $\ln 82=\frac{\ln\left(\frac{20}{3}\right)}{2}t\ \ln e$ $t=4.65$ years (to 3sf)	M1 to simplify to e expression A1

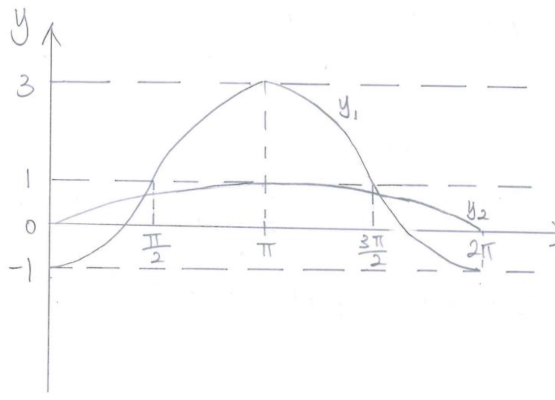
5i [2]	$h = -4t^2 + 16t$ $= -4\left[t^2 - 4t\right]$ $= -4\left[t^2 - 4t + 2^2 - 2^2\right]$ $= -4\left[(t-2)^2 - 4\right]$ $= -4(t-2)^2 + 16$	M1 for $\pm 2^2$ A1
5ii [2]	Max height = 16m Occurs when t = 2s	B1 B1
5iii [3]	Sub h = 2.5 $2.5 = -4(t-2)^2 + 16$ $\dfrac{-13.5}{-4} = (t-2)^2$ $t = 2 \pm \sqrt{\dfrac{13.5}{4}}$ $t = 3.84 \text{ or } 0.163$(NA as ball going upwards) Sub h = 0 $4t(-t+4) = 0$ $t = 0 \text{ or } t = 4$ $3.84 \le t \le 4$  	 M1 to calculate t when h =2.5 M1 to calculate t when h = 0 A1

6i [5]	$y=\dfrac{x^2+6}{2x+1}$ $u=x^2+6 \qquad v=2x+1$ $\dfrac{du}{dx}=2x \qquad \dfrac{dv}{dx}=2$ $\dfrac{dy}{dx}=\dfrac{2x(2x+1)-2(x^2+6)}{(2x+1)^2}$ $=\dfrac{4x^2+2x-2x^2-12}{(2x+1)^2}$ $=\dfrac{2x^2+2x-12}{(2x+1)^2}$ Sub $\dfrac{dy}{dx}=0$, $\dfrac{2x^2+2x-12}{(2x+1)^2}=0$ $2(x^2+x-6)=0$ $(x-2)(x+3)=0$ $x=2$ *OR* $x=-3$ (NA as $x\geq 0$) $y=\dfrac{10}{5}$ Turning point $(2,\ 2)$	M1 for either correct differentiation M1 for correct quotient rule M1 for = 0 M1 for solve x A1
6ii [3]	$\dfrac{dy}{dx}=\dfrac{2x^2+2x-12}{(2x+1)^2}$ Sub $x=0, \dfrac{dy}{dx}=-12$ $y=6$ Equation of normal: $y=\dfrac{1}{12}x+6$	M1 gradient M1 for y-intercept A1

7a [4]	Term in $x^2 = {}^6C_2(4)^4\left(\frac{x}{2}\right)^2 + {}^8C_2(1)^6\left(-mx\right)^2$ $=15(256)\frac{x^2}{4}+28m^2x^2$ $=960x^2+28m^2x^2$ $960+28m^2=1212$ $m^2=9$ $m=3$ (reject -3 since m is positive)	M2 for correct term of each expression M1 equate to 1212 A1
7b [5]	$(1+kx)^n=1+{}^nC_1(1)^{n-1}(kx)^1+{}^nC_2(1)^{n-2}(kx)^2+...$ $=1+nkx+\frac{n(n-1)}{2}k^2x^2+...$ Compare coefficient of term in x $nk=-27$ $k=\frac{-27}{n}\quad ---(1)$ Compare coefficient of term in x^2 $\frac{n(n-1)k^2}{2}=324\quad ---(2)$ Sub (1) into (2) $\frac{n(n-1)(729)}{2n^2}=324$ $n(n-1)=\frac{8}{9}n^2$ $n^2-n-\frac{8}{9}n^2=0$ $\frac{1}{9}n^2-n=0$ $n\left(\frac{1}{9}n-1\right)=0$ $n=0\ (NA)$ or $n=9$ $k=-3$	M1 for expansion M1 form 2 eqn by equating coeff M1 substitution mtd only if 1 linear and 1 quad eq A1 A1

8i [3]	Let $\angle CAB = a$ $\angle DAB = 180 - a$ (adjacent angles on a straight line) $\angle DXC = 180 - \angle DAB$ (angles in opposite segment) $= 180 - (180 - a)$ $= a$ So $\angle DXC = \angle CAB$ $\angle ACB = \angle XCD$ (common angle) Since two corresponding angles are equal, ΔABC and ΔXDC are similar (AA Similarity Test)	M1 for reason M1 for reason M1 for reason Minus 1m if no proper conclusion
8ii [3]	$\angle CBY = \angle CAB$ (angles in alternate segment, tangent YD) $\angle CBY = \angle DBX$ (vertically opposite angles) So $\angle DBX = \angle CAB$ From 8i, $\angle CAB = \angle CXD = \angle BXD$ So $\angle DBX = \angle BXD$ (base angles of isosceles triangle BDX) Since ΔBDX is an isosceles triangle, $DB = DX$.	M1 for reason M1 for reason M1 for reason base angles

9i [1]	In ΔABC, R is midpoint of AC and Q is midpoint of BC. Using midpoint theorem, RQ is parallel to AB.	B1 with correct quote of ONLY TWO midpoints and midpoint theorem.
9ii [2]	Gradient $RQ = \dfrac{4-0}{-2-2} = -1$ Equation of $AB: y = -x + c$ Sub (1, 5) $5 = -1 + c$ $c = 6$ $y = -x + 6$	M1 for gradient A1
9iii [2]	$AB: y = -x + 6$ $AC: y = -5x - 6$ Solve simultaneous eq $-x + 6 = -5x - 6$ $12 = -4x$ $x = -3$ $y = 9$ $A(-3, 9)$	 M1 A1
9iv [2]	$Area = \dfrac{1}{2}\begin{vmatrix} -3 & -2 & 2 & 1 & -3 \\ 9 & 4 & 0 & 5 & 9 \end{vmatrix}$ $= \dfrac{1}{2}(-12 + 10 + 9 + 18 - 8 + 15)$ $= 16 \text{ units}^2$	M1 for 5 coordinates with correct "shoe-lace" method A1 only if coord in formula in anti-clockwise direction

10i [2]	Amplitude = 2 Period = 2π	B1 B1
10ii [4]	Each graph – B1 correct shape, B1 correct y-intercept  cosine graph – B1 correct shape – B1 correct y-intercept sine graph – B1 correct shape – B1 correct y-intercept minus 1m if max pts of both graph not aligned at $x=\pi$	B2 B2 Minus 1m if max point of both graphs not aligned at $x=\pi$
10iii [4]	$1-2\cos x=\sin\frac{1}{2}x$ $1-2\left(1-2\sin^2\left(\frac{1}{2}x\right)\right)=\sin\frac{1}{2}x$ Let $u=\sin\frac{1}{2}x$ $1-2(1-2u^2)=u$ $1-2+4u^2=u$ $4u^2-u-1=0$ $u=\frac{1\pm\sqrt{1-4(4)(-1)}}{8}$ $=\frac{1\pm\sqrt{17}}{8}$ $\sin\frac{1}{2}x=\frac{1+\sqrt{17}}{8}$ *or* $\frac{1-\sqrt{17}}{8}$ (reject since negative as $0\le\frac{1}{2}x\le\pi$) Basic angle = 0.695004 $\frac{1}{2}x=0.695004,\ \pi-0.695004$ $x=1.39,\ 4.89$ (to 3sf)	M1 for use of formula M1 form quad eq M1 for solving for trigo fn A1 for both values
10iv [1]	For Graph y_1 above Graph y_2 $1.39\le x\le 4.89$	B1

11i [2]	$a = \frac{dv}{dt}$ $= 9 - 6t$ Sub $t = 1,\ a = 3\ m/s^2$	M1 for correct differentiation A1
11ii [3]	For max v, $\frac{dv}{dt} = 0$ $9 - 6t = 0$ $t = 1.5$ Check $\frac{d^2v}{dt^2} = -6 < 0$ (max v) $\max v = 9(1.5) - 3(1.5)^2$ $= 6.75\ m/s$	M1 for t M1 for check max v A1
11iii [5]	$s = \int v\, dt$ $= \int 9t - 3t^2 dt$ $= \frac{9t^2}{2} - \frac{3t^3}{3} + c$ Sub $t = 0, s = 3$ $3 = c$ $s = 4.5t^2 - t^3 + 3$ At rest, $v = 0$ $9t - 3t^2 = 0$ $3t(3 - t) = 0$ $t = 0$ *or* $t = 3$ Sub $t = 3,\ s = 4.5(3)^2 - (3)^3 + 3 = 16.5m$ Distance $OP = 16.5m$	M1 for integration M1 for c-value OR for definite integral M1 for set $v = 0$ M1 for t A1 No A1 if earlier c=0 and +3 in final step

12i [3]	$\frac{dy}{dx}=9(2x-1)^{-2}$ $y=\frac{9(2x-1)^{-1}}{(-1)(2)}+c$ Sub (2,12) $12=\frac{9}{-2(2\times 2-1)}+c$ $12=\frac{9}{-6}+c$ $c=13.5$ $y=-\frac{9}{2(2x-1)}+13\frac{1}{2}$	M1 for correct integration M1 for sub point A1
12ii [2]	$\frac{dy}{dx}=9(2x-1)^{-2}$ $\frac{d^2y}{dx^2}=-18(2x-1)^{-3}(2)$ $=\frac{-36}{(2x-1)^3}$ Sub $x=0$, $\frac{d^2y}{dx^2}=36$	M1 for chain rule A1
12iii [2]	At a turning point, $\frac{dy}{dx}=\frac{9}{(2x-1)^2}=0$ Since <u>there is no solution for x</u>, there is no turning point. The conclusion is wrong. OR Sub $x=0, \frac{dy}{dx}=\frac{9}{(0-1)^2}=9\neq 0$ Since at $x=0, \frac{dy}{dx}\neq 0,$ there is no stationary point. OR Since $(2x-1)^2>0$, so $\frac{dy}{dx}>0$ Since $\frac{dy}{dx}$ is always positive, there is no turning point.	M1 only if followed by explanation M1 M1 only if followed by explanation M1 M1 M1

13a [4]	Base Area $=\left(2\sqrt{3}+3\right)^2$ $=4\times3+12\sqrt{3}+9$ $=21+12\sqrt{3}$ Volume $=\dfrac{1}{3}\times$ base area $\times$ height $15+\sqrt{48}=\dfrac{1}{3}\times\left(21+12\sqrt{3}\right)\times$ height $15+4\sqrt{3}=\left(7+4\sqrt{3}\right)\times$ height height $=\dfrac{15+4\sqrt{3}}{7+4\sqrt{3}}\times\dfrac{7-4\sqrt{3}}{7-4\sqrt{3}}$ $=\dfrac{105-60\sqrt{3}+28\sqrt{3}-16\times3}{49-16\times3}$ $=\dfrac{57-32\sqrt{3}}{1}$ $=57-32\sqrt{3}$	M1 for square base area M1 for correct relation M1 for rationalising A1
13b [4]	$\dfrac{5^{x-3}\left(5^3\right)^{x+2}}{\left(5^4\right)^{x+1}-10\left(5^2\right)^{2x}}$ $=\dfrac{5^{x-3+3x+6}}{5^{4x+4}-10\left(5^{4x}\right)}$ $=\dfrac{5^{4x+3}}{\left(5^{4x}\right)5^4-10\left(5^{4x}\right)}$ $=\dfrac{\left(5^{4x}\right)5^3}{\left(5^{4x}\right)\left(5^4-10\right)}$ $=\dfrac{125}{625-10}$ $=\dfrac{125}{615}$ $=\dfrac{25}{123}$	M1 change to base 5 M1 split numerator power M1 factorise denominator A1